The ICON model: actual state and first steps towards a new dynamical core based on the Discontinuous Galerkin method

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Outline

- The ICON dyn. core, recent developments: deep atmosphere
- Discontinuous Galerkin scheme as a possible alternative dynamical core for ICON - results from a 2D toy model
  - Introduction
  - The HEVI approach
  - DG on the sphere

(short break)

- First steps towards an ICON-prototype based on DG: BRIDGE

Properties of the dynamical core of ICON

- uses non-hydrostatic, compressible Euler eqns.
- exactly mass- and tracer mass-conserving
- true 2nd order scheme (as long as parameterizations are switched off)
- stable in very steep mountainous regions
- useable both for global and regional applications
- computationally very efficient and scales well on current parallel computers

(Zängl et al. (2015) QJRMS, Zängl (2012) MWR, Baldauf, Reinert, Zängl (2014) QJRMS)

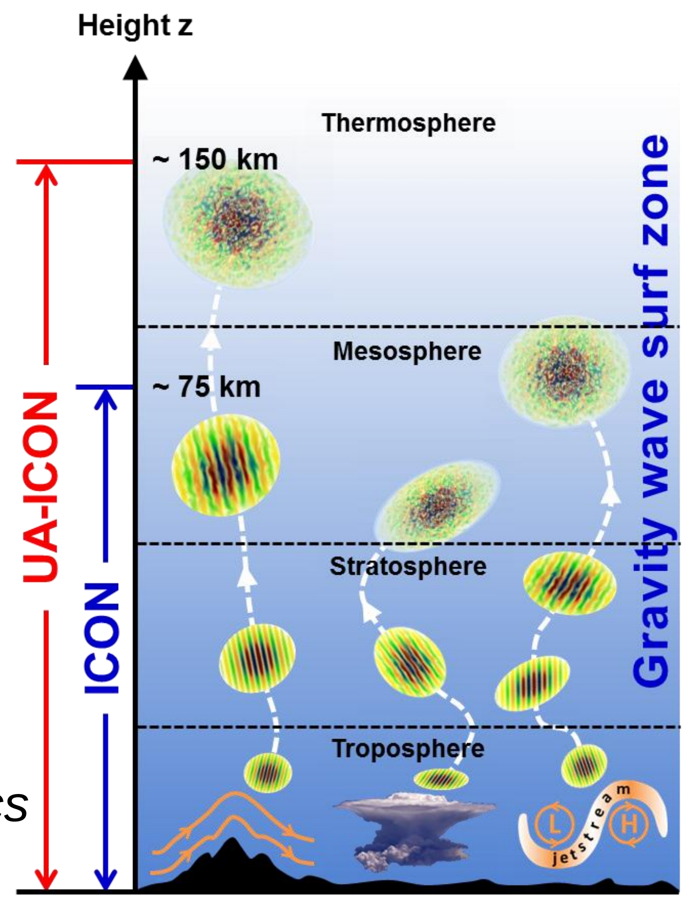
Some numerical details:

- staggering: horizontal: icosahedral, triangle C-grid, vertical: Lorenz-grid
- mixed finite-volume / finite-difference
- predictor-corrector time-integration, horiz. explicit – vertic. implicit (HEVI)
- several damping mechanisms are used (off-centering, divergence damping, horizontal Smagorinsky diffusion and 4th order artificial diffusion, …)

Modification of ICON* for the deep atmosphere: Motivation

S. Borchert

- Work is part of DFG research group: **Multiscale Dynamics of Gravity Waves** (MS-GWaves**)
- Collaboration with colleagues from Max Planck Institute for Meteorology (Hamburg)
- Goal: simulation of large part of gravity wave life cycle, from sources in Troposphere to wave breaking in upper Mesosphere lower Thermosphere
- Implementation of upper-atmosphere physics package (by MPI-M)
- Implementation of *deep-atmosphere dynamics* (by DWD)

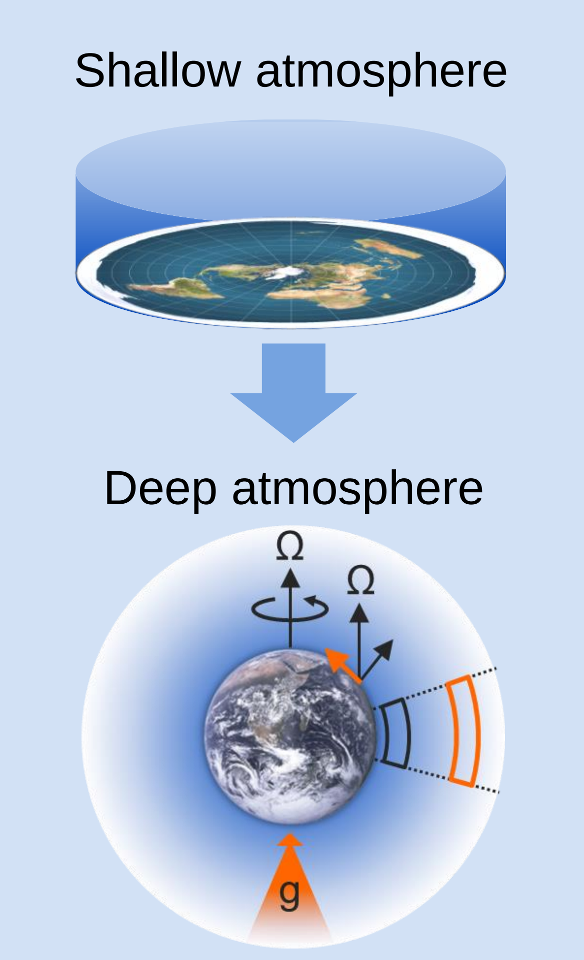


* Zängl et al (2015) Q. J. R. Met. Soc. **https://ms-gwaves.iau.uni-frankfurt.de/

Modification of ICON for the deep atmosphere: Overview

- **Shallow atmosphere***
  - Standard configuration in ICON
  - In particular, replace prefactors $1/r \rightarrow 1/a$

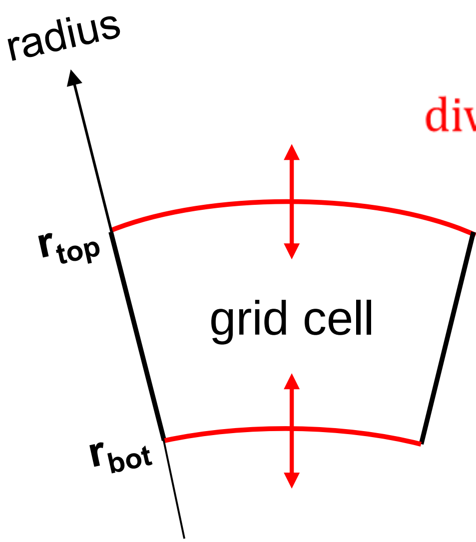
- **Deep-atmosphere modifications**
  - Abandon *shallow-atmosphere approximation*
    - Increase of grid cell extension with height
    - Gravitational field strength $|\boldsymbol{g}|$ decreases with height
  - Abandon *traditional approximation*
    - Coriolis acceleration due to $\boldsymbol{\Omega}_h$
    - take all metric terms in advection



* See *Thuburn & White (2013) Q. J. R. Met. Soc.*

Example: vertical divergence of some flux ξ

$$\mathrm{div}_z(\xi) = \frac{\xi_{\mathrm{top}}\left\{3\left[1+\frac{r_{\mathrm{bot}}}{r_{\mathrm{top}}}+\left(\frac{r_{\mathrm{bot}}}{r_{\mathrm{top}}}\right)^2\right]^{-1}\right\} - \xi_{\mathrm{bot}}\left\{3\left[1+\frac{r_{\mathrm{top}}}{r_{\mathrm{bot}}}+\left(\frac{r_{\mathrm{top}}}{r_{\mathrm{bot}}}\right)^2\right]^{-1}\right\}}{z_{\mathrm{top}} - z_{\mathrm{bot}}}$$

