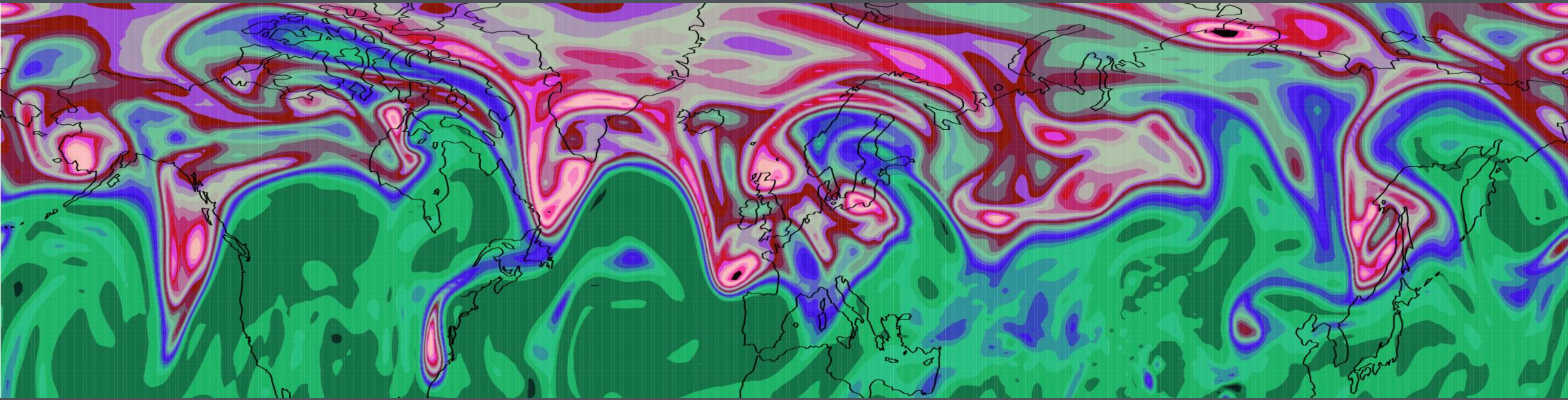
Department of Meteorology

University of Reading

How do diabatic processes in WCBs affect circulation and Rossby waves?



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Claudio Sanchez, Met Office

Influences of latent heat release on development of weather systems

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1. Diabatic heating intensifies ascent, vortex stretching and baroclinic growth rate

2. Majority of ascent occurs in "*warm conveyor belt*" (WCB) of cyclones (poleward moving, warm, moist air)

3. Heating enables ascent across surfaces of constant potential temperature (θ) - ***diabatic mass transport***

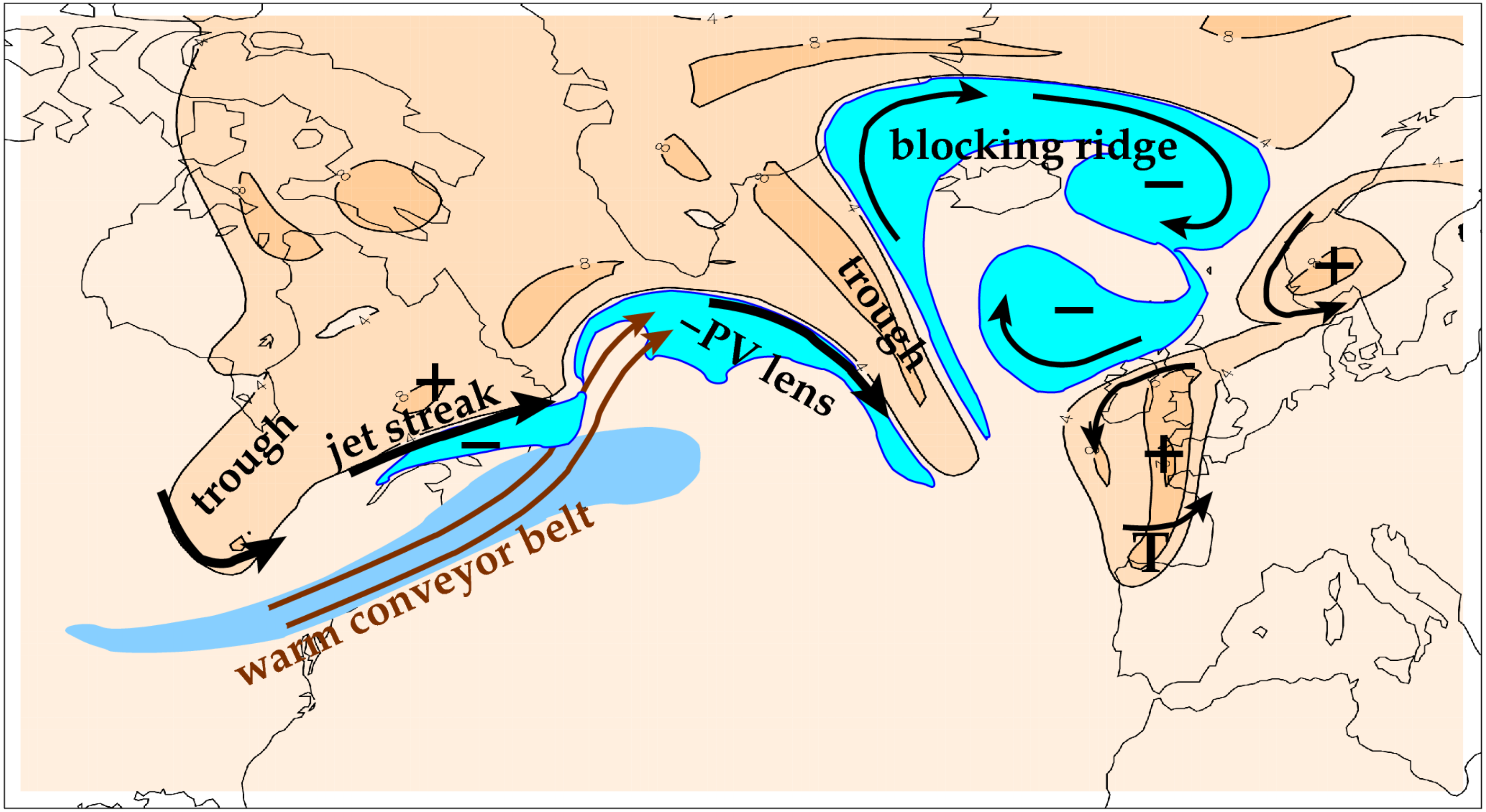
4. Mass outflow of WCBs into the upper troposphere in the *ridges of meandering jet stream*

Q.1 What fraction of mass in a ridge arrives by diabatic transport?

Q.2 What influence can it have on Rossby wave behaviour?

Overarching scientific aim of NAWDEX:

to quantify the effects of diabatic processes on disturbances to the jet stream near North America, their influence on downstream propagation across the North Atlantic, and consequences for high-impact weather in Europe.



Features related to the meandering tropopause and jet stream

(orange is stratospheric air; cyan marks upper tropospheric PV anomalies).

