Temperature discretizations for the IFS, horizontal to vertical resolution aspect ratio and their importance for accurate weather prediction

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ECMWF

Outline

Brief overview of the IFS dynamical and ECMWF forecasts

Part I: Horizontal to vertical resolution aspect ratio

Part II: Potential temperature as a prognostic variable in IFS

Background: Integrated Forecast System (IFS) dynamical core

- The operational IFS is a **semi-Lagrangian, semi-implicit, hydrostatic** dynamical core. **Spectral** in the horizontal, **vertical finite elements** with hybrid eta-coordinate in the vertical.

$$\frac{D\mathbf{V}}{Dt} + f\mathbf{k} \times \mathbf{V} + \nabla_h \Phi + R_d T_V \nabla_h \ln p = P_V + K_V,$$

$$\frac{DT}{Dt} - \frac{\kappa T_V \omega}{[1 + (\delta - 1)q]p} = P_T + K_T,$$

$$\frac{\partial}{\partial t}\left(\frac{\partial p}{\partial \eta}\right) + \nabla_h \cdot \left(\mathbf{V}\frac{\partial p}{\partial \eta}\right) + \frac{\partial}{\partial \eta}\left(\dot{\eta}\frac{\partial p}{\partial \eta}\right) = 0,$$

$$\frac{Dq}{Dt} = P_q,$$

$$\frac{\partial \Phi}{\partial \eta} = -R_d T_V \frac{\partial \ln p}{\partial \eta},$$

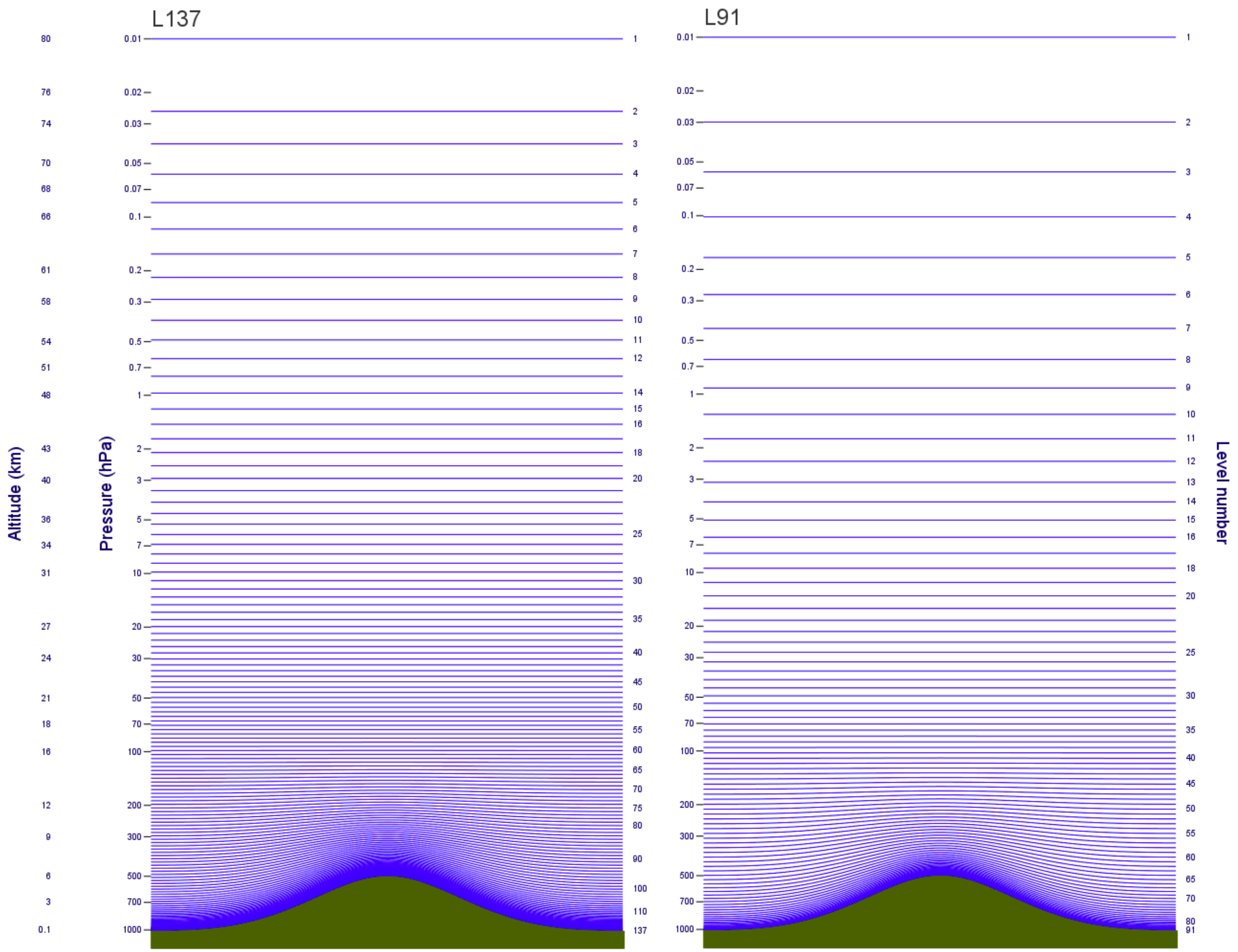
$$T_V = T\left[1 + \left(\frac{R}{R_d} - 1\right) q\right]$$

Background: Forecast configuration at ECMWF

- **Medium-range (10 days ahead):** High resolution forecasts at **TCo1279L137** resolution (9 km in the horizontal) + 51 member ensemble at **TCo639L91** resolution (18 km in the horizontal).

- **Extended-range (46 days ahead):** 51 member ensemble at TCo639L91 resolution for days 0-15, followed by **TCo319L91** (36 km in the horizontal) for days 16-46.

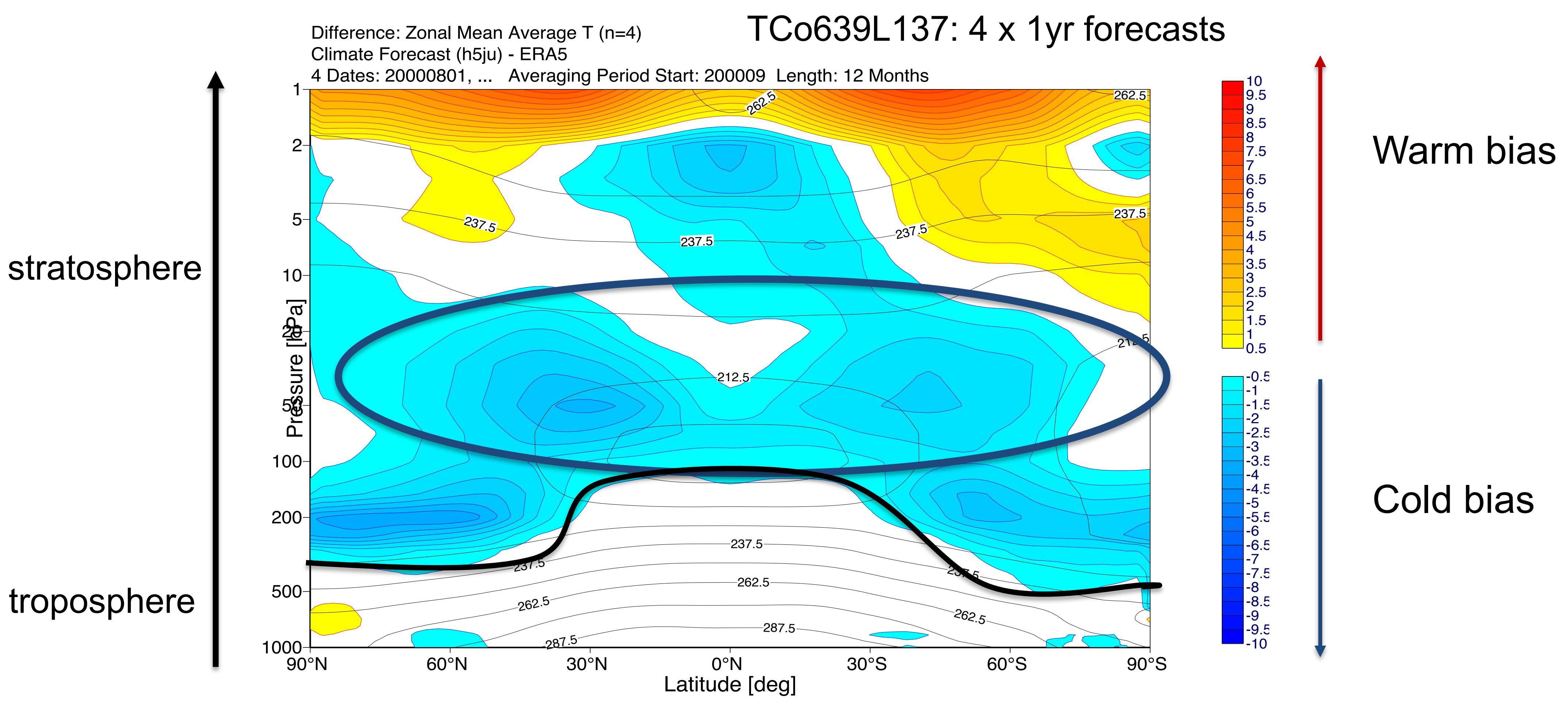
- **Seasonal (7 months ahead):** 51 member ensemble at TCo319L91 resolution.



