EUMETSAT/ECMWF NWP-SAF Satellite data assimilation

A practical guide to IR and MW radiative transfer using the RTTOV model and GUI

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**Why learn about radiative transfer**

The exploitation of satellite data requires a radiative transfer model (also referred to as the “observation operator” in data assimilation) to predict a first guess observation from the NWP model fields corresponding to the observation.

The radiative transfer model (and its adjoint) is a key element in the assimilation of satellite data into a NWP system.
Satellite observations

The radiative transfer model simulates satellite observations using the equation of radiative transfer

Assimilation of satellite data

Y(n) → Quality control → Bias tuning → Y(n)

Model fields u,v,T,q ... X_{FG}

Interpolation etc.

Observation operator H(x)

Radiative transfer model

H(x) FG observation

(Y - H(x)) \Delta y sat

Radiance monitoring

Other observations \Delta a, \Delta b...

Minimise \Delta's globally

Observation and FG errors (O+F) (B)

Analysis X_{an}
Satellite instruments measure the upwelling electromagnetic radiation at the top of the atmosphere. A fundamental quantity associated to a radiation field is the intensity of the radiation field or radiance.

The monochromatic radiance, $L_\nu$, is defined as the amount of energy crossing, in a time interval $dt$ and in the frequency interval $\nu$ to $\nu + d\nu$, a differential area $dA$ at an angle $\theta$ to the normal to $dA$, the pencil of radiation being confined to a solid angle $d\Omega$.

$$L_\nu = \frac{dE_\nu}{\cos(\theta) dA dt d\Omega d\nu} \left[ \frac{W}{m^2 \text{sr} \text{s}^{-1}} \right]$$

The radiance can also be defined for a unit wavelength, $\lambda$, or wave number, $\tilde{\nu}$, interval. If, $c$, is the speed of light in vacuum, the relation between these quantities is:

$$\lambda = \frac{c}{\nu} = \frac{1}{\tilde{\nu}}$$

Wavelengths are usually expressed in units of microns (1µm=10^{-6} m) whereas wave numbers are expressed in units of cm^{-1}. 
In many applications, the radiance, $L(\nu)$, is expressed in units of equivalent brightness temperature, $T_b(\nu)$.

The brightness temperature is computed using the inverse Planck function.

$$B_\lambda (T) = \frac{c_1 \lambda^{-5}}{\pi (e^{\frac{c_2}{\lambda T}} - 1)}$$

The Planck function is used to compute the radiance emitted by a black body at temperature $T$.

$$T_b(\lambda) = \frac{c_2}{\lambda \ln\left(\frac{c_1}{\pi \lambda^5 L(\lambda)} + 1\right)}$$

The brightness temperature
### Spectrum of Electromagnetic Radiation

<table>
<thead>
<tr>
<th>Name of Region</th>
<th>Wavelength (cm)</th>
<th>Frequency (cps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gamma rays</td>
<td>$10^{-9}$</td>
<td>$3 \times 10^{19}$</td>
</tr>
<tr>
<td>X-rays</td>
<td>$10^{-6}$</td>
<td>$3 \times 10^{16}$</td>
</tr>
<tr>
<td>Ultraviolet</td>
<td>$3 \times 10^{-5}$</td>
<td>$10^{15}$</td>
</tr>
<tr>
<td>Visible</td>
<td>$10^{-4}$</td>
<td>$3 \times 10^{11}$</td>
</tr>
<tr>
<td>Infrared</td>
<td>$10^{-1}$</td>
<td>$3 \times 10^{10}$</td>
</tr>
<tr>
<td>Microwaves</td>
<td>1</td>
<td>$3 \times 10^{8}$</td>
</tr>
<tr>
<td>Spacecraft</td>
<td>$10^2$</td>
<td>$3 \times 10^{8}$</td>
</tr>
<tr>
<td>Television &amp; FM</td>
<td>$10^3$</td>
<td>$3 \times 10^{7}$</td>
</tr>
<tr>
<td>Shortwave</td>
<td>$10^4$</td>
<td>$3 \times 10^{6}$</td>
</tr>
<tr>
<td>AM</td>
<td>$10^5$</td>
<td>$3 \times 10^{5}$</td>
</tr>
</tbody>
</table>

After Liou (2002)
Satellite radiometers make measurements over a finite spectral interval. They respond to radiation in a non-uniform way as a function of frequency. To represent the outgoing radiance as viewed by a radiometer, the spectrum of monochromatic radiance must be convolved with the appropriate instrument response function. This yields the polychromatic (or channel) radiance.
Transmittance and optical depth

Electromagnetic radiation interacts with atmospheric molecules and aerosol/cloud/hydrometeors particles through two main mechanisms.