Air mass changes following WCB

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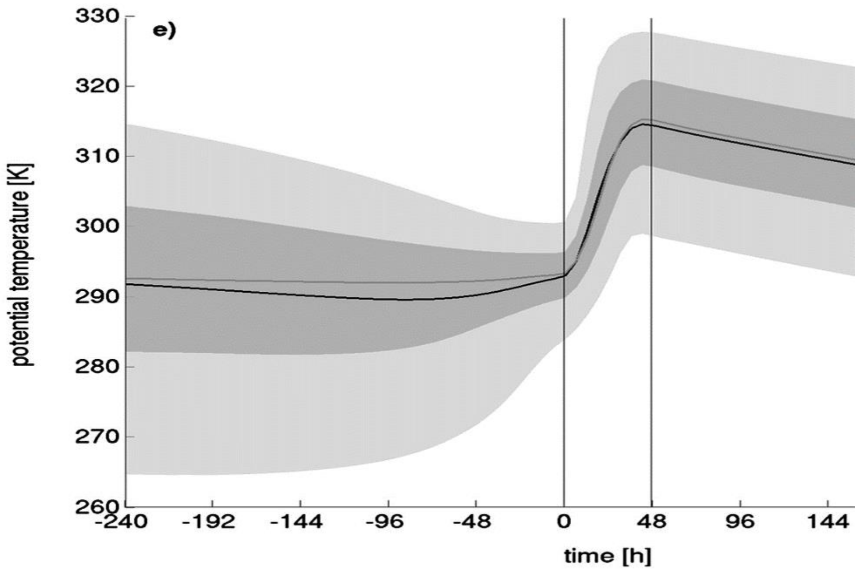

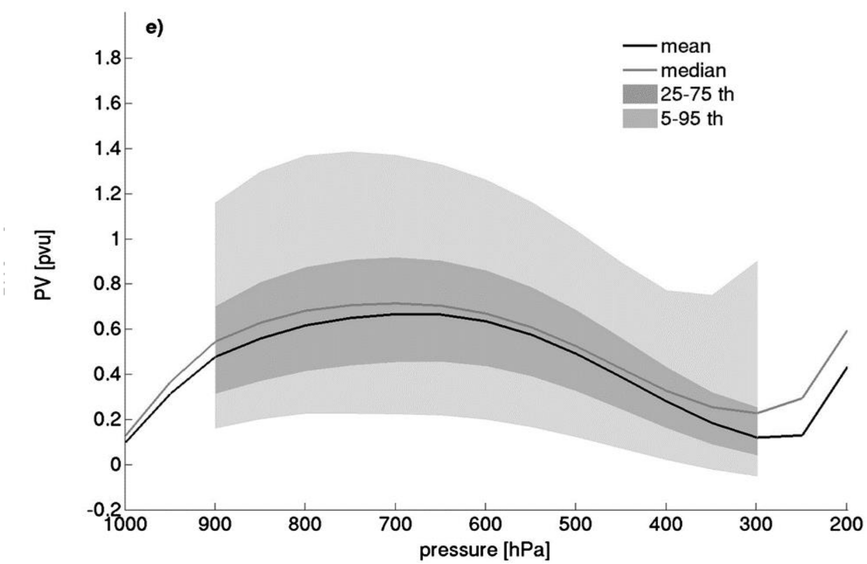

- WCB frequently defined as a *coherent ensemble of trajectories*
  Wernli and Davies, 1997, QJRMetS
- Climatology from
  Madonna et al, 2014, J. Climate

A. θ increases by greater than isentropic spread of the inflow or outflow layer of the CET

B. Although **potential vorticity** (PV) increases below heating maximum, it decreases again above.
PV of outflow ≈ PV of inflow

- Why is PV constrained in this way?
- Implications for role of heating?

PV evolution equation (I)

Lagrangian form of Ertel PV equation *(change following 3-D trajectories):*

$$\rho \frac{DP}{Dt} = \boldsymbol{\zeta} \cdot \nabla \dot{\theta} + \nabla \times \boldsymbol{F} \cdot \nabla \theta$$

PV increases below heating maximum (and decreases above it).

However, there is not an obvious constraint on PV values (P).

First step: re-write as flux (or local conservation) form of PV equation:

$$\frac{\partial}{\partial t}(\rho P) + \nabla \cdot (\rho P \boldsymbol{u}) = \nabla \cdot \left(\boldsymbol{\zeta}\dot{\theta} + \boldsymbol{F} \times \nabla \theta\right)$$

$\Rightarrow$ RHS is *"non-advective PV-flux divergence"* arising from non-conservative processes (heating, $\dot{\theta}$, and friction, $\boldsymbol{F}$)

(Haynes & McIntyre, 1987, JAS)

PV evolution equation (II)

Impermeability theorem

First, define the local normal vector to local isentropic surfaces as

$$\boldsymbol{n} = \nabla\theta / |\nabla\theta|$$

and split the absolute vorticity into components that are normal and parallel to the local isentropic surface:

$$\boldsymbol{\zeta} = (\boldsymbol{\zeta} \cdot \boldsymbol{n})\boldsymbol{n} + \boldsymbol{\zeta}_{//}$$

Second step: re-write the flux form PV equation as:

$$\frac{\partial}{\partial t}(\rho P) + \nabla \cdot (\rho P \tilde{\boldsymbol{u}}) = \nabla \cdot \left(\boldsymbol{\zeta}_{//}\dot{\theta} + \boldsymbol{F} \times \nabla\theta\right) = -\nabla . \boldsymbol{J}$$

Using $\rho P = (\boldsymbol{\zeta} \cdot \boldsymbol{n})|\nabla\theta|$ and partitioning velocity into cross-isentropic and along-isentropic components :

$$\boldsymbol{u}_J = \frac{\dot{\theta}}{|\nabla\theta|}\boldsymbol{n} \qquad \tilde{\boldsymbol{u}} = \boldsymbol{u} - \boldsymbol{u}_J$$

$\Rightarrow$ ***PV impermeability theorem*** *(Haynes & McIntyre, 1990, JAS)*

$(\rho P \tilde{\boldsymbol{u}} + \boldsymbol{J}). \boldsymbol{n} = 0$ **There can be no PV flux across isentropic surfaces**

Integral PV conservation (circulation)

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Integrate **PV equation** over control volume (lateral boundary velocity $\mathbf{V}_b$)

$$\frac{\mathrm{d}}{\mathrm{d}t}\iiint rq\,\mathrm{d}A\,\mathrm{d}\theta + \int\oint \left\{rq\left(\mathbf{V}-\mathbf{V}_\mathrm{b}\right)+\mathbf{J}\right\}.\mathbf{n}\,\mathrm{d}l\,\mathrm{d}\theta = 0,$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(C\Delta\theta) = \Delta\theta\frac{\mathrm{d}C}{\mathrm{d}t} = 0$$

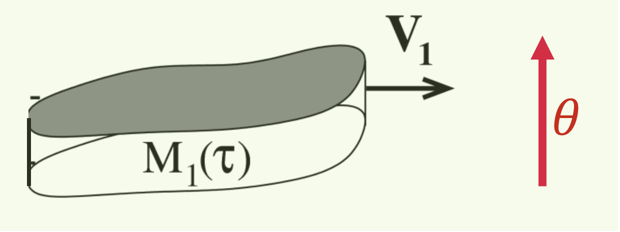
Conservation of circulation, C
(if $V_\mathrm{b}=V$ & J-integral $= 0$)