Part I:
Horizontal to vertical resolution aspect ratio

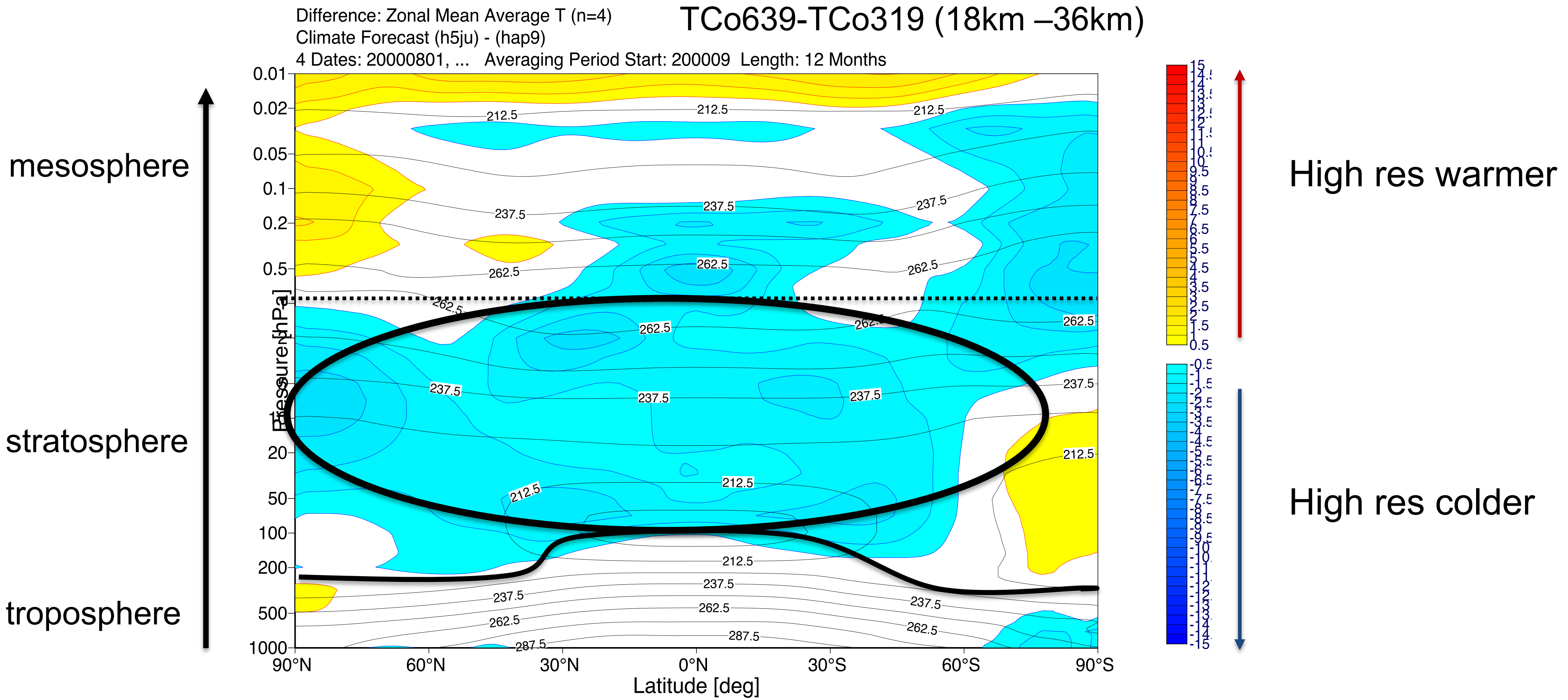
Stratosphere in the IFS: Temperature biases against ERA-5

- Cold mid- to lower stratosphere bias of ~3K. Worse at higher horizontal resolution due to the **cooling in the global mean** of the stratosphere in the **dynamical core** at higher horizontal resolution.



Stratosphere in the IFS: Horizontal resolution sensitivity

- **Question:** Does the cooling at higher horizontal resolution arise due to inadequate **horizontal to vertical resolution aspect ratio**? OR: Is the vertical resolution **too coarse**?



Theoretical considerations: Horizontal to vertical resolution aspect ratio

- Aspect ratio relevant for balanced **quasi-geostrophic** dynamics (Lindzen & Fox-Rabinovitz, 1989) and **inertia-gravity** wave dispersion relation

$$\frac{\Delta z}{\Delta x} \sim \frac{f}{N}$$

Δz ~ 200 m, for Δx ~18 km

- **Stratified turbulence** develops shear layer of thickness (e.g., Waite & Bartello, 2004)

$$L_b \equiv 2\pi U / N$$

In the stratosphere, L_b ~ 1 km, need 4-6 Δz to resolve → Δz ~ 200 m

- **Gravity wave propagation**:

Dispersion relation (medium-frequency)

$$| \lambda_z | \sim | c - U | / N$$

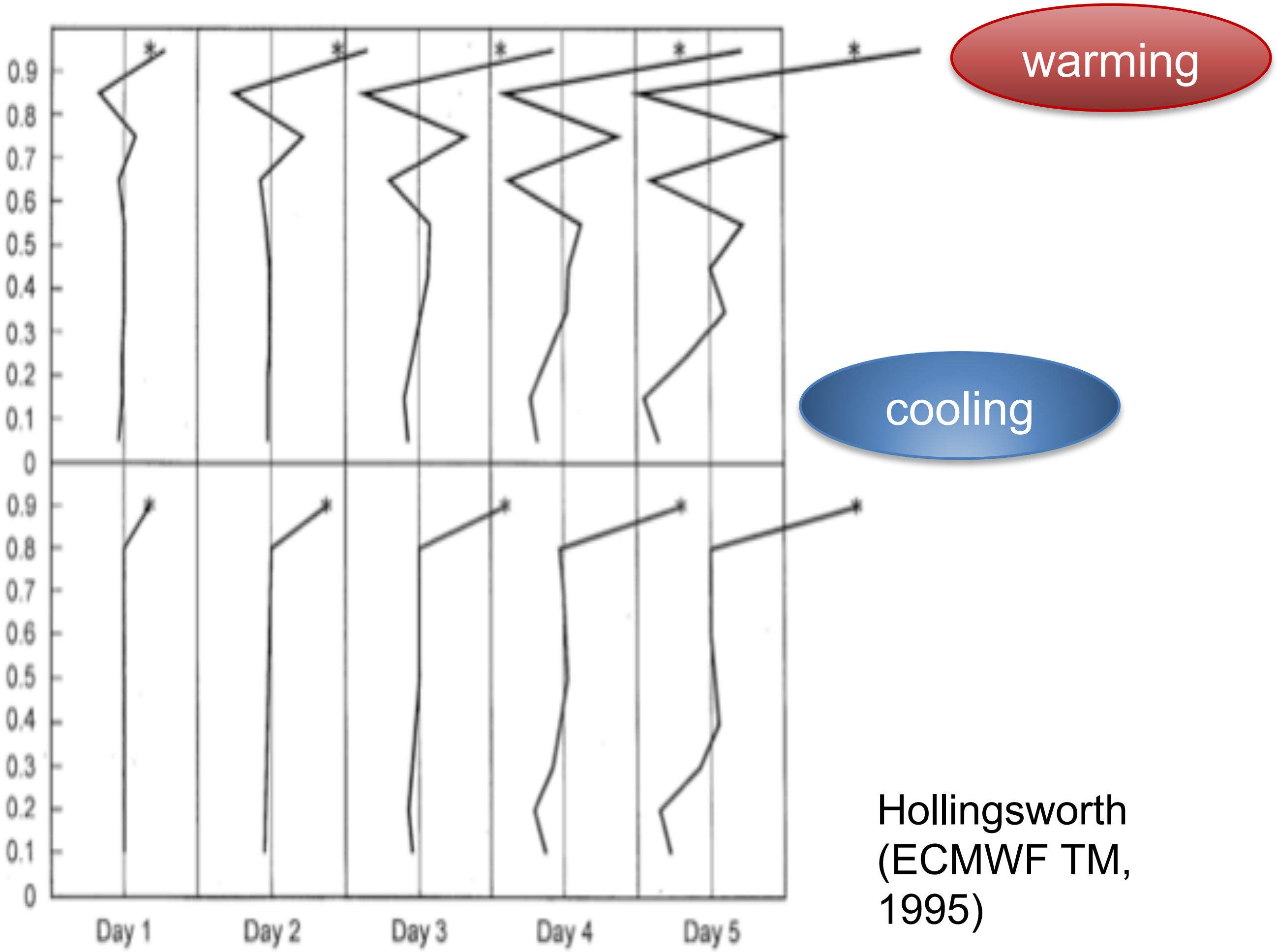
If vertical resolution not adequate to resolve λ_z, discretization errors occur

Question: Why insufficient vertical resolution leads to a thermal response?

Historical background: Vertical discretization in the IFS prior to 2003 was **finite difference** with **Lorenz staggering**. Lorenz staggering permits $2\Delta z$ noise **in temperature.** If excited, can generate spurious cooling/heating.

Lorenz staggering produces spurious heating/cooling, due to $2\Delta z$ noise

Charney-Phillips staggering, which eliminates $2\Delta z$ noise, produces correct response

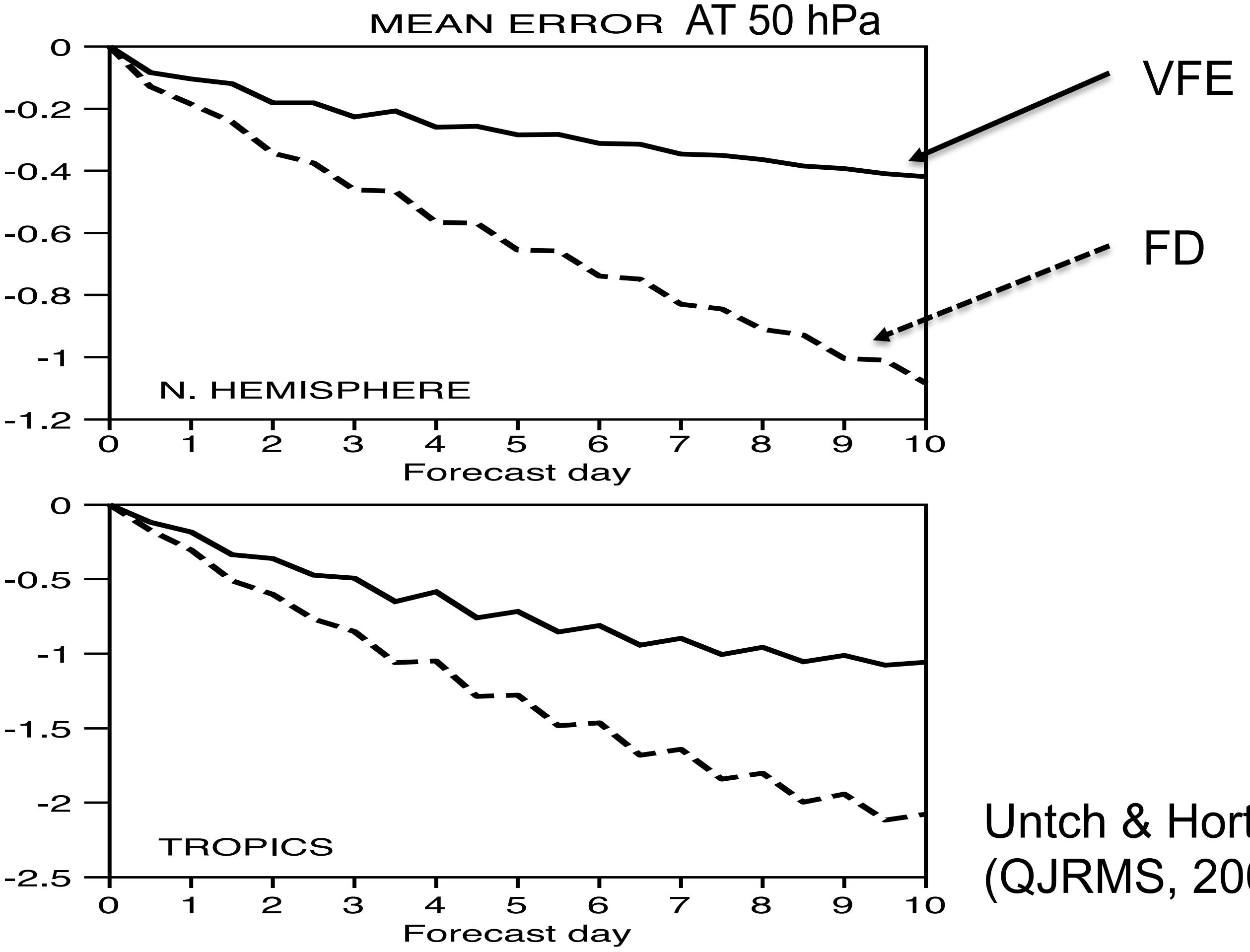


Hollingsworth (ECMWF TM, 1995)

Question: Why insufficient vertical resolution leads to thermal response?

