Particle filters for (convective-scale) data assimilation

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Shunji Kotsuki
Peter-Jan van Leeuwen

LAPF Development Summary

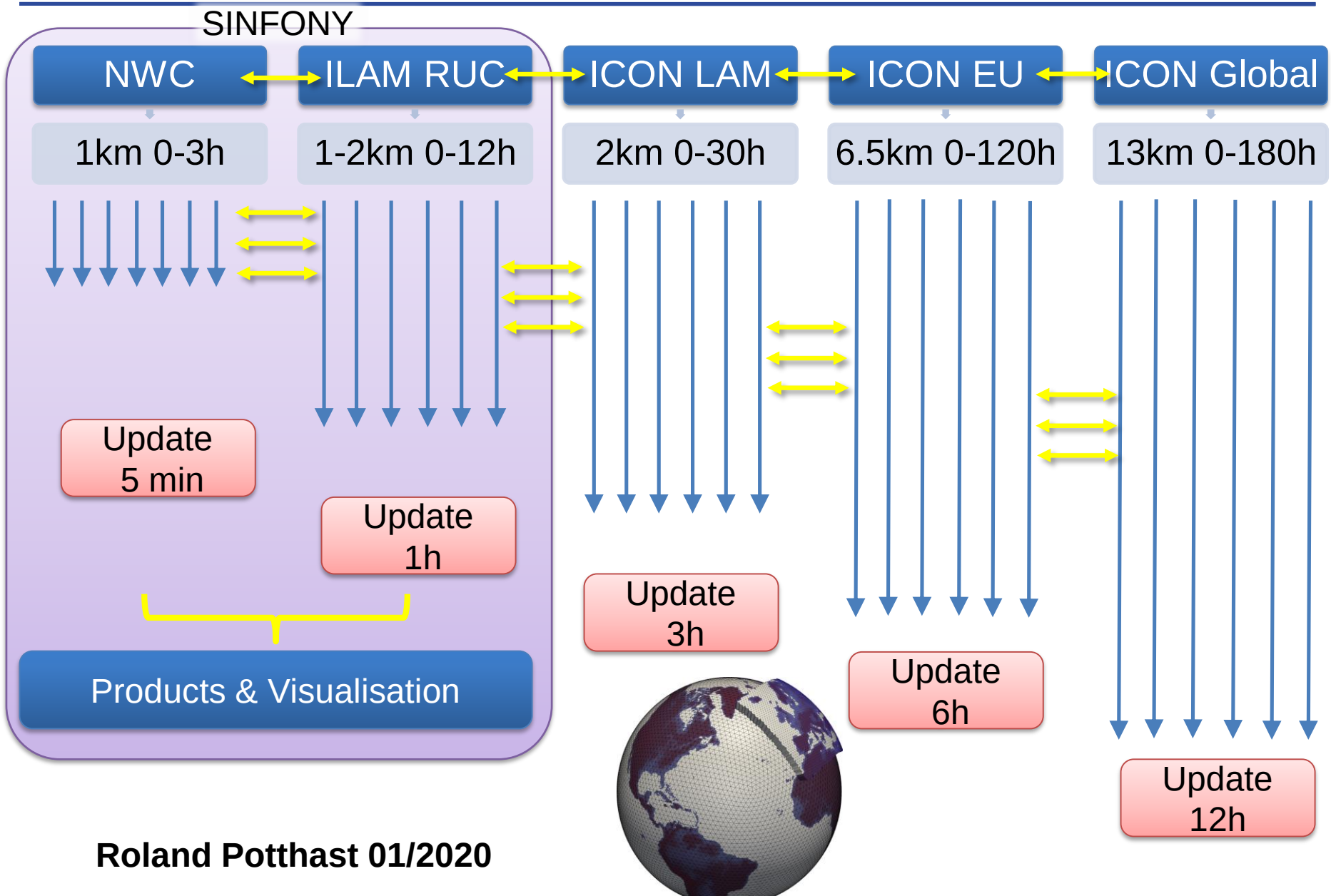
Deutscher Wetterdienst
Wetter und Klima aus einer Hand

DWD

1. Localized Adaptive Particle Filter (**LAPF**) first high-dimensional sampling tests in Summer 2015 (c.f. Inverse Modeling by Nakamura & Potthast)
2. Implemented in **DACE** (Data Assimilation Coding Environment) for the *global ICON Model,* first stable runs in 2016 by Anne Walter (c.f. Potthast, Walter, Rhodin MWR 2019)
3. Extension to Localized Adaptive Mixture Coefficient Particle Filter (**LMCPF**) 2018 by Walter and RP
4. Implemented **LMCPF** for *global ICON Model* since 2018 (Walter, RP), Diskussions with Peter-Jan van Leeuwen, Alternative PF for ICON+DACE
5. LAPF and LMCPF implemented in DACE for *COSMO Convective Scale Model*, first stable runs in Nov 2018 by RP
6. Implemented **LMCPF** for *ICON D2* (convective scale), stable runs in Nov 2019, by RP
7. Lorenz 63 and 96 Implementations of **LAPF/LMCPF** and Testing (Nora Schenk & RP 2019 & 2020)
8. Implementation of **LMCPF** in *Speedy Model* by Miyoshi and Kotsuki, RIKEN (2019)
9. Testing **LMCPF** with SEVIRI VIS Reflectance Assimilation (2019/2020), Lilo Bach & RP

1) DWD Model Configurations

Deutscher Wetterdienst
Wetter und Klima aus einer Hand



Roland Potthast 01/2020

1) ICON D2

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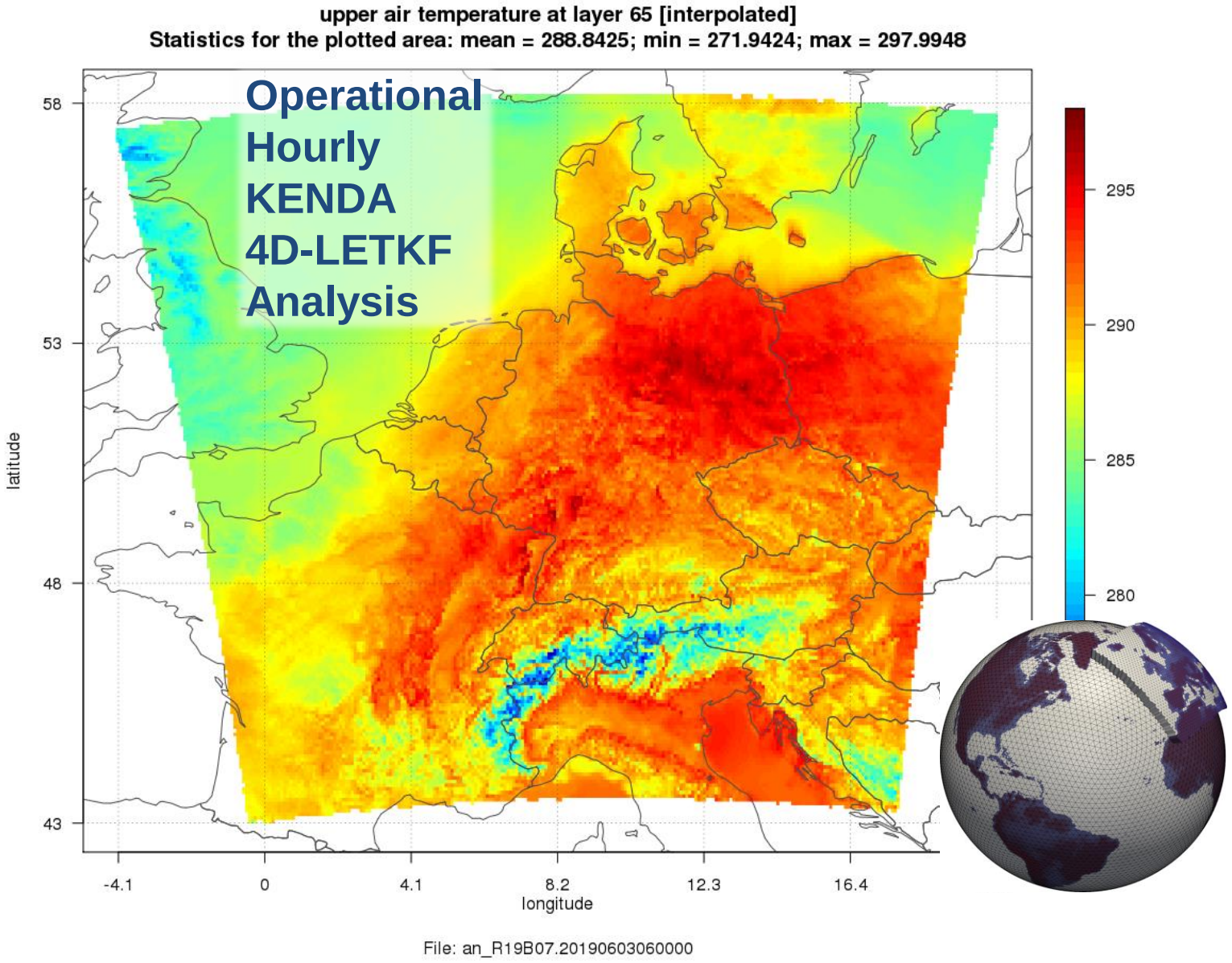
Wetter und Klima aus einer Hand



OPERATIONS

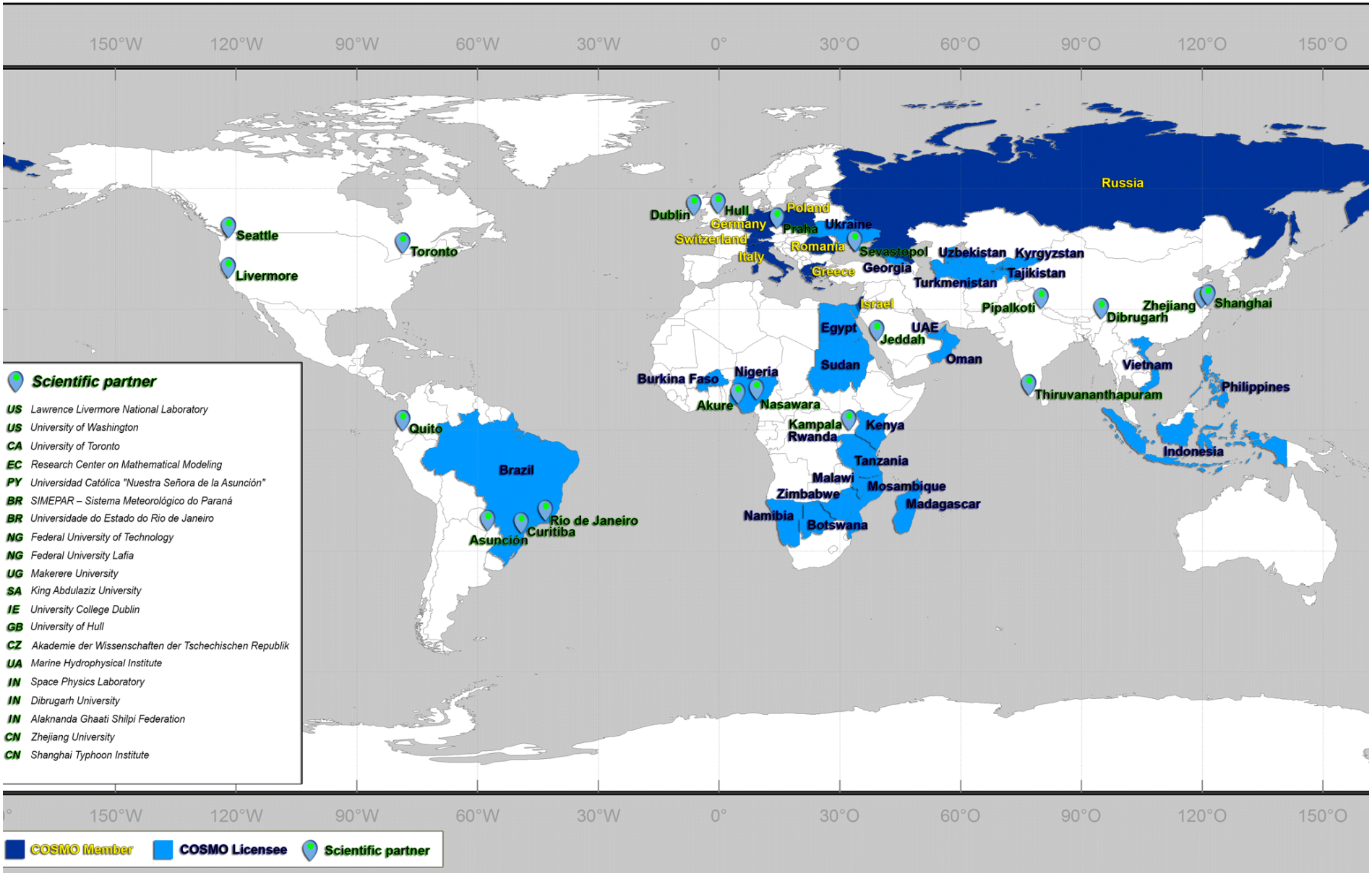
AIREP
MODE-S
PILOT
TEMP
DRIBU
SYNOP
RADAR LHN
RADAR RW

Development

RADAR 3D
SYNOP T,RH
RADAR OBJ
SEVIRI IR
SEVIRI VIS
Lightning LPI
MWR
GB Remote
Crowd Source
RADAR DualP

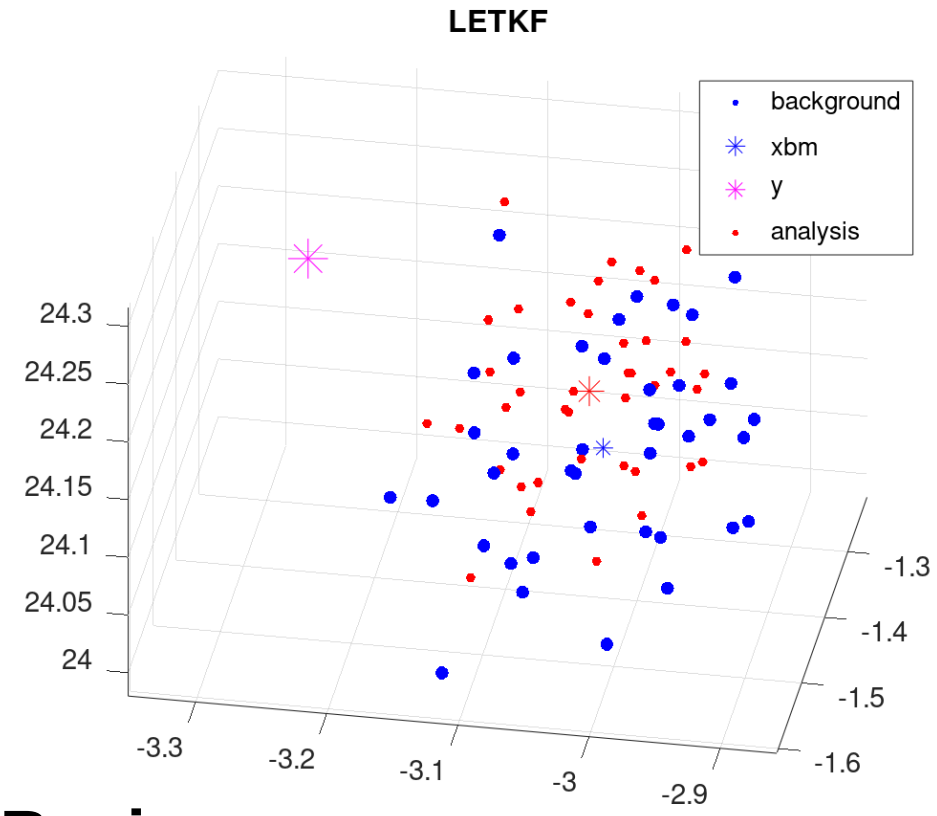
1) COSMO Consortium

Deutscher Wetterdienst
Wetter und Klima aus einer Hand



2) Ensemble Methods & PF

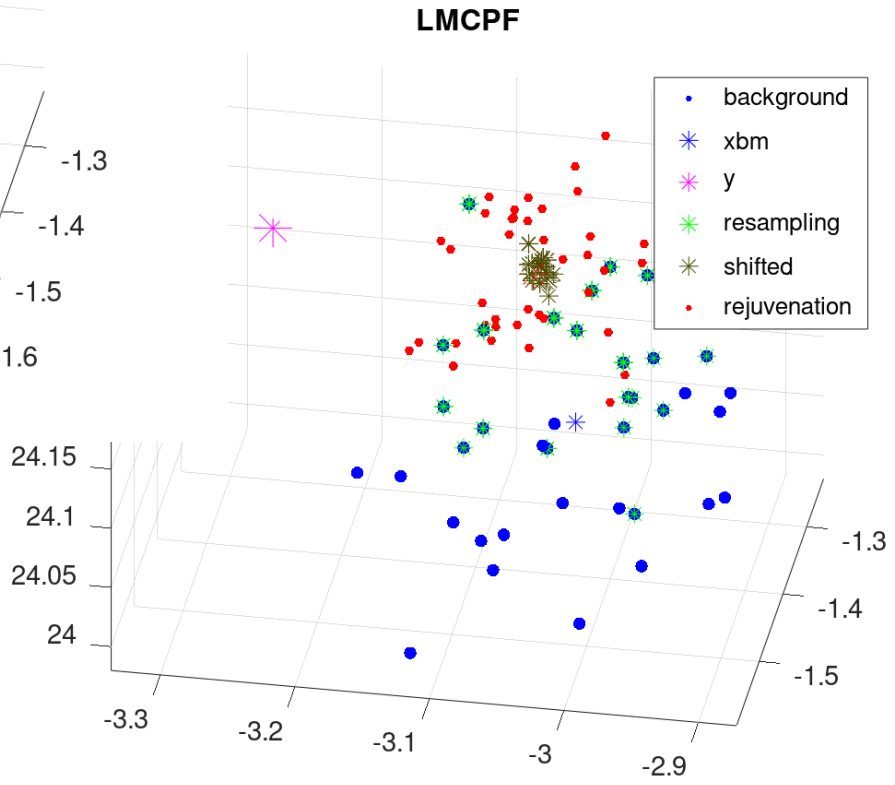
Deutscher Wetterdienst
Wetter und Klima aus einer Hand



What does a particle stand for?