Integrate **mass continuity** over control volume

$$\frac{d}{dt}\iiint r\,dA\,d\theta \quad + \quad \int\oint r\left(\mathbf{V}-\mathbf{V}_b\right).\mathbf{n}\,dl\,d\theta$$

$$+\iint\left[r\dot{\theta}\right]_{bot}^{top} dA \quad = \quad 0$$

$$\frac{dM}{dt} \quad = \quad -D_{top} + D_{bot}$$



Diabatic mass flux convergence "dilutes" average PV

Mass-weighted average PV or
amount of PV substance divided by mass
(Haynes & McIntyre, 1990)

$$\langle q\rangle = \frac{\iiint rq\,dA\,d\theta}{\iiint r\,dA\,d\theta} = \frac{C\Delta\theta}{M}$$

PV in warm conveyor belts

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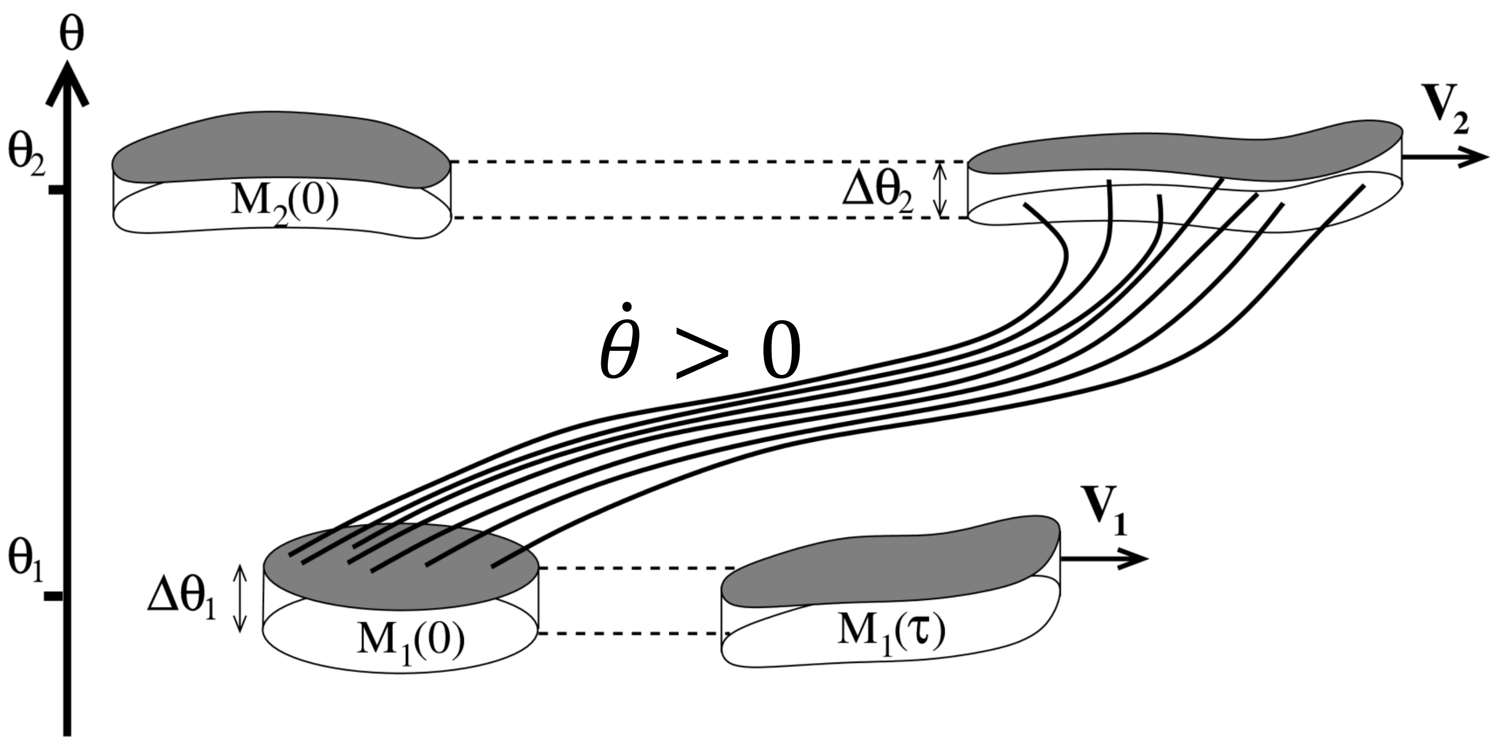
Consider two such volumes, in isentropic layers representing the inflow and outflow of a warm conveyor belt.

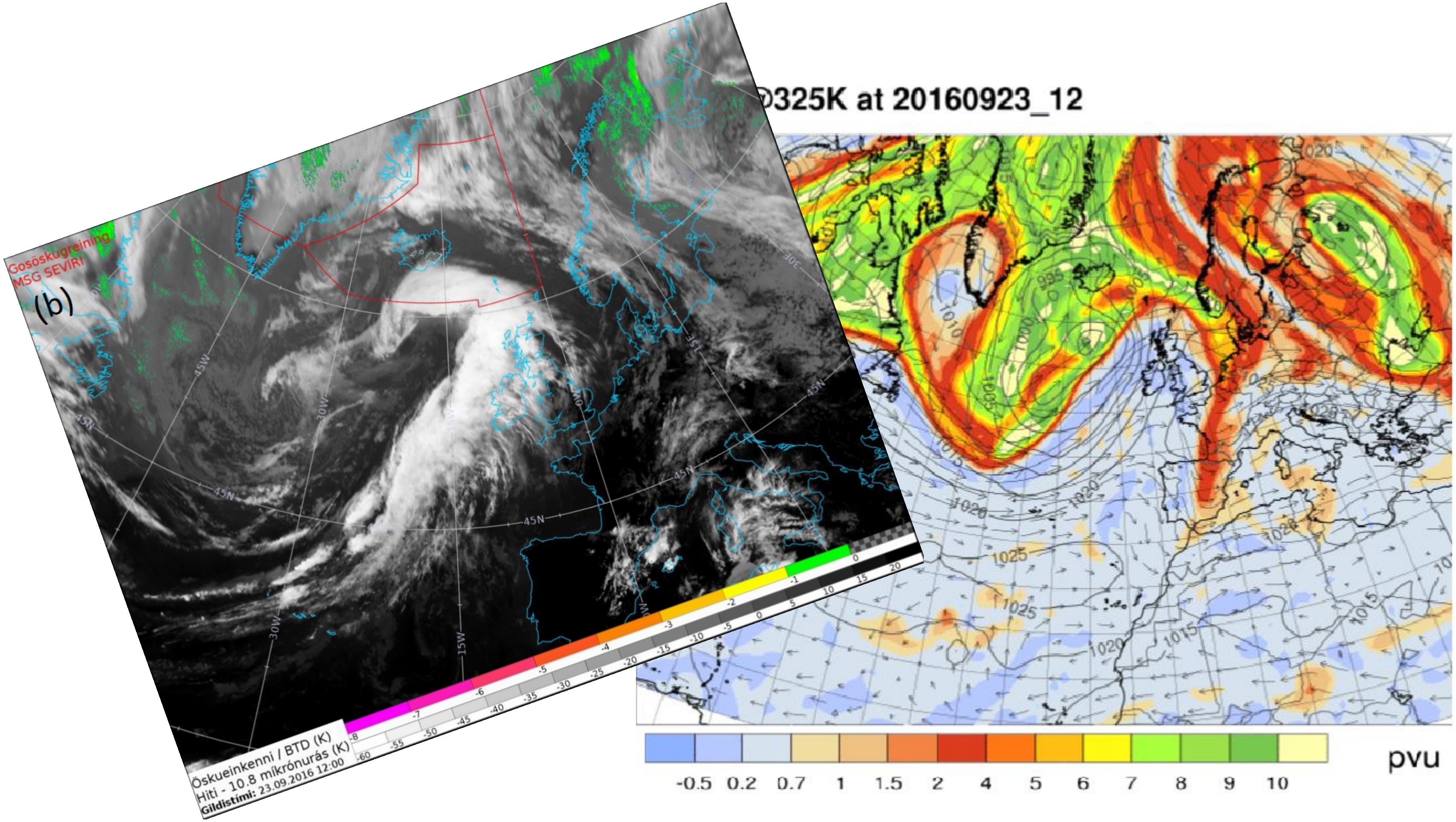
Heating $\Rightarrow$ **diabatic mass transport** from lower to upper volume