Mechanism 1: Extinction of radiation

Extinction is the attenuation of the intensity of the radiation field. Extinction is the sum of absorption and scattering.

When we have **absorption**, radiant energy is converted into other forms of energy (e.g., kinetic energy of the medium).

When we have **scattering**, radiant energy is redirected from its original direction.

Mechanism 2: Emission of radiation

Emission is the conversion of molecular kinetic energy into electromagnetic energy.

Absorbing medium

Scattering medium

Scattering particles (aerosols/cloud/hydrometeors)

After McCartney (1983)
Transmittance and optical depth

The mechanism of extinction is governed by the Beer-Bouguer-Lambert law. It states that extinction is linear in the amount of matter and in the intensity of radiation.

The attenuation of radiation intensity along a path $ds$ is:

$$dL_{\nu} = -L_{\nu} k^{e}_{\nu} \rho(s) ds$$

The extinction coefficient is the sum of the absorption coefficient, $k^{a}_{\nu}$, and the scattering coefficient, $k^{s}_{\nu}$.

The optical depth of the medium between points $s_1$ and $s_2$ is defined as:

$$\tau_{\nu} = \int_{s_1}^{s_2} k^{e}_{\nu} \rho(s) ds$$

The transmittance of the medium between points $s_1$ and $s_2$ is defined as:

$$\Gamma_{\nu} = \exp(-\tau_{\nu})$$

A completely transparent medium has a transmittance of 1.

A completely opaque medium has a transmittance of 0.
The equation of radiative transfer in clear sky (i.e. no scattering)

To compute the upwelling monochromatic radiance at the top of the atmosphere we divide the atmosphere into $N$ homogeneous layers bounded by $N+1$ pressure levels.

\[ L_\nu^\uparrow = \varepsilon_\nu B_\nu (T_s) \Gamma_{\nu,s} + (1 - \varepsilon_\nu) L_\nu \Gamma_{\nu,s} + \sum_{j=1}^{N} B_\nu (T_j) (\Gamma_{\nu,j} - \Gamma_{\nu,j+1}) \]

\[ L_\nu^\downarrow = L_\nu^\uparrow (top) \Gamma_{\nu,s} + \sum_{j=1}^{N} B_\nu (T_j) \frac{\Gamma_{\nu,j} (\Gamma_{\nu,j} - \Gamma_{\nu,j+1})}{\Gamma_{\nu,j} \Gamma_{\nu,j+1}} \]

Downwelling radiance at the top of the atmosphere. It is omitted in the infrared. In the microwave it corresponds to the emission of the cosmic background radiation.
The upwelling atmospheric emission term can be rewritten as:

\[
\sum_{j=1}^{N} B_v(T_j) \left( -\frac{\Gamma_{v,j} - \Gamma_{v,j+1}}{\ln(p_j) - \ln(p_{j+1})} \right) (\ln(p_{j+1}) - \ln(p_j))
\]

The contribution of the Planck function to the upwelling atmospheric emission is weighted by \(w(p)\).

The weighting function has a peak in the region where the transmittance varies most rapidly with height.
Example of radiance/brightness temperature spectrum

IASI brightness temperature spectrum (8461 channels)

- Atmosphere window
- CO absorption
- CH₄ absorption
- N₂O absorption
- CO₂ absorption
- H₂O absorption
- O₃ absorption
- Equivalrent brightness temperature spectrum
The accurate computation of atmospheric transmittances/radiances is carried out using physical models based on first principles. These models are called line-by-line models (e.g. LBLRTM).

Line-by-line models, however, are too slow to be used operationally in NWP.