Drawn from some distribution:

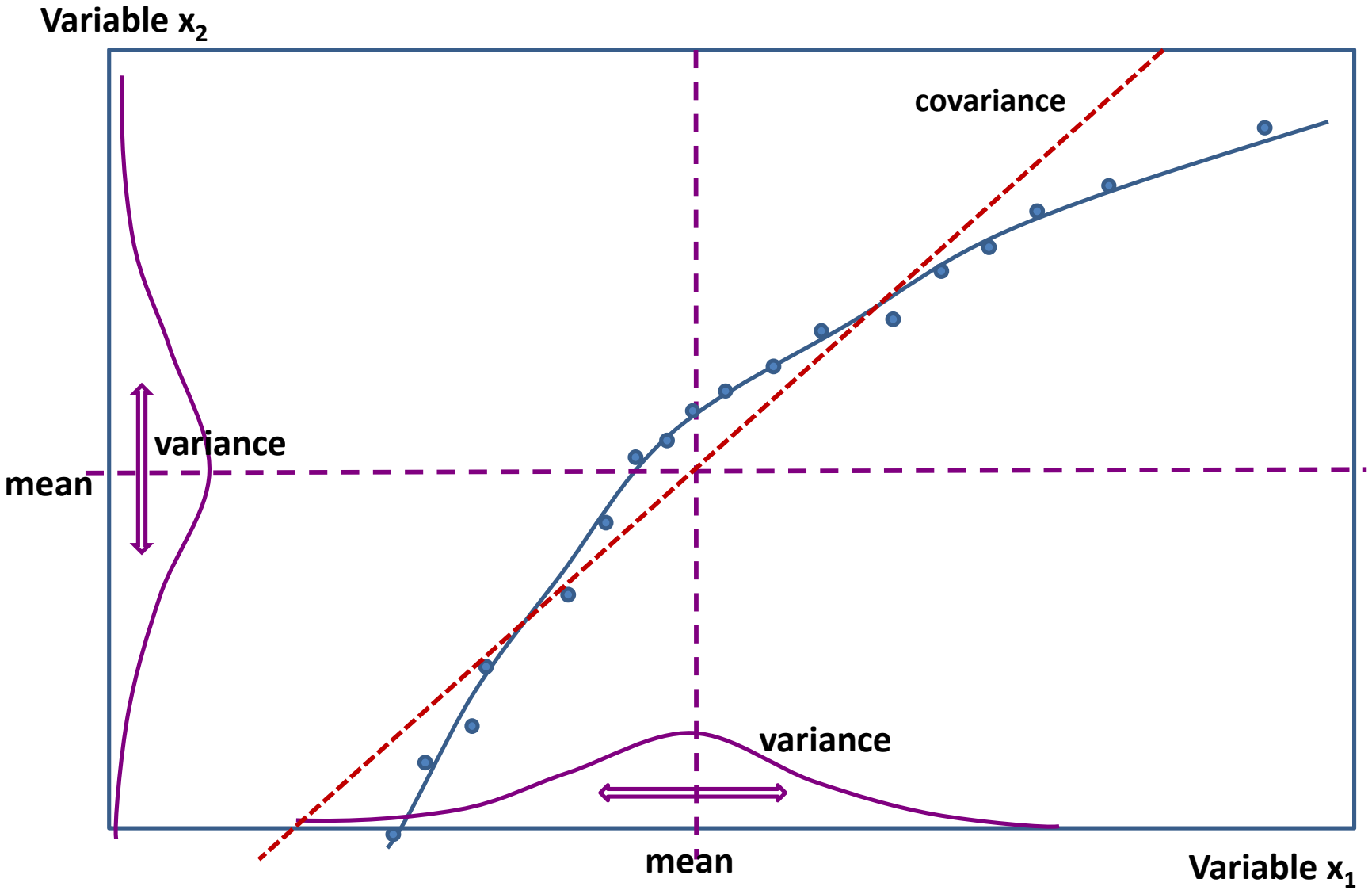
Representing the Distribution



Basics

- Background
- **Observation**
- Weights
- **Resampling**

2) 3D-VAR, EnKF and PF



2) 3D-VAR, EnKF and PF

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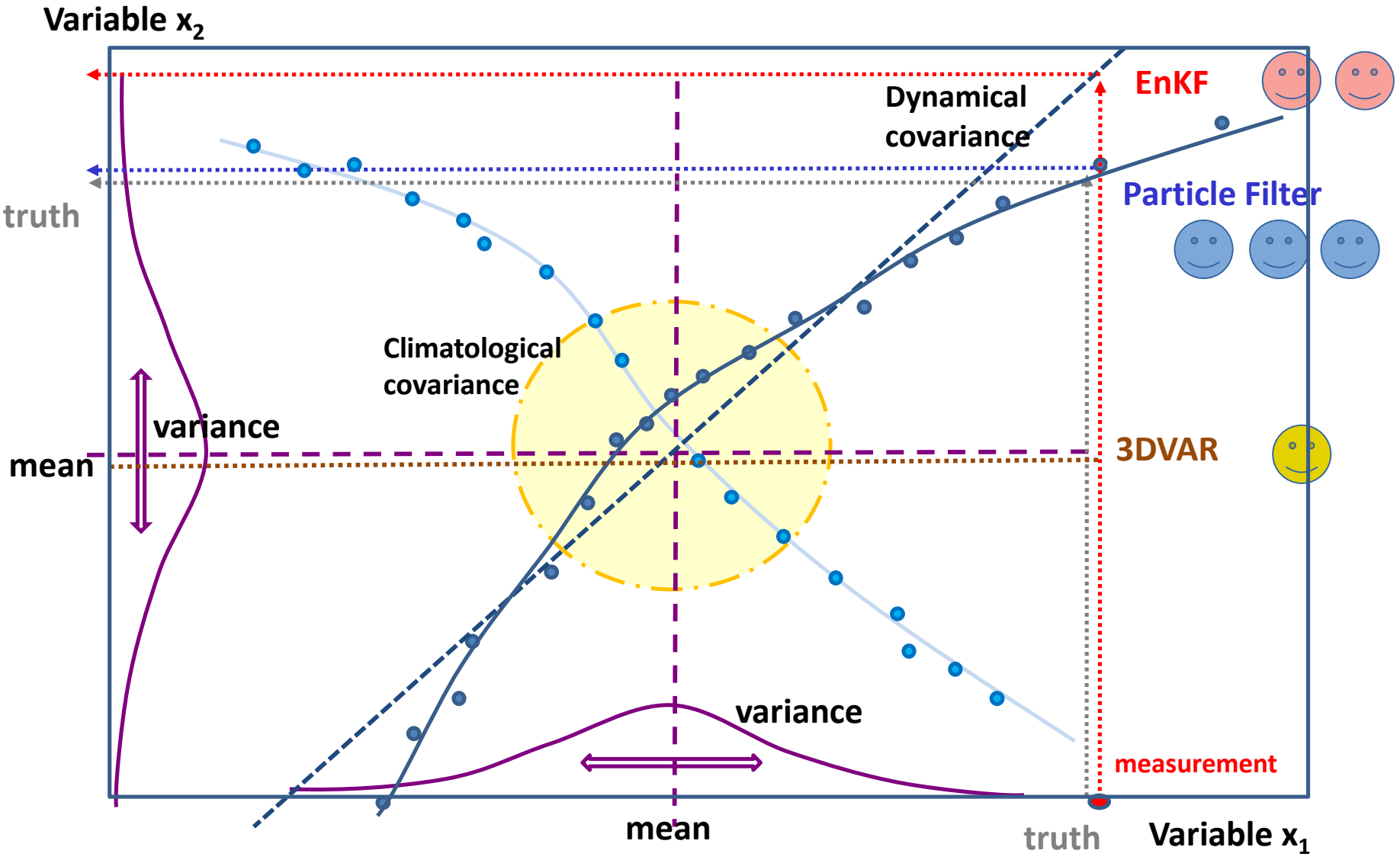
Wetter und Klima aus einer Hand



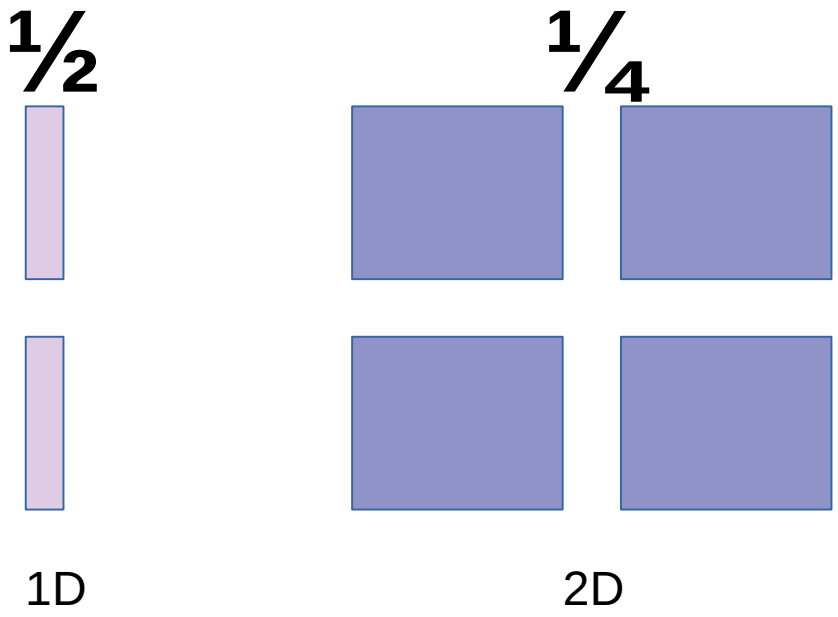
2) Curse of Dimensionality

1) Curse of Dimensionality

2) Low Ensemble Number

3) Dynamical System Errors

Practically you cannot sample from high-dimensional spaces!