Concentrates PV substance of "inflow volume" and dilutes outflow PVS



Methven, *Q. J. Royal Met. Soc. (2015)*

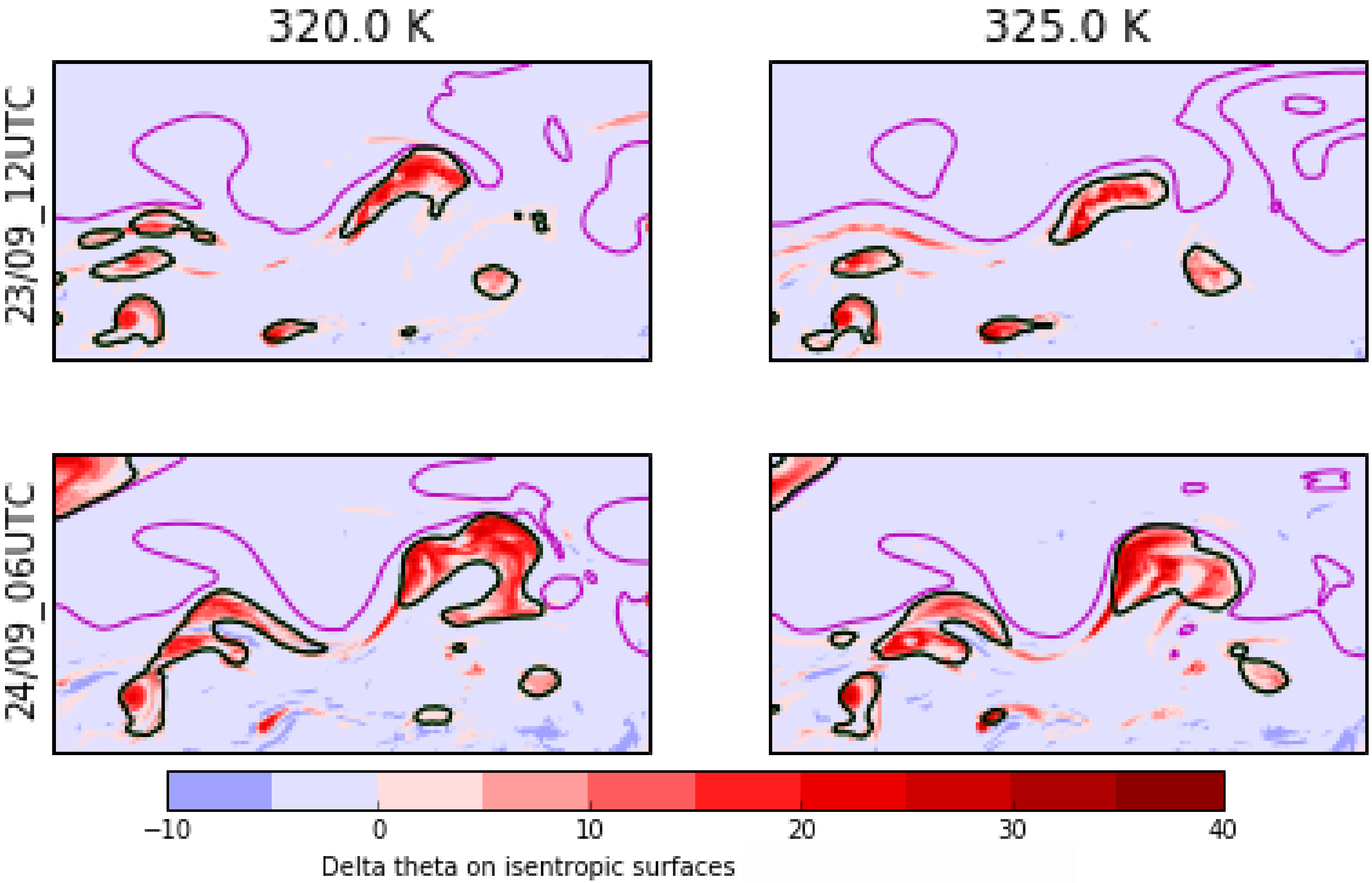
Is this conceptual picture realised?