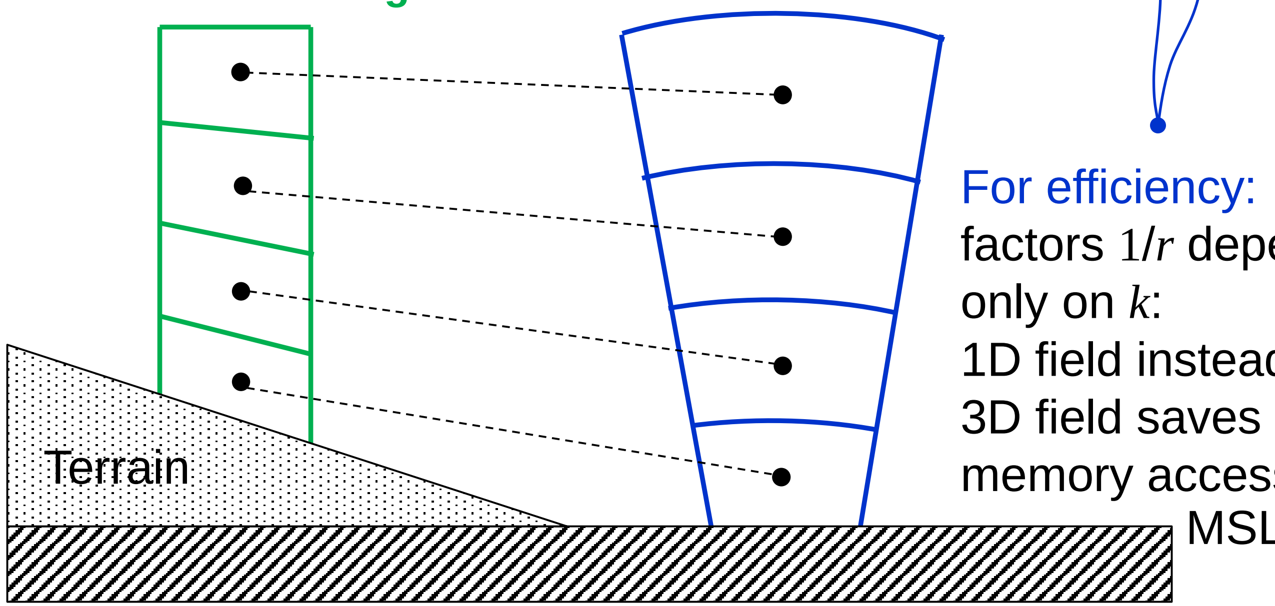


$r = a + z$

$a =$ Earth radius

Shallow-atmosphere formulation:
terrain-following

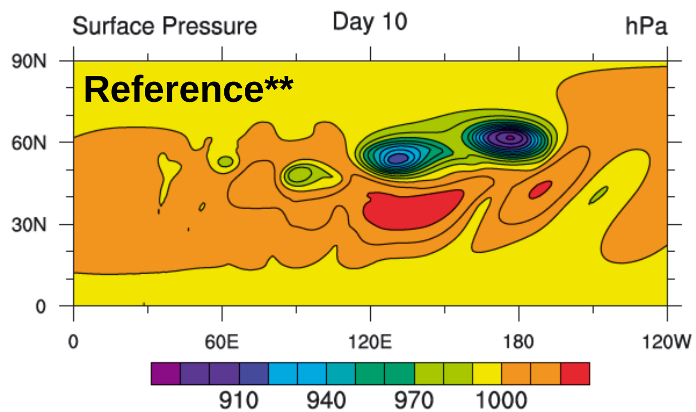
Deep-atmosphere modification:
as if there is no terrain!

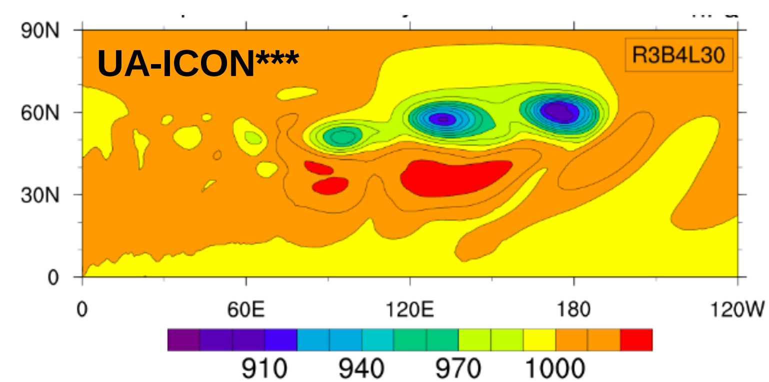


For efficiency:
factors $1/r$ depend only on k:
1D field instead of 3D field saves memory access

Modification of ICON for the deep atmosphere: Test II

- Jablonowski-Williamson baroclinic instability test case*
in its extension for deep-atmosphere dynamical cores**
- Focus on hydrostatic balance and baroclinic waves as important atmospheric synoptic-scale features
- No analytic solution available: model intercomparison





➔ If you are interested in the upper-atmosphere extension of ICON:
*** Borchert, Zhou, Baldauf, Schmidt, Zängl, Reinert (2019) *The upper-atmosphere extension of the ICON general circulation model (version ua-icon-1.0),* GMD

* Jablonowski & Williamson (2006) Q. J. R. Met. Soc. ** Ullrich et al (2014) Q. J. R. Met. Soc.

Time table (at DWD)

Jan. 2015: ICON (13km) replaces GME as a global forecast model

Jan. 2016: ICON-EU-nest (6.5km) replaces COSMO-EU (7km)

Jan. 2021: ICON-D2 (2.1km) replaces the convection-permitting COSMO-D2 (2.2km)
similar replacements at COSMO partner countries until ~2023

Current, ongoing dynamical core developments:

- Replace continuity equation for total mass by those for dry mass
  → this has a lot of implications in dyn. core itself, in physics-dyn.-coupling, in boundary conditions, …
  (D. Reinert (DWD), … KIT)

Discontinuous Galerkin solver as a possible alternative dynamical core for ICON

- Results from a 2D toy model

Michael Baldauf (DWD)

Discontinuous Galerkin (DG) methods in a nutshell (I)

$$\frac{\partial q^{(k)}}{\partial t} + \nabla \cdot \mathbf{f}^{(k)}(q) = S^{(k)}(q), \qquad k = 1, ..., K$$

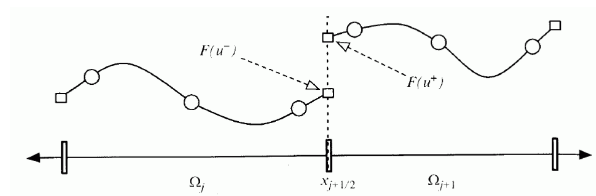
e.g.
Cockburn, Shu (1989) Math. Comput.
Cockburn et al. (1989) JCP
Hesthaven, Warburton (2008)

1.) weak formulation $\int_{\Omega_j} dx \; v(\mathbf{x}) \cdot \ldots$

$$\rightarrow \quad \frac{d}{dt} \int_{\Omega_j} q^{(k)} \, v \, dV + \int_{\partial \Omega_j} f^{(k)num,\perp} \, v \, da - \int_{\Omega_j} \mathbf{f}^{(k)} \cdot \nabla v \, dV = \int_{\Omega_j} S^{(k)} \, v \, dV$$

2.) Finite-element ingredient

$$q^{(k)}(x,t) = \sum_{l=0}^{p} q_{j,l}^{(k)}(t) \; p_l(x - x_j)$$



From *Nair et al. (2011)* in ‚*Numerical techniques for global atm. models'*

Galerkin-idea: identify $v \equiv p_l$

Modal base: orthogonal functions e.g. Legendre-Polynomials
Nodal base: interpolation (Lagrange) polynomials

Discontinuous Galerkin (DG) methods in a nutshell (II)

Weak formulation

$$\frac{d}{dt}\int_{\Omega_j} q^{(k)}\, v\, dV + \int_{\partial\Omega_j} f^{(k)num,\perp}\, v\, da - \int_{\Omega_j} \mathbf{f}^{(k)} \cdot \nabla v\, dV = \int_{\Omega_j} S^{(k)}\, v\, dV$$

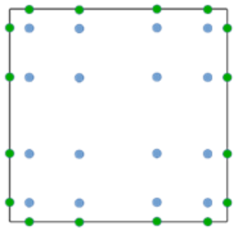
3.) Finite-volume ingredient:

Replace physical flux by a numerical flux in the surface integral
→ couple two neighbouring cells

Often used: simple Lax-Friedrichs flux

$$\mathbf{f}(q) \to f^{num,\perp}(q^+, q^-) = \frac{1}{2}\left(\mathbf{f}(q^+) + \mathbf{f}(q^-)\right) \cdot \mathbf{n} - \frac{\alpha}{2}(q^+ - q^-)$$

4.) Gaussian quadrature for the volume and surface integrals



→ ODE-system for $q^{(k)}{}_{jl}(t)$

5.) Use a time-integration scheme (Runge-Kutta, …)

DG – Pros …

- **local conservation** of every prognostic variable
- any **order of approximation (convergence)** possible
- flexible application on **unstructured grids** (also dynamic adaptation is possible, h-/p-adaptivity)
- very good **scalability** on massively-parallel computers (compact data transfer and no extensive halos)
- **separation** between (analytical) equations and numerical implementation
- **boundary conditions** are easily prescribed (fluxes or values in weak form) → coupling with other subcomponents (ocean model, …) should be easy
- higher accuracy helps to **avoid several awkward approaches** of standard 2nd order schemes: staggered grids (on triangles/hexagons, vertically heavily stretched), numerical hydrostatic balancing, grid imprints by pentagon points or along cubed sphere lines, …
- **unified numerical treatment** of all flux terms and source terms
- **explicit** schemes are relatively easy to build and are quite well understood

DG – … and Cons

Deutscher Wetterdienst
Wetter und Klima aus einer Hand

- **high computational costs** due to
  - (apparently) **small Courant numbers** → small time steps
  - higher number of **degrees of freedom**
    - variables ‚live' both on interior *and* on edge quadrature points
    - this holds additionally for **parabolic problems (diffusion)**
  - HEVI approach leads to **band diagonal matrices** with many bands
- **well-balancing** (hydrostatic, perhaps also geostrophic?) in Euler equations is an issue (can be solved!)
- basically ‚only' an **A-grid-method**, however, the ‚spurious pressure mode' is very selectively damped!