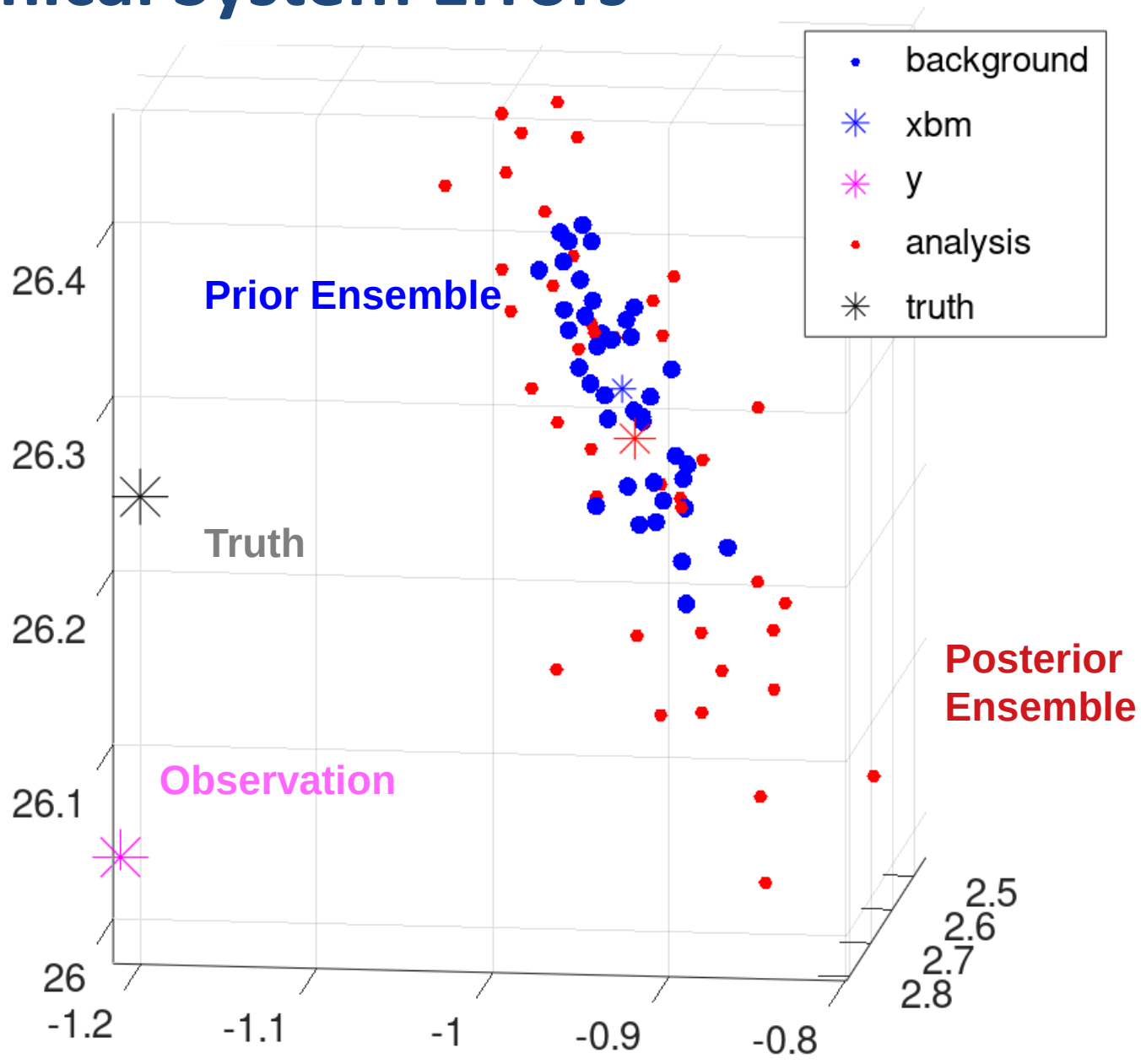
2) Dynamical System Errors

Lorenz 63 System Simulation

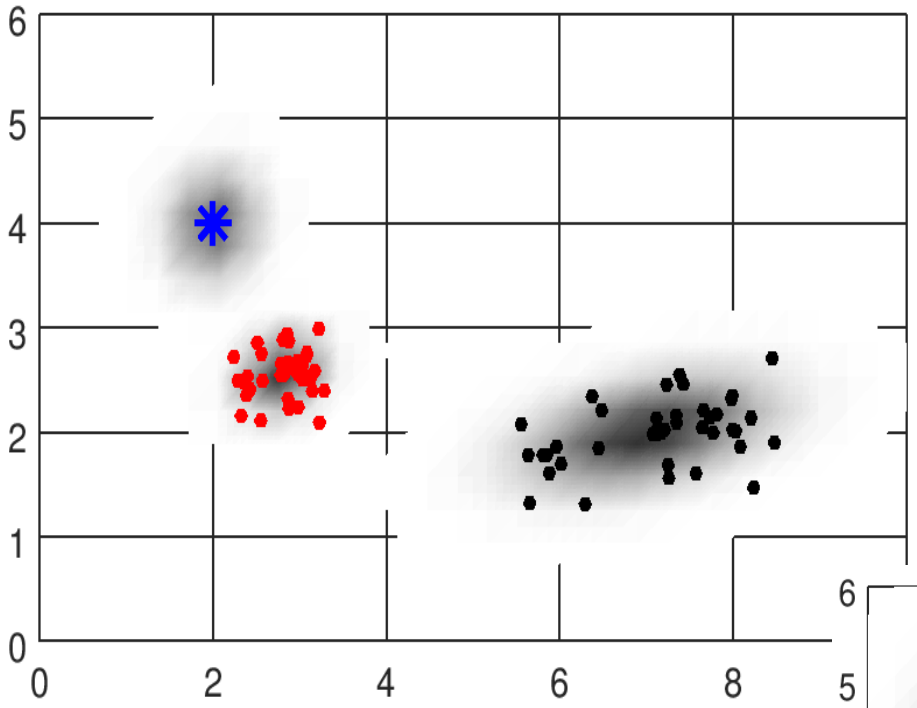
40 Members LETKF

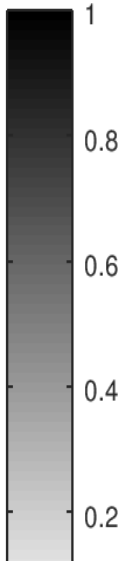
Natural Run Sigma = 10

Assimilation Run Sigma = 10.5



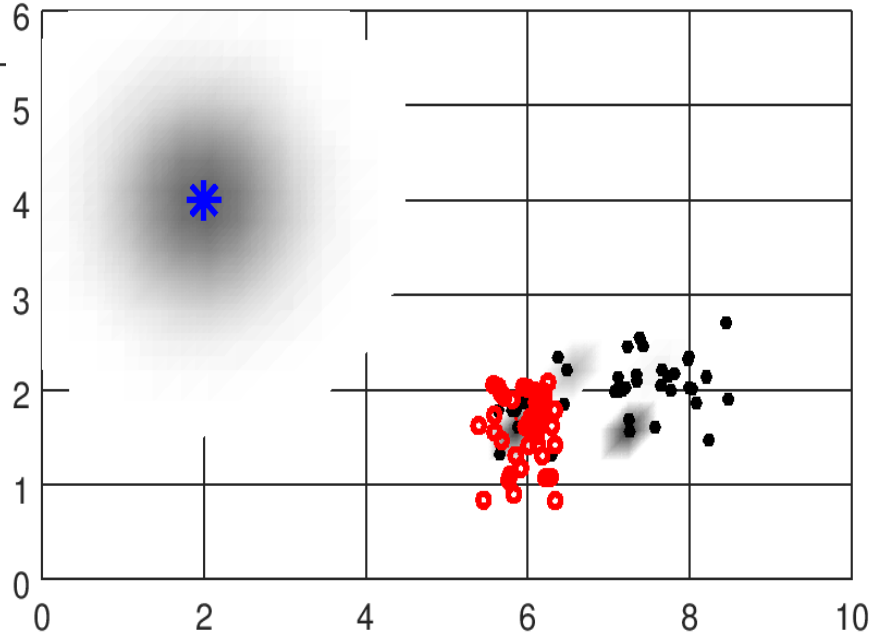
3) LAPF Basic Idea

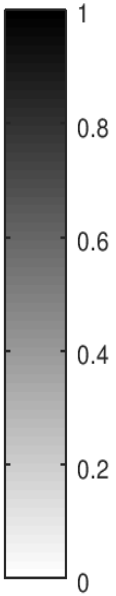




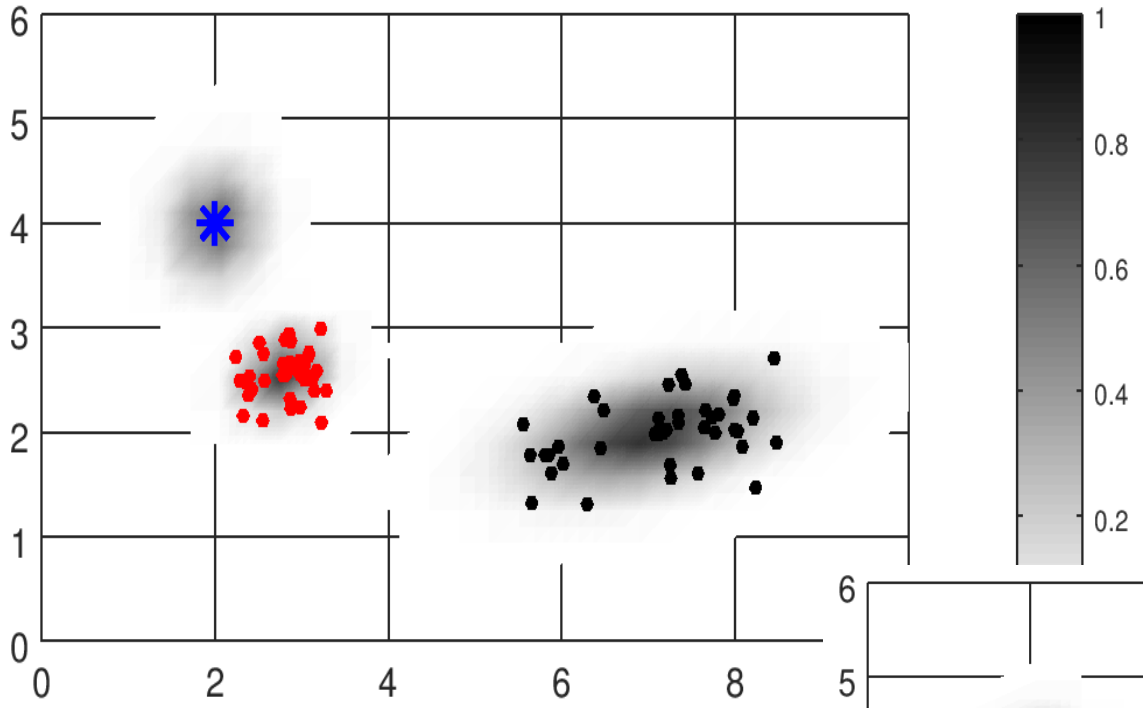
LETKF =
Move ensemble as for Gaussian distribution

Classical PF =
Select Particle according to its weight



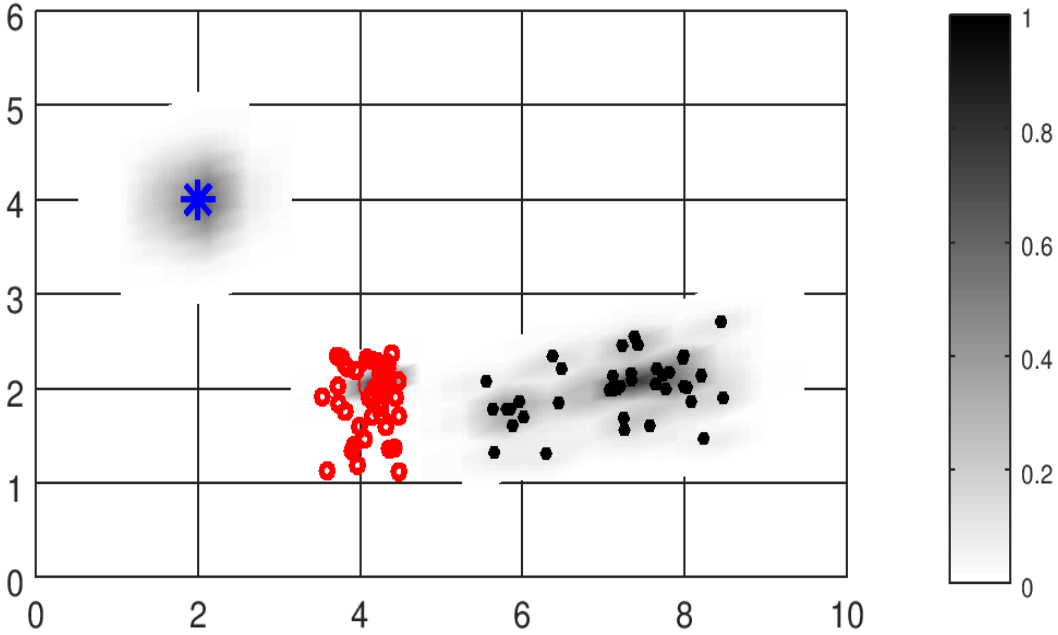


3) LAPF/LMCPF Basic Idea



LETKF =
Move ensemble as for Gaussian distribution

LMCPF =
Each particle a Gaussian + PF weights



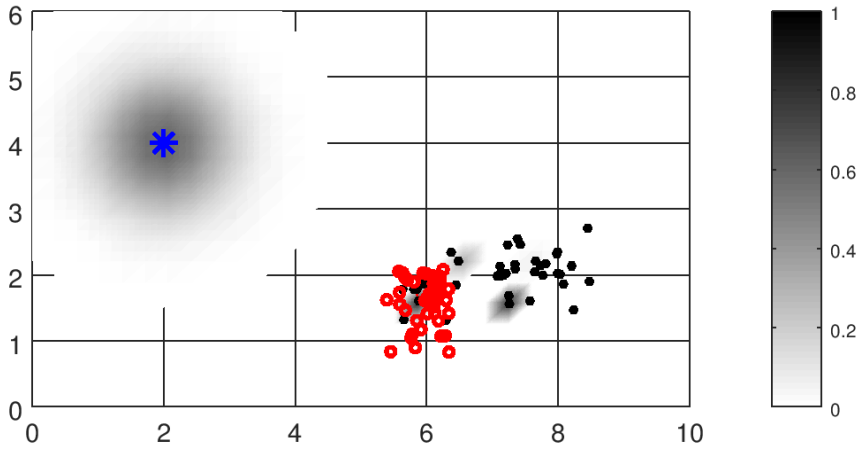
3) LAPF & LMCPF: Weights

$$w_\ell := e^{-\frac{1}{2}(y-Hx^{(\ell)})^T R^{-1}(y-Hx^{(\ell)})}, \quad \ell = 1, \ldots, L$$

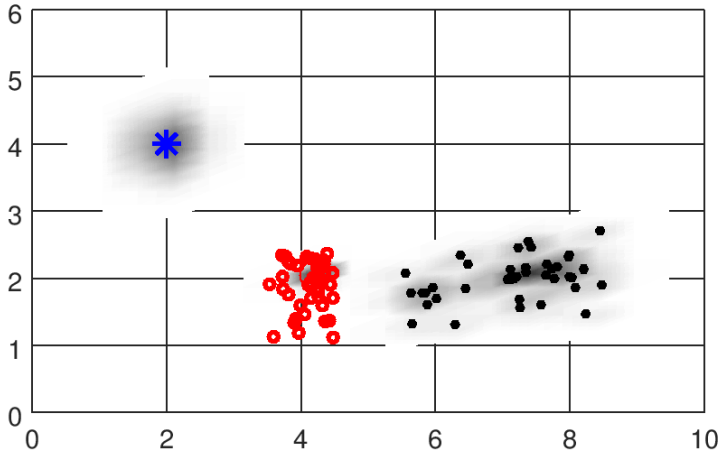
$$w_{tot} := \sum_{\ell=1}^{L} w_\ell.$$

We need a selection based on relative weights!

LAPF



LMCPF



3) LAPF & LMCPF: Ansatz

- **Kalman Filter**

Best Estimator

$$x^{(a)} = x^{(b)} + BH^T(R + HBH^T)^{-1}(y - Hx^{(b)})$$

$$K = BH^T(R + HBH^T)^{-1}$$

Kalman Matrix

$$\tilde{B} = (I - KH)B$$

Update B Matrix

Ensemble B Estimator

$$\bar{x} := \frac{1}{L}\sum_{\ell=1}^{L} x^{(\ell)}$$

$$B = \frac{1}{L-1} X X^T$$

$$X = \left(x^{(1)} - \bar{x}, ..., x^{(L)} - \bar{x}\right) \in \mathbb{R}^{n \times L}.$$

3) LAPF & LMCPF: Transform

- **Transformed Kalman Filter**

$$
\begin{aligned}
x^{(a)} &= x^{(b)} + BH^T(R + HBH^T)^{-1}(y - Hx^{(b)}) \\
&= x^{(b)} + \gamma XX^T H^T(R + \gamma H XX^T H^T)^{-1}(y - Hx^{(b)}) \\
&= x^{(b)} + \gamma X Y^T(R + \gamma YY^T)^{-1}(y - Hx^{(b)}) \\
&= x^{(b)} + \gamma X (I + \gamma Y R^{-1} Y^T)^{-1} Y^T R^{-1}(y - Hx^{(b)})
\end{aligned}
$$

$$A = YR^{-1}Y^T$$

$$B = \frac{1}{L-1}XX^T$$

$$Y := HX$$

$$Y^T(R + \gamma YY^T)^{-1} = (I + \gamma Y^T R^{-1} Y)^{-1} Y^T R^{-1}$$

3) LAPF & LMCPF: Transform

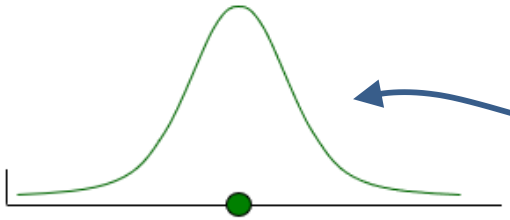
$$Y := HX$$

$$\begin{aligned}
\tilde{B} &= (I - KH)B \\
&= (I - BH^T(R + HBH^T)^{-1}H)B \\
&= \left(I - \gamma X X^T H^T (R + \gamma H X X^T H^T)^{-1} H\right)\gamma X X^T \\
&= X\left(I - \gamma Y^T (R + \gamma Y Y^T)^{-1} Y\right)\gamma X^T \\
&= X\left(I - \gamma (I + \gamma Y^T R^{-1} Y)^{-1} Y^T R^{-1} Y\right)\gamma X^T \\
&= X\left((I + \gamma Y^T R^{-1} Y)^{-1}(I + \gamma Y^T R^{-1} Y - \gamma Y^T R^{-1} Y\right)\gamma X^T \\
&= X(I + \gamma Y^T R^{-1} Y)^{-1}\gamma X^T \\
&= X(\frac{1}{\gamma} I + Y^T R^{-1} Y)^{-1} X^T
\end{aligned}$$

Transform for any Gaussian Particle

$$Y^T(R + \gamma Y Y^T)^{-1} = (I + \gamma Y^T R^{-1} Y)^{-1} Y^T R^{-1}$$

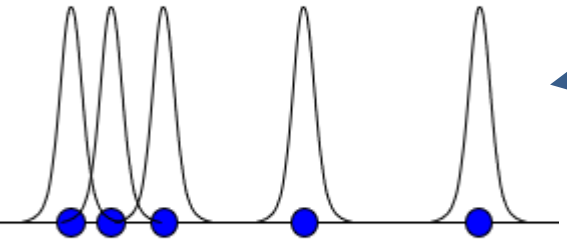
3) LAPF & LMCPF: Mixture



$$p(x|y) = cp(y|x) \cdot p(x)$$

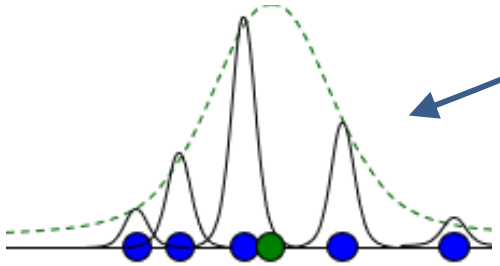
Superposition p(x)

$$= ce^{-\frac{1}{2}(y-Hx)^T R^{-1}(y-Hx)} \sum_{\ell=1}^{L} p_G\left(x - x^{(\ell)}\right)$$



$$= c \sum_{\ell=1}^{L} e^{-\frac{1}{2}(y-Hx)^T R^{-1}(y-Hx)} p_G\left(x - x^{(\ell)}\right),$$

Gaussian Mixture Case



$$p_G\left(x - x^{(\ell)}\right) = \tilde{c}e^{-\frac{1}{2}(x-x^{(\ell)})^T G^{-1}(x-x^{(\ell)})}$$

Explicit Calculations possible for each term

We need a selection based on relative weights!