Examine NAWDEX case 3 **(Jake Bland's MSc)**

IR satellite image and corresponding PV map: 12UT 23/9/2016

Defining outflow volume using net heating



Black contour = lateral boundary of “outflow volume” on θ-surface

Outflow volume at outflow time

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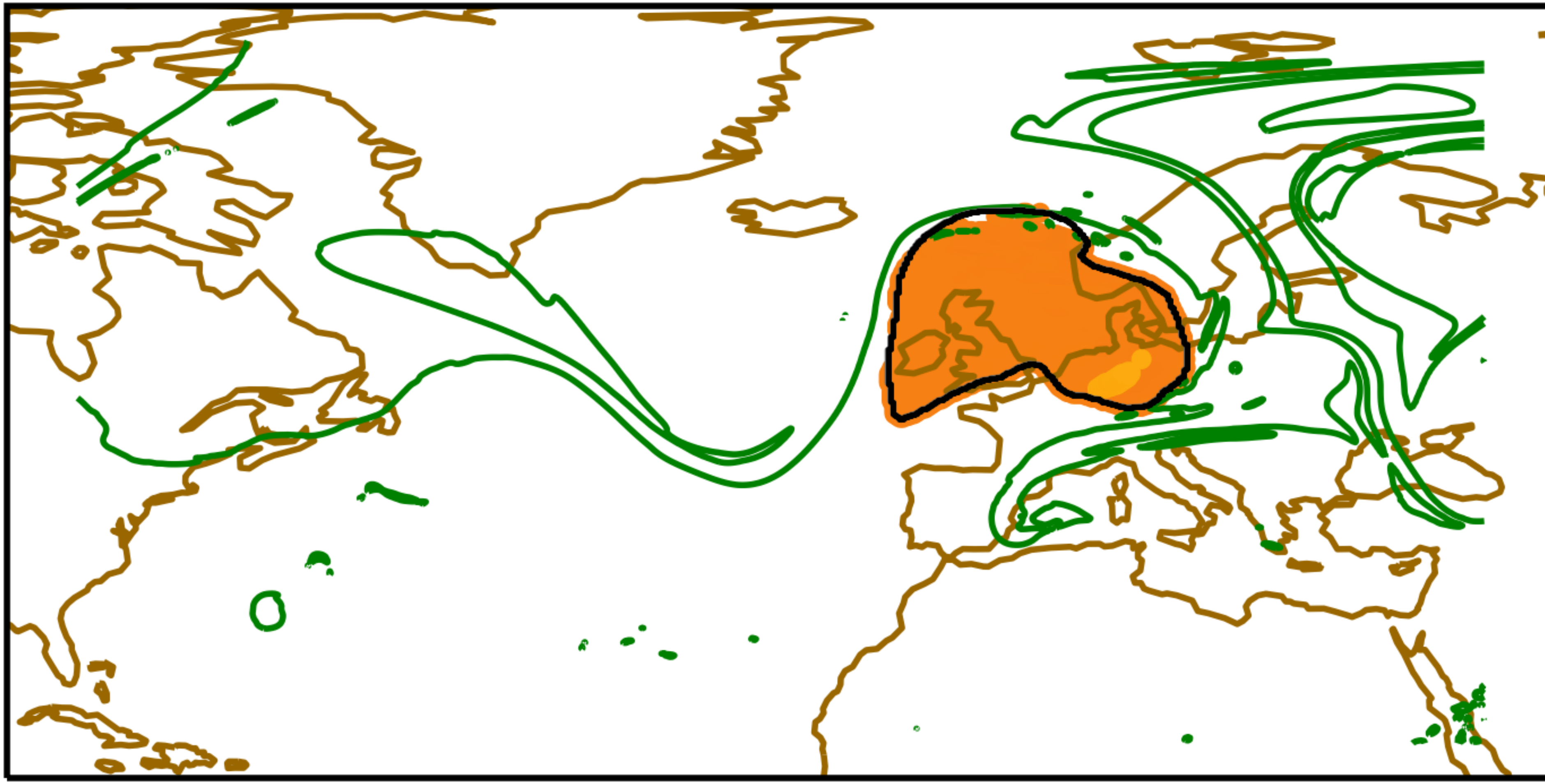

Black contour = lateral boundary of “outflow volume” on 325K surface

Yellow = release locations of 3D back trajectories from outflow volume

Green contour = tropopause on 325K

Following backwards (T-18)

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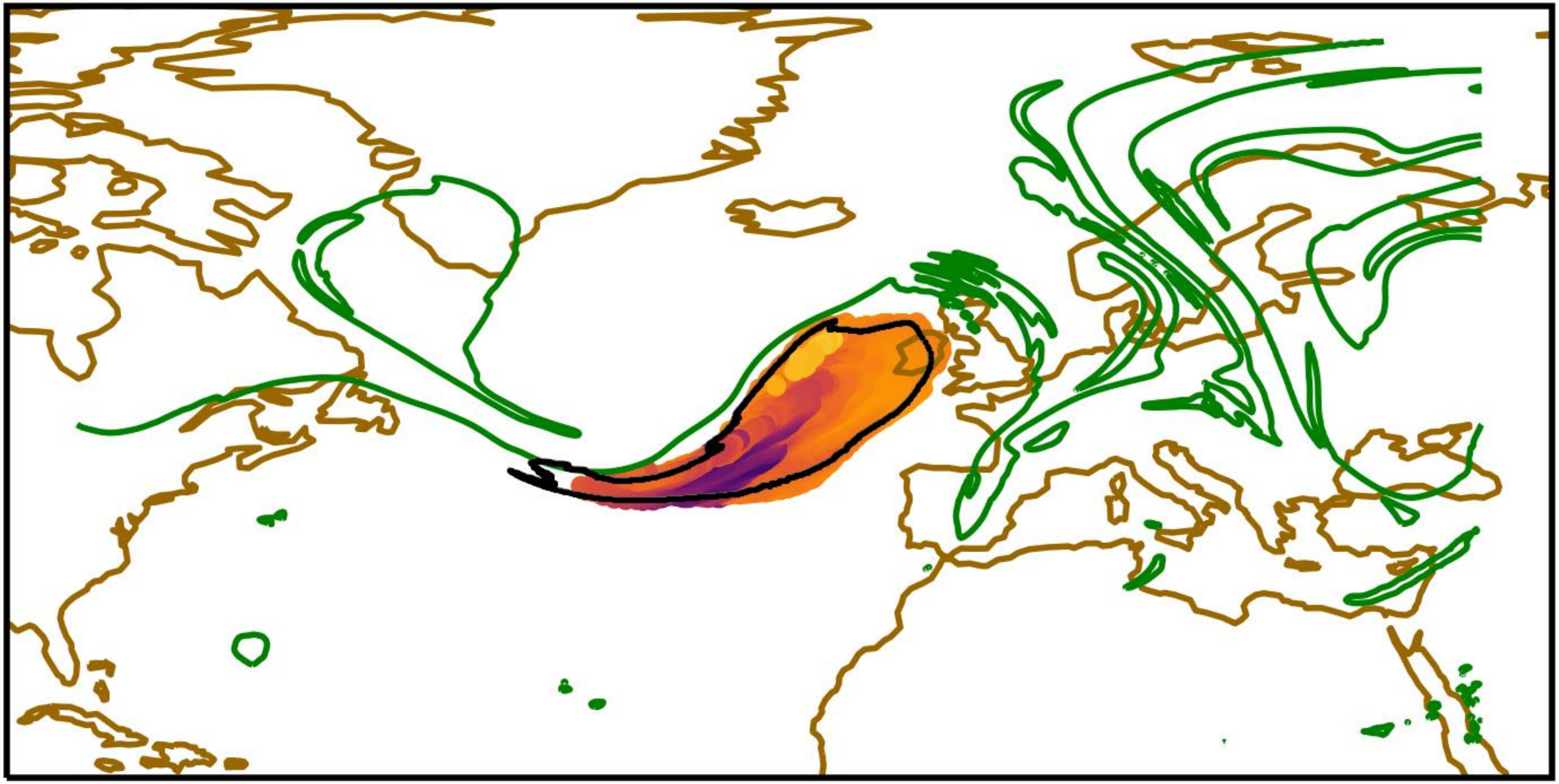

Contour = lateral boundary of “outflow volume” on 325K surface

Colour dots = θ at locations of 3D back trajectories from outflow volume

Green contour = tropopause on 325K

Following backwards (T-30)

