Treatment, Estimation, and Issues with Representation Error Modelling

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Motivation

- Representation error arises from an incompatibility between coarse model grids (prior estimate **x**) and observations (**y°**) which observe a **higher resolution state**
- Includes contributions from errors in the **observation operator**
- The best state that the model can represent is a **smoothed version of reality**
- The state that we want to estimate is defined by the model: the pdf of a **discrete model state given the observation**, not the pdf of the **true state given the observations**

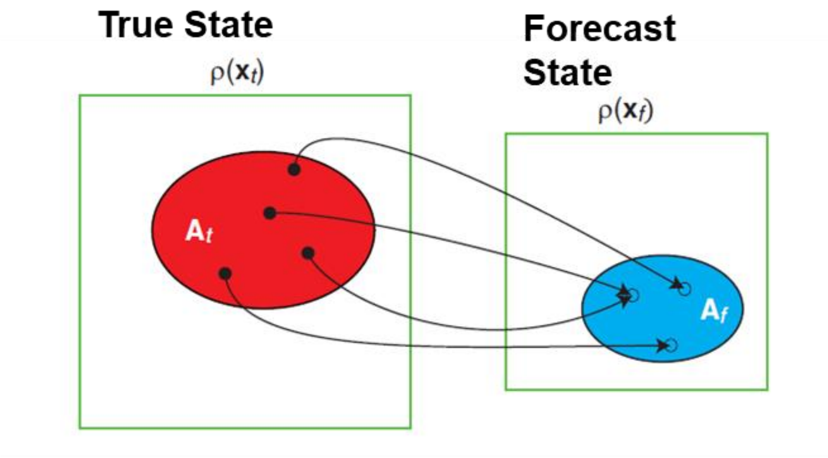

Image credit: Hodyss and Nichols, 2015

Outline of this Talk

1. **Definitions, Sources and Characteristics**
2. **Diagnosing observation error along with the focus on representation error**

   -Overview methods to diagnose observation error and discuss their shortcomings

   -Discuss the ability of these methods to estimate representation error
3. **Discuss current practice for handling representation error**
4. **Methods for incorporating a flow dependent model of representation error**
5. **Concluding Remarks/Discussion**

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Components of Observation Error

- Observation error, broadly speaking, has two components, the **representation error** and the **instrument (or measurement) error**

$$\boxed{\boldsymbol{\varepsilon}^{o} = \boldsymbol{\varepsilon}^{i} + \boldsymbol{\varepsilon}^{r}}$$

- This means that even if the instrument error is small, the observation and the prior can still be quite different.

Sources of Representation Error (Janjić et al. 2018)

1. Error Due to Unresolved Scales and Processes: Discretization and Parameterizations

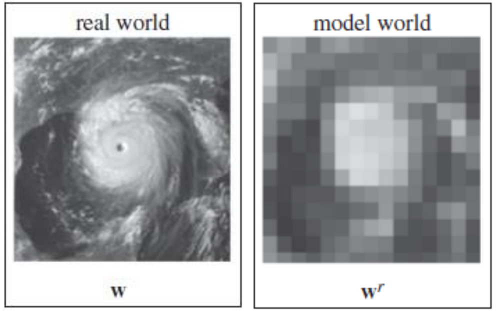
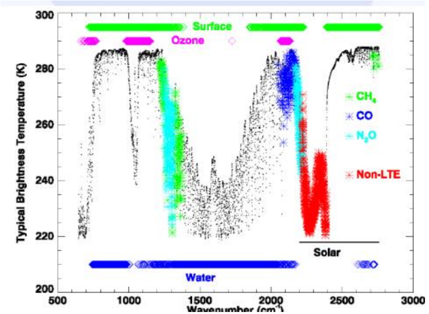


Image credit: Janjić et al 2018

2. Observation Operator: For non-local variables requires the use of complicated observation operators that introduce their own (inseparable) errors.



3. Preprocessing or QC: Imperfections in procedures that remove observations that cannot be adequately modeled or derive a state variable quantity from an observed variable.