3) LAPF & LMCPF: Weights

Deutscher Wetterdienst
Wetter und Klima aus einer Hand

Projection onto Ensemble Space

Derivation by Anne Walter, Andreas Rhodin and RP (MWR 2019)

Abbreviating $A := \mathbf{Y}^T\mathbf{R}^{-1}\mathbf{Y}$ and $C := A^{-1}\mathbf{Y}^T\mathbf{R}^{-1}(\mathbf{y}^o - \overline{\mathbf{y}}^b)$

Projection Operator

$$P(\mathbf{y}^o - \overline{\mathbf{y}}^b) = \mathbf{Y}(\mathbf{Y}^T\mathbf{R}^{-1}\mathbf{Y})^{-1}\mathbf{Y}^T\mathbf{R}^{-1}(\mathbf{y}^o - \overline{\mathbf{y}}^b),$$

Projected discrepancy

$$P(\mathbf{y}^o - H\mathbf{x}^{(\ell)}) = \mathbf{Y}A^{-1}\mathbf{Y}^T\mathbf{R}^{-1}((\mathbf{y}^o - \overline{\mathbf{y}}^b) - \mathbf{Y}e_\ell)$$

Exponent

$$= \mathbf{Y}(C - e_\ell), \quad \ell = 1, \ldots, L.$$

$$P(\mathbf{y}^o - H\mathbf{x}^{(\ell)})]^T\mathbf{R}^{-1}P(\mathbf{y}^o - H\mathbf{x}^{(\ell)}) = [C - e_\ell]^T A[C - e_\ell], \quad \ell = 1, \ldots, L,$$

Weights

$$w_{k,\ell} = ce^{-\frac{1}{2}[C-e_\ell]^T A[C-e_\ell]}, \quad \ell = 1, \ldots, L.$$

3) LAPF & LMCPF: Ingredients

Deutscher Wetterdienst
Wetter und Klima aus einer Hand

Derivation by Anne Walter, Andreas Rhodin and RP (MWR 2019)

Ensemble Transform

Ensemble Transform as for LETKF X w_m

Localization

Localization on R as for LETKF

$$A := \mathbf{Y}^T\mathbf{R}^{-1}\mathbf{Y} \text{ and } C := A^{-1}\mathbf{Y}^T\mathbf{R}^{-1}(\mathbf{y}^o - \bar{\mathbf{y}}^b)$$

Move Particle (Shift)

Move each particle as LETKF moves the mean, but now individually calculated

Adaptivity via O-B

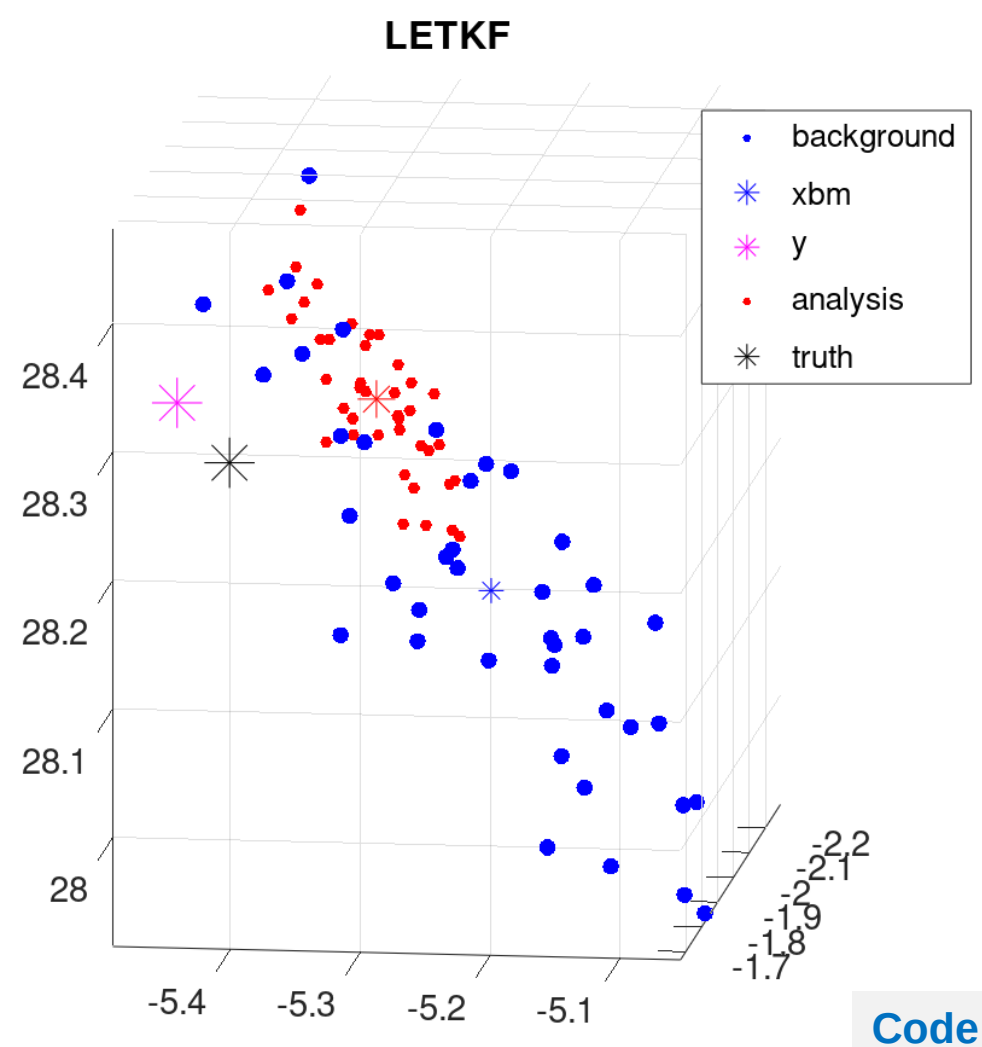
$$\mathrm{E}\left[\boldsymbol{d}_{o-b}^T\boldsymbol{d}_{o-b}\right] = \mathrm{Tr}(\mathbf{R}) + \rho\,\mathrm{Tr}(\boldsymbol{H}\boldsymbol{P}^b\boldsymbol{H}^T)$$

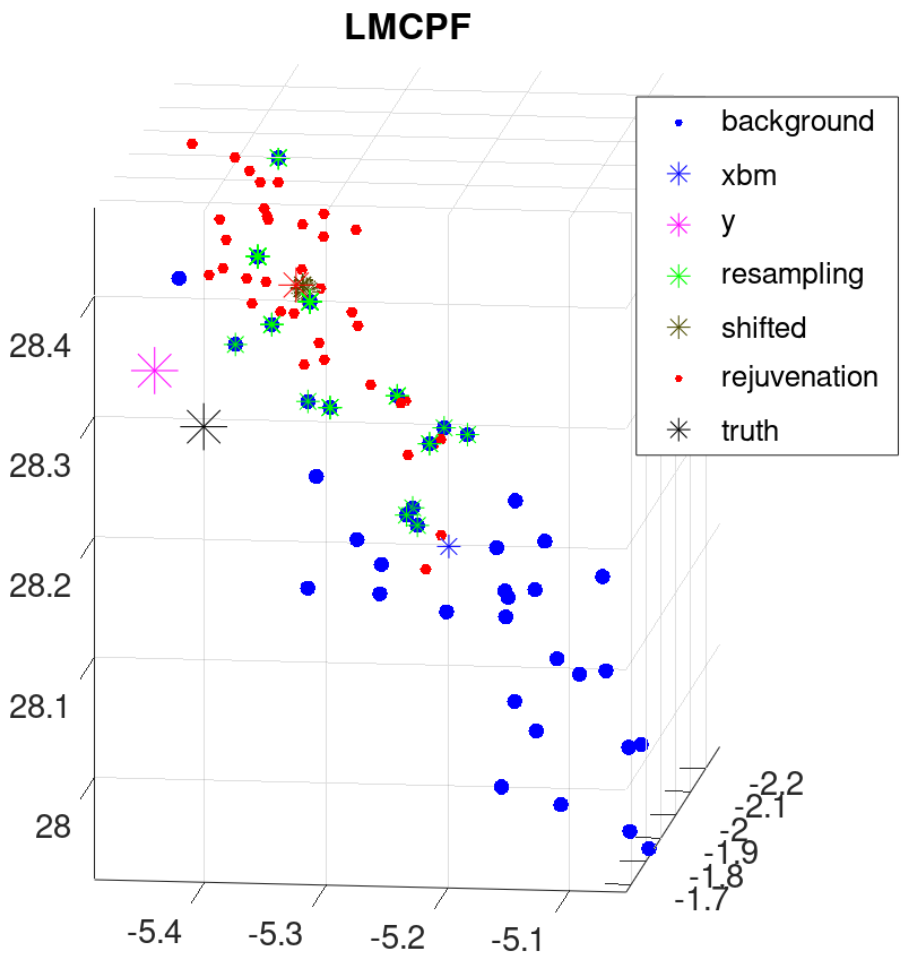
Resampling

Modulated resampling in ensemble space around each remaining particle adaptively based on ρ

4) LMCPF for Lorenz Examples

Deutscher Wetterdienst
Wetter und Klima aus einer Hand

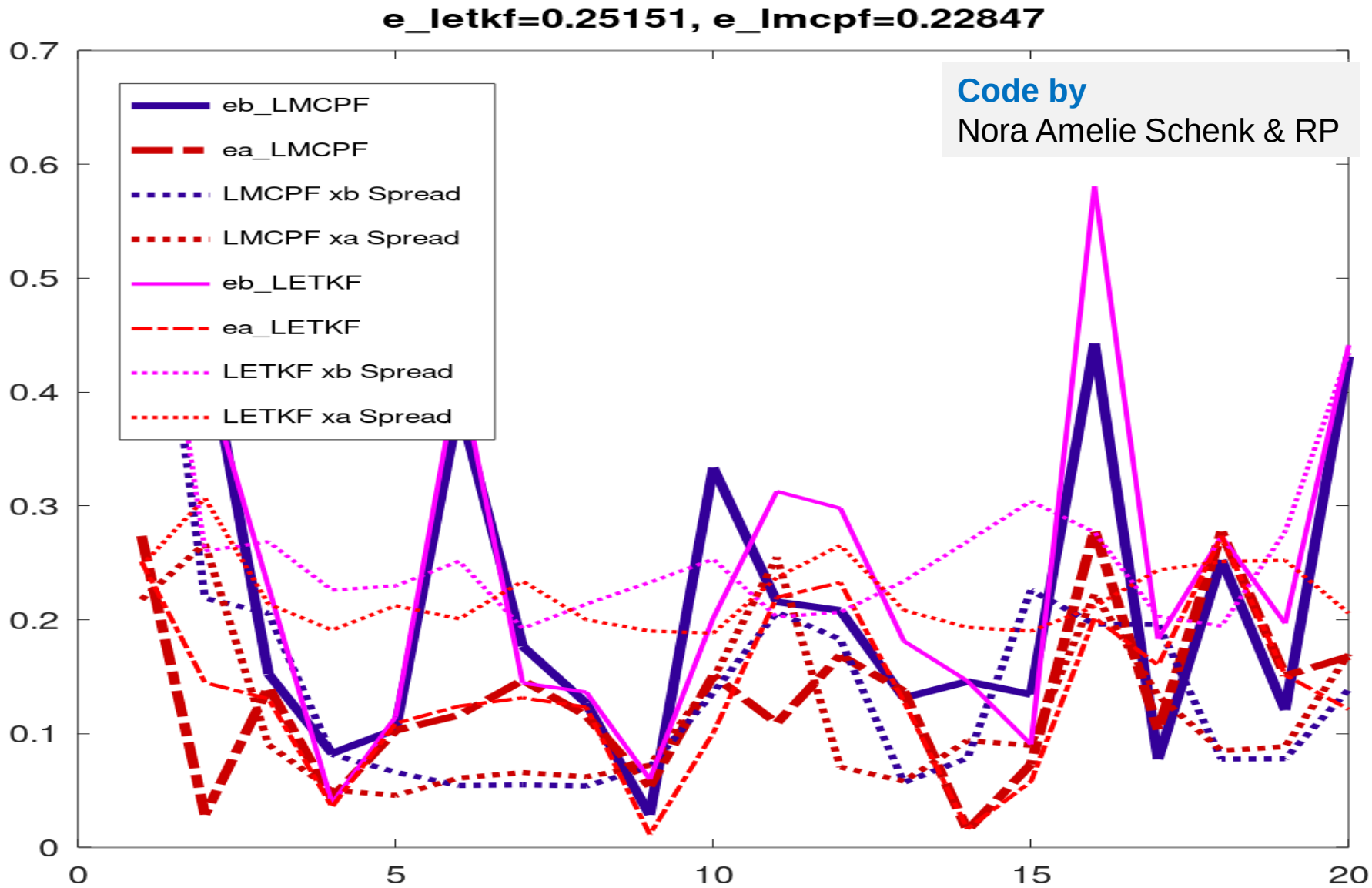




Code by
Nora Amelie Schenk & RP

4) LMCPF for Lorenz Examples

Deutscher Wetterdienst
Wetter und Klima aus einer Hand



Code by
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4) LMCPF for Lorenz Examples

Deutscher Wetterdienst
Wetter und Klima aus einer Hand

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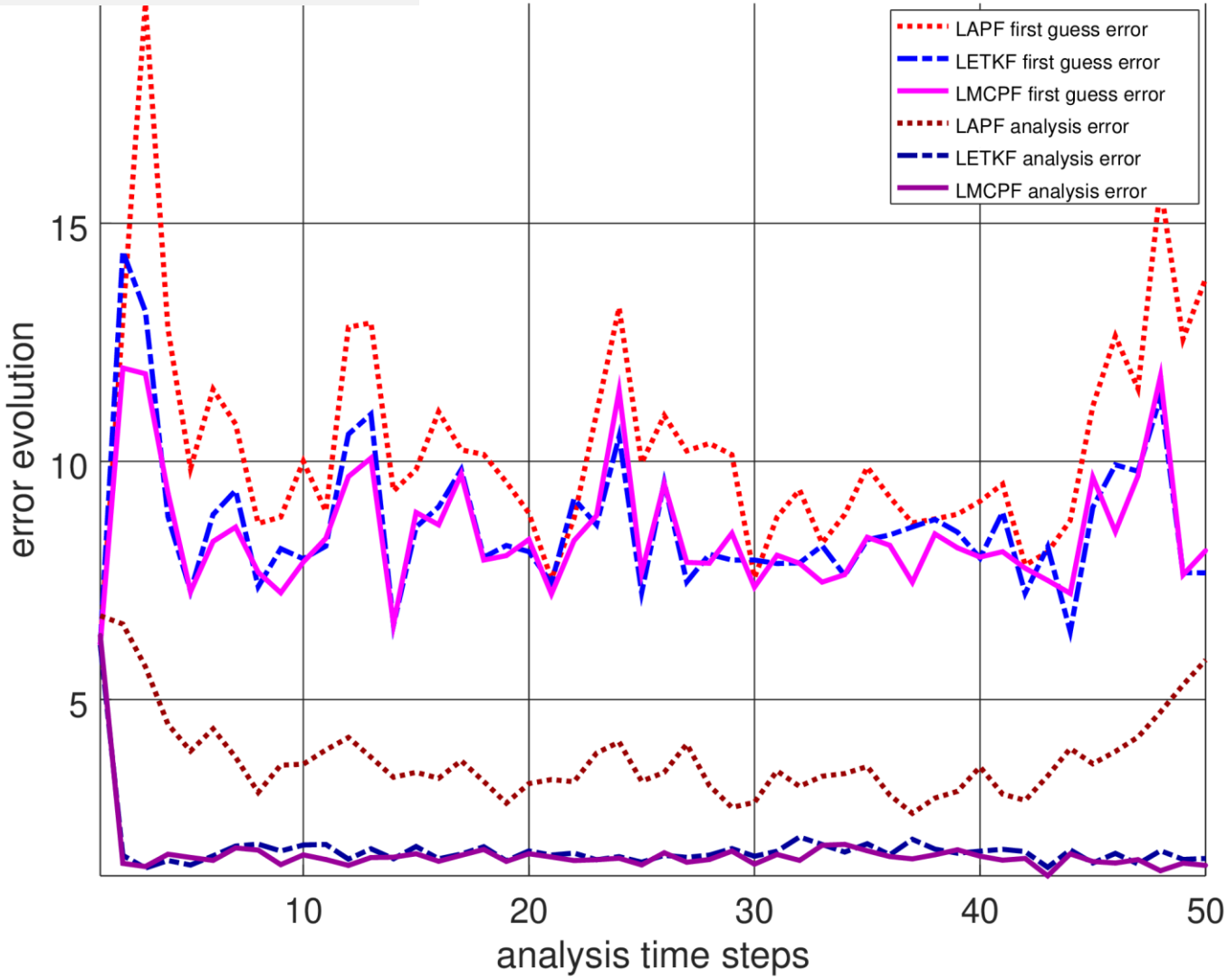