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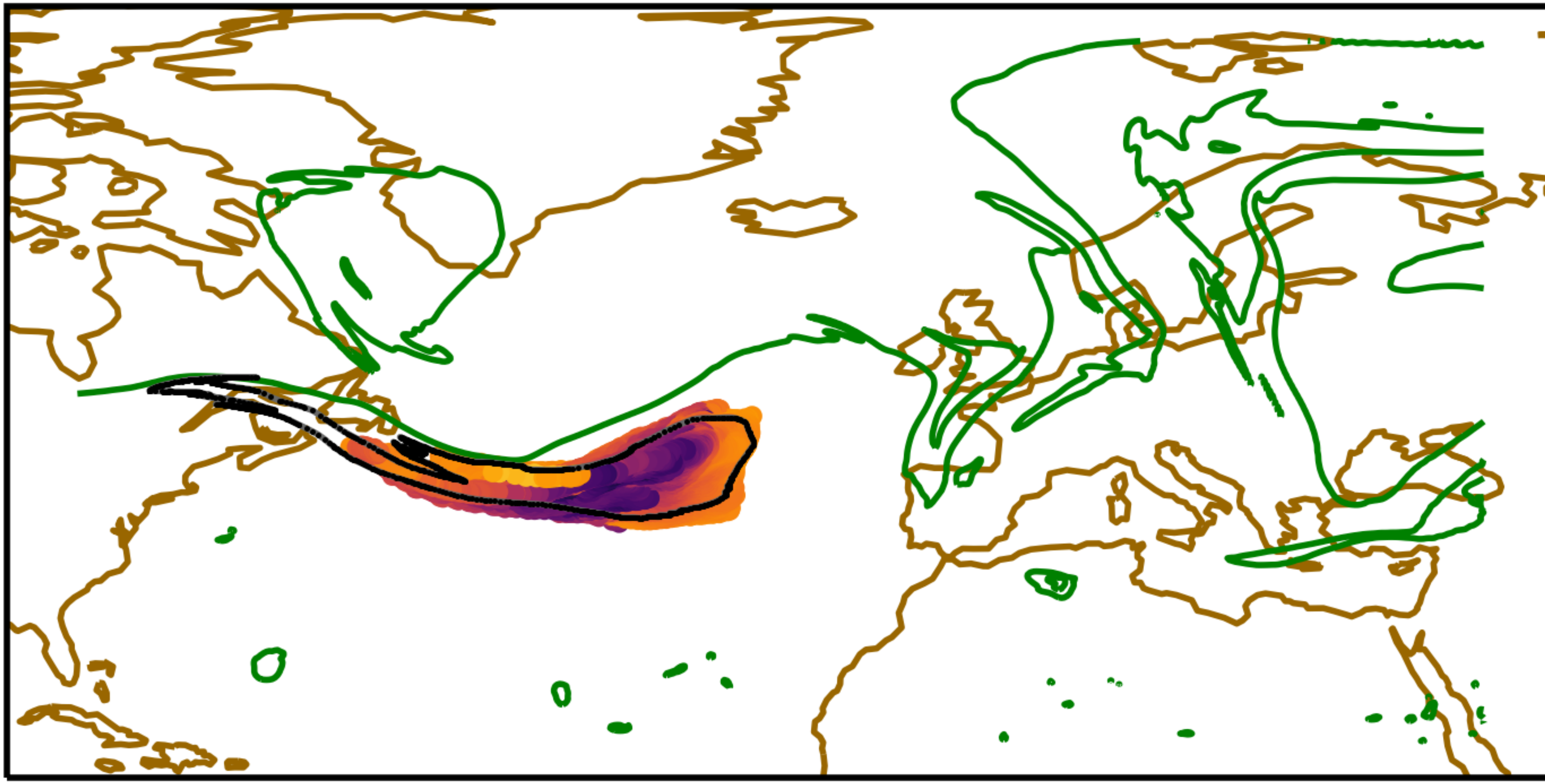

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Colour dots = θ at locations of 3D back trajectories from outflow volume

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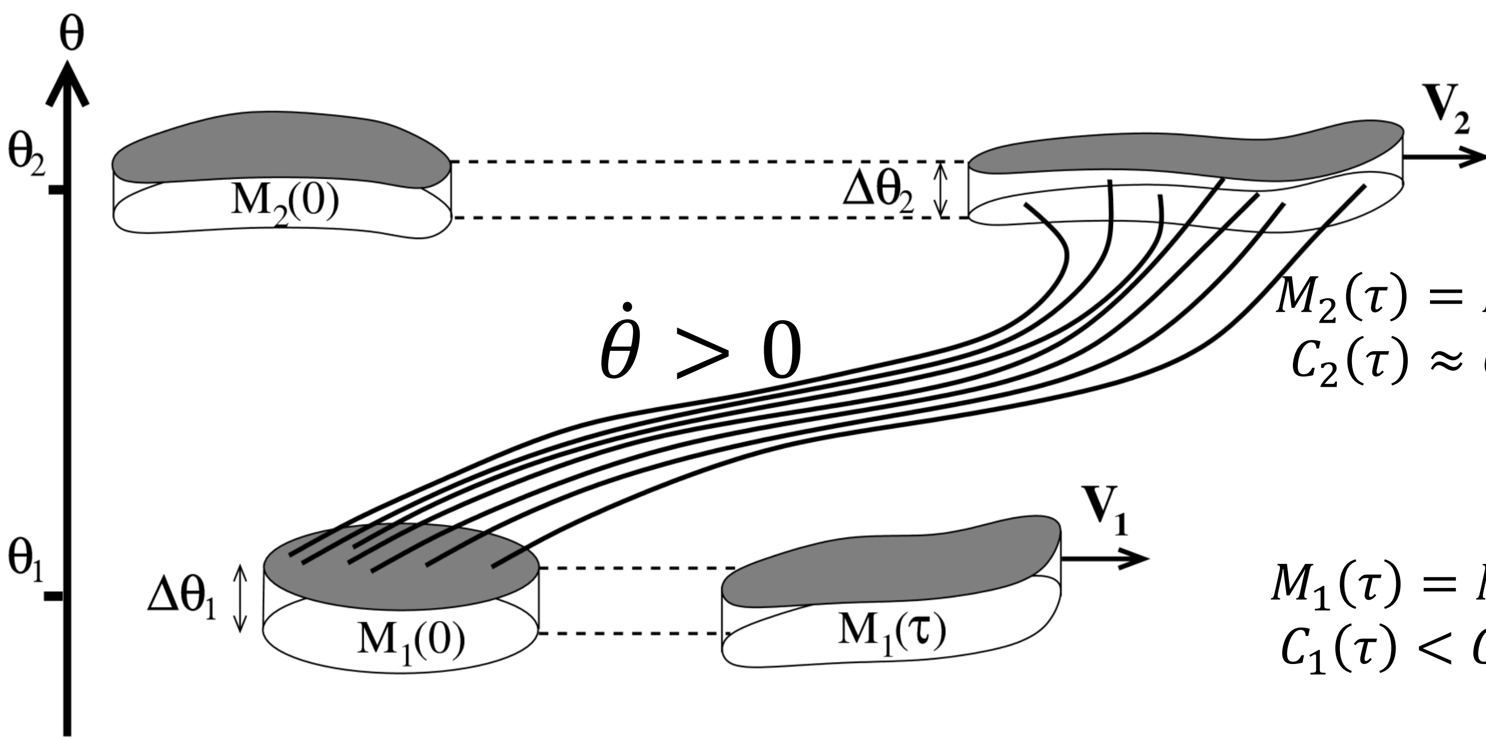
Concentrates PV substance of "inflow volume" and dilutes outflow PVS