Historical background: From 2003 IFS uses vertical finite element vertical discretization, which **damps** the $2\Delta z$ noise, but **does not fully eliminate** it (Untch & Hortal, 2004) → Aliasing of vertically unresolved waves onto the $2\Delta z$ noise can still generate spurious thermal response.

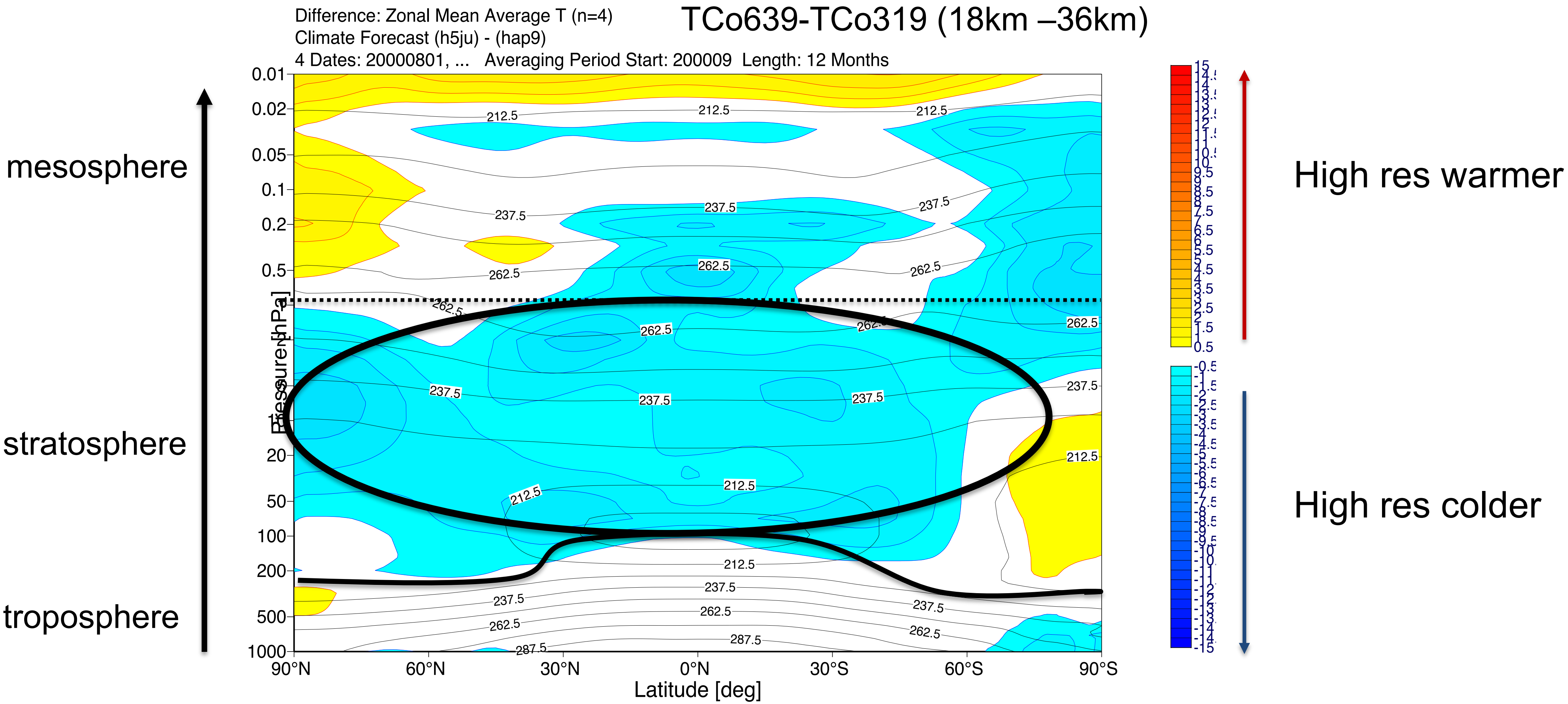
- Damping the computational mode via VFE reduced spurious global-mean cooling in the lower to mid stratosphere in IFS in comparison to FD with Lorenz staggering.



Untch & Hortal (QJRMS, 2004)

Stratosphere in the IFS: Horizontal resolution sensitivity

- **Question:** Does the cooling at higher horizontal resolution arise due to inadequate **horizontal to vertical resolution aspect ratio**? Is the vertical resolution too coarse?



Stratosphere in the IFS: Horizontal to vertical resolution aspect ratio

- Perform **10-day forecast** experiments at **high (9 km)** and **low (80 km)** horizontal resolution and gradually increasing **vertical resolution** in the lower to mid-stratosphere.

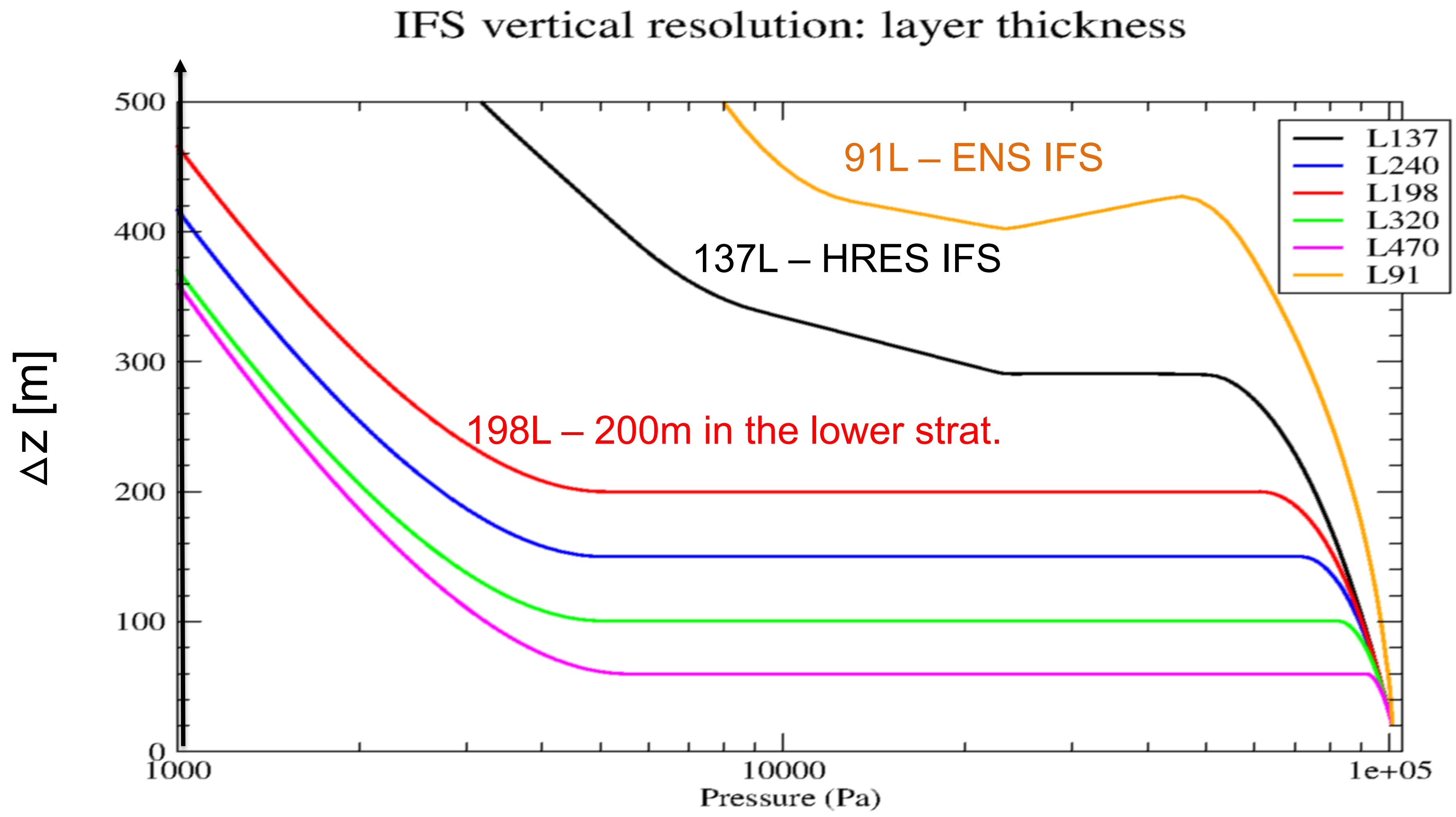
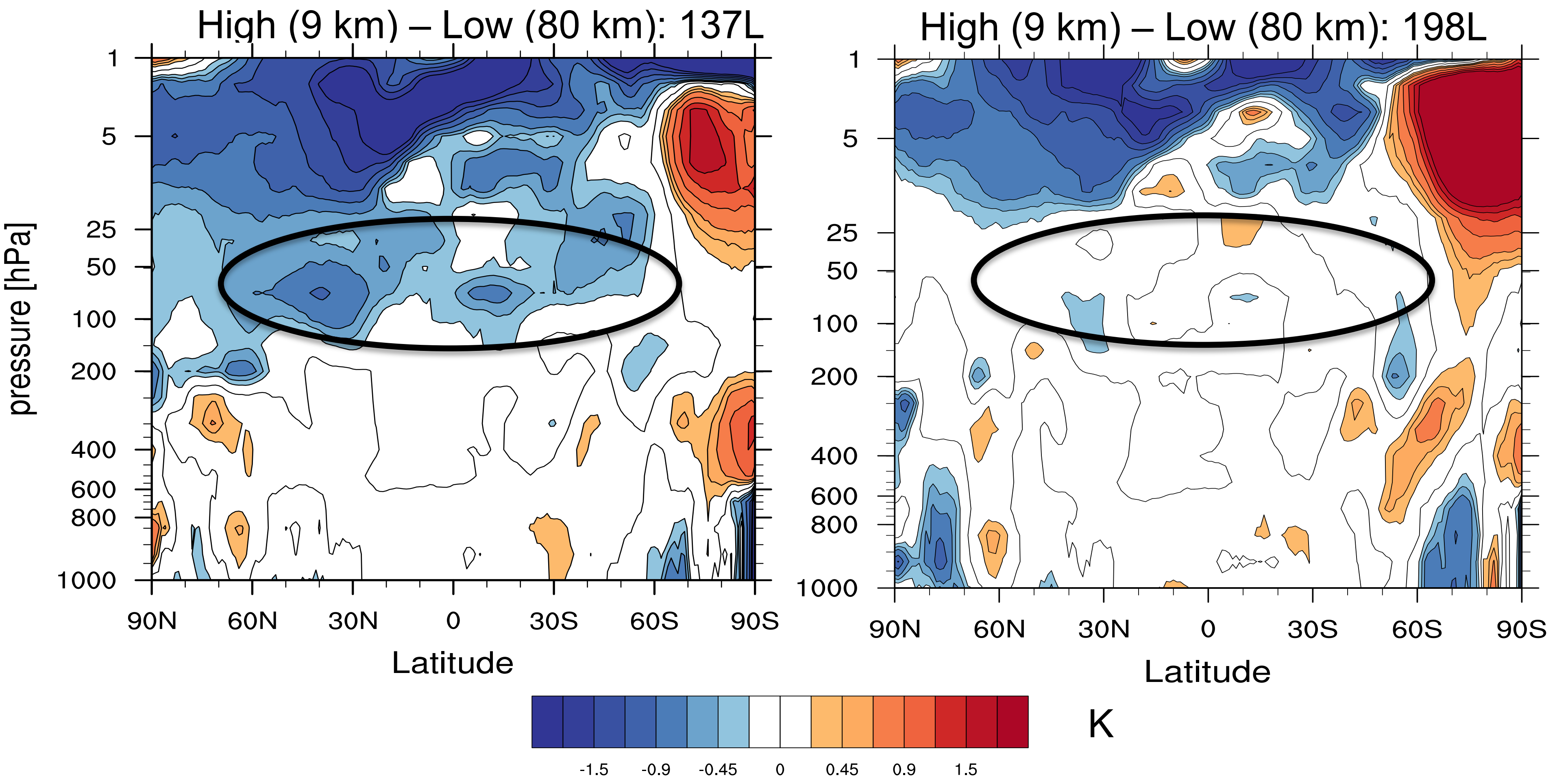