Lorenz 96

100 Variables

20 Ensemble Members

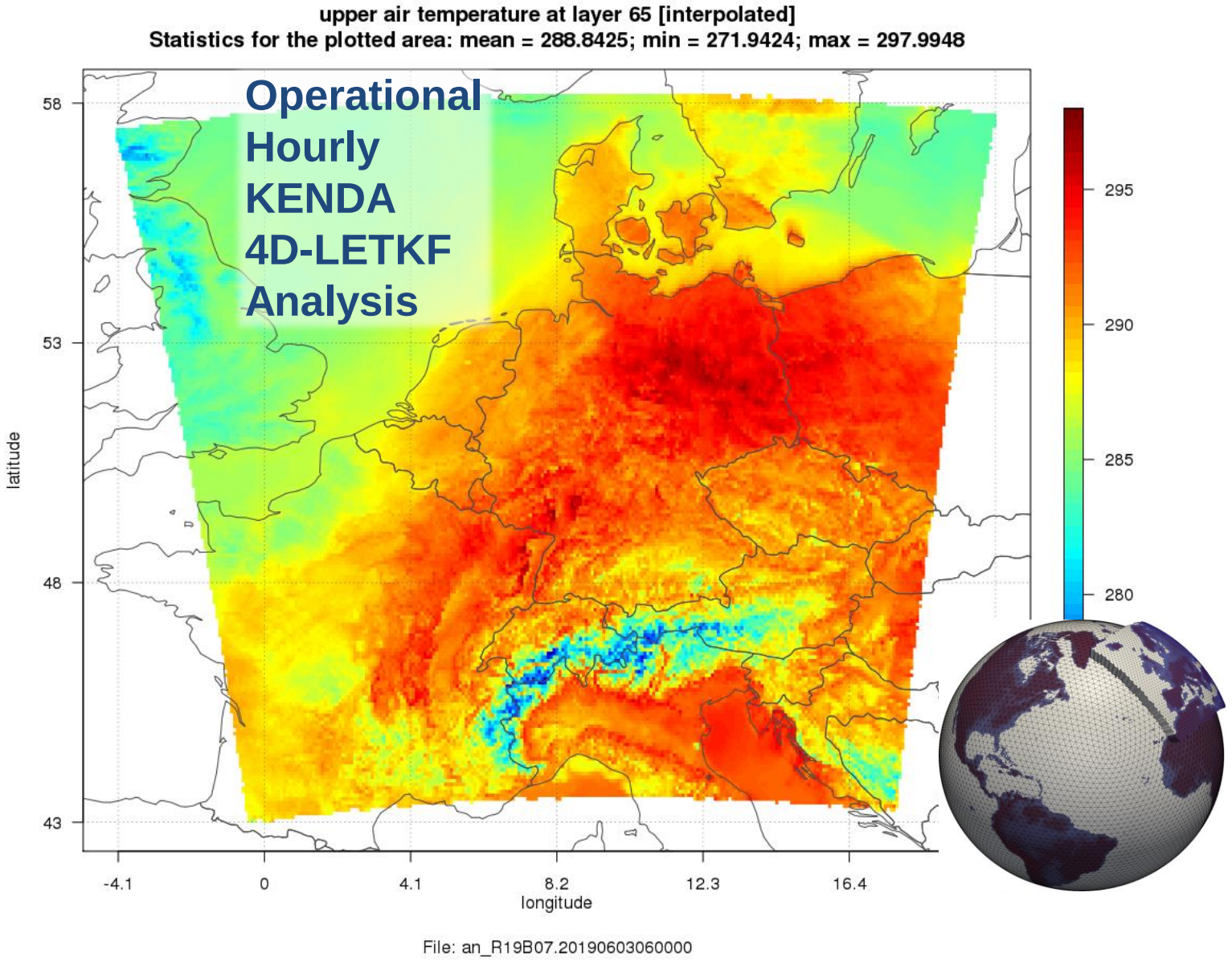
50 Assimilation Cycles

Comparing

LAPF
LMCPF
LETKF

5) LMCPF vs LETKF for ICON D2

Deutscher Wetterdienst
Wetter und Klima aus einer Hand



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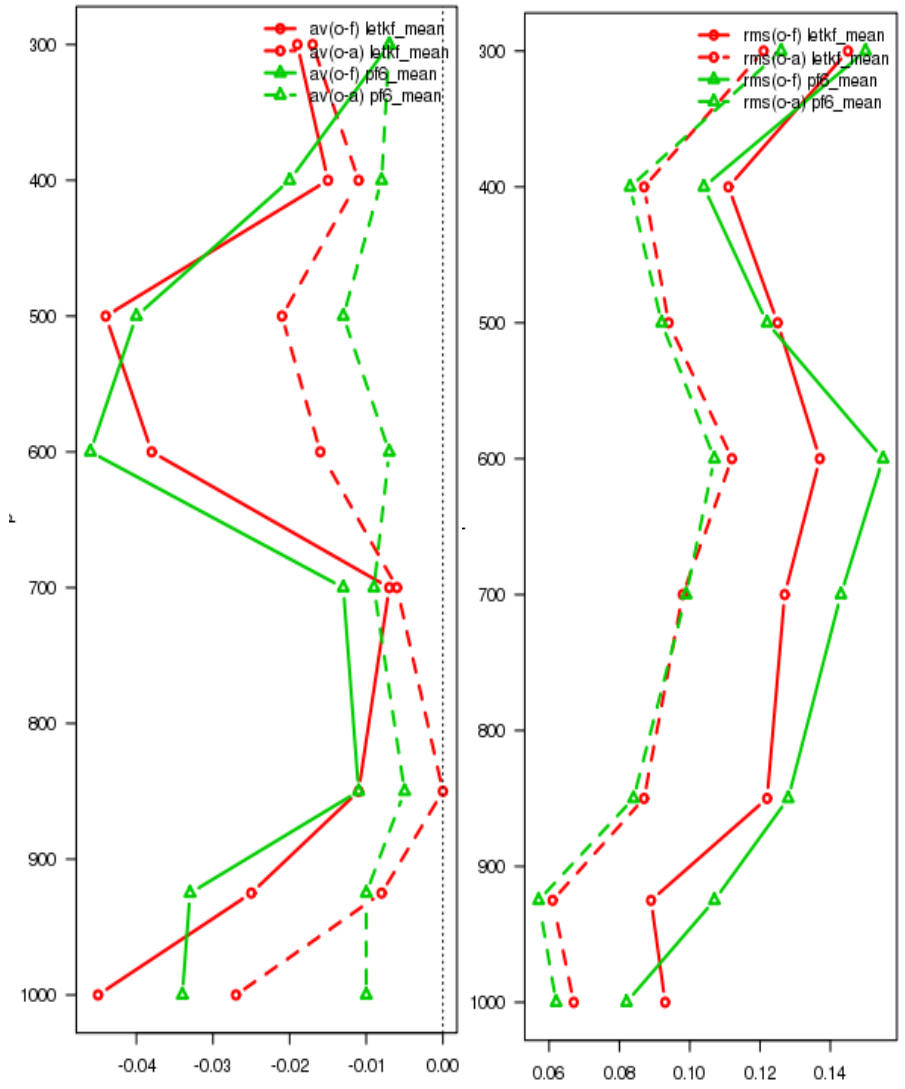
5) LMCPF vs LETKF for ICON D2

Conventional + all-sky SEVIRI VIS

Deutscher Wetterdienst
Wetter und Klima aus einer Hand

RH



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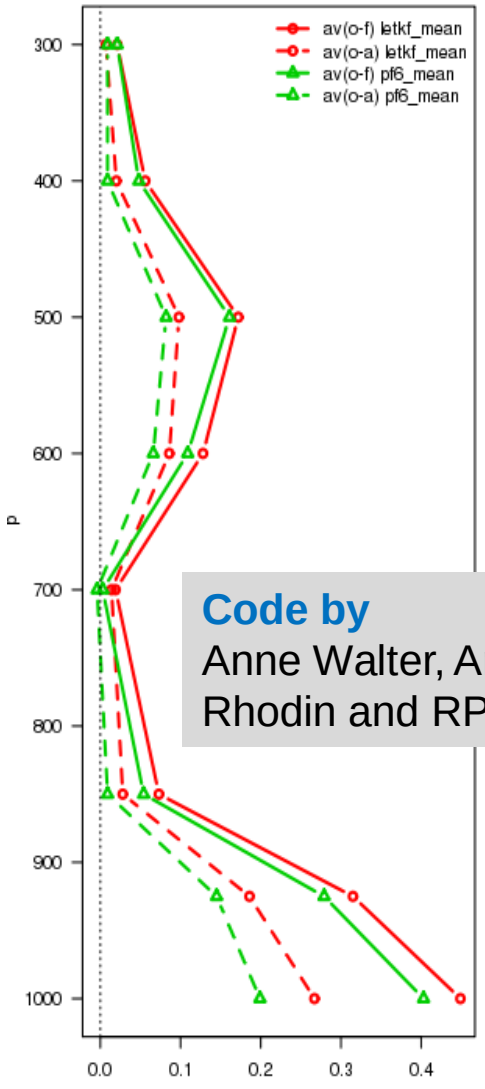
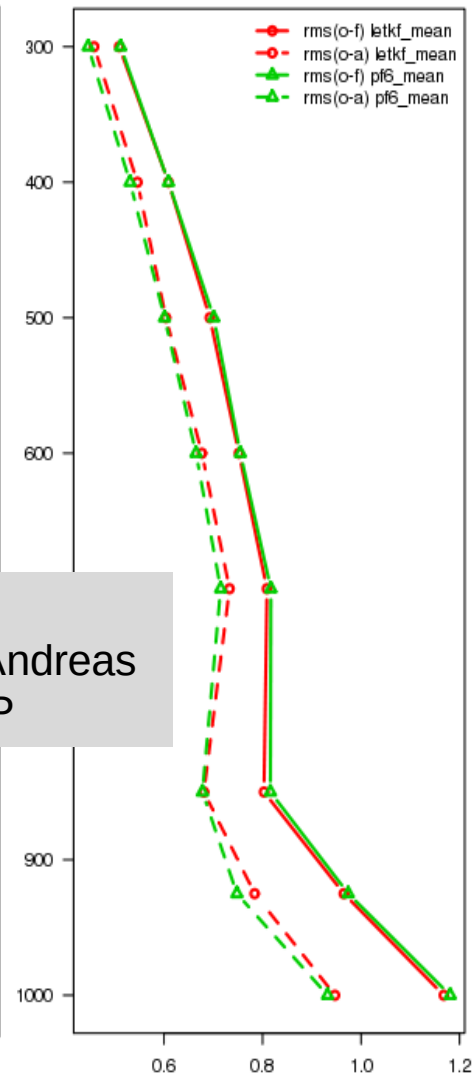