Current status:
we are still relatively far away from a full-fledged meteorological model,
only a **toy model for 2D problems exists** with:

- explicit time integration DG-RK (with Runge-Kutta schemes) or horizontally explicit-vertically implicit (DG-HEVI) (with IMEX-Runge-Kutta)
- ‚local DG' (LDG) option for PDEs with higher spatial derivatives (e.g. diffusion)
- use of a triangle grid (also on the sphere) is optional

Some examples on the next slides …

2D Euler-equations (non-hydrostatic, compressible) with diffusion (=Navier-Stokes eqns.) with terrain-following coordinates (x, z‘) on a plane

$$\begin{aligned}
\frac{\partial}{\partial t}\sqrt{G'}\rho' + \frac{\partial}{\partial x}\sqrt{G'}M^{*x} + \frac{\partial}{\partial z'}\sqrt{G'}\left(\frac{\partial z'}{\partial x}M^{*x} + \frac{\partial z'}{\partial z}M^{*z}\right) &= 0, \\
\frac{\partial}{\partial t}\sqrt{G'}M^{*x} + \frac{\partial}{\partial x}\sqrt{G'}T^{*x*x} + \frac{\partial}{\partial z'}\sqrt{G'}\left(\frac{\partial z'}{\partial x}T^{*x*x} + \frac{\partial z'}{\partial z}T^{*x*z}\right) &= 0, \\
\frac{\partial}{\partial t}\sqrt{G'}M^{*z} + \frac{\partial}{\partial x}\sqrt{G'}T^{*z*x} + \frac{\partial}{\partial z'}\sqrt{G'}\left(\frac{\partial z'}{\partial x}T^{*z*x} + \frac{\partial z'}{\partial z}T^{*z*z}\right) &= -\sqrt{G'}g\rho' - \sqrt{G'}\frac{M^{*z}}{\tau}, \\
\frac{\partial}{\partial t}\sqrt{G'}\eta' + \frac{\partial}{\partial x}\sqrt{G'}H^{*x} + \frac{\partial}{\partial z'}\sqrt{G'}\left(\frac{\partial z'}{\partial x}H^{*x} + \frac{\partial z'}{\partial z}H^{*z}\right) &= 0,
\end{aligned}$$

$$p = p_{ref}\left(\frac{\eta R_d}{p_{ref}}\right)^{cp/cv},$$

$M^{*i} = \rho\, v^i$

$\eta = \rho\theta$

Momentum fluxes:

$$\begin{aligned}
T^{*x*x} &= \frac{M^{*x}M^{*x}}{\rho} + p' - 2K\rho\left(\frac{\partial v^{*x}}{\partial x} + \frac{\partial z'}{\partial x}\frac{\partial v^{*x}}{\partial z'}\right), \\
T^{*x*z} \equiv T^{*z*x} &= \frac{M^{*x}M^{*z}}{\rho} - K\rho\left(\frac{\partial z'}{\partial z}\frac{\partial v^{*x}}{\partial z'} + \frac{\partial v^{*z}}{\partial x} + \frac{\partial z'}{\partial x}\frac{\partial v^{*z}}{\partial z'}\right), \\
T^{*z*z} &= \frac{M^{*z}M^{*z}}{\rho} + p' - 2K\rho\frac{\partial z'}{\partial z}\frac{\partial v^{*z}}{\partial z'},
\end{aligned}$$

Heat fluxes:

$$\begin{aligned}
H^{*x} &= \frac{\eta M^{*x}}{\rho} - K\rho\left(\frac{\partial\Theta}{\partial x} + \frac{\partial z'}{\partial x}\frac{\partial\Theta}{\partial z'}\right), \\
H^{*z} &= \frac{\eta M^{*z}}{\rho} - K\rho\frac{\partial z'}{\partial z}\frac{\partial\Theta}{\partial z'}.
\end{aligned}$$

strong conservation form with terrain following coordinates but cartesian base vectors (*Wedi, Smolarkiewicz (2003) JCP*, *Schuster et al. (2014) MetZ, appendix*, for the sphere)

Horizontally explicit - vertically implicit (HEVI)-scheme with DG

Motivation: get rid of the **strong time step restriction** by vertical sound wave expansion in **flat grid cells** (in particular near the ground)

$$\frac{\partial q^{(s)}}{\partial t} + \underbrace{\nabla \cdot \mathbf{f}^{(s)}_{slow}}_{\text{explicit}} + \underbrace{\nabla \cdot \mathbf{f}^{(s)}_{fast}}_{\text{implicit}} = \underbrace{S^{(s)}_{slow}}_{\text{explicit}} + \underbrace{S^{(s)}_{fast}}_{\text{implicit}}$$

$$\mathbf{f}^{(s)}_{fast} = f^{(s)}_{z,fast} \mathbf{e}_z$$

$$f^{(s)}_{z,fast} = \sum_{s'} H^{ss'} q^{(s')}$$

- Use of **IMEX-Runge-Kutta** (SDIRK) schemes: SSP3(3,3,2), SSP3(4,3,3) (*Pareschi, Russo (2005) JSC*)
- The implicit part leads to several band diagonal matrices
  → here a direct solver is used (expensive!)

References:
Giraldo et al. (2010) SIAM JSC: propose a HEVI semi-implicit scheme
Bao, Klöfkorn, Nair (2015) MWR: use of an iterative solver for HEVI-DG
Blaise et al. (2016) IJNMF: use of IMEX-RK schemes in HEVI-DG
Abdi et al. (2017) arXiv: use of multi-step or multi-stage IMEX for HEVI-DG

IMEX-Runge-Kutta

- general stability function for the Dahlquist problem is known
- general order conditions are known
- described by double Butcher tableaus
  e.g. SSP3(3,3,2) by *Pareschi, Russo (2005) JSC:*

$$
\begin{array}{c|ccc}
0 & 0 & & \\
1 & 1 & 0 & \\
1/2 & 1/4 & 1/4 & 0 \\
\hline
 & 1/6 & 1/6 & 2/3
\end{array}
\qquad
\begin{array}{c|ccc}
a & a & & \\
1\text{-}a & 1\text{-}2a & a & \\
1/2 & 1/2\text{-}a & 0 & a \\
\hline
 & 1/6 & 1/6 & 2/3
\end{array}
$$

$$a = 1 - 1/\sqrt{2}$$

- practically SDIRK schemes are preferred

Lock, Wood, Weller (2014) QJRMS
Pareschi, Russo (2005) JSC: SSP3(3,3,2), SSP3(4,3,3)
Giraldo et al. (2012) Siam JSC: ARK2(2,3,2)
Kang, Giraldo, Bui-Thanh (2020) JCP: IMEX-RK in hybridiz. DG

2D linear advection equation – DG-HEVI

maximum Courant numbers:
for a modal basis with Legendre polynomials of degree p
(i.e. convergence order p+1)

RK-IMEX scheme	time order	p=0	p=1	p=2	p=3
RK1-IMEX	1	1.0	0	0	0
Trap2	2	1.0	0.198	0.081	
SSP3(3,3,2)	2	1.25	0.378	0.199	
ARK2(2,3,2)	2		0.219	0.110	
SSP3(4,3,3)	3		0.256	0.131	0.079

more specific: $\boldsymbol{U_0^{slow}=1}$, $V_0^{slow}=0$, $U_0^{fast}=0$, $\boldsymbol{V_0^{fast}=10}$, $dx=dy=1$

Compare: explicit DG p=3 /4th order RK: CFL=0.136

A problem with the Euler equations …

Approximate hydrostatic balance

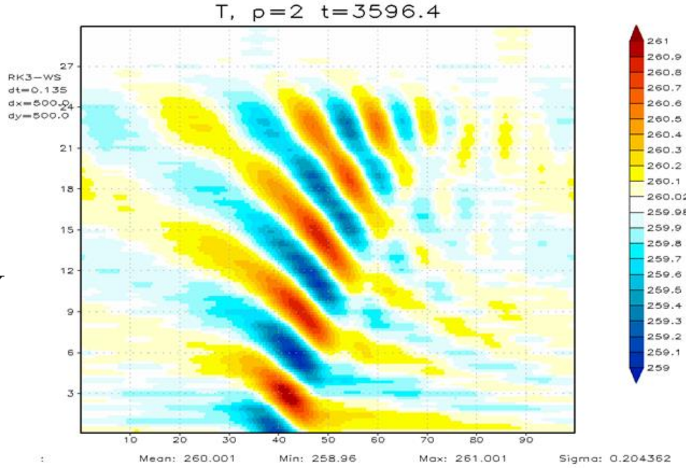
pressure gradient (→flux div.) ≅ buoyancy term (→ source term)

is crucial for the Euler equations in the atmosphere.

However, the source term integral contains base polynomials themselves, whereas the flux div. term integral uses *derivatives* of base polynomials:

$$\frac{d}{dt}\int_{\Omega_j} q^{(k)}\, v\, dV + \int_{\partial\Omega_j} f^{(k)num,\perp}\, v\, da - \int_{\Omega_j} \mathbf{f}^{(k)} \cdot \nabla v\, dV = \int_{\Omega_j} S^{(k)}\, v\, dV$$

→ no proper balance possible.



… and its solution:

Blaise et al. (2016) IJNMF, Orgis et al. (2017) JCP:

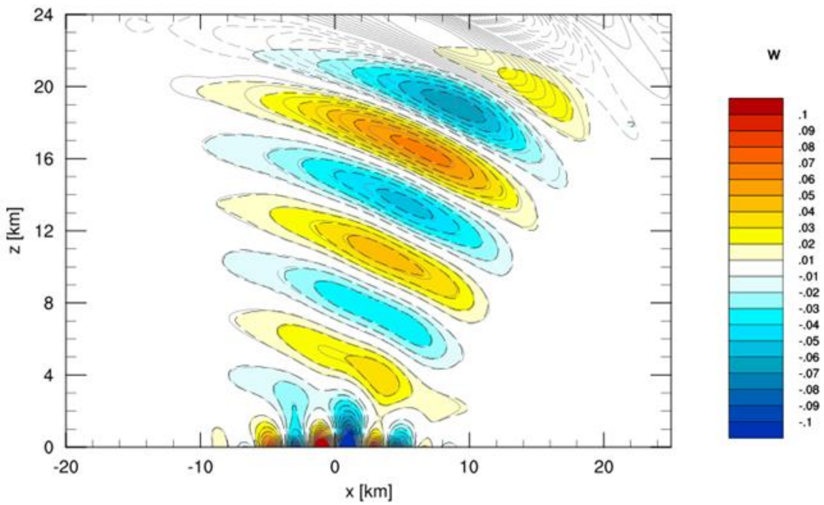
use vertically a reduced base (one polynomial degree less in a modal base) for the calculation of the source term.