Figure courtesy:
Tim Stockdale

Stratosphere in the IFS: Horizontal to vertical resolution aspect ratio

- **Vertical resolution** of **200** m in the lower to mid stratosphere eliminates global mean cooling there at high horizontal resolution.

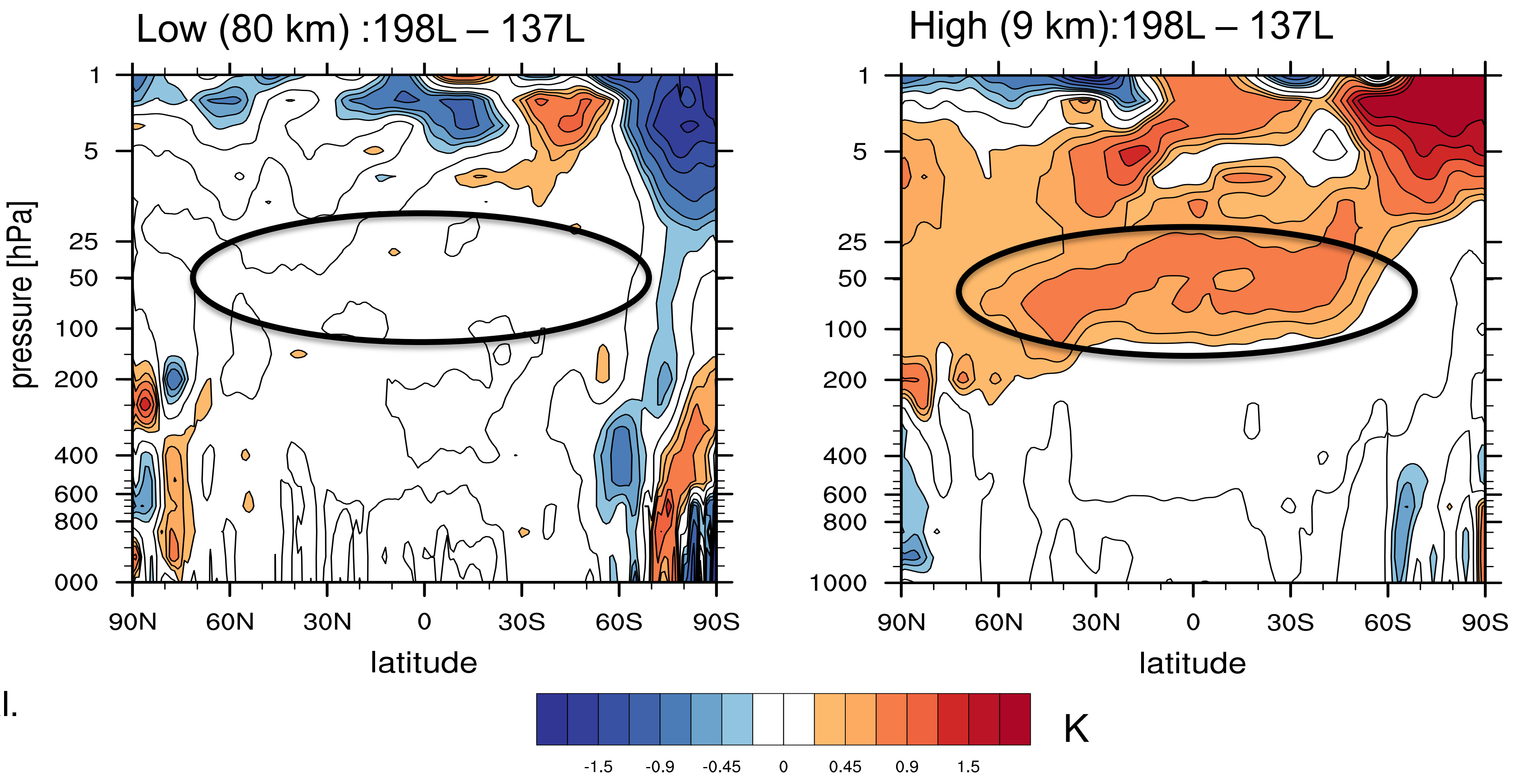
Average over 31 forecasts in July at 10-day lead time.



Polichtchouk et al. (2019, TM)

Stratosphere in the IFS: Horizontal to vertical resolution aspect ratio

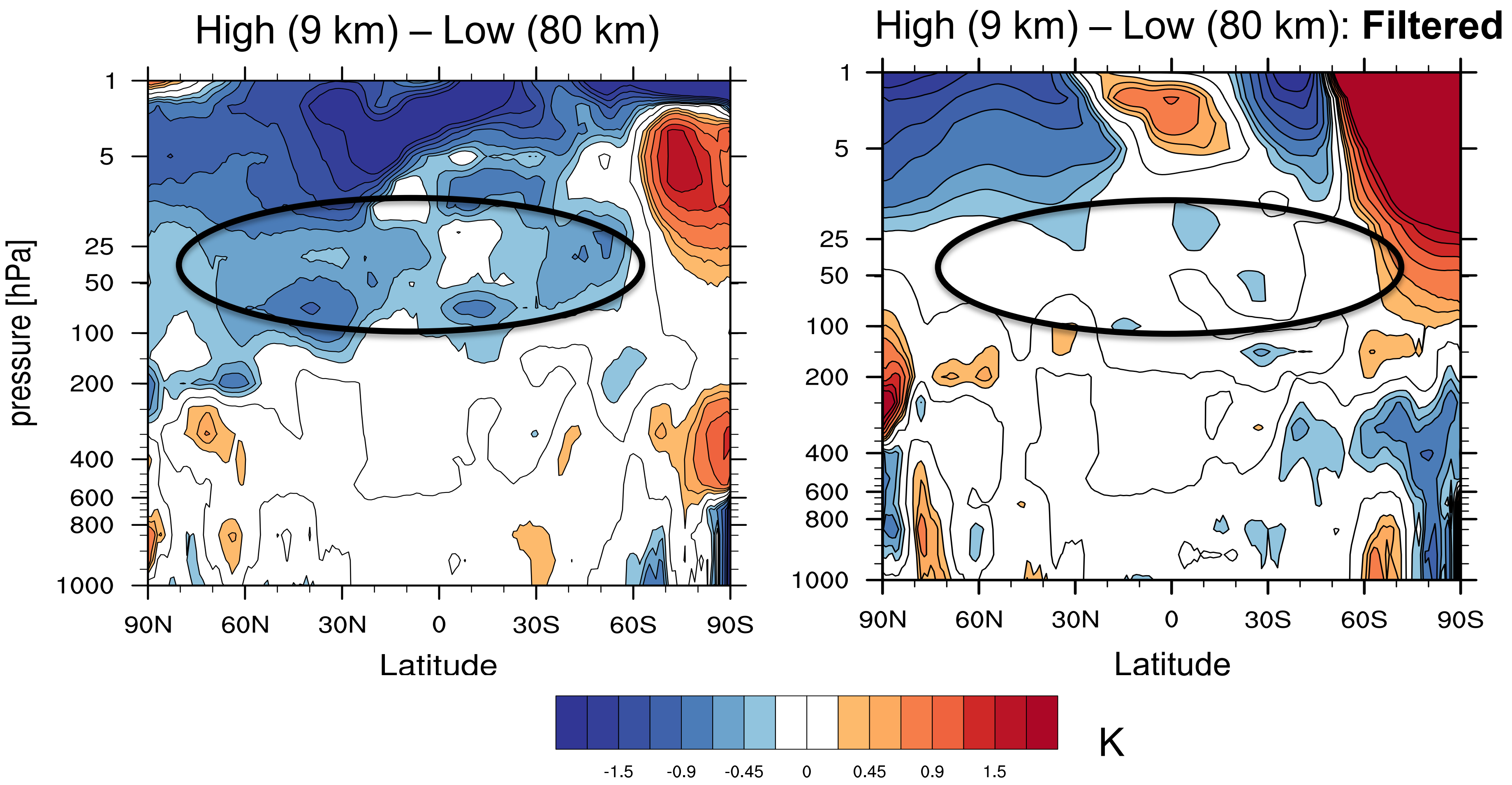
- Increase in the **vertical resolution** leads to warming in the stratosphere at **high horizontal resolution. No impact at low horizontal resolution**.