Dominant source for cloudy assimilation

Dominant source for clear-sky assimilation

Defining the Components of Observation Error (Satterfield et al. 2017)

- If the NWP model were "perfect", the **instrument error** can be expressed as,

$$\boxed{\boldsymbol{\varepsilon}^{i} = \mathbf{y}^{o} - \mathbf{H}_{H}(\mathbf{x}^{t}{}_{H})}$$

- Where, $\mathbf{H}_H$ is the high-resolution observation operator that maps the high-resolution true state $\mathbf{x}^t$ to the observation location

- The **representation error** can be expressed by considering a (spectral truncation based) smoother $\mathbf{S}_{SC}$ and an imperfect low resolution observation operator.

$$\boxed{\boldsymbol{\varepsilon}^{r} = \mathbf{H}_{H}(\mathbf{x}^{t}{}_{H}) - \hat{\mathbf{H}}_{L}(\mathbf{S}_{SC}\mathbf{x}^{t}{}_{H})}$$

Characteristics of Representation Error

- **Modeling representation error** is not a straightforward problem
- Separation of representation error into **individual components** seldom possible
- It can also be difficult to distinguish between **model error** and **representation error**
- Representation error is **flow dependent** and a function of **model resolution**
- Key, often dominant, contributor to spatially and temporally **correlated observation errors**
- Needs to be considered for applications of **data assimilation**, **observation based verification**, as well as **observation inter-comparison**.

Methods to Recover Observation Error Statistics

1. **Estimate variances using innovation (observation-minus-background) data:**
   - Split innovation(o-b) statistics into observation and background error covariances (e.g. Rutherford, 1972; Höllingsworth and Lonnberg, 1986)
   - Iterative procedures based on assumptions of data assimilation (e.g optimality diagnostic of Desroziers and Ivanov, 2001; Desroziers et al., 2005)
   - Lag innovation statistics (e.g. Berry and Sayer 2013)

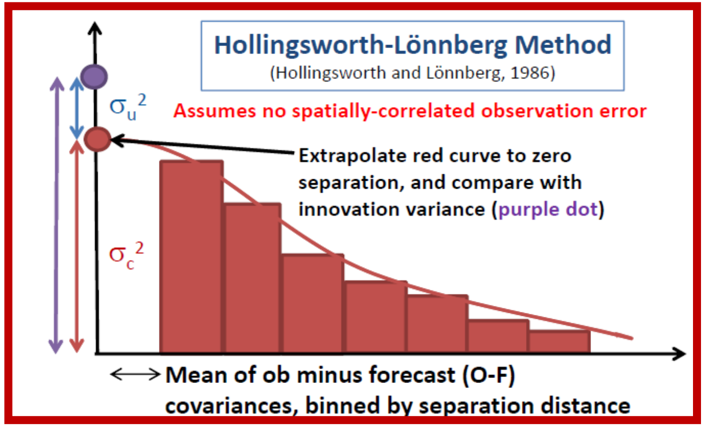
2. **Methods based on estimating a high resolution true state** using observations (e.g. Oke and Sakov, 2007) or model states (e.g. Etherton and Bishop, 2004)

3. Analysis and forecast **Sensitivity diagnostics**, such as those developed at NRL (Daescu and Langland, 2013)

4. **Ensemble Based Approaches** (e.g. Karspeck, 2016)

5. **Maximum likelihood** (e.g. Dee, 1995; Dee et al. 1999) or **Bayesian approaches** (e.g. Stroud and Bengtsson, 2007) to tune parameters of probability density functions (pdfs) using observed data.

Standard Diagnostics: Estimating Covariance Matrices



$$E[\mathbf{d}_b^o(\mathbf{d}_b^o)^T] = \mathbf{R} + \mathbf{HBH}^T$$

$$E[\mathbf{d}_b^a(\mathbf{d}_b^o)^T] = \mathbf{HBH}^T$$

$$E[\mathbf{d}_a^o(\mathbf{d}_b^o)^T] = \mathbf{R}$$

1. **Split Observation-minus-Background statistics into observation and background error covariances** (e.g. Höllingsworth and Lonnberg, 1986)
   - Requires a dense observing network
   - Dependent on the chosen correlation function
   - Assumes observation errors are uncorrelated or requires additional assumptions on background error statistics

2. **(Iterative) procedures based on assumptions of data assimilation,** Desroziers et al. (2006)
   - Easy to implement
   - Dependent on prescribed error covariance matrices
   - Iterative procedure may be required
   - Iteration on background and observation errors concurrently results in convergence in one step that may be in error

Can Innovation Based Diagnostics Estimate Representation Error?