This leads to an additional filtering procedure in the implicit solver.

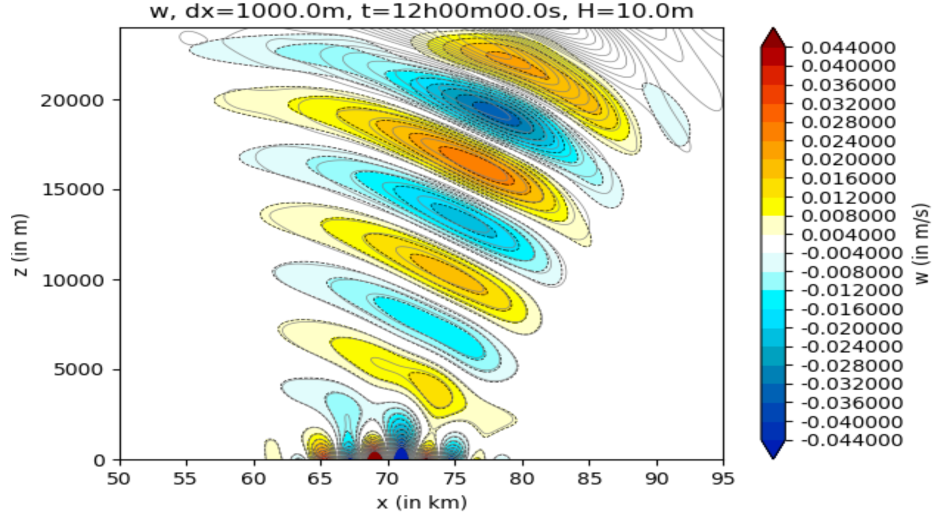
Flow over mountain with the HEVI-solver

ICON Δx=250m Δz=150m

ICON, w t=01d00:00:00 dx=250m H=25m



HEVI-DG 4th order Δx=1000m Δz= 600m



colors and black dotted lines: model

grey lines: analytic solution *(Baldauf, 2008, COSMO-Newsl.)*

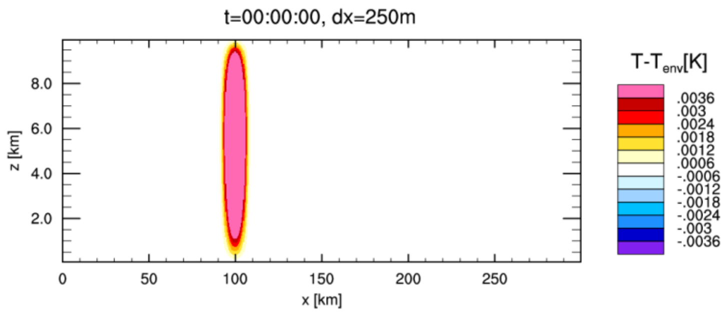
Computing time on 160 processors
(Cray XC40 Broadwell)
for t_{total}=24h, 26min
→on 1 processor: 69h

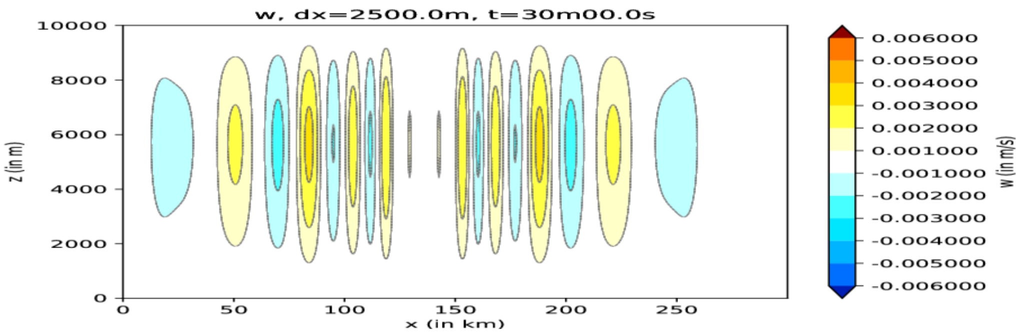
Computing time on 1 processor:
(Intel(R) Core(TM) i7-4790 CPU @ 3.60GHz)
for t_{total}=24h, 4th order DG: 160h

But don‘t take the computing times too serious!

Linear gravity/sound wave expansion in a channel

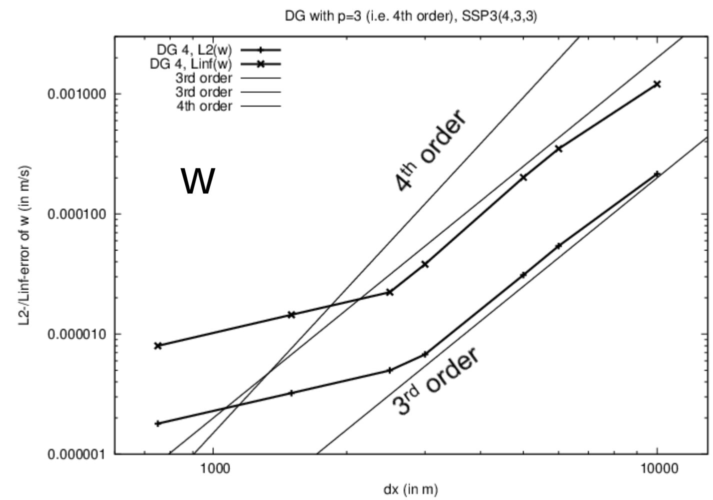
setup similar to *Skamarock, Klemp (1994) MWR*

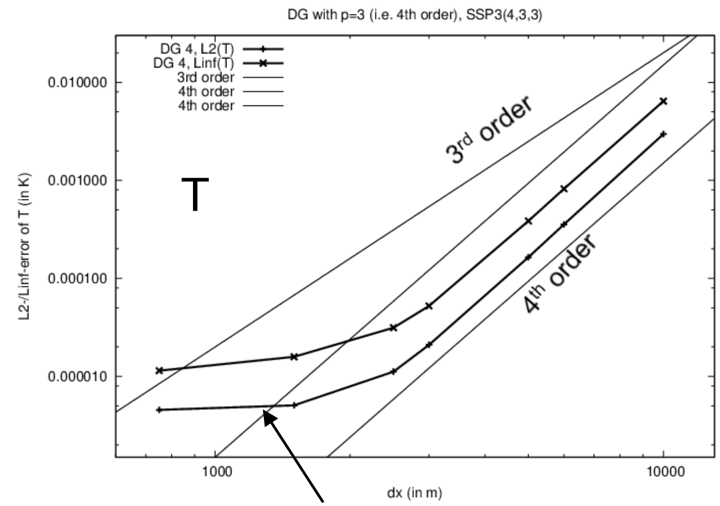




colors : simulation with p=3/SSP3(4,3,3)
dx=2500m, dz=1250m

black lines: analytic solution for compressible, non-hydrostatic Euler eqns. (*Baldauf, Brdar (2013) QJRMS*)





Nonlinear effect?

General treatment of diffusion in DG

Simple replacement of a flux $f(q)$ by $f(q, \partial q/\partial x)$, where $\partial q/\partial x$ is directly calculated from q, does not work.
Instead (*Bassi, Rebay, 1997*, and similar for ‚local DG‘):

- define a new variable $d = \partial q/\partial x$
- treat q as a flux and apply the DG formalism to this equation, too.
- Replace $f(q) \rightarrow f(q) + f_{\text{diffus}}(q,d)$
  Remark: the numerical flux for f_{diffus} does not need additional numerical diffusion.

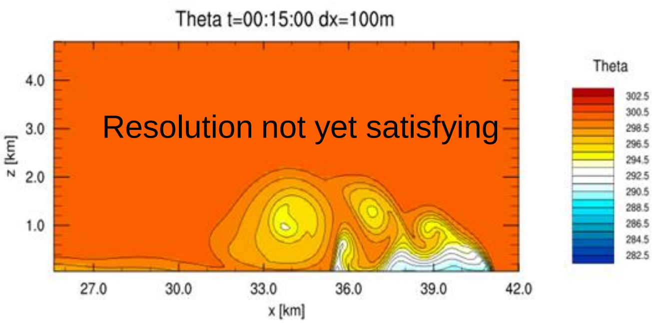
3D-diffusion in terrain-following coordinates

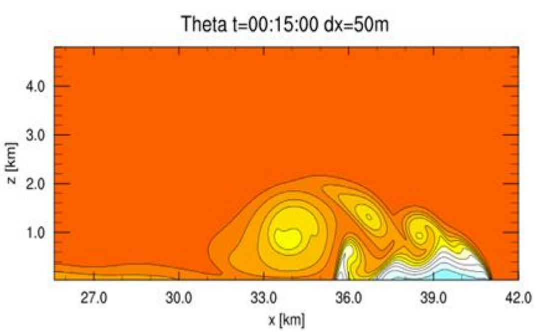
New developments:

- In terrain-following coord. there are several choices for d possible.
  here: covariant derivatives of the prognostic variables
- If diffusion is treated vertically implicit (i.e. HEVI), too
  → extension of the band diagonal matrix necessary
- Currently: ‚HEVI-diffusion‘ is done in every sound time-step
  → expensive! Appropriate time-integration necessary.