Polichtchouk et al. (2019, TM)

Question: Which horizontal scales contribute to the global mean cooling?

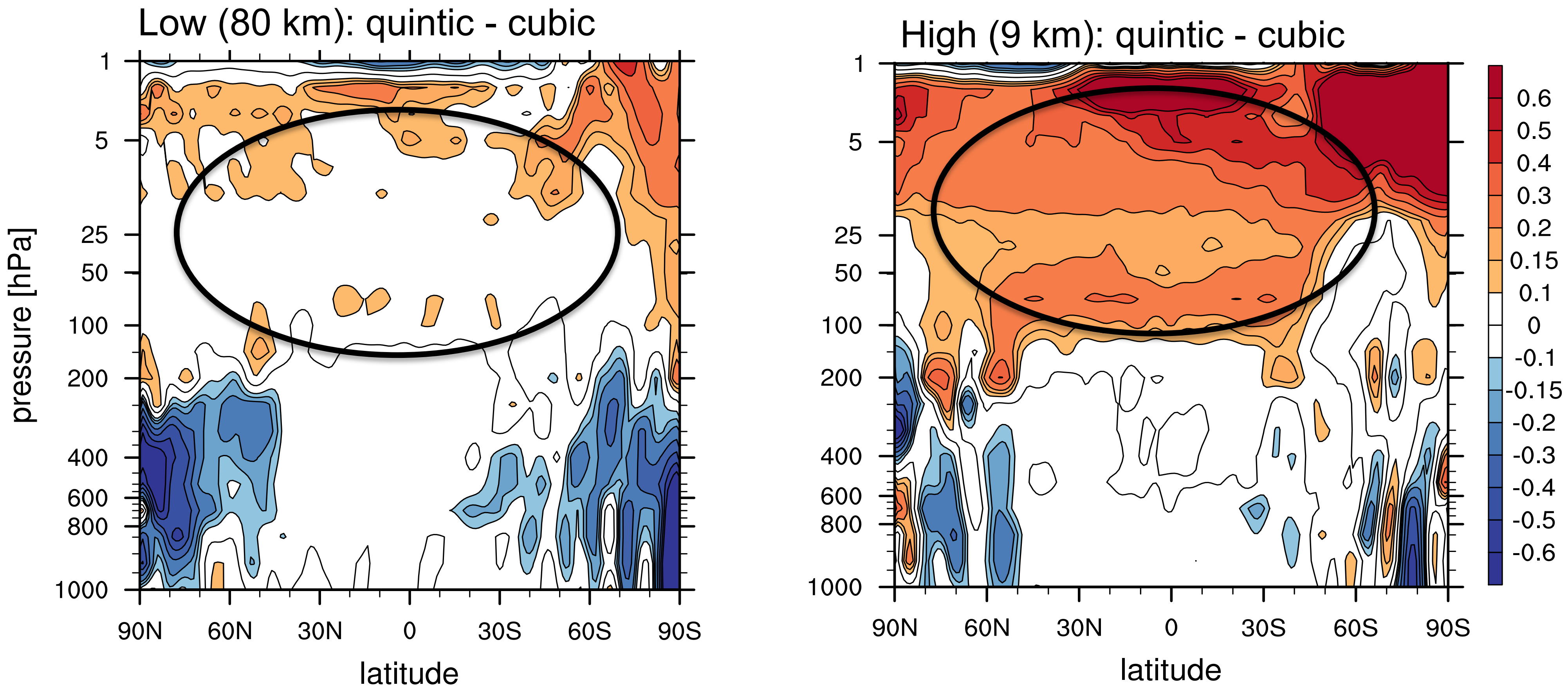
- **Filtering gravity** waves out with **horizontal wavelengths 100-1000 km** reduces global mean cooling at high horizontal resolution in the mid- to lower stratosphere.
- Such scales are well resolved at 18-9 km horizontal resolution → **200 m** vertical resolution should be enough for horizontal resolutions finer than18 km.



Polichtchouk et al. (2019, TM)

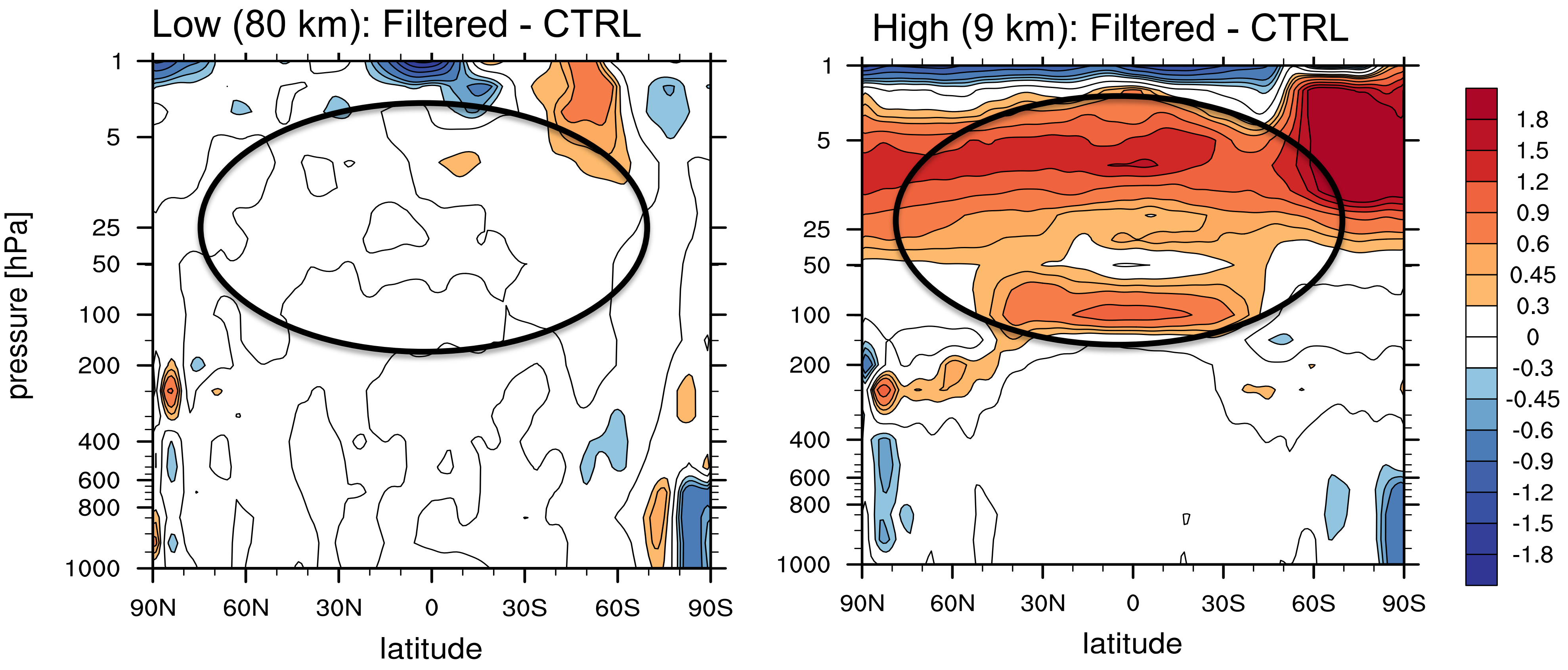
Higher order vertical Semi-Lagrangian interpolation

- Increasing vertical resolution is expensive → seek cheaper solutions to solve the problem.
- **Question:** Does improving the accuracy of the vertical semi-Lagrangian interpolation help?
- Going from **3rd** to **5th** order vertical interpolation helps → Stratosphere warms with higher order interpolation at high horizontal resolution.



Filtering $2\Delta z$ noise during semi-Lagrangian interpolation

- **Question:** Does **filtering $2\Delta z$**-noise in temperature via semi-Lagrangian vertical filter help horizontal resolution sensitivity?
- Filtering warms high horizontal resolution forecasts, **no impact** on **low horizontal resolution**.



Impact of alleviating the cold bias on the tropospheric skill

- **Quintic vertical interpolation** applied on temperature and specific humidity alleviates the bias.

- Alleviating the bias has **no statistically significant impact on extended range** forecast skill scores (RPSS) in the Northern Hemisphere or on **medium-range** skill scores in the troposphere.

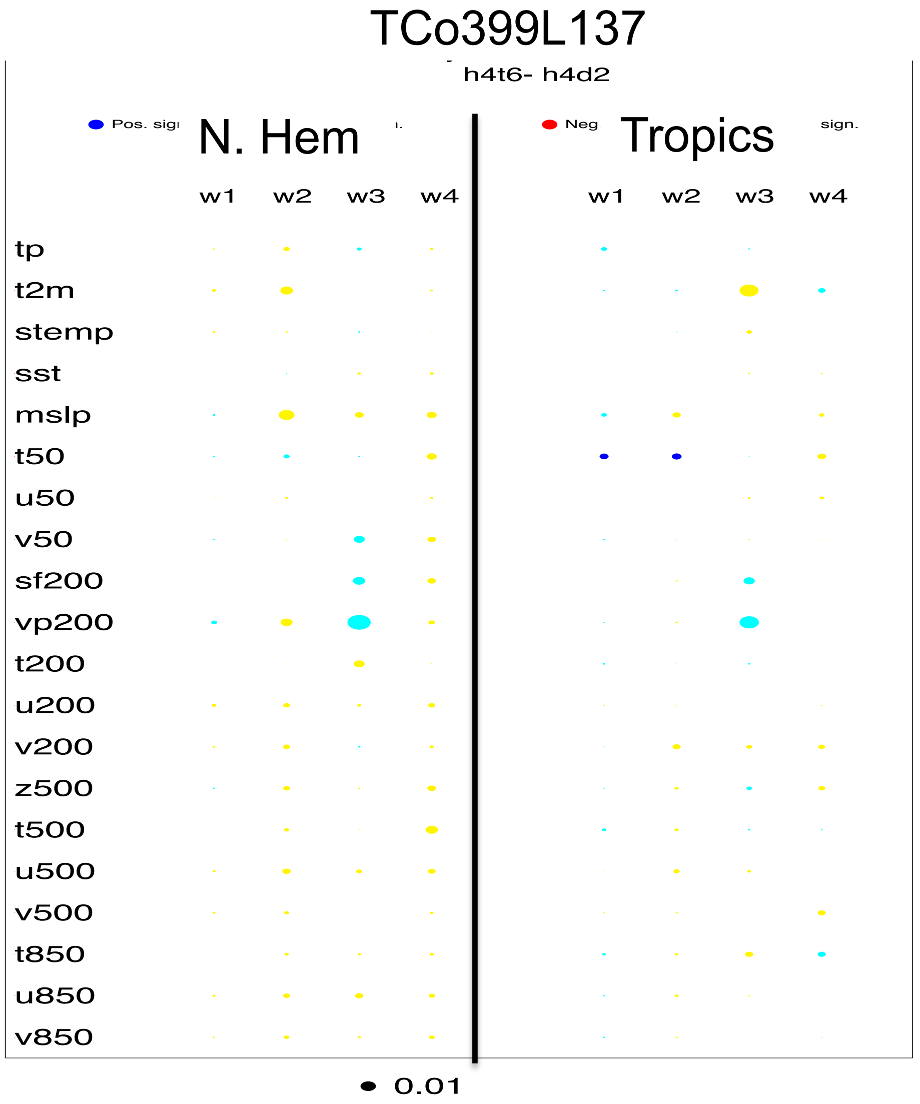


Figure courtesy: Frederic Vitart

Part I: Summary

- At **higher horizontal resolution**, IFS dynamical core **cools in the global mean** in the stratosphere → cold bias in the 150-50hPa region worse. Affects all forecast ranges.

