Turbulence parametrization  
(with a focus on the boundary layer)  

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Why studying the Planetary Boundary Layer (PBL) ?

为什么不研究行星边界层（PBL）？

- 自然环境为人类活动提供条件
- 理解和预测其结构
  - 农业、航空、电信通讯、地球能预算
- 天气预报、污染物扩散、气候变化预测
Definition

Turbulence/Equations

Stability

Classification

Clear convective boundary layers

Cloudy boundary layers (stratocumulus and cumulus)

Summary
PBL: Definitions

The PBL is the layer close to the surface within which vertical transports by turbulence play dominant roles in the momentum, heat and moisture budgets.

- The layer where the flow is turbulent.
- The fluxes of momentum, heat or matter are carried by turbulent motions on a scale of the order of the depth of the boundary layer or less.
- The surface effects (friction, cooling, heating or moistening) are felt on times scales $< 1$ day.

![Diagram of the atmosphere layers with temperature ($\theta$) and vapor pressure ($q_v$) profiles.]:

**Composition**

- atmospheric gases ($N_2$, $O_2$, water vapor, ...)
- aerosol particles
- clouds (condensed water)
### Governing equations for the mean state

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<td>momentum (Navier Stokes)</td>
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<td>continuity eq. (conservation of mass)</td>
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<td>heat (first principle of thermodynamics)</td>
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Reynolds averaging \( A = \overline{A} + A' \)

Averaging (overbar) is over grid box, i.e. sub-grid turbulent motion is averaged out.

### Simplifications

- **Boussinesq** approximation (density fluctuations non-negligible only in buoyancy terms)
- **Hydrostatic** approximation (balance of pressure gradient and gravity forces)
- **Incompressibility** approximation (changes in density are negligible)
Governing equations for the mean state

Reynolds averaging \[ A = \bar{A} + A' \]

\( \bar{p} = \bar{\rho} \, R_d \, \bar{T_v} \)

\( \bar{T_v} = T(1 + 0.61q_v - q_l) \)

\( \frac{\partial \bar{u}_i}{\partial t} + u_j \frac{\partial \bar{u}_i}{\partial x_j} = -\delta_{i3}g + f_c \, \varepsilon_{ij3} \, \bar{u}_j - \frac{1}{\bar{\rho}} \frac{\partial \bar{P}}{\partial x_i} + \nu \frac{\partial^2 \bar{u}_i}{\partial x_j^2} - \frac{\partial (u'_i u'_j)}{\partial x_j} \)

\( \frac{\partial u_i}{\partial x_j} = 0 \)

\( \frac{\partial \bar{\theta}}{\partial t} + u_j \frac{\partial \bar{\theta}}{\partial x_j} = -\frac{1}{\rho c_p} \frac{\partial F_j}{\partial x_j} - \frac{\partial u'_j \theta'}{\partial x_j} - \frac{L_v E}{\rho c_p} \)

\( \frac{\partial \bar{q}_t}{\partial t} + u_j \frac{\partial \bar{q}_t}{\partial x_j} = \frac{S_{q_t}}{\rho} - \frac{\partial u'_j q'_t}{\partial x_j} \)

Gas law

Momentum

Continuity equation

Heat

Total water

Mean advection

Gravity

Coriolis

Pressure gradient

Viscous stress

Turbulent transport

Need to be parameterized!

2nd order

Latent heat release
Turbulent kinetic energy equation

- $e = \frac{1}{2} (u'^2 + v'^2 + w'^2)$

\[ \frac{\partial e}{\partial t} + u_j \frac{\partial e}{\partial x_j} = g \frac{w' \theta'_v}{\theta_0} - u_i' u_j' \frac{\partial u_i}{\partial x_j} - \frac{\partial u_j' e}{\partial x_j} - \frac{1}{\rho} \frac{\partial u_i' p'}{\partial x_i} - \varepsilon \]

\[ \theta_v = \theta \left(1 + 0.61 q_v - q_l\right) \]

An example:

- $\theta_v' < 0$, $w' < 0$ or $\theta_v' > 0$, $w' > 0 \rightarrow w' \theta_v' > 0$ source

- $\theta_v' < 0$, $w' > 0$ or $\theta_v' > 0$, $w' < 0 \rightarrow w' \theta_v' < 0$ sink
Traditionally stability is defined using the temperature gradient

\[ \frac{\partial \theta_v}{\partial z} \]

\( \theta_v \) gradient (local definition):

\[ \frac{\partial \theta_v}{\partial z} > 0 \] stable layer

\[ \frac{\partial \theta_v}{\partial z} < 0 \] unstable layer

\[ \frac{\partial \theta_v}{\partial z} = 0 \] neutral layer

How to determine the stability of the PBL taken as a whole?