The near real-time simulation of satellite data is carried out using fast radiative transfer models. These models are very computationally efficient and are able to reproduce line-by-line “exact” calculations very closely.
Operational satellite data assimilation at ECMWF (and many other NWP centres) is carried out using the RTTOV fast radiative transfer model.

RTTOV is an acronym for Radiative Transfer for TOVS

\[ \text{TOVS} = \text{TIROS Operational Vertical Sounder} \]

RTTOV was originally developed ~25 years ago and is funded by EUMETSAT through the NWP SAF, developed by ECMWF, Met Office, Météo-France, and DWD.

The RTTOV suite comprises direct, tangent linear (TL), adjoint (AD) and K routines.

\textit{RTTOV has >1000 users worldwide}
RTTOV solves the radiative transfer equation using the so-called polychromatic approximation.

Ideally, one should solve the radiative transfer equation at many monochromatic frequencies and convolve the resulting radiances with the instrument spectral response function to obtain the channel radiance.

In RTTOV, the channel radiance is computed solving the radiative transfer equation for polychromatic transmittances.
How does RTTOV compute the polychromatic transmittances?

RTTOV is a *regression based* fast model where channel optical depths are parameterised using profile dependent predictors. This allows the very fast calculation of optical depths for each channel.

In RTTOV, the atmosphere is divided into $N$ homogeneous layers bounded by $N+1$ fixed pressure levels. The total channel optical depth for layer $j$ is written as:

$$\hat{\tau}_{j,v^*} = \sum_{k=1}^{M} a_{j,k,v^*} X_{k,j}$$

here $M$ is the number of predictors, $a_{j,k,v^*}$ are *expansion coefficients* and $X_{k,j}$ are *profile-dependent predictors*. 
The RTTOV fast radiative transfer model: optical depth simulation

The total channel optical depth is the sum of different terms

\[ \hat{\tau} = \sum_{j,v} a_{\text{Mixed}, j,k,v} X_{\text{Mixed}, k,j} + \sum_{k=1}^{M_{\text{WV}}} a_{\text{WV}, j,k,v} X_{\text{WV}, k,j} + \sum_{k=1}^{M_{\text{OZ}}} a_{\text{OZ}, j,k,v} X_{\text{OZ}, k,j} \]

Optical depth due to fixed gases
Optical depth due to water vapour
Optical depth due to ozone

In general, the predictors are functions of temperature, gas absorber amount, pressure and viewing angle.
The RTTOV fast radiative transfer model: optical depth simulation