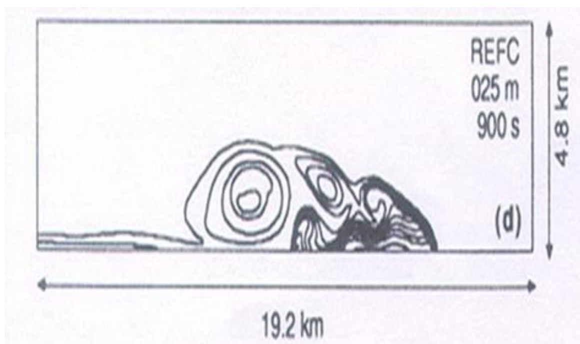
Falling bubble test case

ICON

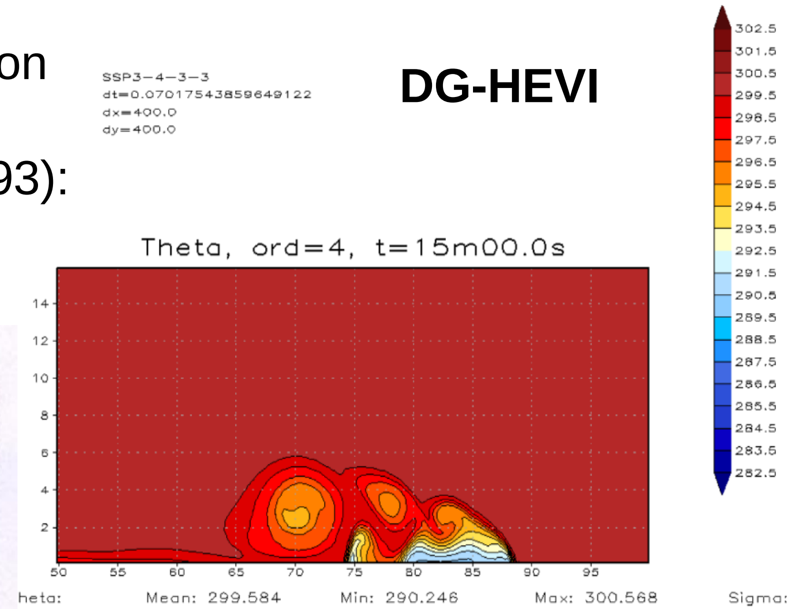




Reference solution for θ
Straka et al. (1993):



DG-HEVI



Computing time on 40 processors
(Cray XC40 Broadwell)
dx=dz=50m: 5min
→ on 1 processor: 3h30min

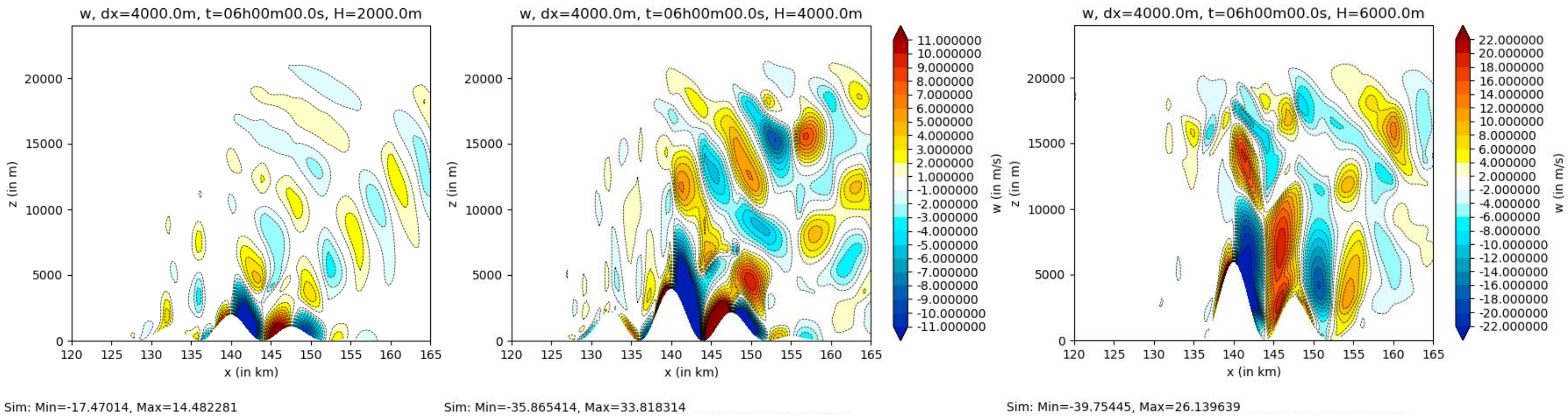
Computing time on 1 processor
(Intel(R) Core(TM) i7-4790 CPU @ 3.60GHz)
dx=dz=400m, 4th order DG
Euler HEVI, diffusion explicit: 1h40min
Euler + diffusion in HEVI: 3h50min

But don‘t take the computing times too serious!

Flow over mountain with steep slopes and vertical grid stretching

Schaer et al. (2002) MWR, test case 5b: U_0=10m/s, N=0.01 1/s, but a=10km

H_{oro}= 2000m, α_{max} = 38° | H_{oro}= 4000m, α_{max} = 58° | H_{oro}= 6000m, α_{max} = 67°



Δx=4 km; vertical grid stretching: Δz_{min}~46m, Δz_{max}~736m, $z_{lowest\ QP}$~10.3m

HEVI-DG simulation (4th order) remains stable even for steeper slopes!

to avoid instability by strong gravity wave breaking, vertically implicit ‚3D' diffusion with K=100m²/s was used

Efficiency of the implicit solver

The vertically implicit problem leads to the solution of J **block tridiagonal LES**, i.e J band diagonal matrices with $KSMN$ rows and colums and $2SMN$ diagonals both above and below the main diagonal

J = number of grid cells in the horizontal direction
K = number of grid cells in the vertical direction
S = number of variables
M = number of horizontal base functions
N = number of vertical base functions

- Use of a **direct band diagonal solver**:
  - LU decomposition: complexity $O(JK(SMN)^2)$
    → at most useable with low order DG
    efficiency gain: only once every several dozen time steps
  - Solution of the LES: one matrix-vector-mult. per column in every RK-substep: complexity $O(JKSMN)$
  - compare with a tridiagonal solver in our ‚standard' FD/FV codes: $O(JM KN)$
- Use of an **iterative solver** (*Bao, Klöfkorn, Nair (2015) MWR*)

How to bring DG on the sphere

annoying fact: the sphere doesn‘t allow a single coordinate system without singularities → leads to **pole problem** in lat-lon coordinates

→ usual remedies:

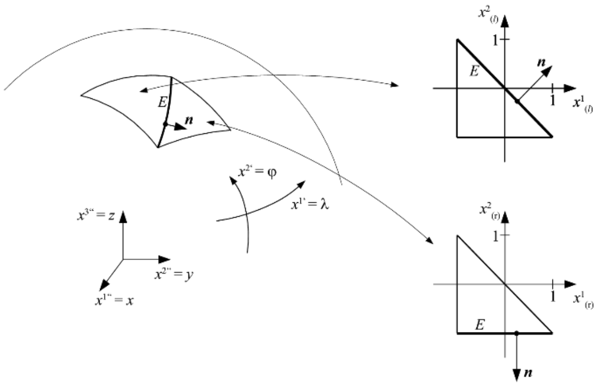
- *Yin-Yang* grid (often with quadrilateral cells)
- *cubed-sphere* grid (often with quadrilateral cells)
- *Icosahedron* with hexagonal (e.g. MPAS) or **triangle** cells (e.g. ICON)

additionally: allow higher order (i.e. >2nd) discretization
→ use **local coordinates** for every (triangle) grid cell,
= locally rotated gnomonial projection (*Läuter, Giraldo, … (2008) JCP*)
→ geometry is treated exactly!

Idea here: fields components are expressed in local base vectors, too
→ no pole problem at all
→ transform fluxes between neighbouring cells
→ use covariant formulation of the equations

Shallow water equations, covariant formulation

$$\begin{aligned}
\frac{\partial \sqrt{g} H}{\partial t} + \frac{\partial}{\partial x^i} \sqrt{g} M^i &= 0 \\
\frac{\partial \sqrt{g} M^i}{\partial t} + \frac{\partial}{\partial x^j} \sqrt{g} T^{ij} &= \sqrt{g} \left(-g_{grav} H g^{ij} \frac{\partial h_B}{\partial x^j} + F^i_{Cor} - \Gamma^i_{jk} T^{jk} \right) \\
T^{ij} &= \frac{M^i M^j}{H} + \frac{1}{2} g^{ij} g_{grav} H^2, \quad M^i = H v^i
\end{aligned}$$



x^1, x^2 are arbitrary local coordinates on each triangle.
This description is valid on arbitrary 2D manifolds.
→ extension to an ellipsoid is easy and without additional costs.