- Behaviour linked to **2Δz-noise** in the vertical arising from **inconsistent vertical to horizontal resolution aspect ratio** and inability to resolve gravity waves in the vertical.

- Increasing the **vertical resolution** to **200 m** eliminates the global mean cooling at **higher horizontal resolution**.

- **Filtering out 2Δz-noise** or increasing the **order of vertical SL interpolation** also alleviate the global mean cooling at **high horizontal resolution**.

- Alleviating the bias via **quintic vertical interpolation** or modest vertical resolution increase has **no statistically significant** impact on tropospheric skill scores.

Part II:
Potential temperature as a prognostic variable in IFS dynamical core

IFS: Temperature and potential temperature formulation

- **Question:** Can using the **potential temperature** $\boldsymbol{\theta = T\left(\frac{p_0}{p}\right)^{\kappa}}$ instead of **temperature** $\boldsymbol{T}$ as a prognostic variable improve the temperature biases and the behavior of the IFS?

Original, T-formulation

$$\frac{D\mathbf{V}}{Dt} + f\mathbf{k}\times\mathbf{V} + \nabla_h\Phi + R_d T_V \nabla_h \ln p = P_V + K_V,$$

$$\frac{DT}{Dt} - \frac{\kappa T_V \omega}{[1+(\delta-1)q]p} = P_T + K_T,$$

$$\frac{\partial}{\partial t}\left(\frac{\partial p}{\partial \eta}\right) + \nabla_h \cdot \left(\mathbf{V}\frac{\partial p}{\partial \eta}\right) + \frac{\partial}{\partial \eta}\left(\dot{\eta}\frac{\partial p}{\partial \eta}\right) = 0,$$

$$\frac{Dq}{Dt} = P_q,$$

$$\frac{\partial \Phi}{\partial \eta} = -R_d T_V \frac{\partial \ln p}{\partial \eta},$$

$$T_V = T\left[1 + \left(\frac{R}{R_d} - 1\right) q\right]$$

$\boldsymbol{\theta}$-formulation

$$\frac{D\mathbf{V}}{Dt} + f\mathbf{k}\times\mathbf{V} + \nabla_h\Phi + R_d \theta_v \left(\frac{p}{p_0}\right)^{\kappa} \nabla_h \ln p = P_V + K_V,$$

$$\frac{D\theta}{Dt} = P_T + K_T,$$

$$\frac{\partial}{\partial t}\left(\frac{\partial p}{\partial \eta}\right) + \nabla_h \cdot \left(\mathbf{V}\frac{\partial p}{\partial \eta}\right) + \frac{\partial}{\partial \eta}\left(\dot{\eta}\frac{\partial p}{\partial \eta}\right) = 0,$$

$$\frac{Dq}{Dt} = P_q,$$

$$\frac{\partial \Phi}{\partial \eta} = -R_d \theta_v \left(\frac{p}{p_0}\right)^{\kappa} \frac{\partial \ln p}{\partial \eta},$$

$$\theta_v = \theta\left[1 + \left(\frac{R}{R_d} - 1\right) q\right]$$

Potential temperature as a prognostic variable in IFS

- **Benefits of θ-formulation**:
  - Thermodynamic equation is simpler. No non-linear 'Tw' term.
  - **Potential temperature is** materially **conserved** in adiabatic scenario. **Temperature** is **not**.

- **Note:** In practice, the semi-Lagrangian advection in the T-formulation is performed on T-T_b with:

$$T_\mathrm{b} = -\left(p_\mathrm{s}\frac{\partial p}{\partial p_\mathrm{s}}\frac{\partial T}{\partial p}\right)_\mathrm{ref} \cdot \phi_\mathrm{s}/(R_\mathrm{dry}\bar{T})$$

  This means that the advected thermodynamic variable is essentially **independent of orography**. A compensating expression is then added to the right-hand side:

$$-\underset{\sim}{\mathbf{v}}_\mathrm{H}\cdot\nabla T_\mathrm{b} - \dot{\eta}\frac{\partial T_\mathrm{b}}{\partial \eta}$$

  This procedure is redundant in θ-formulation, because θ naturally follows orography.

Potential temperature as a prognostic variable: Semi-implicit formulation

- The **semi-implicit** formulation for $\boldsymbol{\theta}$ as a prognostic variable can be achieved by considering a perturbation $\boldsymbol{\theta}$ from a pressure dependent basic state θ_0 (p). Details can be found in Polichtchouk, Malardel & Diamantakis (ECMWF TM, 2020)

$$\frac{\partial D}{\partial t} = -\nabla_h^2(\boldsymbol{\gamma}\theta + \boldsymbol{\mu} \ln p_s)$$

$$\frac{\partial \theta}{\partial t} = -\boldsymbol{\tau} D,$$

$$\frac{\partial \ln p_s}{\partial t} = -\boldsymbol{\nu} D.$$

$$\boldsymbol{\gamma}(\theta) = \frac{R_d}{p_{oo}^{\kappa}} \int_\eta^1 \theta \frac{(p^R)^\kappa}{p^R} m_R^* d\eta, \Longrightarrow \boldsymbol{\gamma}_{OPER}(T) \equiv R \int_\eta^1 T \frac{d(\ln p^R)}{d\eta} d\eta$$

$$\boldsymbol{\mu}(\ln p_s) = \ln p_s R_d T^R,$$

$$\boldsymbol{\tau}(D) = \frac{\partial \theta_0}{\partial p} \int_\eta^0 D \frac{dp^R}{d\eta} d\eta, \Longrightarrow \boldsymbol{\tau}_{OPER}(D) \equiv \frac{\kappa T^R}{p^R} \int_0^\eta D \frac{\partial p^R}{d\eta} d\eta,$$

$$\boldsymbol{\nu}(D) = \frac{1}{p_s^R} \int_0^1 D \frac{dp^R}{d\eta} d\eta.$$

$$\frac{\partial^2 D}{\partial t^2} - \nabla_h^2 \boldsymbol{\Gamma}(D) = 0,$$

$$\boldsymbol{\Gamma} \equiv \boldsymbol{\gamma\tau} + \boldsymbol{\mu\nu}.$$

IFS: Temperature and potential temperature formulation

- **Question:** Can using the **potential temperature** $\boldsymbol{\theta = T\left(\frac{p_0}{p}\right)^{\kappa}}$ instead of **temperature** ***T*** as a prognostic variable improve the temperature biases and the behavior of the IFS?

- To answer this question, perform several **simulations in increasing complexity** with θ-formulation and *T*-formulation:

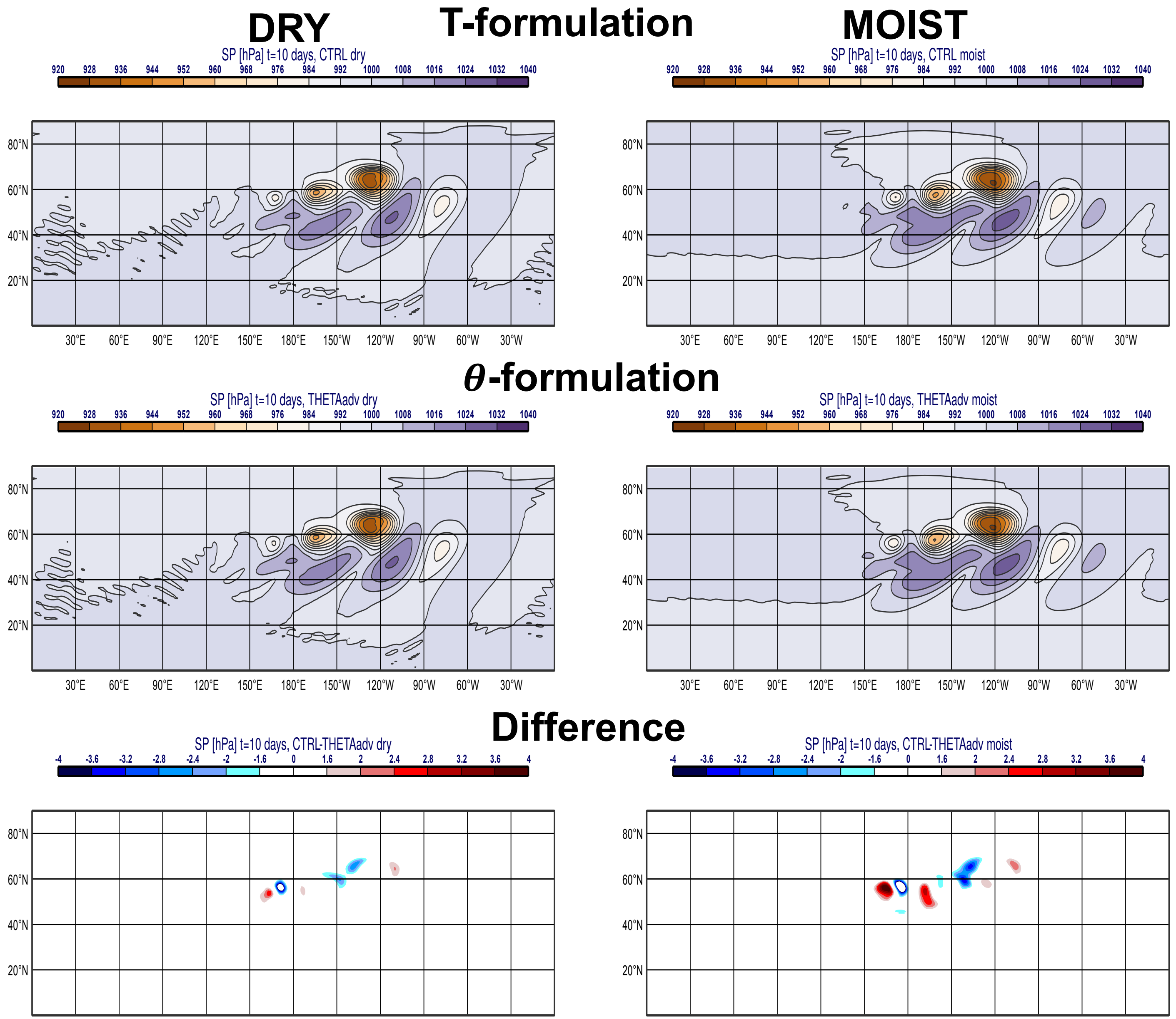
  1. Dry and moist **baroclinic instability** adiabatic test cases (see Ullrich et al, 2012)

  2. Uniform 10m/s **flow over idealized Schär mountain** in an isothermal atmosphere and on a small planet test case

  3. Full **complexity** simulations with physical parametrizations. Longer 1-year forecasts and medium range forecasts are performed at varying horizontal resolutions.