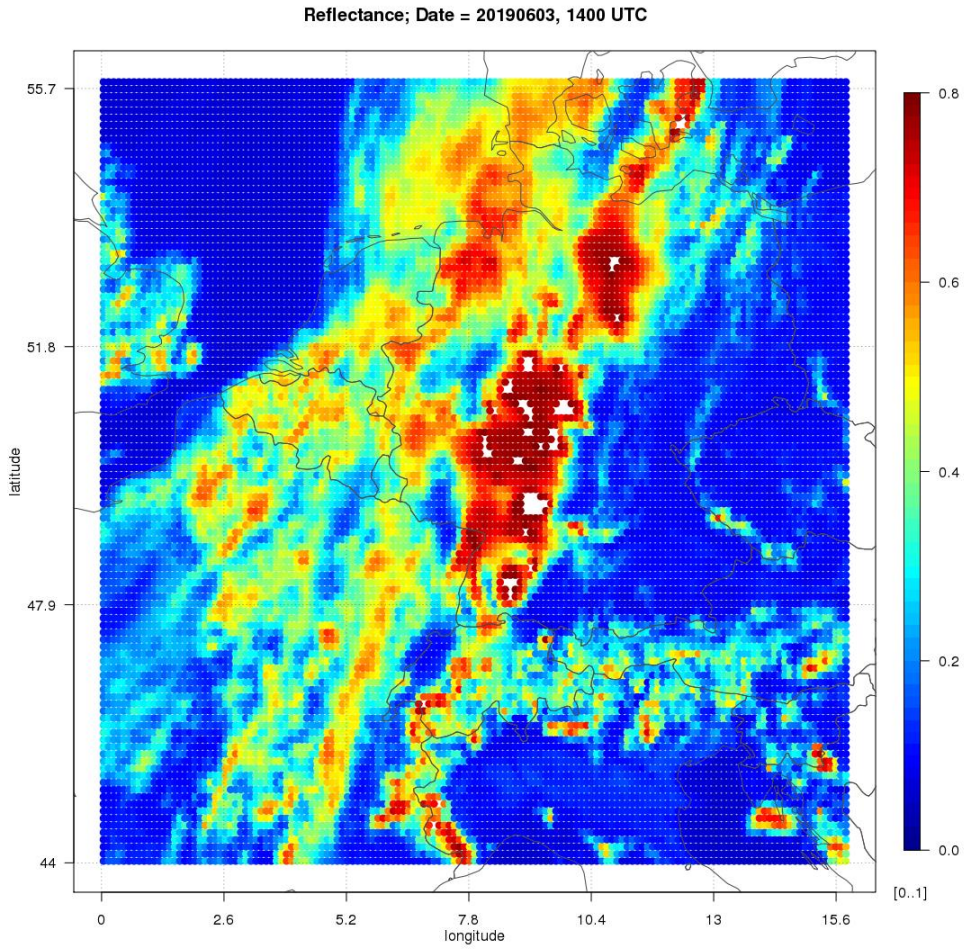


Code by
Anne Walter, Andreas Rhodin and RP

5) LMCPF vs LETKF for ICON D2

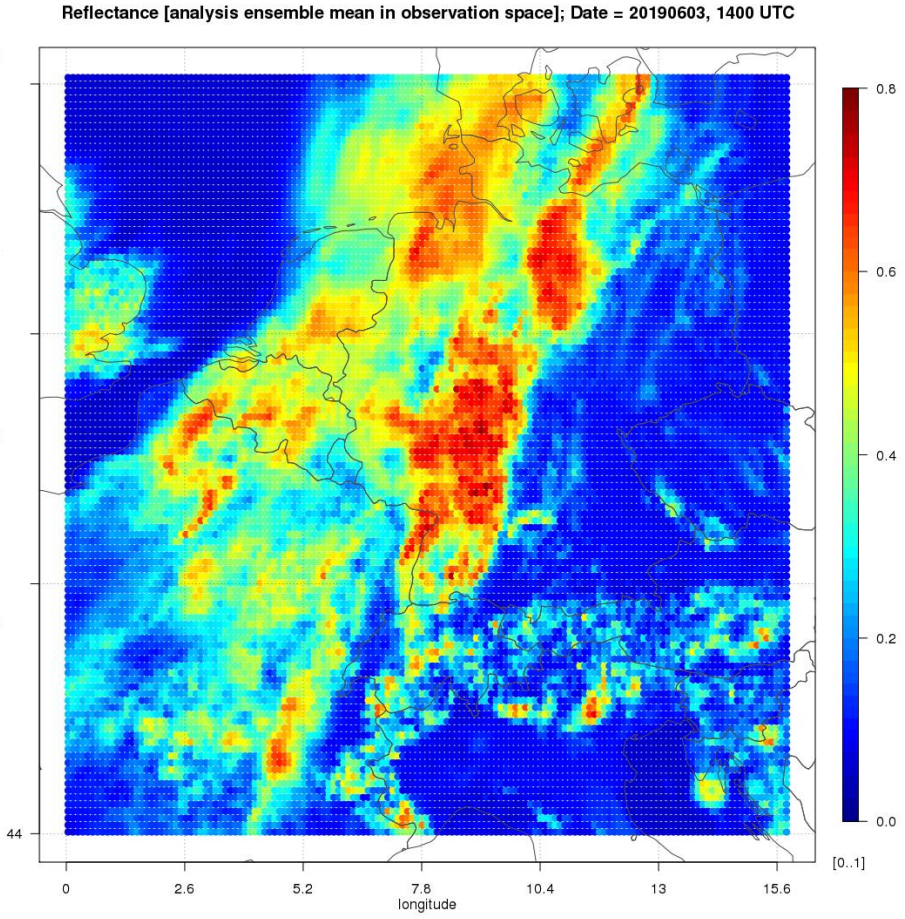
Conventional + all-sky SEVIRI VIS

Deutscher Wetterdienst
Wetter und Klima aus einer Hand



Observations

LETKF Analysis Ensemble Mean

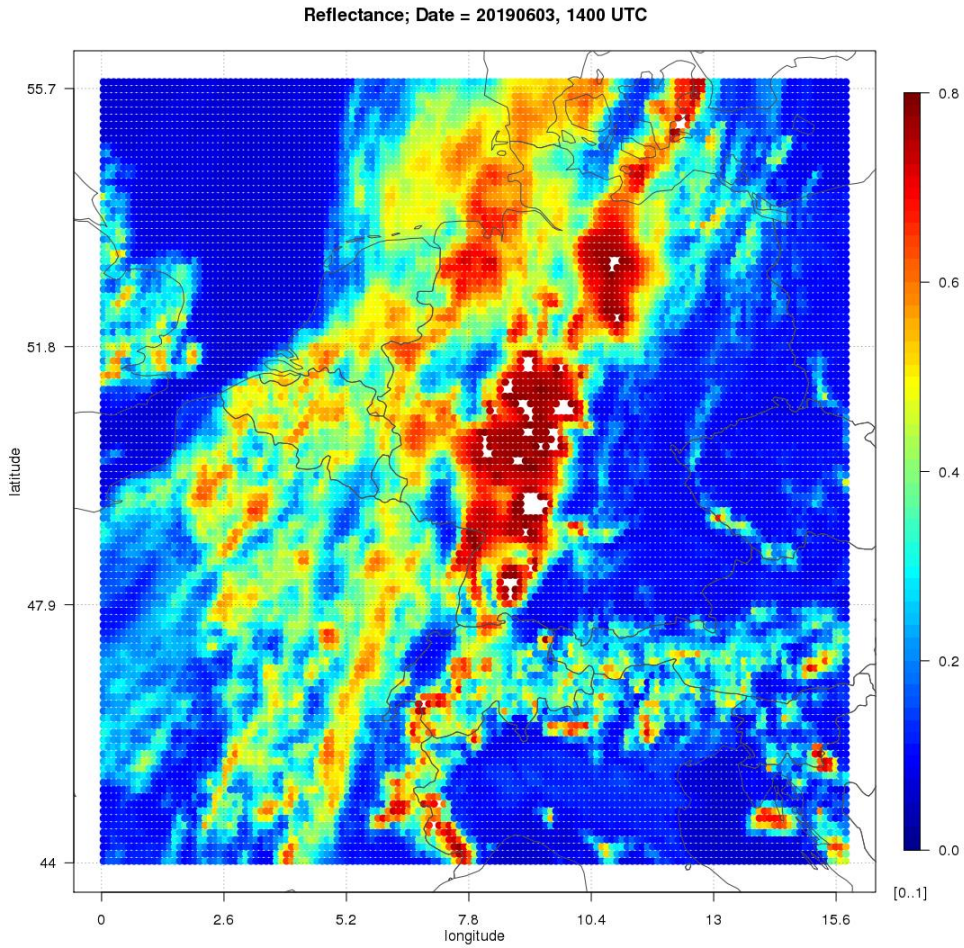


Evaluation by
Liselotte Bach and RP

5) LMCPF vs LETKF for ICON D2

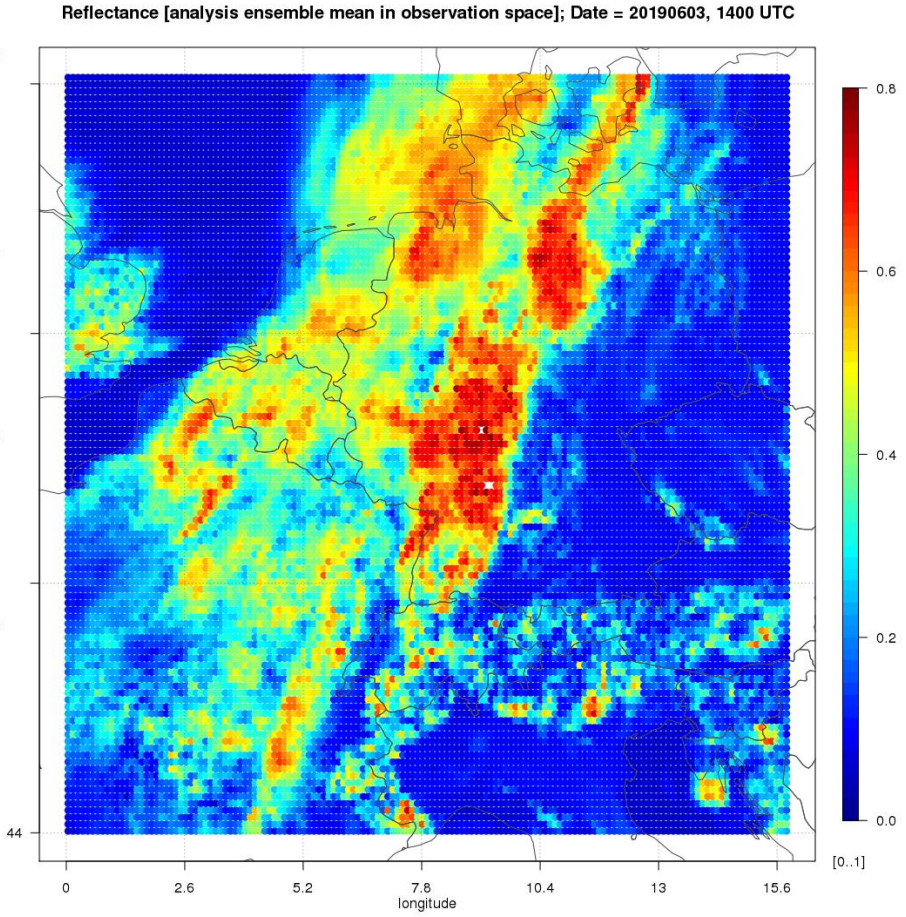
Conventional + all-sky SEVIRI VIS

Deutscher Wetterdienst
Wetter und Klima aus einer Hand



Observations

LMCPF Analysis Ensemble Mean

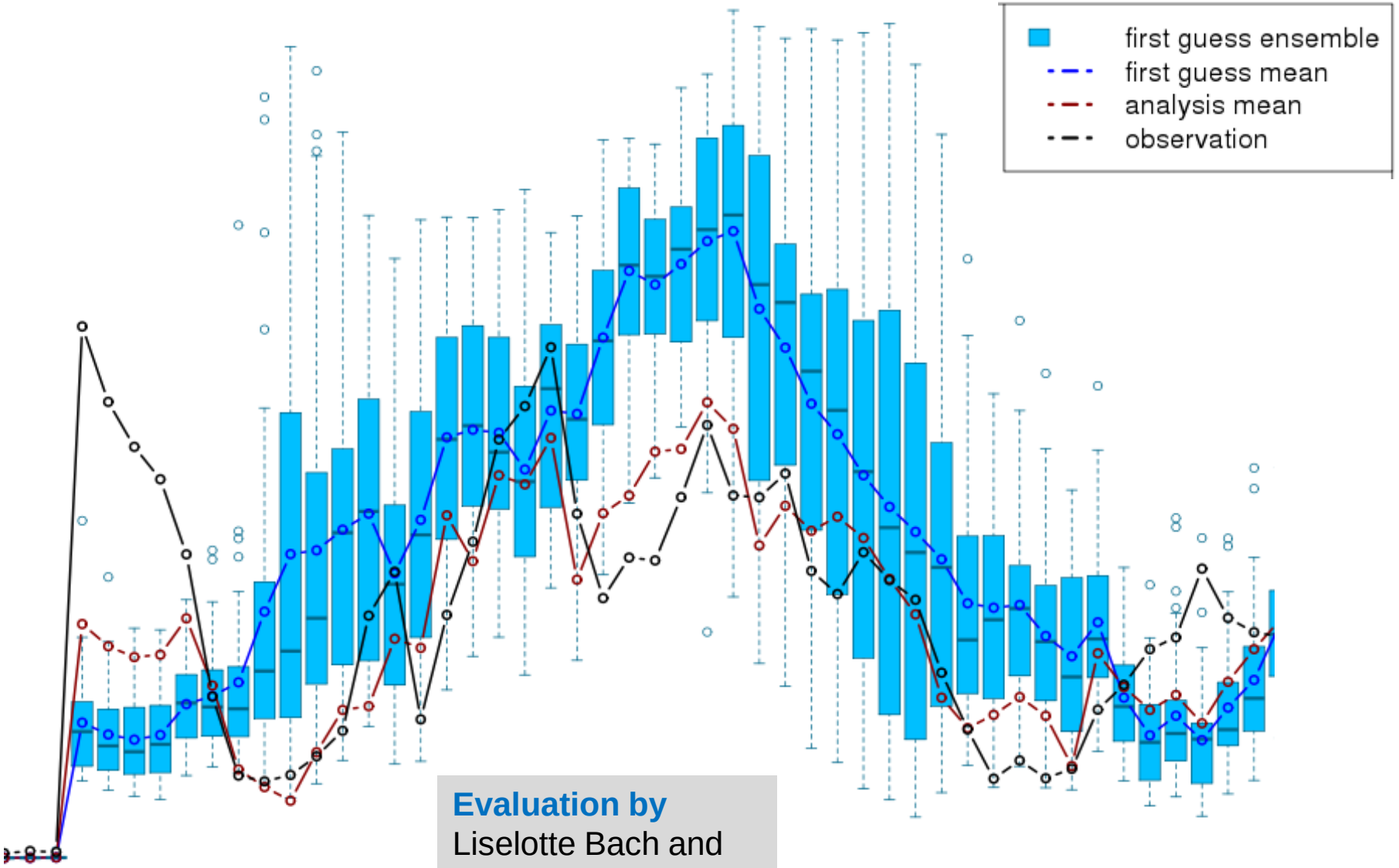


Evaluation by
Liselotte Bach and RP

5) LMCPF vs LETKF for ICON D2

Conventional + all-sky SEVIRI VIS

Deutscher Wetterdienst
Wetter und Klima aus einer Hand

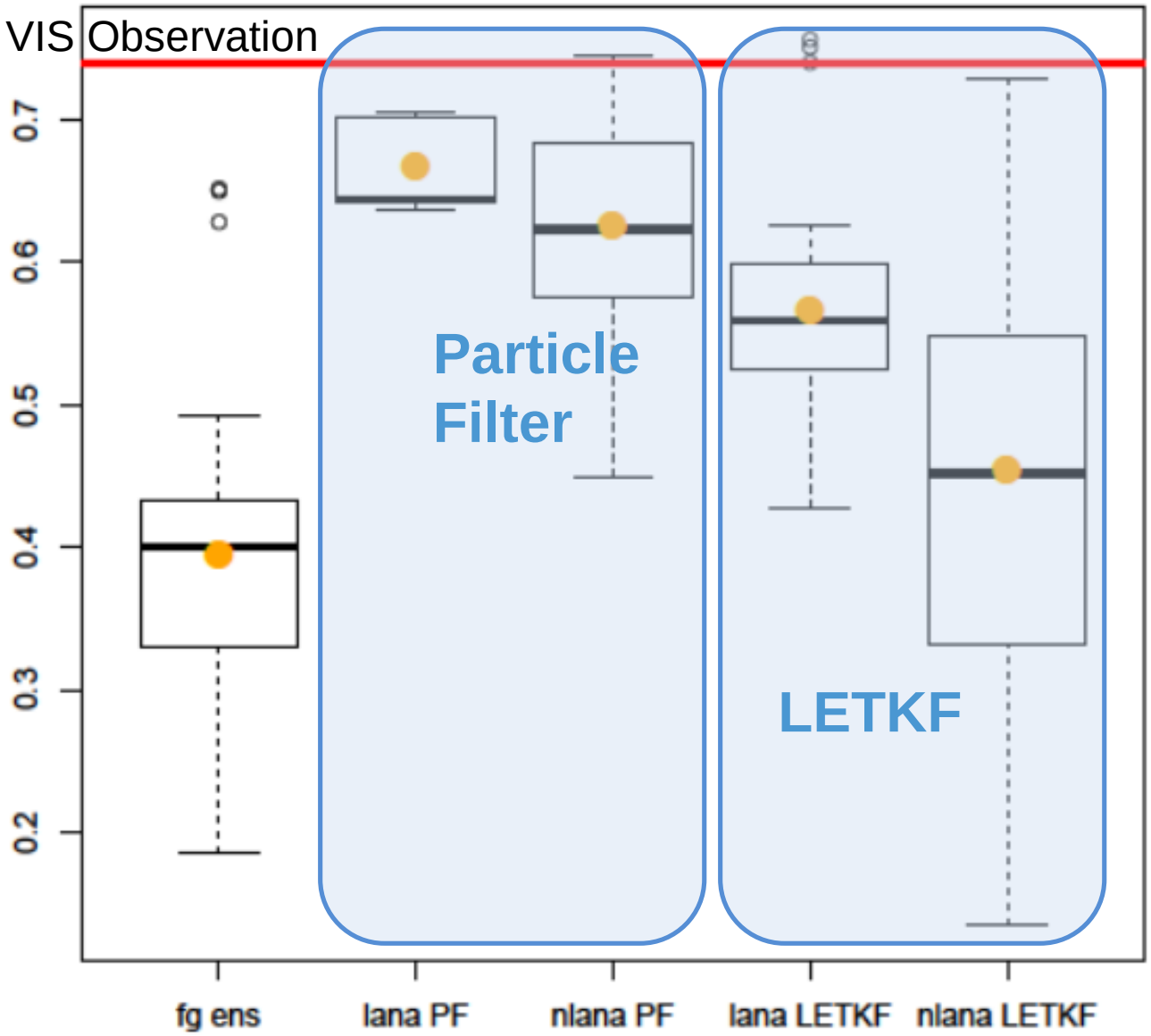


Evaluation by
Liselotte Bach and
Nora Amelie Schenk

5) LMCPF vs LETKF for ICON D2

Conventional + all-sky SEVIRI VIS

Deutscher Wetterdienst
Wetter und Klima aus einer Hand



Graphics by
Liselotte Bach
Nora Amelie Schenk
Christian Welzbacher

Linear Analysis Distributions

vs

Non-linear Analysis Distributions

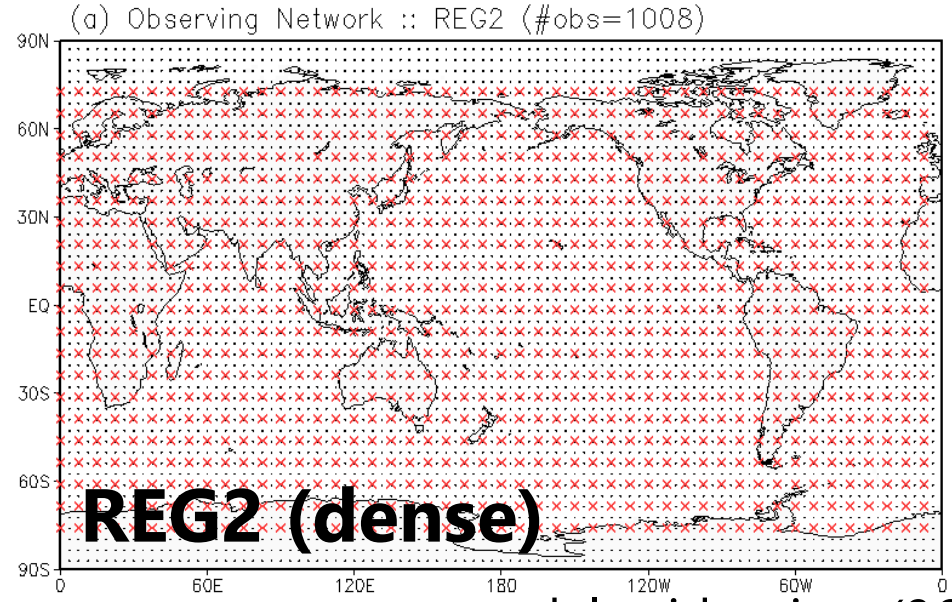
PF vs LETKF

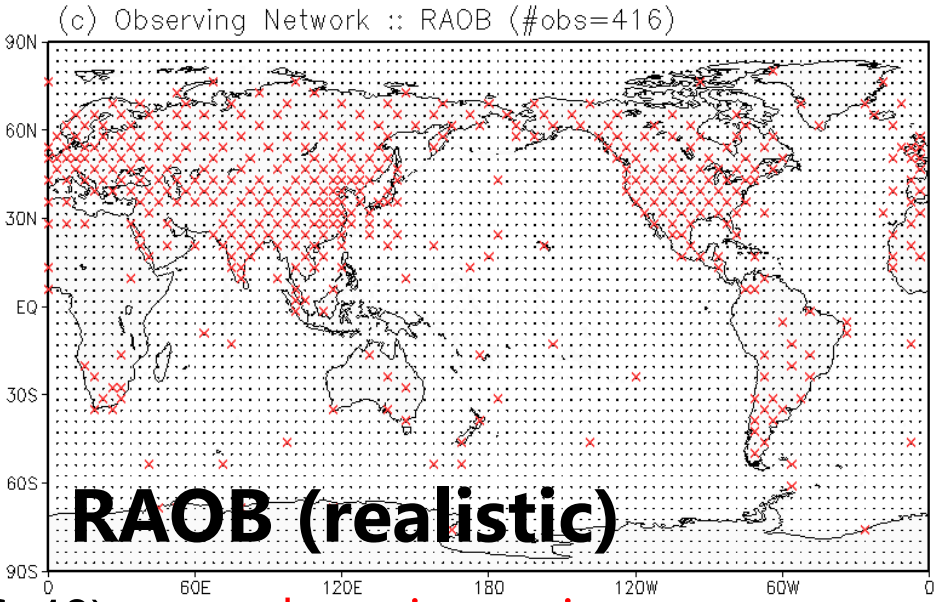