\( \theta_v \) or \( w' \theta_v' \) profiles are needed to determine the PBL stability state.

In a mixed layer the gradient of temperature is practically zero.
Bouyancy flux at the surface:

- $w'\theta'_v > 0$: unstable PBL (convective)
- $w'\theta'_v < 0$: stable PBL
- $w'\theta'_v = 0$: neutral PBL

Or dynamic production of TKE integrated over the PBL depth stronger than thermal production

Monin-Obukhov length:

$$L = \frac{-\bar{\theta}_v u_*^3}{kg(w'\theta'_v)_s}, \quad u_*^2 = (u'w'_v)_s$$

- $-10^5 m \leq L \leq -100 m$: unstable PBL
- $-100 m < L < 0$: strongly unstable PBL
- $0 < L < 10$: strongly stable PBL
- $10 m \leq L \leq 10^5 m$: stable PBL
- $|L| > 10^5 m$: neutral PBL
Neutral PBL:
- Turbulence scale $l \sim 0.07 \, H$, $H$ being the PBL depth
- Quasi-isotropic turbulence
- Scaling - adimensional parameters: $z_0$, $H$, $u_*$

Stable PBL:
- $l << H$ (stability embeds turbulent motion)
- Turbulence is local (no influence from surface), stronger on horizontal
- Scaling: $\frac{(w'\theta')_3}{(u'w')_s}, H$

Unstable (convective) PBL
- $l \sim H$ (large eddies)
- Turbulence associated mostly to thermal production
- Turbulence is non-homogeneous and asymmetric (top-down, bottom-up)
- Scaling: $H$, $w_* = \left(\frac{g}{\theta_v} \frac{(w'\theta')_s}{H} \right)^{1/3}$
- $z_H, q_* = \frac{E_0}{w_*}, \theta_* = \frac{Q_0}{w_*}$
PBL: Diurnal variation

- Clear convective PBL
- Cloudy convective PBL
- Inversion
- Residual Layer
- Convective Mixed Layer
- Stable (nocturnal) Layer

Adapted from Introduction to Boundary Layer Meteorology - R.B. Stull, 1988
PBL: State variables

**Clear PBL**

- **Specific humidity**
  \[ q_v = \frac{m_v}{m_d + m_v} \]

- **Potential temperature**
  \[ \theta = T \left( \frac{p}{p_0} \right)^{-\frac{R_d}{c_p}} \]

**No liquid water is condensed** \( (q_l = 0) \)

**Cloudy PBL**

- **Total water content**
  \[ q_t = \frac{m_v + m_c}{m_d + m_v + m_c} \]

- **Liquid water potential temperature**
  \[ \theta_l \approx \theta - \frac{L_v}{c_p} q_l \]

**Evaporation temperature**

**Conserved variables**
Clear Convective PBL

$q_t$, $q_{sat}$, $\theta_v$

Free atmosphere

Inversion layer

mixed layer

surface layer

Stephan de Roode, Ph.D thesis
Clear Convective PBL

- Buoyantly-driven from surface

**PBL height:**
\[ \frac{dH}{dt} = \bar{w} + w_e \]

**Entrainment rate:**
\[ w_e = A \left( \frac{w^3}{g \Delta \theta_v H} \right) \]

(a possible parameterization)

**Fluxes at PBL top:**
\[ w' \psi'_H = -w_e \Delta \psi \]

**Key parameters:** \( w_e, \Delta \theta_v, H, (w' \theta'_v)_0 \)
Clouds effects on climate

Greenhouse effect: warming

High clouds, like cirrus

Infrared radiation

Umbrella effect: cooling

Boundary layer clouds (low clouds)

Solar radiation

Shallow cumulus

Stratocumulus
Boundary layer clouds over oceans

Cloud Clusters

Hadley/Walker Circulation

Land/Sea Circulation

EQ warm → cold

Sandu, Stevens, Pincus, 2010
Cloudy boundary layers

Stephan de Roode, Ph.D thesis

Stratocumulus topped PBL

Cumulus PBL

Dry-adiabatic lapse rate

Free atmosphere

Inversion layer

mixed layer

surface layer

Free atmosphere

Inversion layer

Conditionally unstable cloud layer

Subcloud mixed surface layer

Dry-adiabatic lapse rate

wet-adiabatic lapse rate

\( q_t \)

\( q_{\text{sat}} \)

\( \theta_v \)
Stratocumulus – Why are they important?