- Physical parametrizations in the θ-formulation are not changed and use temperature as input. This means that $\boldsymbol{\theta}$→T conversion is performed before the call to physics and T→$\boldsymbol{\theta}$ conversion after physics.

Results: Dry and moist baroclinic instability idealized, adiabatic test cases



Surface pressure at t=10 days for the dry (left) and moist (right) baroclinic instability.

Both formulations **very similar.**

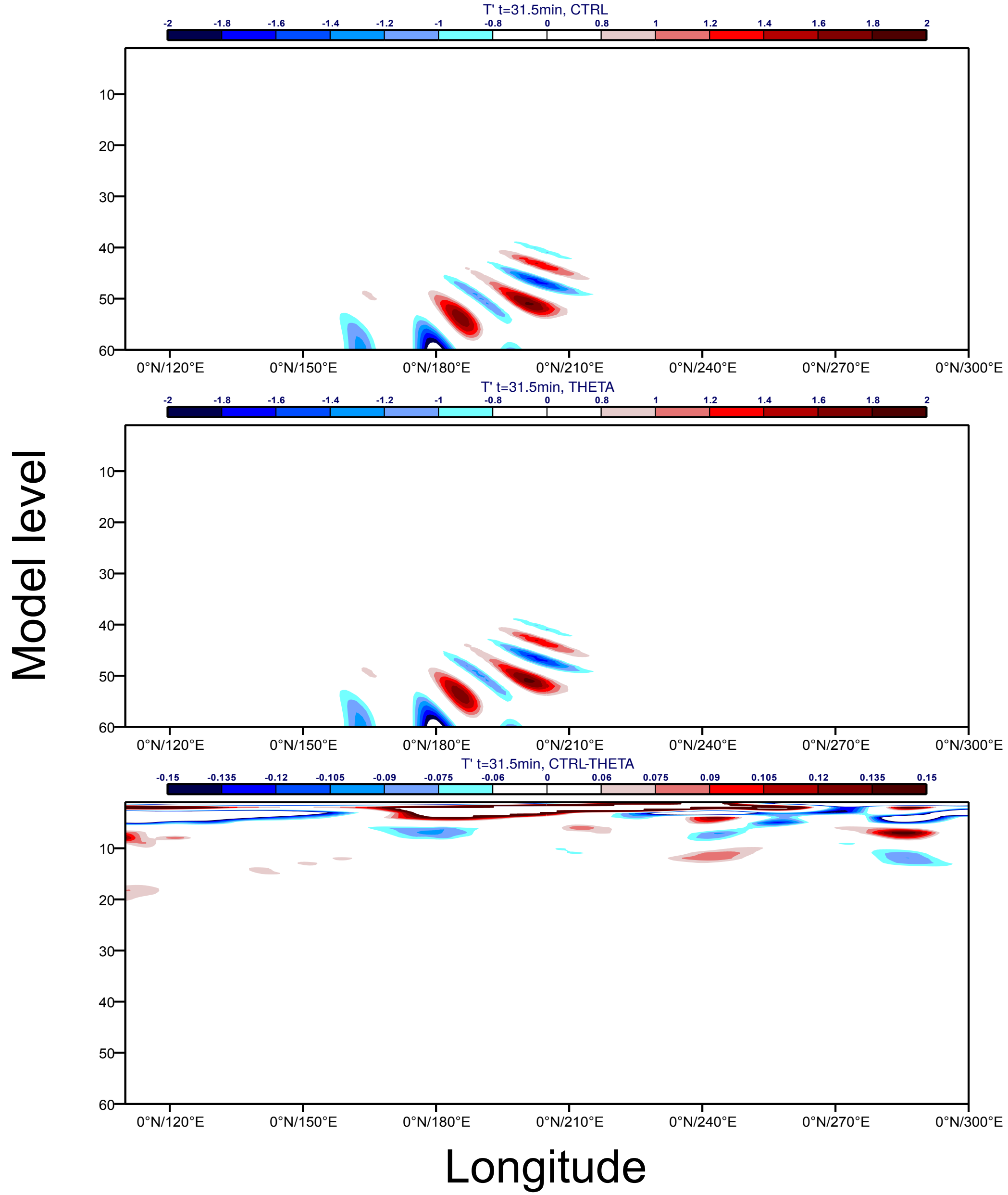
Polichtchouk, Malardel & Diamantakis (2019, ECMWF TM)

Results: Flow over idealized topography test case

T-formulation

***θ*-formulation**

Difference
(CI 14x smaller)



Temperature perturbation at the equator at t=32 mins in the flow over topography test case.

Both formulations **very similar**.

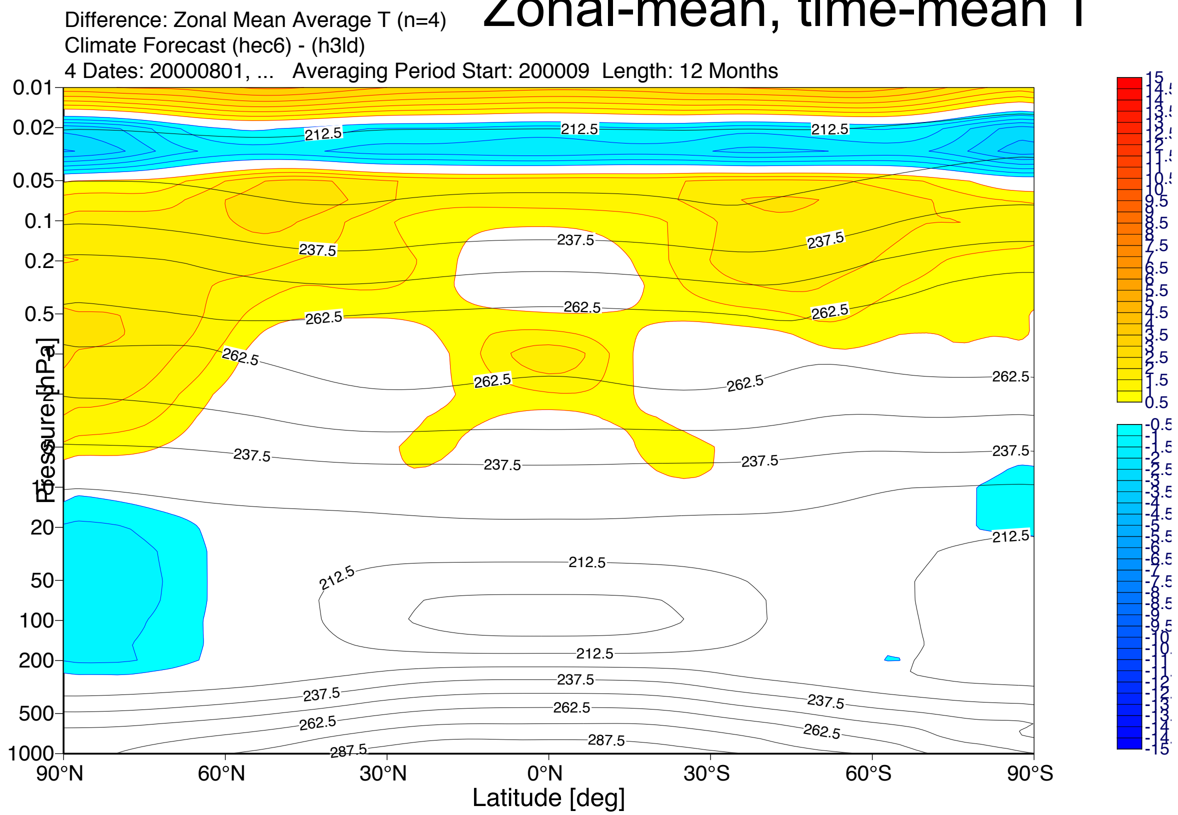
Polichtchouk, Malardel & Diamantakis (2019, ECMWF TM)

Results: Full complexity four 1-year forecasts at TL255L137 resolution

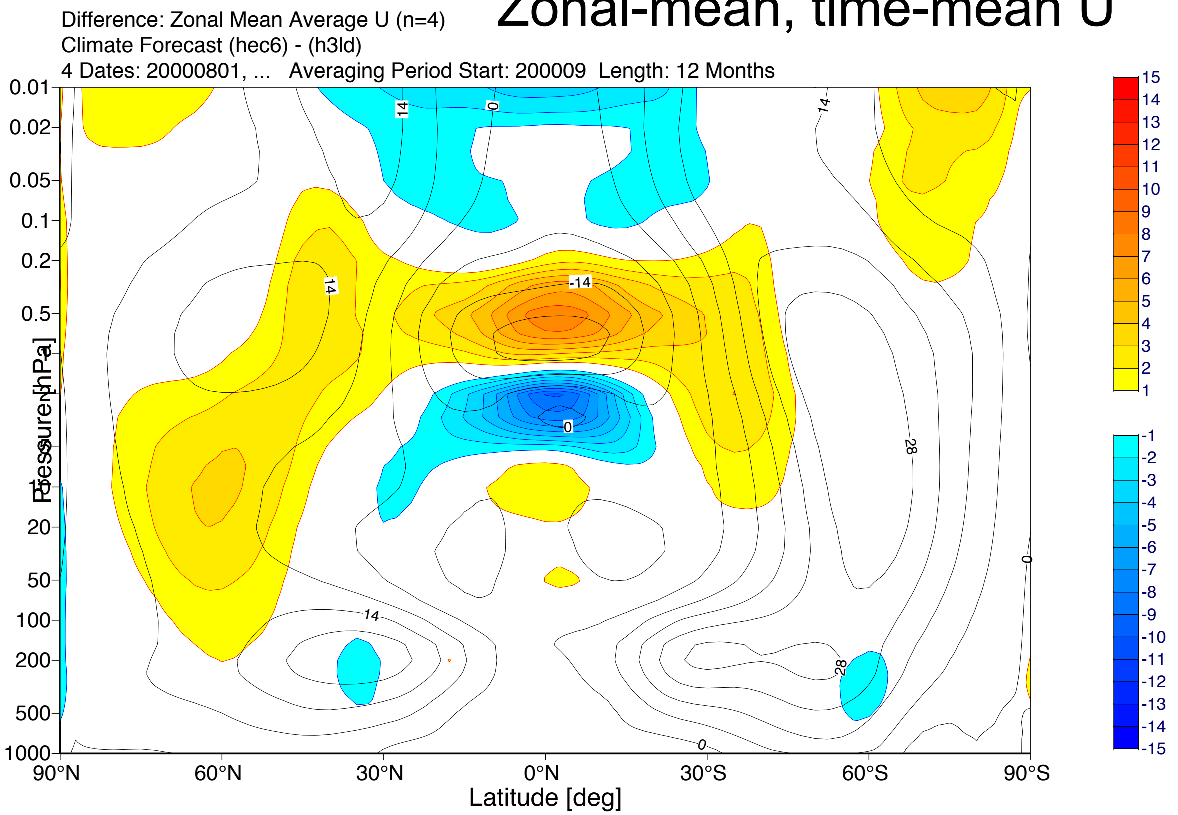
- Both formulations produce **very similar** zonal-mean climate in the troposphere and stratosphere. Some global-mean warming signal evident in the θ-formulation in the mesosphere.