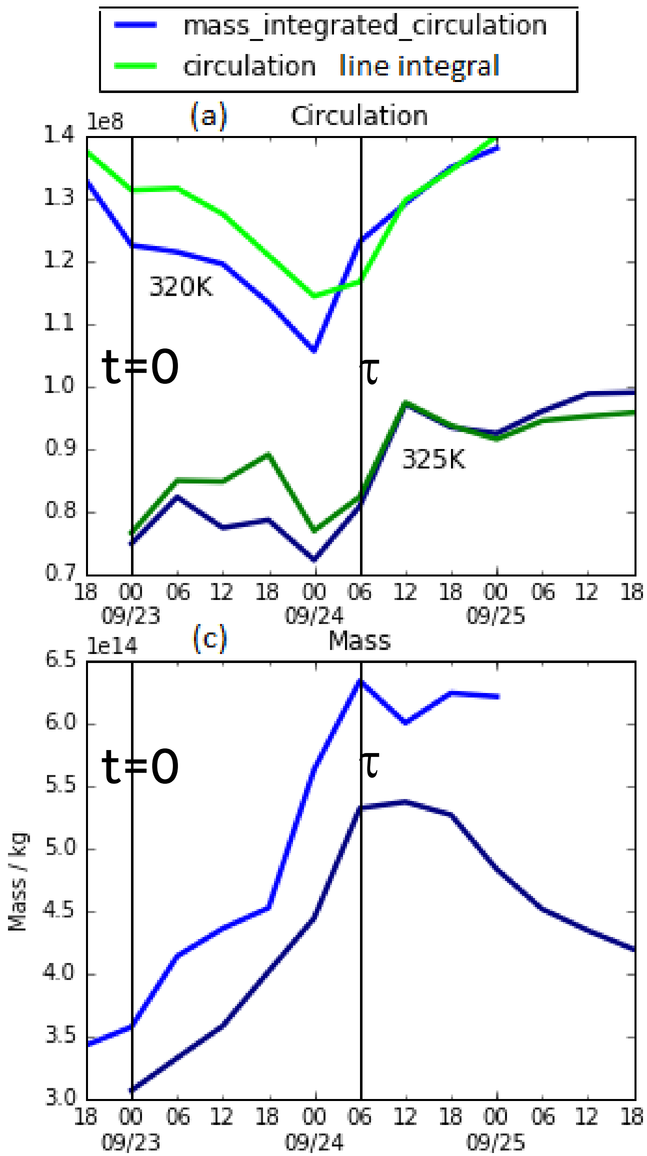


$$M_2(\tau) = M_2(0) + \Delta M$$
$$C_2(\tau) \approx C_2(0)$$

$$M_1(\tau) = M_1(0) - \Delta M$$
$$C_1(\tau) < C_1(0)$$

Methven, *Q. J. Royal Met. Soc. (2015)*

Circulation and mass of outflow



Fractional variation in circulation of outflow volume is small

$$\frac{\Delta C}{C_2(\tau)} < 0.05$$

Verifying that circulation is conserved even though diabatic processes are strong within WCB

Large increase in mass of outflow volume (x2) by diabatic mass transport from below:

$$\frac{\Delta M}{M_2(\tau)} = 0.44 \pm 0.02$$

Consistent with Pfahl *et al* (2015), *Nat. Geo*.

Circulation and mass of outflow

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Large **increase in mass of outflow volume** (x2) by diabatic mass transport
But, **circulation is almost invariant**.
Consequence of **PV impermeability theorem** (HM 1987, 1990).

Then, why is outflow size important to Rossby wave evolution?

Can write Kelvin's circulation as an area integral of vorticity:

$$C = \int_{\partial S} \underline{u}.d\underline{l} = \iint_{S} (\nabla \times \underline{u}).d\underline{S} = \langle f + \xi \rangle S$$

and mass of outflow in terms of average isentropic density: $M = \langle r \rangle S \, \Delta\theta$

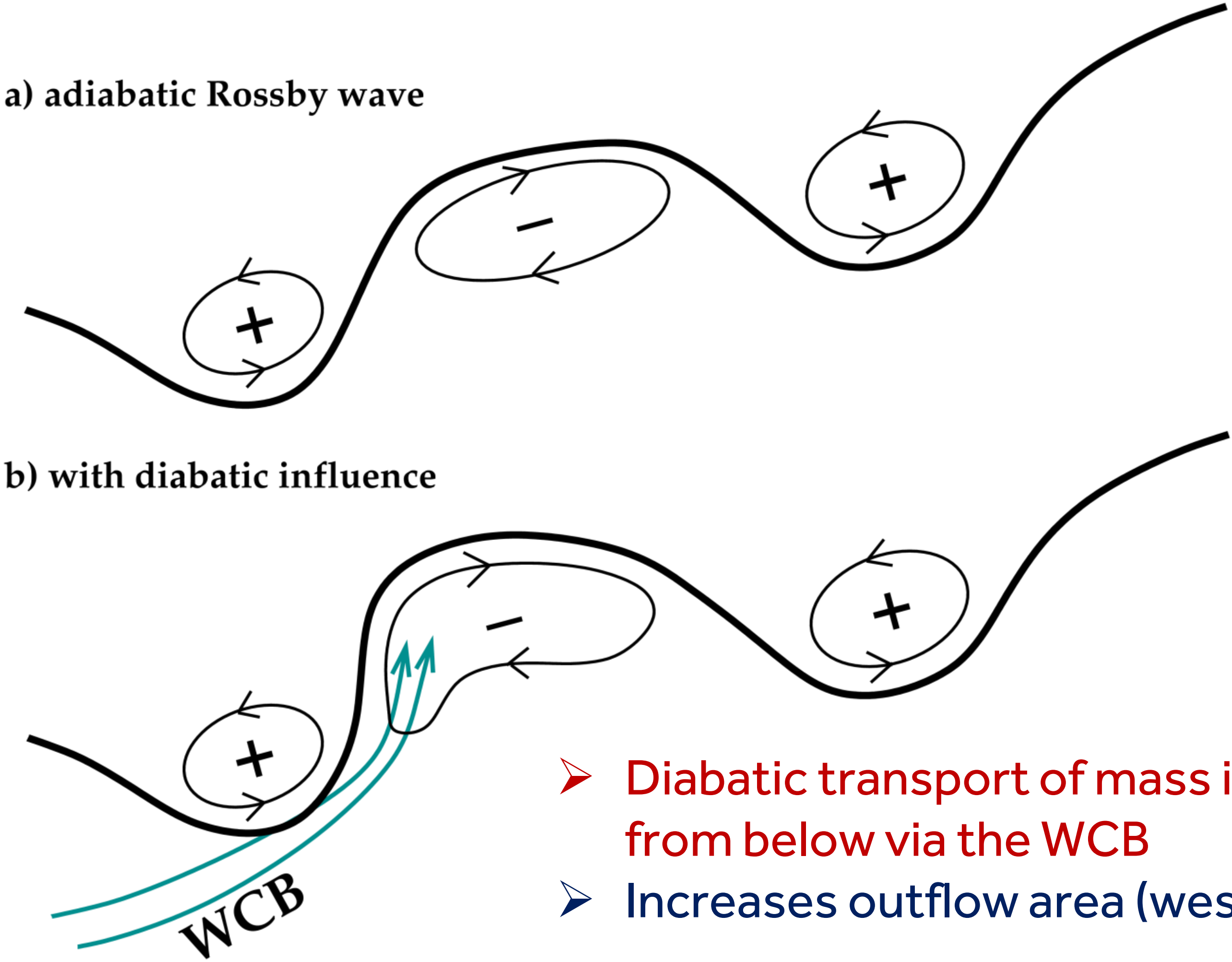
Fractional changes *due to diabatic mass transport* can be related:

$\frac{\Delta M}{M_{ad}} = \frac{\Delta S}{S_{ad}} + \frac{\Delta r}{r_{ad}}$ and using $\Delta C = 0$ we also have $\frac{\Delta \zeta}{\zeta_{ad}} = -\frac{\Delta S}{S_{ad}}$ and

$\frac{\Delta P}{P_{ad}} = \frac{\Delta \zeta}{\zeta_{ad}} - \frac{\Delta r}{r_{ad}} = -\frac{\Delta M}{M_{ad}}$ Inversion of PV anomalies $\Rightarrow$ *equipartition* between

vorticity (area) anomalies and stratification (*r*) anomalies.