<table>
<thead>
<tr>
<th>Predictor</th>
<th>Fixed gases</th>
<th>Water vapour</th>
<th>Ozone</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_{j,1}$</td>
<td>$\sec(\theta)$</td>
<td>$\sec^2(\theta)W_r^2(j)$</td>
<td>$\sec(\theta)O_r(j)$</td>
</tr>
<tr>
<td>$X_{j,2}$</td>
<td>$\sec^2(\theta)$</td>
<td>$(\sec(\theta)W_w(j))^2$</td>
<td>$\sqrt{\sec(\theta)O_r(j)}$</td>
</tr>
<tr>
<td>$X_{j,3}$</td>
<td>$\sec(\theta)T_r(j)$</td>
<td>$(\sec(\theta)W_w(j))^4$</td>
<td>$\sec(\theta)O_r(j)\delta T(j)$</td>
</tr>
<tr>
<td>$X_{j,4}$</td>
<td>$\sec(\theta)T_r^2(j)$</td>
<td>$\sec(\theta)W_r(j)\delta T(j)$</td>
<td>$(\sec(\theta)O_r(j))^2$</td>
</tr>
<tr>
<td>$X_{j,5}$</td>
<td>$T_r(j)$</td>
<td>$\sqrt{\sec(\theta)W_r(j)}$</td>
<td>$\sqrt{\sec(\theta)O_r(j)}\delta T(j)$</td>
</tr>
<tr>
<td>$X_{j,6}$</td>
<td>$T_r^2(j)$</td>
<td>$4\sqrt{\sec(\theta)W_r(j)}$</td>
<td>$\sec(\theta)O_r(j)^2O_w(j)$</td>
</tr>
<tr>
<td>$X_{j,7}$</td>
<td>$\sec(\theta)T_w(j)$</td>
<td>$\sec(\theta)W_r(j)$</td>
<td>$\frac{O_r(j)}{O_w(j)}\sqrt{\sec(\theta)O_r(j)}$</td>
</tr>
<tr>
<td>$X_{j,8}$</td>
<td>$\sec(\theta)\frac{T_w(j)}{T_r(j)}$</td>
<td>$(\sec(\theta)W_r(j))^3$</td>
<td>$\sec(\theta)O_r(j)O_w(j)$</td>
</tr>
<tr>
<td>$X_{j,9}$</td>
<td>$\sqrt{\sec(\theta)}$</td>
<td>$(\sec(\theta)W_r(j))^4$</td>
<td>$O_r(j)\sec(\theta)\sqrt{(O_w(j)\sec(\theta))}$</td>
</tr>
<tr>
<td>$X_{j,10}$</td>
<td>$\sqrt{\sec(\theta)}\frac{4}{\sqrt{T_w(j)}}$</td>
<td>$\sec(\theta)W_r(j)\delta T(j)\delta T(j)$</td>
<td>$\sec(\theta)O_w(j)$</td>
</tr>
<tr>
<td>$X_{j,11}$</td>
<td>0</td>
<td>$(\sqrt{\sec(\theta)}W_r(j))\delta T(j)$</td>
<td>$(\sec(\theta)O_w(j))^2$</td>
</tr>
<tr>
<td>$X_{j,12}$</td>
<td>0</td>
<td>$\sec(\theta)(W_r(j))^2$</td>
<td>0</td>
</tr>
<tr>
<td>$X_{j,13}$</td>
<td>0</td>
<td>$\sqrt{(\sec(\theta)W_r(j))W_r(j)}$</td>
<td>0</td>
</tr>
<tr>
<td>$X_{j,14}$</td>
<td>0</td>
<td>$\frac{W_r^2(j)}{T_r(j)}$</td>
<td>0</td>
</tr>
<tr>
<td>$X_{j,15}$</td>
<td>0</td>
<td>$\frac{\sec(\theta)W_r^2(j)}{T_r^4(j)}$</td>
<td>0</td>
</tr>
</tbody>
</table>

RTTOV predictors for fixed gases, water vapour and ozone
To compute the expansion coefficients, a line-by-line model is used to compute accurate channel optical depths for a diverse set of temperature and atmospheric constituent (typically water vapour and ozone) profiles. The training profiles are chosen to be representative of widely differing atmospheric situations.

The line-by-line optical depths are regressed into the predictors $X_{k,j}$ for each channel. The resulting expansion coefficients are used by RTTOV to compute optical depths for any other input profile.
The training of RTTOV is carried out using 498 profiles, i.e. 83 diverse atmospheric profiles each at 6 zenith angles.
The RTTOV fast radiative transfer model: optical depth simulation

The ability of RTTOV to reproduce line-by-line optical depths

HIRS: High resolution Infrared Radiation Sounder
(\textit{wheel radiometer} with broad channels)

IASI: Infrared Atmospheric Sounding Interferometer
(\textit{hyperspectral sensor} with very narrow channels)

Water vapour sounding channel

Stars indicate the fast model optical depths

Squares indicate the line-by-line optical depths

Optical depth of the path

Water vapour amount in the path
The RTTOV fast radiative transfer model: inputs

Inputs to RTTOV

**Vertical atmospheric profiles**
- Pressure, temperature and water vapour
- Mandatory
- Trace gases: O₃, CO₂, CO, N₂O, CH₄, SO₂
- Optional

**Viewing geometry**
- Zenith and azimuthal angles

**Surface variables**
- Mandatory
  - Skin temperature, surface pressure, 10m u/v wind
- Optional
  - Surface emissivity
The RTTOV fast radiative transfer model: accuracy

Main sources of RTTOV errors

1) Use of polychromatic optical depths
2) Optical depth parameterisation
3) Discretisation of the atmosphere into homogeneous layers and associated interpolation
4) Input profiles values (including zenith angle) lying beyond the limits of the training set
The RTTOV fast radiative transfer model: optical depth parameterisation errors

The error introduced by the parameterisation of the optical depths can be assessed by comparing fast model and line-by-line computed radiances.