- Hodyss and Satterfield (2017) showed that when the observation is at a higher resolution than the model state, innovation based methods will have contributions from representation error as well as errors from resolved scales

$$\mathbf{d}_L^a = \mathbf{y}^o - \mathbf{H}_L \mathbf{x}_L^a$$

- The Desroziers estimates results in

$$\left\langle \mathbf{d}_L^a (\mathbf{d}_L^b)^T \right\rangle = \mathbf{R}_D$$

$$\mathbf{R}_D = \mathbf{R}^i + \underbrace{\mathbf{H}_H \mathbf{P}_H \mathbf{H}_H^T - \mathbf{H}_L \mathbf{P}_L \mathbf{H}^T{}_L}_{\textit{Representation error + model error on resolved scales}}$$

Can deliver the correct covariance estimate if there is no model error on resolved scales

Accounting for Representation Error

- **Attempt to remove correlated (representation) error**
  - Super ob, averaging, thinning, quality control
  - Inflate to reduce effect of neglected correlated error

- **Account for representation error covariance**
  - Use correlated observation error covariance matrices (e.g. Borman, Collard, Bauer 2010; Weston et al. 2014; Waller et al. 2015; Campbell et al. 2017, Simonin et al. 2019)
  - Implementing spatially correlated errors can be a more challenging problem for parallelization (e.g. Simonin et al. 2019)
  - Using spatial correlations can improve smallest scales represented by the model (Rainwater et al. 2015)

- **Modify KF equations to attempt to account for the influence of the unresolved scales on resolved scales** (e.g. Janjić and Cohn, 2006, Janjić et al. 2018, Hodyss and Nichols, 2015)

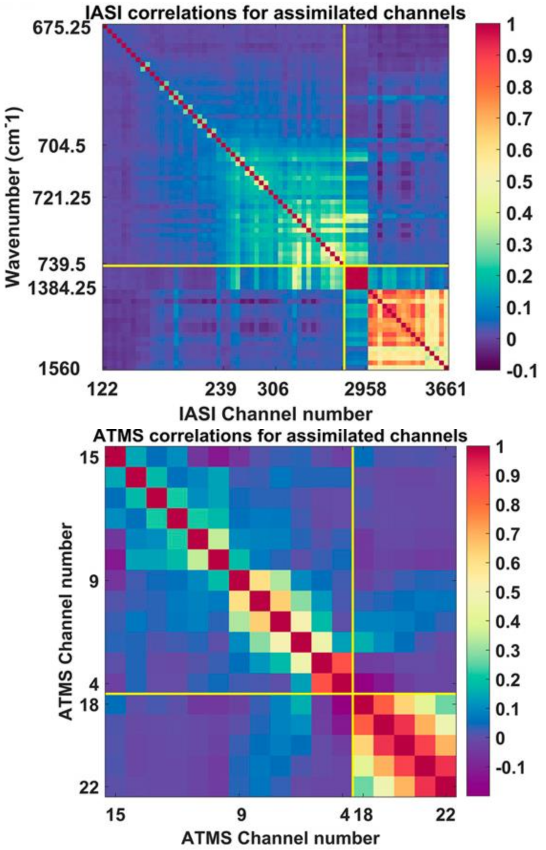


Image credit: Campbell et al. 2017

Flow Dependence Models of Representation Error

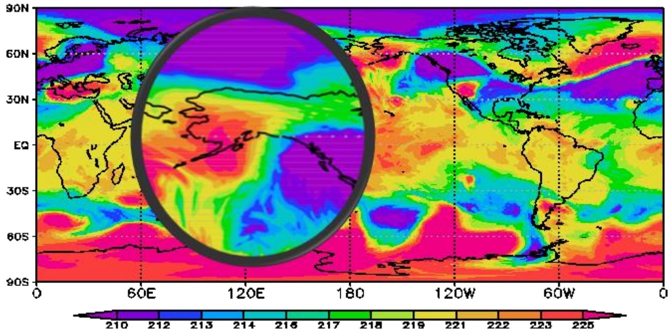
- Address spatial variations by calculating the observation error covariance **based on estimates of local turbulence** (e.g. Frehlich, 2006)