- Cover in (annual) mean 29% of the planet (Klein and Hartmann, 1993)
- Cloud top albedo is 50-80% (in contrast to 7 % at ocean surface).
- A 4% increase in global stratocumulus extend would offset 2-3K global warming from CO$_2$ doubling (Randall et al. 1984).
- Coupled models have large biases in stratocumulus extent and SSTs.

Wood, 2012, based on Han and Waren, 2007
Characterisation of a stratocumulus topped PBL

Subsidence

$\theta_l (K)$

$r_t (g \text{ kg}^{-1})$

$r_{sat} (g \text{ kg}^{-1})$

LWP (kg m$^{-2}$)

(Liquid Water Path)

$h_{STBL} (m)$

$\theta_1 (K)$

0.2 g/kg

20 g/kg
Such a cloudy system is extremely sensitive to thermodynamical conditions.
Such a cloudy system is extremely sensitive to thermodynamical conditions.
Characterisation of a stratocumulus topped PBL

Such a cloudy system is extremely sensitive to thermodynamical conditions.
Processes controlling the evolution of a non-precipitating stratocumulus

Night-time

Entrainment warming, drying

LW radiative cooling

The cloud deepens

Surface heat and moisture fluxes

Courtesy of Bjorn Stevens (data from DYCOMS-II)
Processes controlling the evolution of a non-precipitating stratocumulus

Daytime

Entrainment warming, drying

- Warm, dry, subsiding free-troposphere
- LW radiative cooling
- SW absorption
- The cloud thins

Decoupled

- Stabilisation of the subcloud layer
- Surface heat and moisture fluxes

Courtesy of Bjorn Stevens (data from DYCOMS-II)
Diurnal cycle during observed during FIRE-I experiment

Hignett, 1991 (data from FIRE-I)
Precipitation flux

Under cloud evaporation affects the dynamics of the boundary layer
Stratocumulus topped boundary layer

- Complicated turbulence structure
  - Cloud top entrainment
  - Long-wave cooling
  - Condensation
  - Long-wave warming

- Buoyantly driven by radiative cooling at cloud top
- Surface latent and heat flux play an important role
- Cloud top entrainment an order of magnitude stronger than in clear PBL
- Solar radiation transfer essential for the cloud evolution

Key parameters: \( w_e, \Delta \theta_v, H, (w'\theta'_v)_0 \), \( (w'q'_v)_0, \Delta q_l, z_b, \Delta F \)
Cloudy boundary layers

Stephan de Roode, Ph.D thesis

Free atmosphere
Inversion layer
mixed layer
surface layer

Stratocumulus topped PBL

Dry-adiabatic lapse rate
wet-adiabatic lapse rate

Cumulus PBL

Free atmosphere
Inversion layer
Conditionally unstable cloud layer
Subcloud mixed surface layer
Cumulus capped boundary layers

- Buoyancy is the main mechanism that forces cloud to rise
Cumulus capped boundary layers

- Buoyancy is the main mechanism that forces cloud to rise

- Represented by mass flux convective schemes
  \[ M_c (\psi_u - \psi_d) = kw' \psi' \]

- Decomposition: cloud + environment

- Lateral entrainment/detrainment rates prescribed

- Key parameters:
  \[ H, z_b, (w' \theta'_v)_0, (w' q'_v)_0, \left( \frac{\partial \theta_v}{\partial z} \right)_{environ}, \left( \frac{\partial q_v}{\partial z} \right)_{environ} \]
PBL: Summary

**Characteristics:**
- several thousands of meters – 2-3 km above the surface
- turbulence, mixed layer
- convection
- Reynolds framework

**Classification:**
- neutral (extremely rare)
- stable (nocturnal)
- convective (mostly diurnal)

**Clear convective**

**Cloudy (stratocumulus or cumulus)**
- Importance of boundary layer clouds (Earth radiative budget)
- Small liquid water contents, difficult to measure

P. Bougeault, V. Masson, Processus dynamiques aux interfaces sol-atmosphere et ocean-atmosphere, cours ENM