6) SPEEDY–LETKF (OSSE)

Deutscher Wetterdienst
Wetter und Klima aus einer Hand

- **SPEEDY (Molteni et al. 2003)**
  - An intermediate AGCM (T30/L7 = 96 x 48 x 7)
- **Ensemble Size: 40**
- **Data Assimilation : LETKF, LPF, GM extension (LPFGM)**
- **Observation: upper-air profiles**

Evaluation by
Miyoshi and Kotsuki



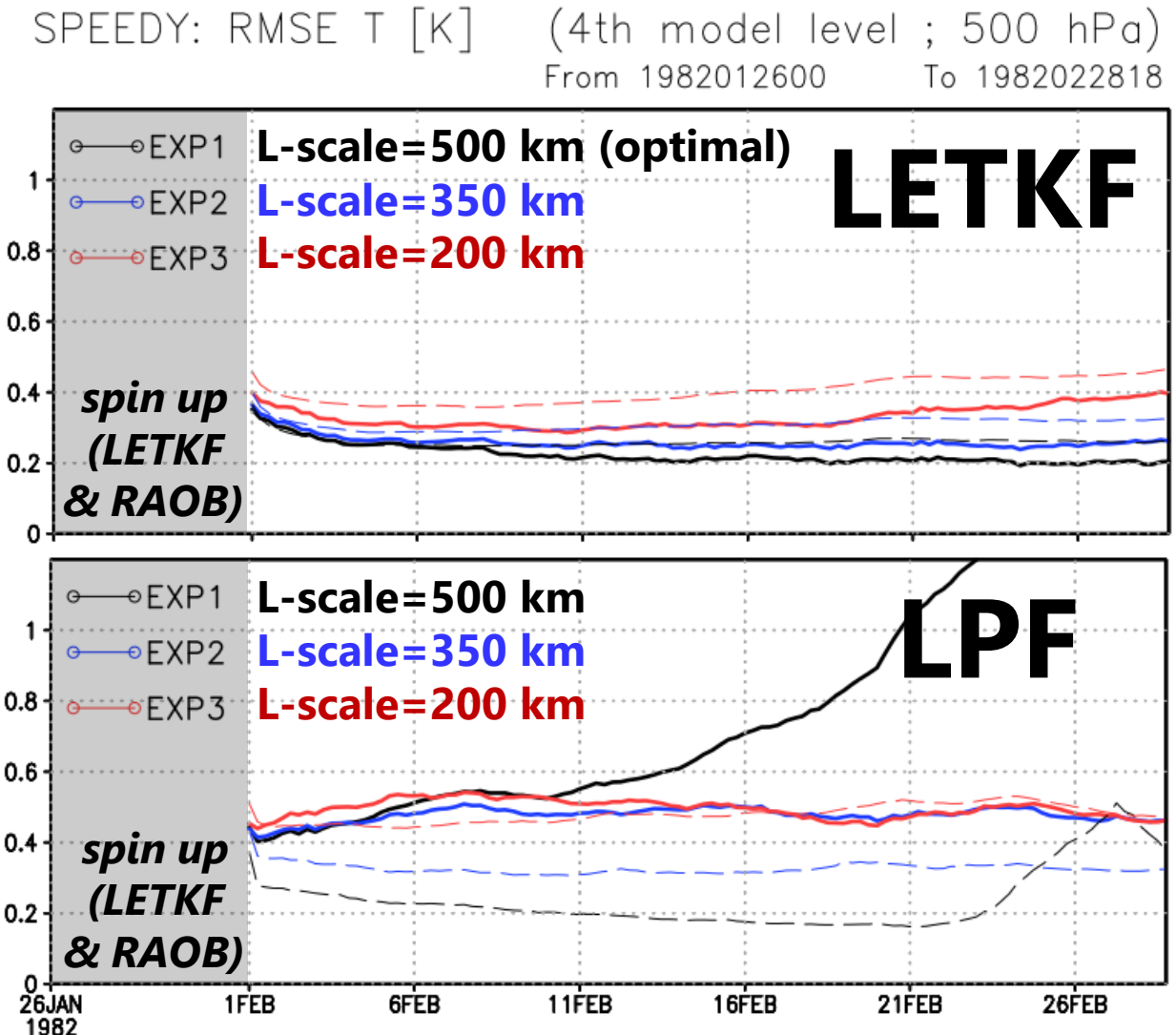


▪ : model grid points (96x48) ×: observing points

6) LETKF vs. LAPF (REG2)

Deutscher Wetterdienst
Wetter und Klima aus einer Hand

(1) Optimal localization scale for LPF seems smaller than LETKF.
(2) LPF works, but would not assimilate dense observations effectively.
(3) Because of smaller localization, LPF is worse than LETKF.



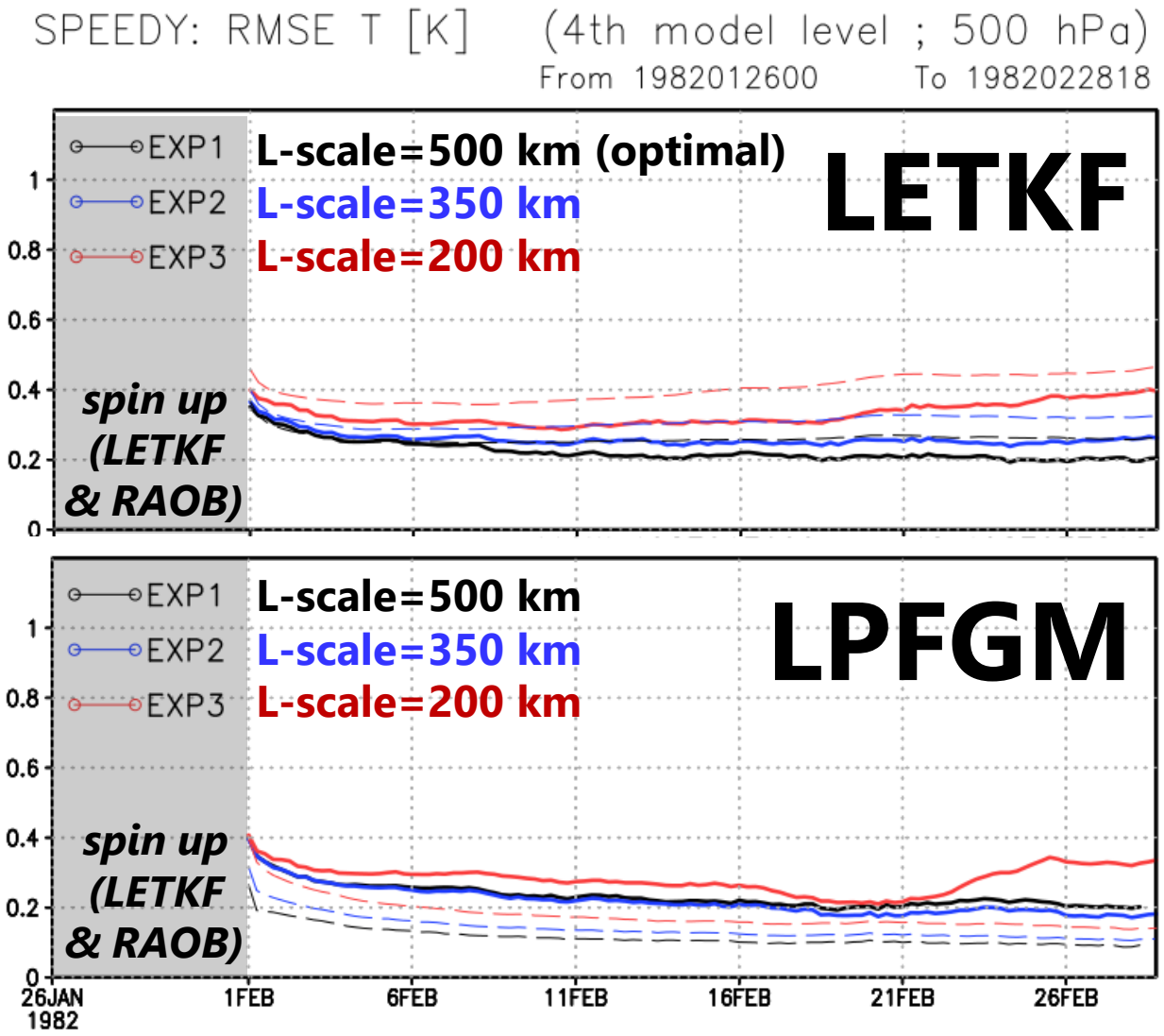
RMSE (solid)
Spread (dashed)

Evaluation by
Miyoshi and Kotsuki

6) LETKF vs. LPFGM (REG2)

Deutscher Wetterdienst
Wetter und Klima aus einer Hand

Optimal localization scale for LPFGM seems smaller than LETKF.



RMSE (solid)

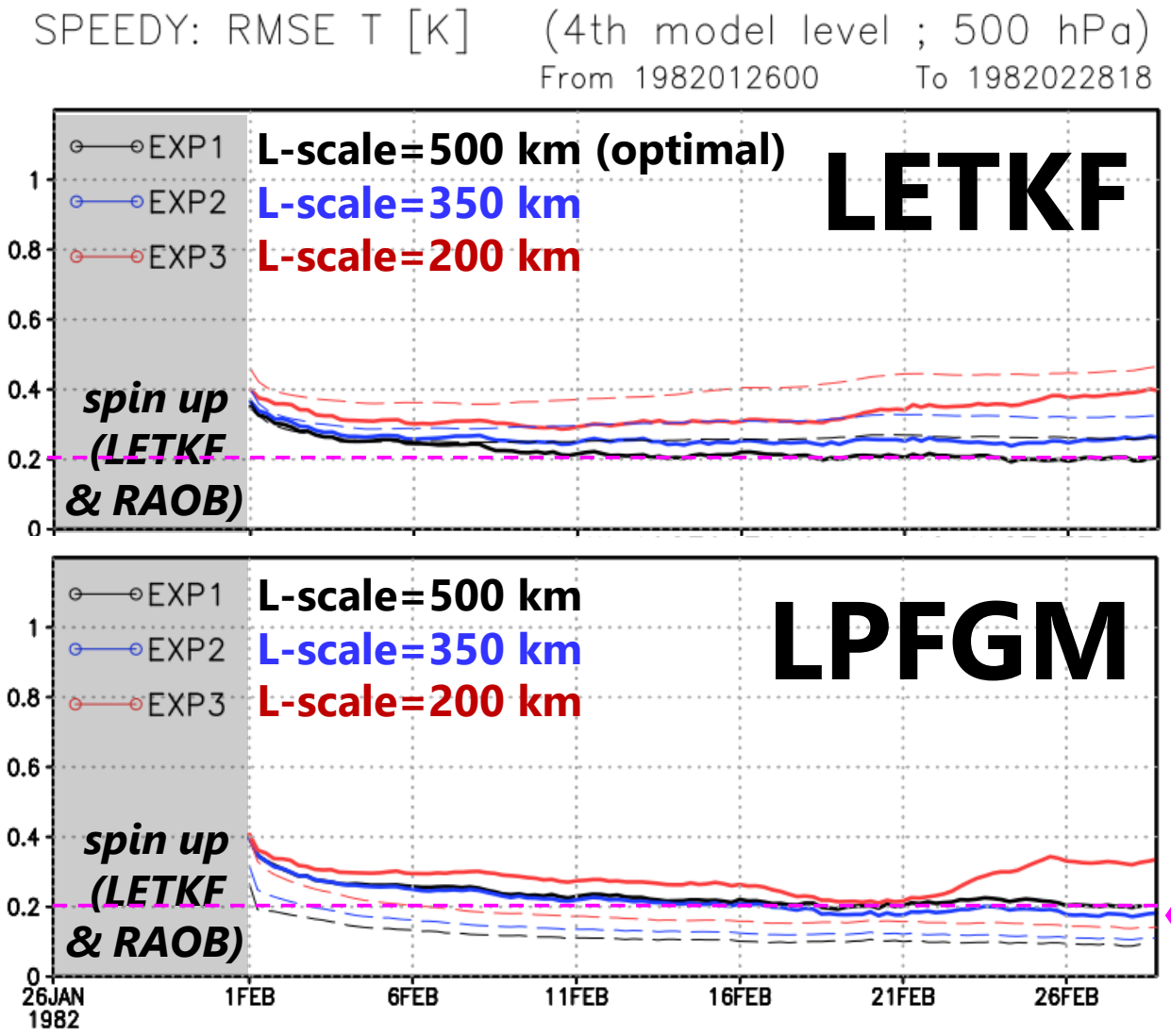
Spread (dashed)

Evaluation by
Miyoshi and Kotsuki

6) LETKF vs. LPFGM (REG2)

Deutscher Wetterdienst
Wetter und Klima aus einer Hand

Optimal localization scale for LPFGM seems smaller than LETKF.