Formulate DG discretization only in these local coordinates
→ ‚standard' DG formulation useable: nodal base, Runge-Kutta time integr., …
however: Transformation of fluxes on the edges between neighbouring cells:

$$f^i_{(r \to l)} = \frac{\partial x^i_{(l)}}{\partial x^j_{(r)}} f^j_{(r)}, \qquad f^i_{(l \to r)} = \frac{\partial x^i_{(r)}}{\partial x^j_{(l)}} f^j_{(l)}$$

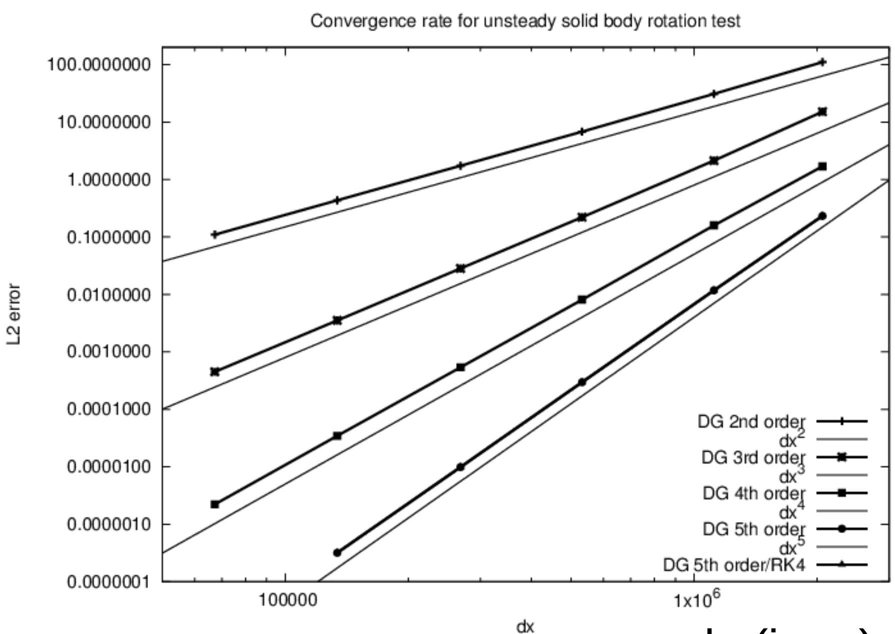
Baldauf, M. (2020): *Discontinuous Galerkin solver for the shallow-water equations in covariant form on the sphere and the ellipsoid,* J. Comp. Phys. 410

Unsteady solid body rotation

Läuter et al. (2005) JCP

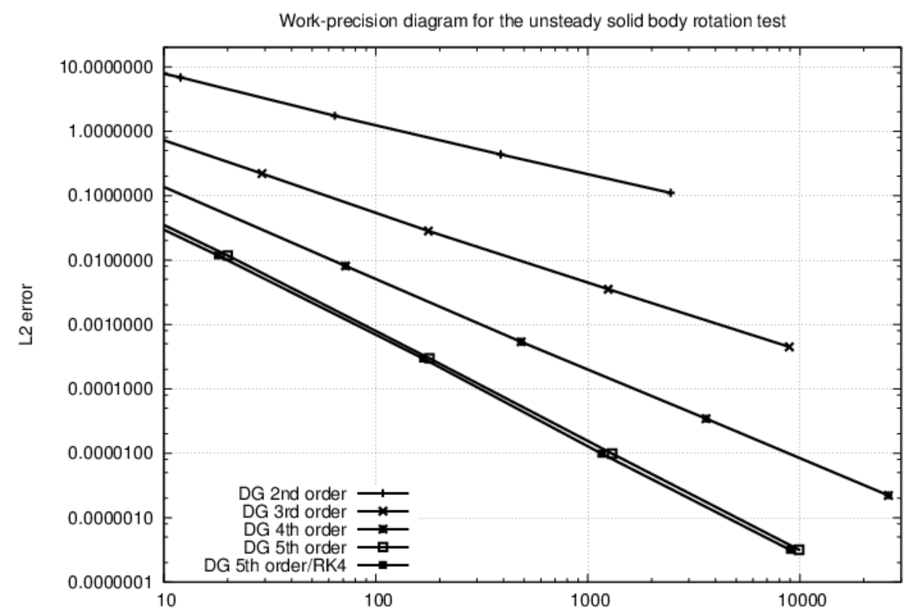
Exact analytic solution available! → calculate error measures L_2 (=RMSE)

Convergence plot:



dx (in m)

Work-precision diagram:

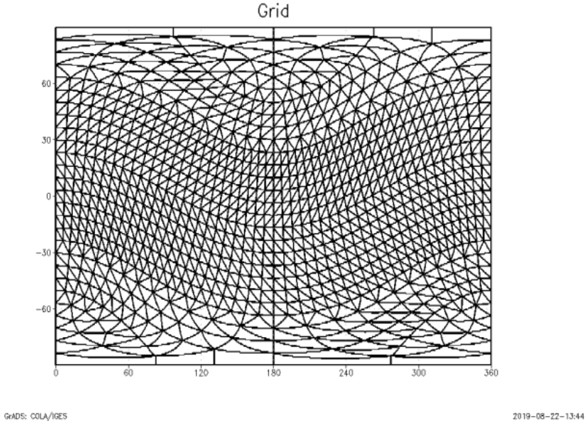


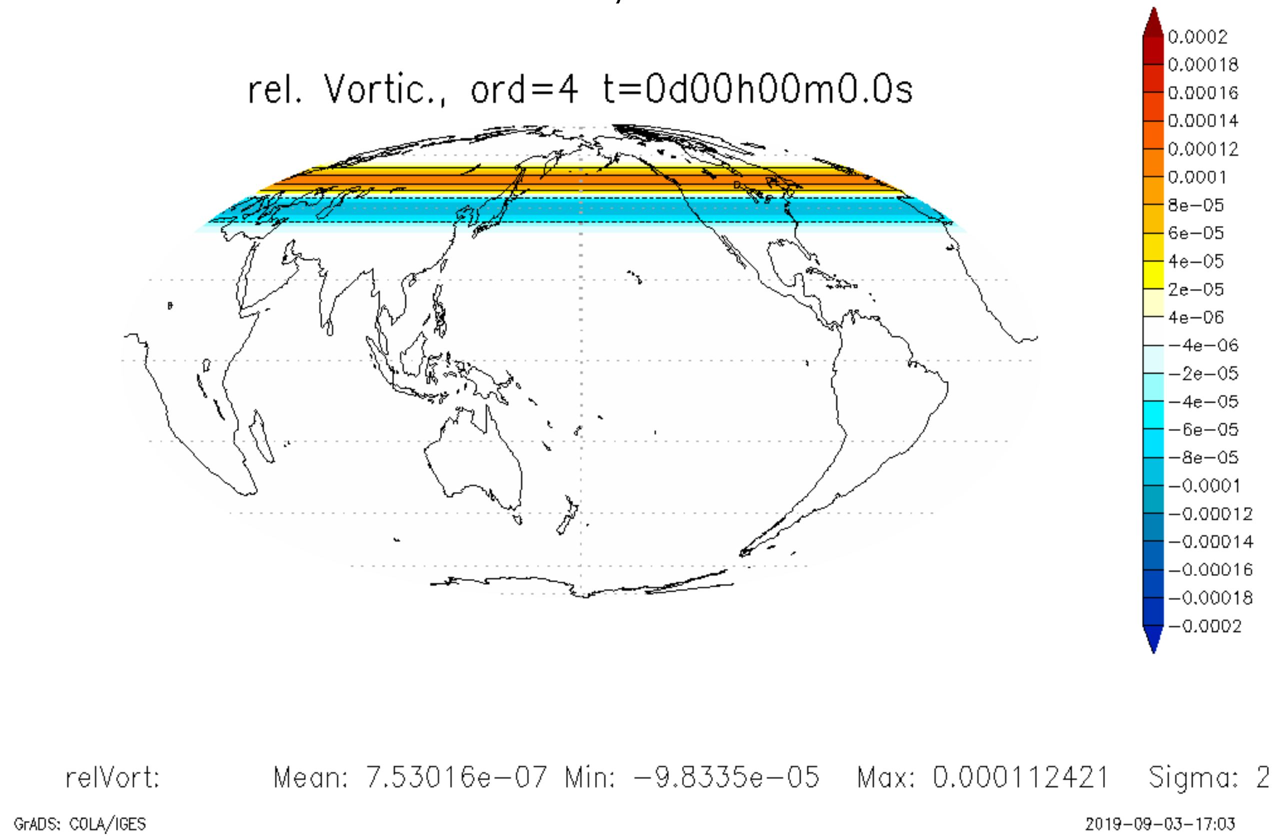
CPU wall clock time (in sec.)

Barotropic instability test
Galewsky et al. (2004)

4th order DG scheme
without additional diffusion
dx~67 km, dt=15 sec.

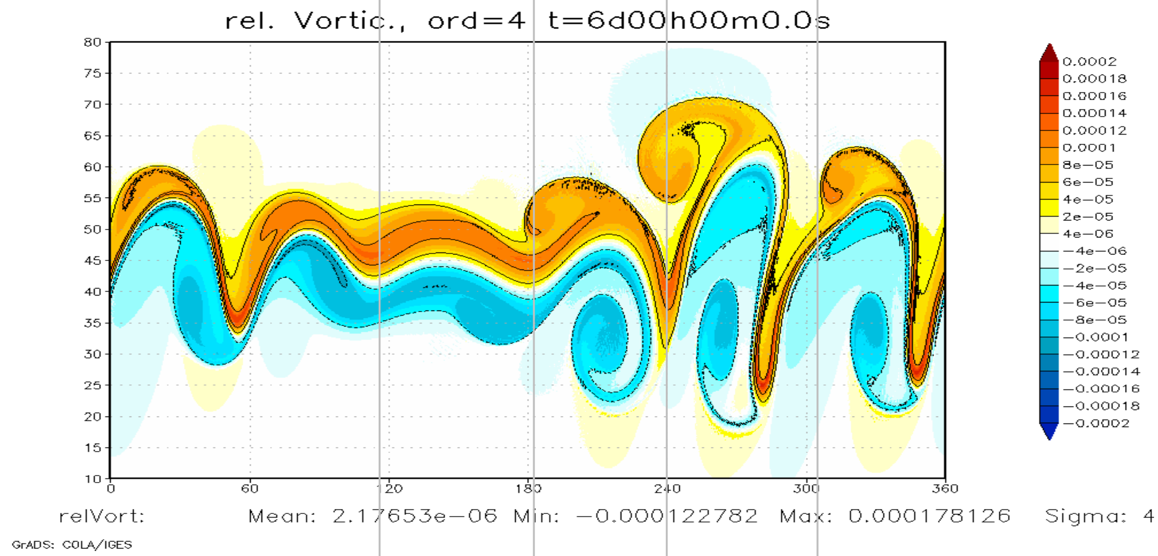
simple triangle grid
on the sphere
dx ~ 500km:



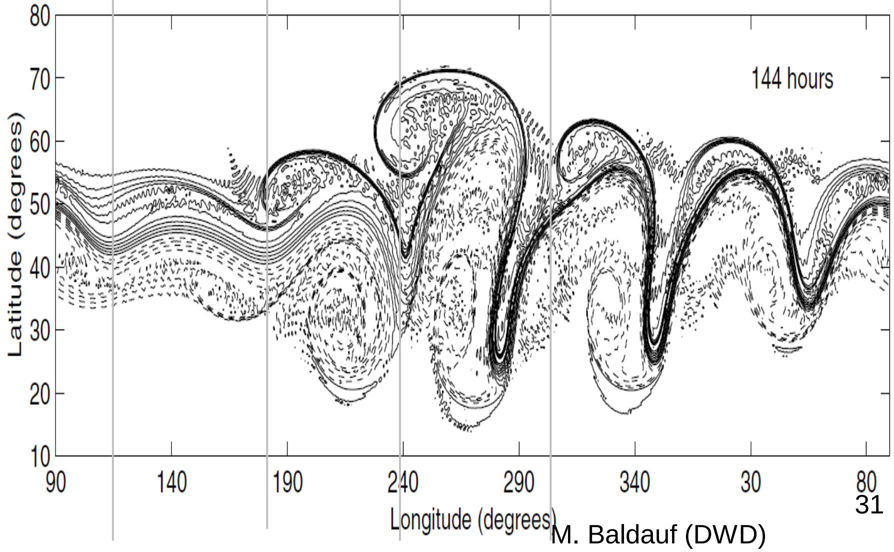


Barotropic instability test
Galewsky et al. (2004)

relative vorticity



4th order DG scheme
without additional diffusion
dx~67 km, dt=15 sec.



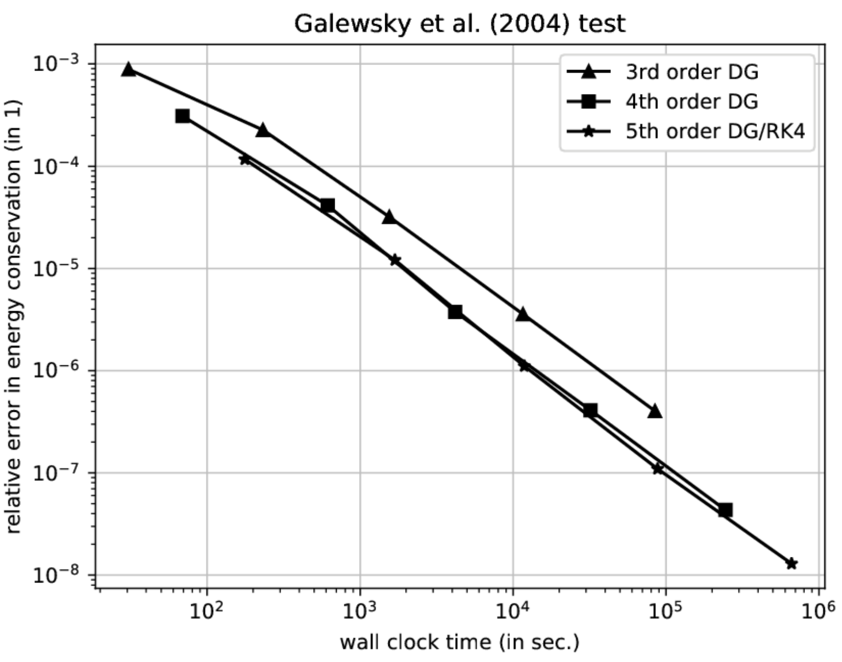
FMS-SWM (Geophys. Fl. Dyn. Lab.)
without additional diffusion
dx~60 km (T341), dt=30 sec.

Fig. 4 from *Galewsky et al. (2004)*

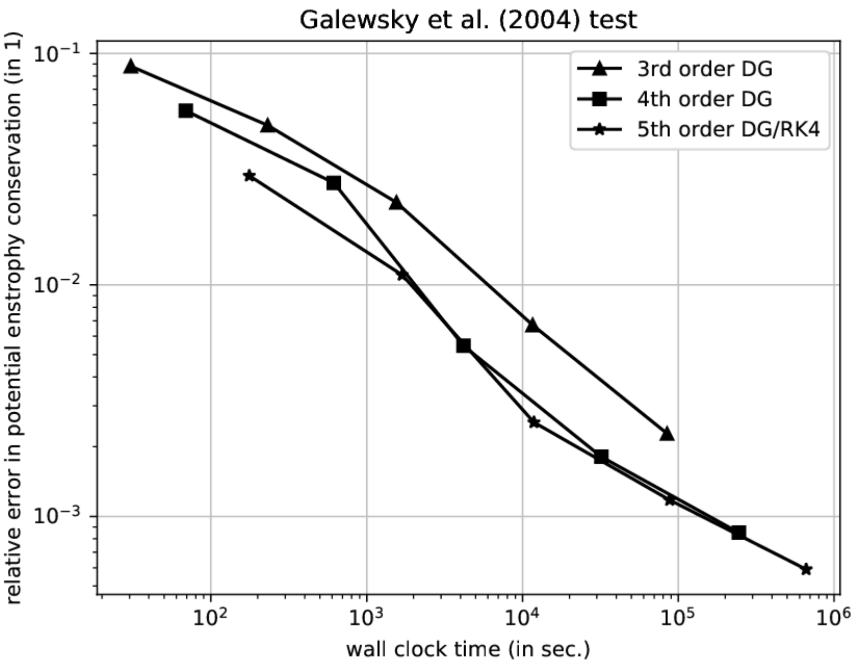
Barotropic instability test
Galewsky et al. (2004)

Work-precision-diagrams

rel. error in energy conservation



rel. error in pot. enstrophy conservation



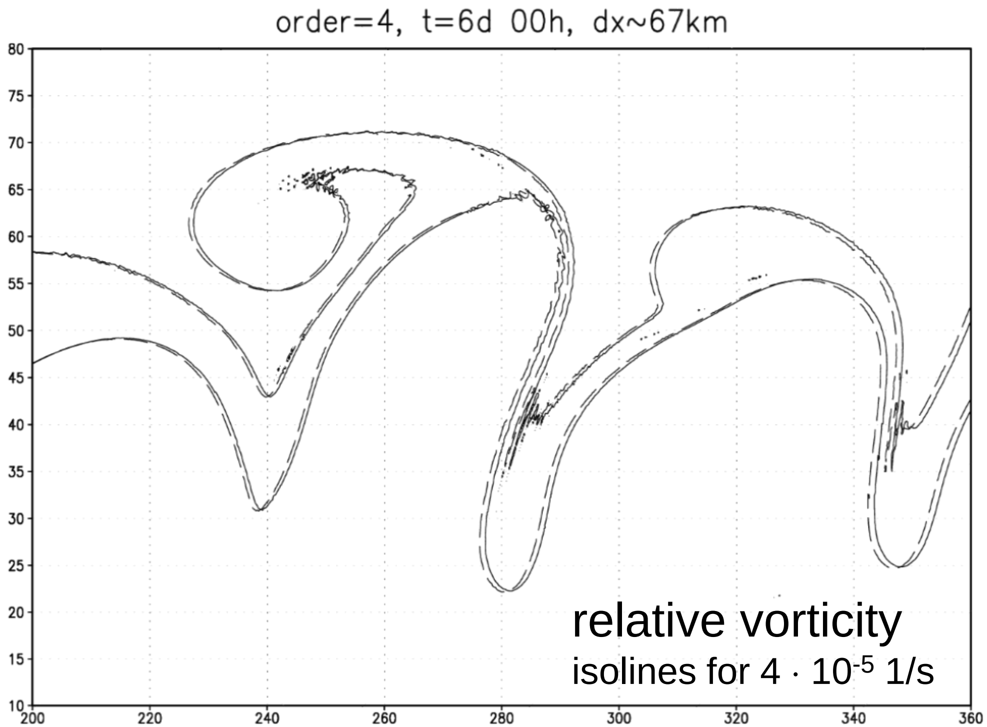
Barotropic instability test
Galewsky et al. (2004)

Comparison between the sphere and the ellipsoid

4th order DG scheme
without additional diffusion
dx~67 km, dt=15 sec.

solid line: sphere
R = 6371.22 km

dashed line: ellipsoid
a = 6378.137 km
c = 6356.752 km
→ numer. excentr. = 0.082



→ ellipsoidal solution shows westward phase shift of ~1° after 6 days
→ is in qualitative agreement with *Bénard (2015) QJRMS*