- Create a time dependent estimate of R using a **temporally smoothed Desrozier estimate** (e.g. Miyoshi et al., 2013; Waller et al. 2014)

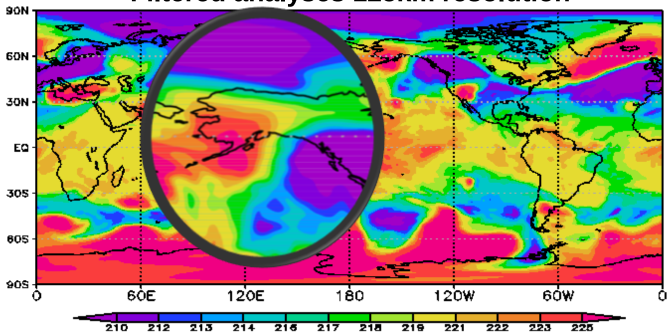
- Use an **ensemble-based estimator** (e.g. Karspeck, 2016)

- Satterfield et al. (2017) hypothesized that **ensemble variance could predict uncertainties associated with unresolved (smoothed) boundaries or gradients** associated with resolved scale features.

200 hPa temperature 1 Jan 2015
ECMWF analyses 32km resolution (TIGGEE)

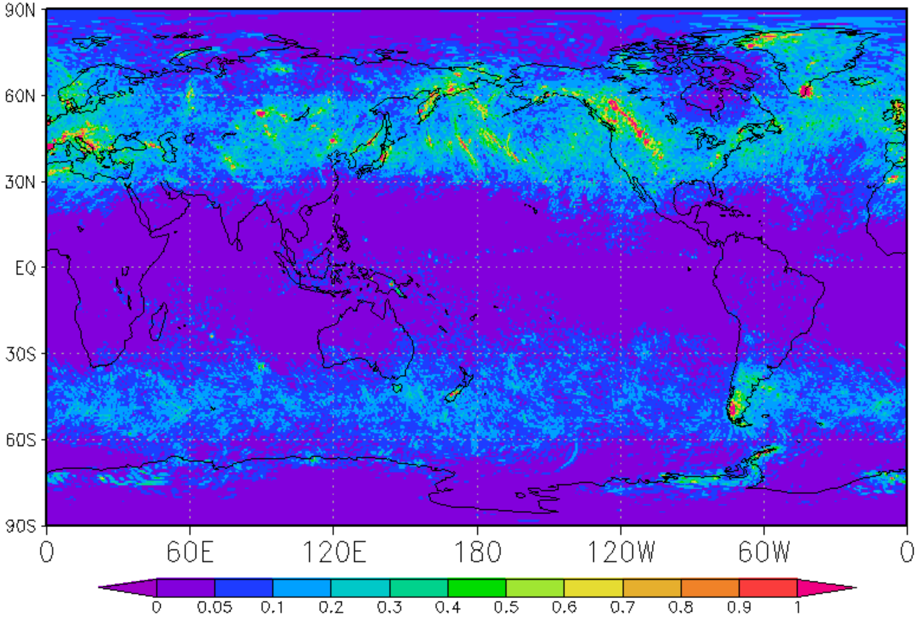


200 hPa temperature 1 Jan 2015
Filtered analyses 125km resolution

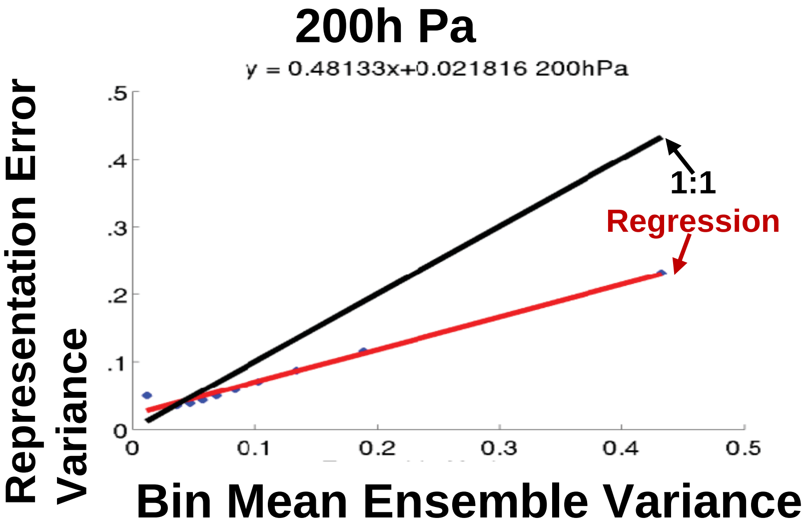


Can Ensemble Variance Predict Representation Error?

Error Variance due to Spectral Truncation for January 2015. Shown for temperature at 200hPa.



We bin ensemble variance and representation error (defined as high-minus-low resolution states) pairs, into equally populated bins as a function of ensemble variance.



Do Desroziers and H-L estimates vary as a function of Ensemble Spread?