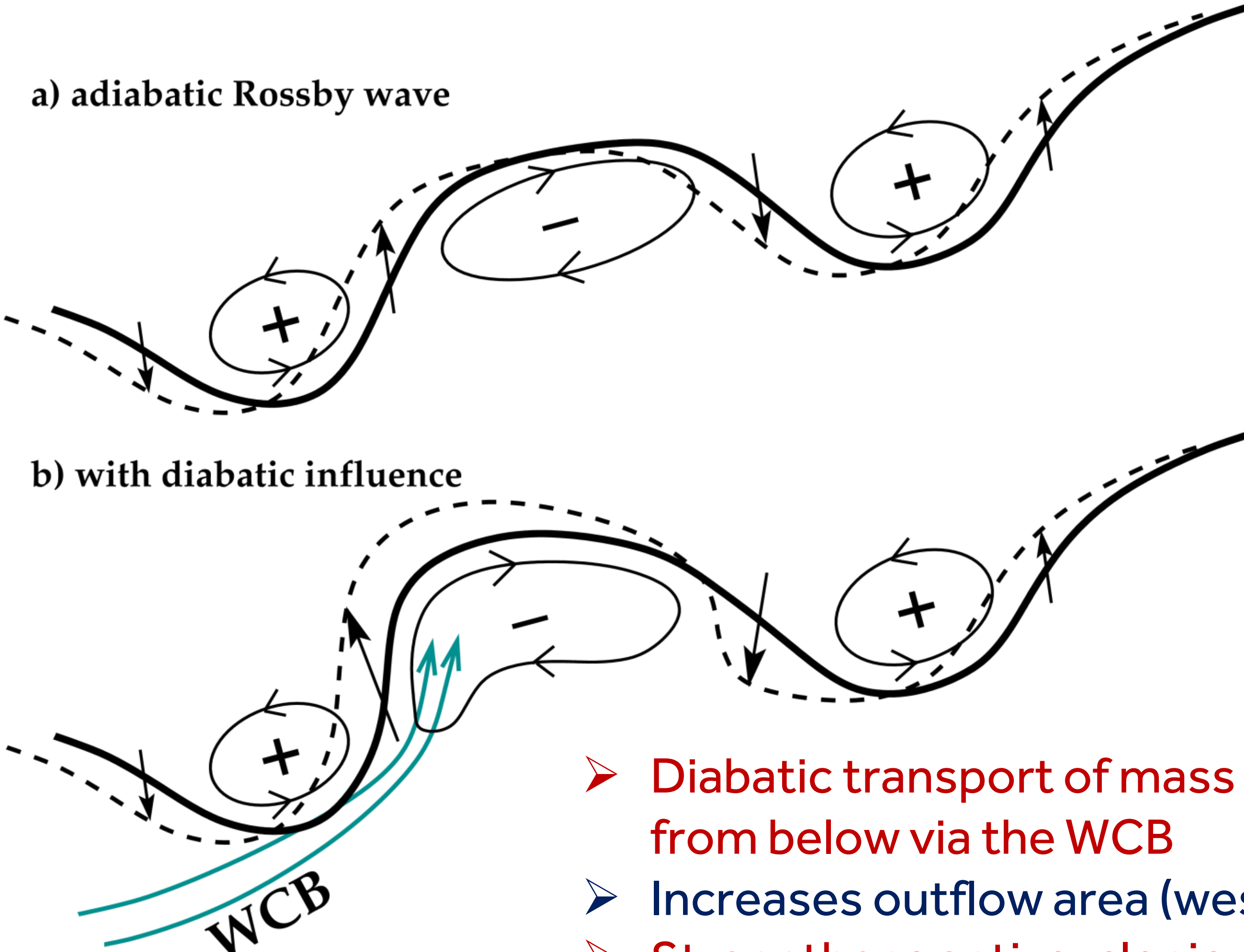
Circulation and mass of outflow



- Diabatic transport of mass into isentropic layer from below via the WCB
- Increases outflow area (west and north flanks)

As described in case study of "forecast bust" by Grams, Magnusson & Madonna (2018), *QJRMetS*

Circulation and mass of outflow



- Diabatic transport of mass into isentropic layer from below via the WCB
- Increases outflow area (west and north flanks)
- Strengthens anti-cyclonic relative flow
- Distorts Rossby wave by advection

Conclusions

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1. **Key ingredients to a WCB** *(even with embedded convection):*
   a. Net heating ($\Delta\theta$) is large, separating "inflow" & "outflow" layers
   b. Horizontal advection on each isentropic surface is dominated by the "balanced flow" *(including ageostrophic motion)*

2. **Influence of latent heating on PV and circulation**
   - PV impermeability theorem is a key constraint *(no PV flux across θ-surface)*
   - ➢ Kelvin's circulation is conserved in the outflow layer *(even with heating)*

3. **Diabatic mass transport and consequences for Rossby waves**
   - WCB net heating determines **diabatic mass transport and outflow level**
   - Can *double* mass of outflow volume
   - Mass increase partitioned between area & density increase (PV inversion)
   - ➢ Anticyclonic relative vorticity increase $\propto$ area increase of divergent outflow
   - Larger –ve anomaly distorts flow downstream $\Rightarrow$ AC wave breaking

Thankyou for your attention

When does PV outflow = PV inflow?

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Under what conditions does average PV of outflow = PV of inflow?

$$\langle q \rangle_2(\tau_2) = \langle q \rangle_1(\tau_1)$$

$$\frac{C_2 \Delta\theta_2}{M_2}(\tau_2) = \frac{C_1 \Delta\theta_1}{M_1}(\tau_1)$$

Define outflow volume to encompass the air that has experienced substantial heating by time τ_2 $\rightarrow \Delta\theta_2,\ M_2(\tau_2),\ C_2(\tau_2)$

Conservation of circulation for upper volume: $C_2(\tau_1) = C_2(\tau_2)$

Define lateral boundaries of inflow and outflow volumes to match, $t \geq \tau_1$ $\rightarrow \mathrm{V_b}$ and thus circulation must match

Appears justified comparing 3D and isentropic trajectories from outflow volume (backwards to time τ_1) $C_1(\tau_1) \approx C_2(\tau_1) = C_2(\tau_2)$

If we choose $M_1(\tau_1) \approx M_2(\tau_2)$ i.e., *initial mass inflow = final mass outflow* then PV relation satisfied subject only to depth $\Delta\theta_1 \approx \Delta\theta_2$