RMSE (solid)

Spread (dashed)

Evaluation by
Miyoshi and Kotsuki

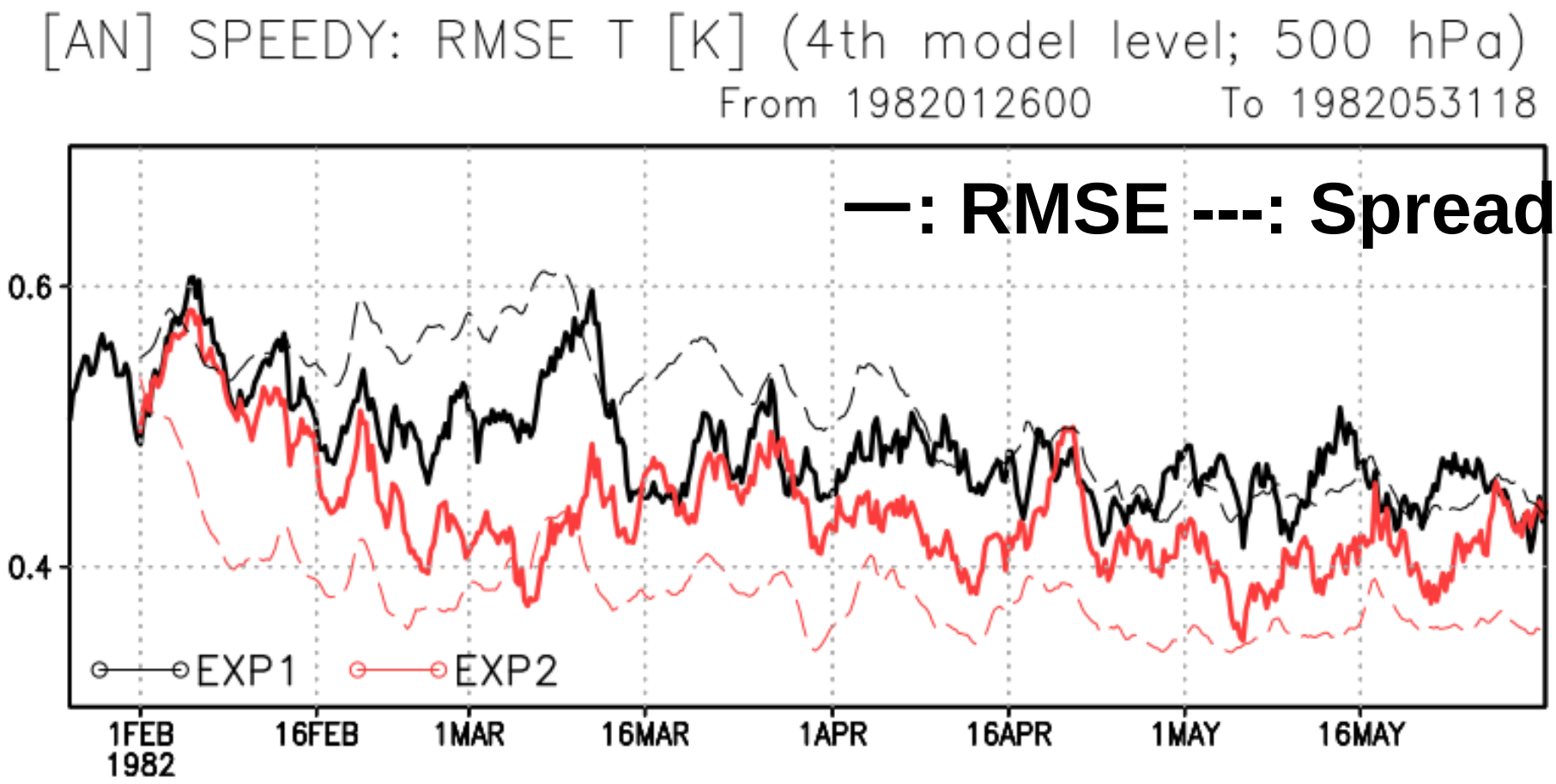
slightly Improved?

6) RAOB, 4th model temperature

Deutscher Wetterdienst
Wetter und Klima aus einer Hand

—: LETKF **LPFGM (RTPS; α=0.55)**
(α manually tuned)

Resampling when Neff>2

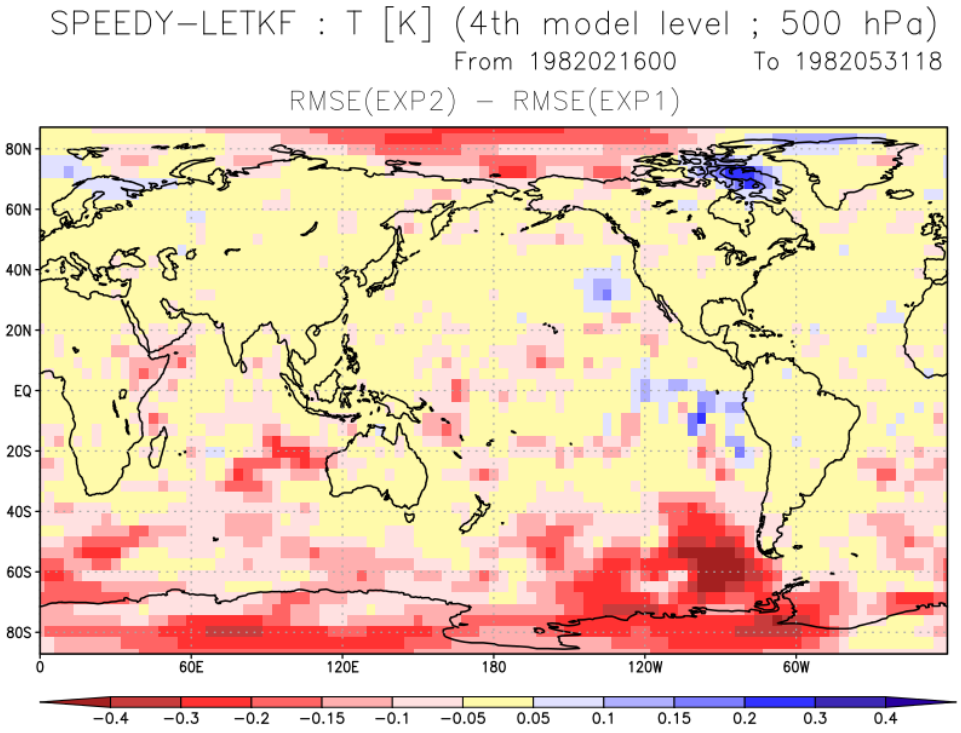


Evaluation by
Miyoshi and Kotsuki

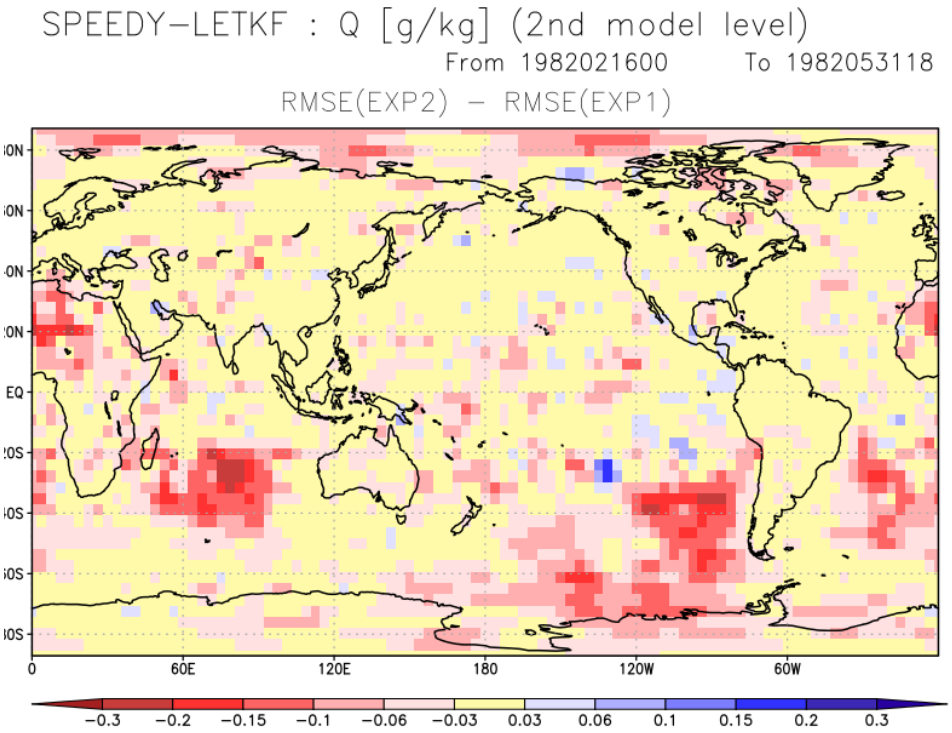
6) RAOB Results

Deutscher Wetterdienst
Wetter und Klima aus einer Hand

4th level temperature [K]



2nd level humidity [g/kg]



← **LPFGM is better**

LETKF is better →

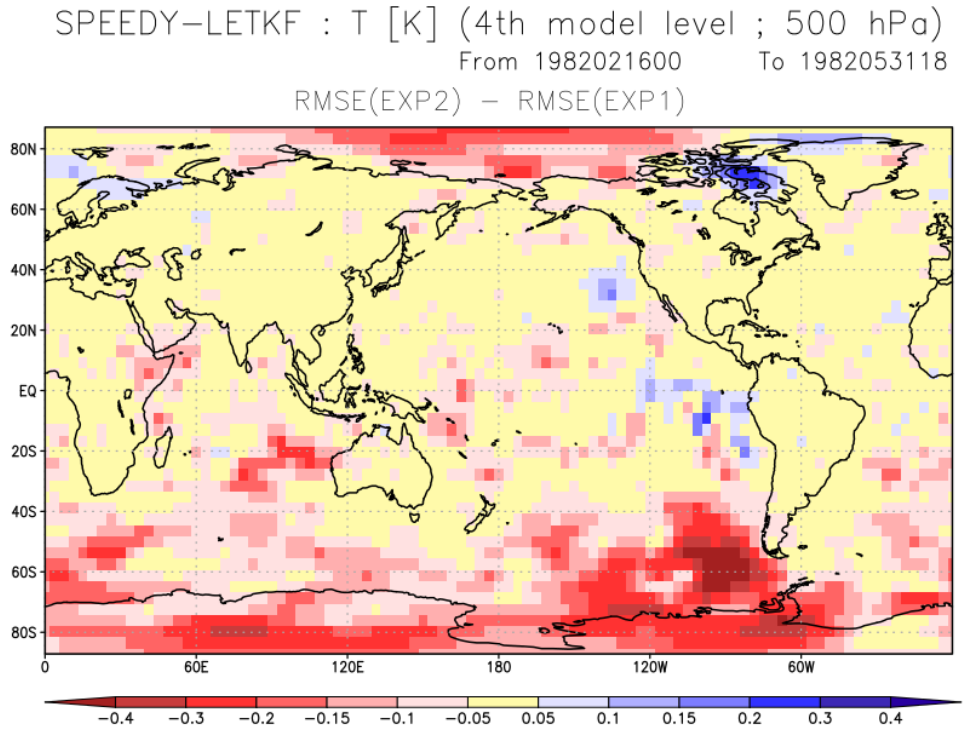
1. Advantage over sparsely observed regions
2. No clear advantage over non-Gaussian regions

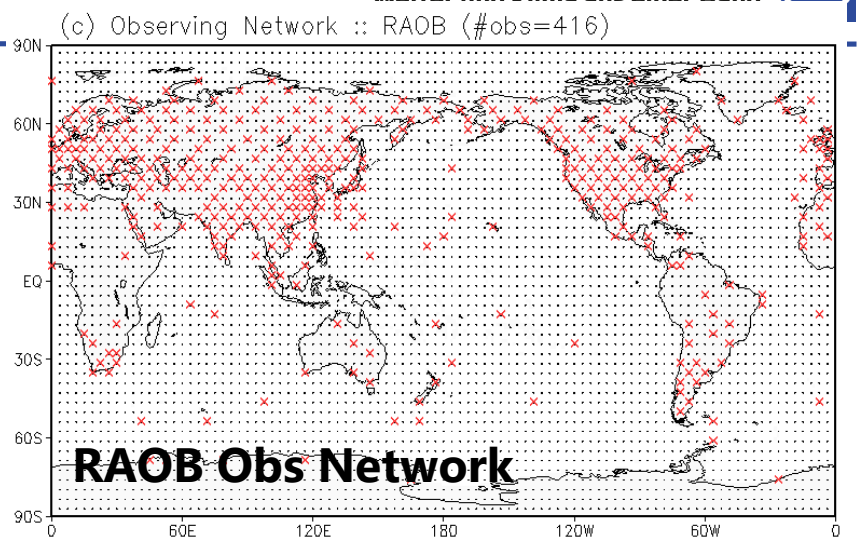
Evaluation by
Miyoshi and Kotsuki

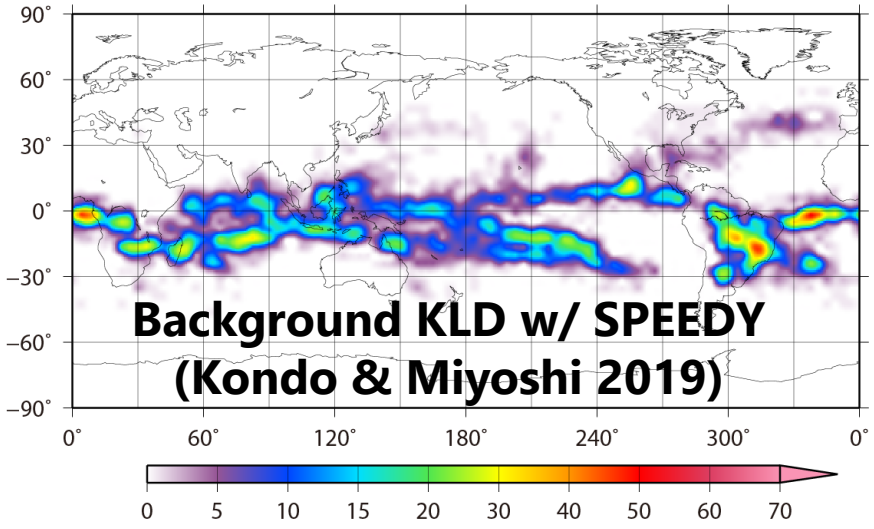
6) RAOB Results

Deutscher Wetterdienst
Wetter und Klima aus einer Hand

4th level temperature [K]







← LPFGM is better

1. Advantage over sparsely observed regions
2. No clear advantage over non-Gaussian regions

Evaluation by
Miyoshi and Kotsuki

7) Summary

Deutscher Wetterdienst
Wetter und Klima aus einer Hand

(1) LAPF and LMCPF extend the capabilities of the LETKF

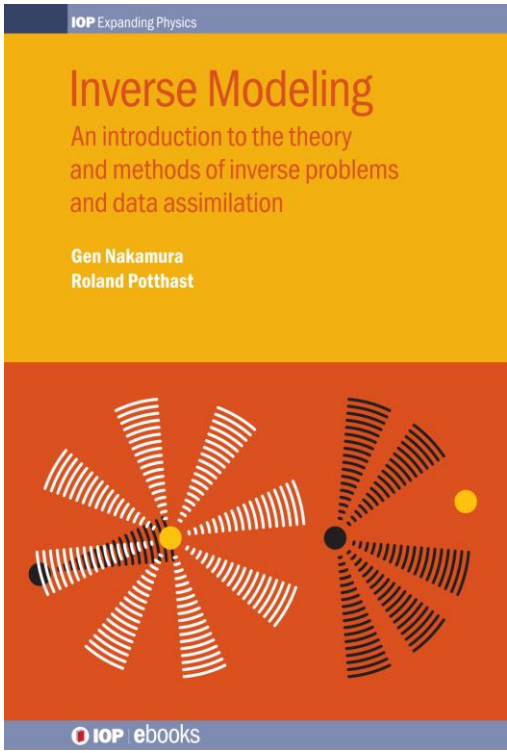
(2) Ensemble Transform, Localization, Adaptive Spread Control – LETKF, LAPF and LMCPF use the same tools

(3) Gaussian Particles allow to move towards Obs in Ensemble Space

(4) More flexible than LETKF

(5) Fully Non-Linear Filtering by LAPF and LMCPF

(6) Stable Particle Filters for global and regional NWP

(7) Simple to Code following LETKF

(8) Promising Features for Cloud Assimilation