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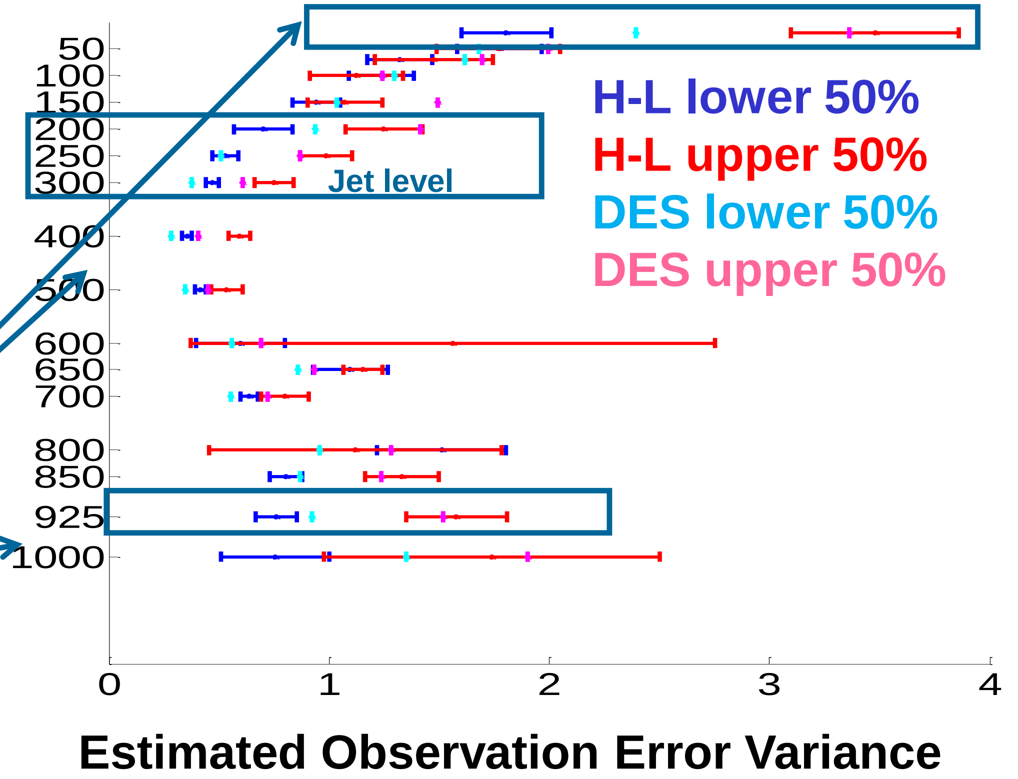
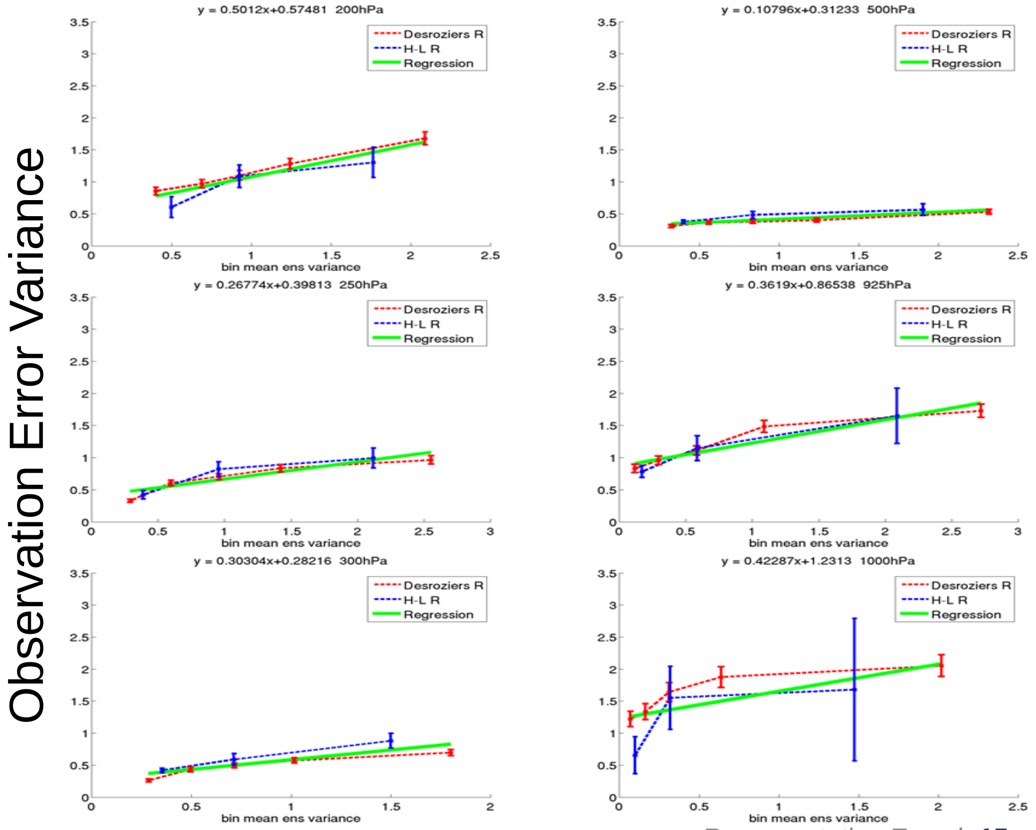
Using radiosonde temperature observations, we examined fluctuations in estimated observation error variances when the Desroziers and H-L methods are applied to subsets of innovations based on binning by ensemble variance.

Largest fluctuations agree with maps of variance due to spectral truncation, suggesting representation error



Accounting for Representation Error

- A regression based method would allow us to prescribe a static error and also allow for representation error to vary as a function of ensemble variance.
- Here we focus on those features that are mostly well represented by coarse model resolution and that have a high degree of spatiotemporal correlation.
- It remains to be determined how well a model like this would work for higher frequency fluctuations.



Discussion

- **Should we be using more quantitative techniques for diagnosing observation error?**
  - Current methods are imperfect, but still offer guidance. Likely more than one method needs to be used.
  - Using spatial correlations changes how observation information is filtered.
  - Capturing flow dependence is challenging, methods using temporal smoothers and ensemble based methods are encouraging.
  - How can existing methods for all sky that inflate observation error in the presence of cloud (e.g Geer, 2011; Minimide and Zhang 2017) be combined with innovation based methods?
  - How can we distinguish what is representation error vs. background error?

- **What are the implications of background error and observation error sharing characteristics and correlations?**
  - If model error and observation error are mutually correlated, the innovation relationship that is at the heart of most operationally used observation error estimation methods, no longer holds
  - Additional difficulty in distinguishing what is representation error and what is model error
  - What is the impact of ignoring the cross correlation in the gain? How much is gained from inflating observation error?

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