In general, RTTOV can reproduce the line-by-line radiances to an accuracy typically below the instrument noise.

The ability of RTTOV to reproduce line-by-line radiances.

Statistics of the difference between RTTOV and line-by-line radiances for the 498 training profiles.

Statistics of the difference between RTTOV and line-by-line radiances for 3000 profiles independent of the regression coefficients.
The RTTOV fast radiative transfer model: optical depth parameterisation errors

The ability of RTTOV to reproduce line-by-line radiances

Note that to characterize the **total RTTOV error**, we must include the error contribution from the underlying line-by-line model.

The average difference between IASI observations and RTTOV simulations using temperature and water vapour forecast fields as inputs.

The CO profile used in the simulations does not represent the variability seen in the real atmosphere.
The RTTOV fast radiative transfer model: capabilities

RTTOV capabilities

- Clear-sky visible/near-infrared, infrared and microwave radiances
- Internal sea surface emissivity and reflectance models
- Land surface emissivity and reflectance atlases
- Aerosol- and cloud-affected infrared radiances
- Cloud- and precipitation-affected microwave radiances
- Simulated Principal Components for high resolution infrared sounders
- *and more...*

It should be stressed that the RTTOV suite comprises *direct* and *gradient* routines \((TL, AD, K)\). This is a prerequisite for a fast model to be used in NWP assimilation.
The Jacobian (K) model calculates the derivatives of the simulated radiances or brightness temperatures with respect to each profile variable.

\[
\frac{\partial L}{\partial T_i}, \frac{\partial L}{\partial q_i}, \frac{\partial L}{\partial O_{3i}}, \ldots \quad \text{for } 1 \leq i \leq n\text{levels}
\]

and surface parameters:

\[
\frac{\partial L}{\partial T_s}, \frac{\partial L}{\partial \varepsilon_s}, \ldots
\]

It tells us how sensitive the satellite-seen radiance is to each individual profile variable.
A Principal component (PC) based version of RTTOV (PC_RTTOV) has been developed for hyperspectral (i.e. for sensors with many thousand channels) remote sensing applications.

PC based fast models parameterise the PC scores of the radiance spectrum.

The PC scores have much smaller dimensions as compared to the number of channels. This optimizations results in significant computational savings and more accurate results.

Statistics of the difference between fast model and line-by-line radiances for 3000 profiles independent of the regression coefficients.
In presence of scattering, the radiative transfer equation cannot be solved analytically.

An “exact” solution for the scattering radiative transfer equation can only be obtained using numerical techniques (e.g. discrete-ordinate, doubling-adding, Monte-Carlo).

An analytical solution, however, can still be sought if approximate methods are used (e.g. two/four-stream approximation, Eddington/Delta-Eddington approximation, single scattering approximation, etc.)

We can parameterise scattering.

For infrared scattering simulations we use the optical depth scaling approach.

For microwave scattering simulations we use the Delta-Eddington approximation.

For visible scattering simulations we use the look-up table approach.

Too slow to be used operationally.
The infrared scattering parameterisation introduced in RTTOV enables to write the radiative transfer equation in a form that is identical to that in clear sky conditions.

In the optical depth scaling approach, the absorption optical depth, $\tau_a$, is replaced by an effective extinction optical depth, $\tilde{\tau}_e$, defined as: $\tilde{\tau}_e = \tau_a + b \tau_s$, where $\tau_s$ is the scattering optical depth.
The RTTOV fast radiative transfer model: infrared scattering capability

The accuracy of the scattering parameterisation

The **red line** denotes the difference between approximate (RTTOV) and exact scattering computations.

The **black line** denotes the difference between clear sky and exact scattering computations performed introducing either aerosol or ice crystal particles.

### Aerosols – Desert
- Number density: global climatological concentration
- Number density: four times the global climatological concentration

### Cirrus – Height of cloud = 12km
- Tropical profile – Single layer
- Double layer
RTTOV is freely available. You can register here: https://nwpsaf.eu/site/register

Coefficients are available here: https://nwpsaf.eu/site/software/rttov/download/coefficients/

RTTOV forum: https://nwpsaf.eu/site/forums/forum/rttov/