Flow over mountain

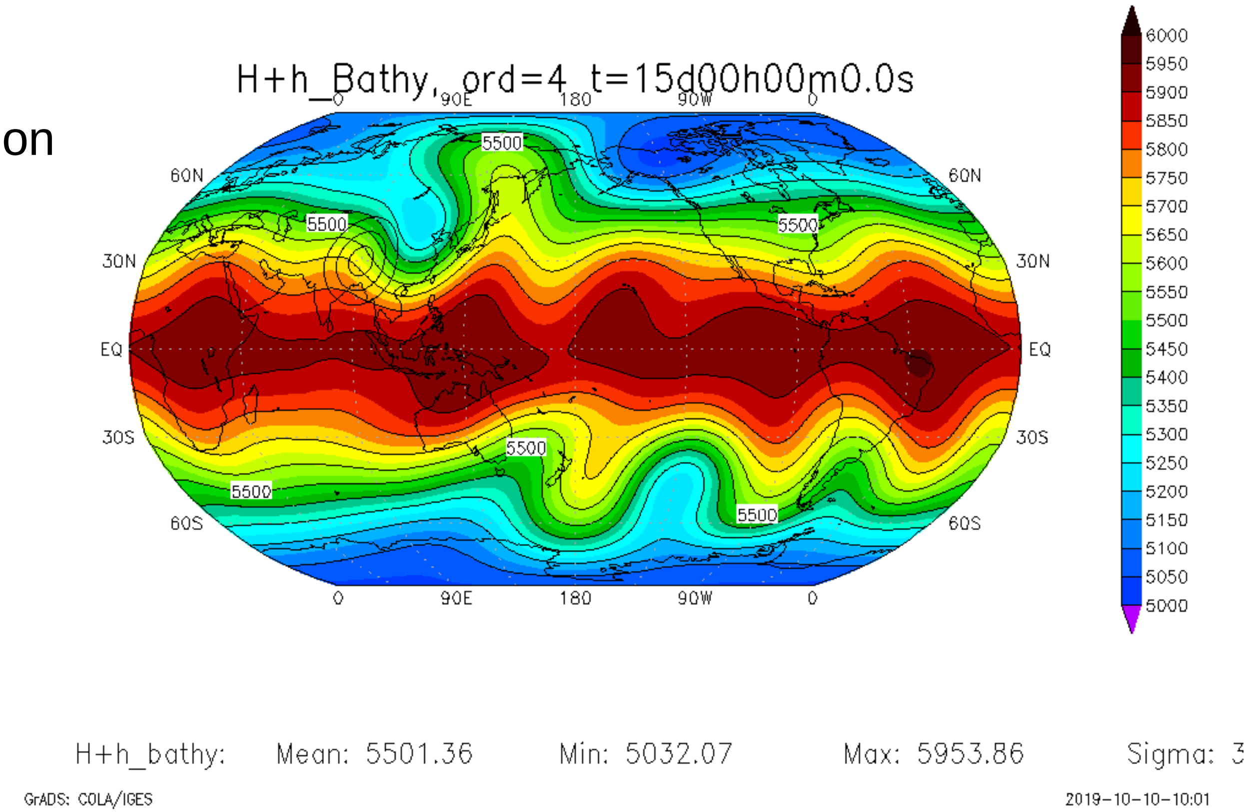
Test case 5 of *Williamson et al. (1992)*

init.: solid body rotation velocity field (u_{max}=20 m/s) in geostrophic balance with H;
mountain height 2000m

4th order DG scheme

without additional diffusion

dx~67 km, dt=15 sec.



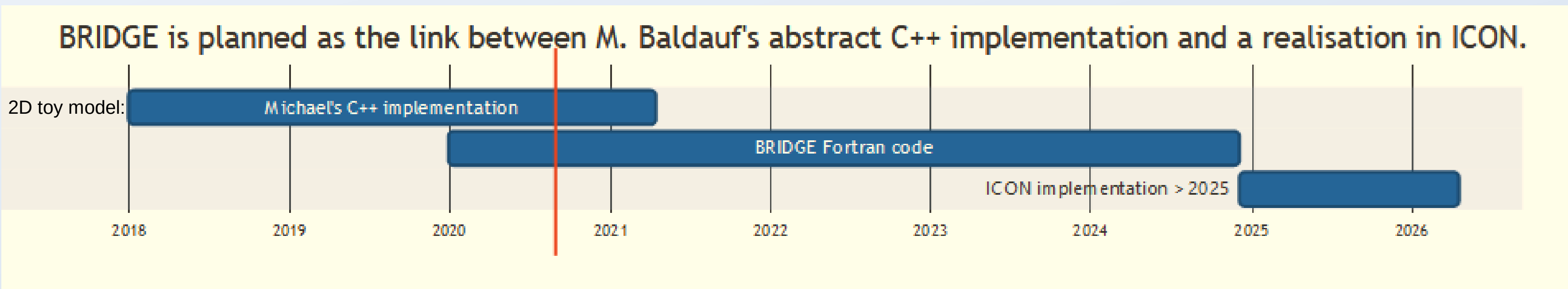
"BRIDGE" - Basic 3D Test Code

Deutscher Wetterdienst
Wetter und Klima aus einer Hand

3D test code with as little infrastructure overhead as possible

Florian Prill (DWD)

- Discontinuous Galerkin discretizations
- Test platform for DSLs and new infrastructure libraries
- Benchmark "dwarfs"



DG Working Group - It's just starting ...

BRIDGE is application-oriented research –
and it's extra work.

M. Baldauf • F. Prill • D. Reinert • S. Borchert • …

Overall Design

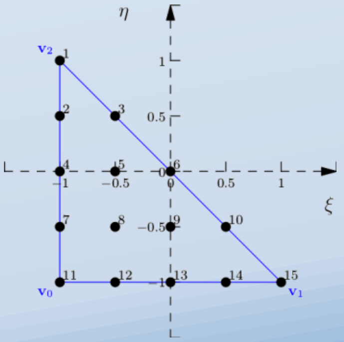
- Fortran 2003
- MPI parallelization
- Modules and interfaces closely resemble ICON

  Example:
  Source tree: **`parallel_infrastructure`**, **`shared`**, **`shr_horizontal`**, ...
  Data types and objects: **`wp`**, **`t_patch`**, **`t_grid_cells`**, **`t_var_list`**, ...
- ICON triangular grid
- Storage layout: MPI parallel domain decomposition, arrays of 2D triangulations are **`nproma`**-blocked

2.5D: Restrict to quadrature points and node sets which are built from 2Dx1D tensor products.

Consequently, restriction to expansions

$$q(r(x^1,x^2),s(x^3),t) = \sum_{l=1}^{M}\sum_{m=1}^{N} Q_{l,m}(t)\phi_l(x^1,x^2)\psi_m(x^3) \ .$$



Restrictions and Simplifications

- MPI parallelization: no "no-MPI" mode,
  only worker tasks present (no detached PEs)
- operates on a single domain only
- no "local parent patch“
  but parent grid may have enlarged halo
- only one precision kind **`wp`**
- only rudimentary field meta-data
- no restarting capability
- no refinement of cells in a column
  (eg. small cells at the bottom, coarse at the top)

Modularization, Encapsulation

Libraries

... as little infrastructure overhead as possible:

- Use of supporting libraries like **YAXT**
  The communication library YAXT (DKRZ, Hamburg) abstracts the communication on MPI level from the application.
- better separation between scientific code and infrastructure:
  two auxiliary libraries have been created: **`libftnbasic`** and **`libcbasic`**.

F2003 object-oriented features

The code uses F2003 objects to a far greater extent than ICON:

- to avoid global variables
- to make the data flow in the model more transparent
  which variable is touched by whom, and when ...

Vectors (FE coefficients, evaluated shape fct., ...) remain **`REAL(wp)`** vectors
= no derived type abstraction.
The user takes care of the index ordering
or the fact if they are global or local, synched or not.

DG and Parameterisations

General principle: spatial transport (advection, sedimentation, diffusion, …) must be treated in the DG-scheme!
Otherwise we loose conservation.

Box-models (e.g. **cloud physics**, **chemistry/aerosol-packages**):
are evaluated and deliver tendencies in every quadrature point
→ at the first place no adaptations necessary!
Nevertheless, the ‚classical' physics/dynamics-coupling questions remain:
overall time integration scheme? how to achieve positive definiteness?

Turbulence:
Remark: diffusion needs special treatment in DG (local DG, compact DG, …)
Advantage of local DG: derivatives of fields are directly available for turbulence modeling!

…

DG and physics perturbations in ensembles

Some recommendations …
… to keep conservation properties of the DG scheme:

- in the **transport terms**, only (physical) fluxes should be perturbed.
- in the **source terms**: e.g. moisture var., perturb in a way that $\rho_{dry} + \rho_v + \rho_r + ... + \rho_g$ is unchanged (while keeping positive def.)

DG and data assimilation

At least adaptations in the forward operators necessary:

- by the modified output grid; better say: to the position of the I/O-grid points (these are probably the quadrature points in the triangle grid)
- different prognostic variables (conserved var.)

Summary

- **2D toy model** for
  - explicit DG-RK (unstructured grids, triangle or quadrilateral grid cells) and
  - **HEVI DG-IMEX-RK**

  now works for *Euler equations* with *terrain-following coordinates* and optionally with *3D diffusion* (explicit or HEVI): several tests show correct convergence behaviour, well-balancing problems solved, …
- Efficiency problems with band diagonal matrix solver strongly improved: the whole implicit part (build coeff. matrix, *LU decomposition*, matrix-vector-mult.) takes ~60% of total run time.
- **DG on the sphere** on a triangle grid possible by the use of local (rotated gnomonial) coordinates and the covariant formulation of the equations. (*Baldauf (2020) JCP*)

→ With respect to the pure dynamical core (=solver for the Euler equations), no show–stopper occured until now
However, total efficiency is still an issue!

Outlook

- Current tasks in the **2D toy model**
  - Consolidation of what has been achieved so far:
    further testing; what are the true limits of the method?

Further **milestones** (for the next years!)

- Development of a **3D prototype,** DG-HEVI on the sphere
  BRIDGE (**B**asic **R**esearch for **I**CON with **DG E**xtension) (start ~mid 2020)
  - **Further design decisions**: nodal vs. modal?,
    local DG vs. interior penalty vs. …?, allow non-conformal grids?,
    efficient data layout, …
  - Coupling of **tracer advection** (mass-consistency)?
  - Develop coupling ideas for parameterizations
    time-integration, preserve pos. def., …
- Implementation into **ICON** (start ~2024)
  - choose optimal convergence order p and grid spacing
    estimated: $p_{horiz} \sim 4 \ldots 6$, $p_{vert} \sim 3 \ldots 4$ ($p_{time} \sim 3 \ldots 4$)
    currently I favor: $p_{horiz} = 4$, $p_{vert} = 4$, $p_{time} = 3$

Thank you very much for your attention!