θ-formulation minus T-formulation

Zonal-mean, time-mean T



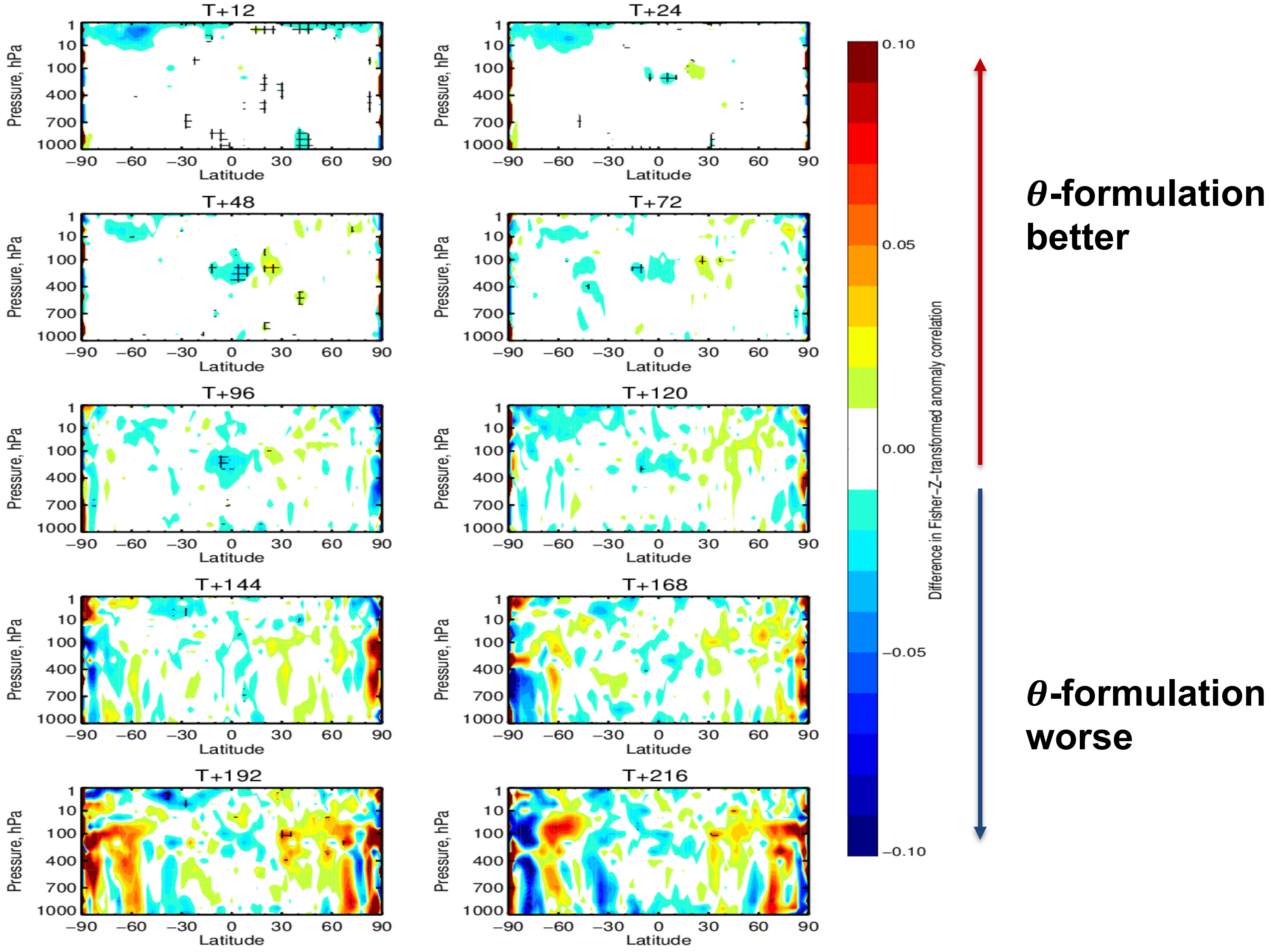
Zonal-mean, time-mean U



Results: Full complexity medium-range weather forecasts

- **No clear signal** in the medium-range forecast scores at either high horizontal resolution (TCo1279 or 9 km) or low horizontal resolution (TCo319 or 36 km).
- Currently **no benefit identified** from moving to potential temperature formulation.

Change in anomaly correlation for T: TCo1279 fcsts in December



Part II: Summary

- IFS **semi-implicit, semi-Lagrangian, hydrostatic** dynamical core re-written to use **potential temperature** instead of temperature as a prognostic variable.

- Under a series of tests in increasing complexity, the potential formulation was shown to perform **very comparably** to the temperature formulation → **no apparent benefit** of adapting potential temperature as a prognostic variable seen as yet. Likely due to the IFS is already formulated to advect temperature, which is essentially independent of orography.

- **Temperature biases** in the stratosphere **not** due to the use of **temperature instead of potential temperature** as a prognostic variable.

- **Horizontal resolution sensitivity** of global-mean temperature **not alleviated** by using potential temperature as a prognostic variable.

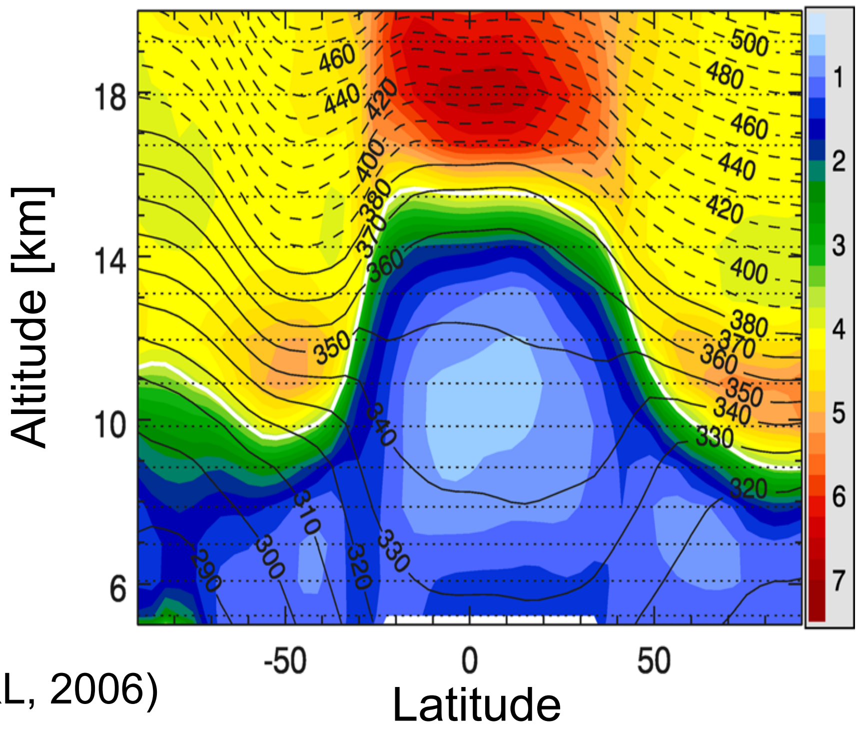
Theoretical considerations: Horizontal to vertical resolution aspect ratio

- Spatial distribution of resolution sensitivity likely a result of background conditions. Large **increase in N** and small changes to **U** in the tropical lower stratosphere.

$$|\lambda_z| \sim |c-U|/N$$

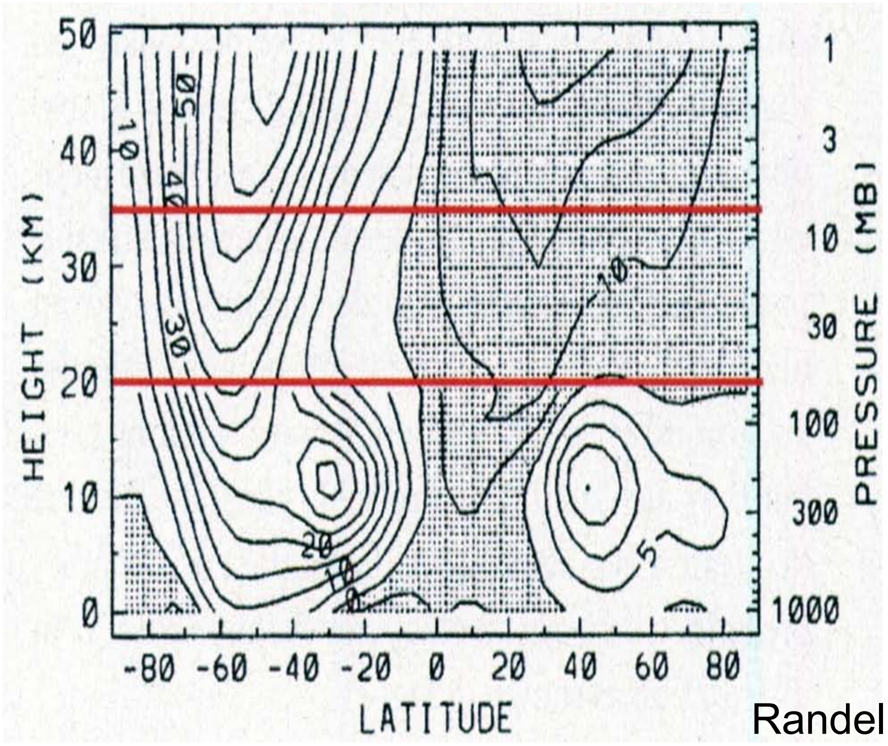
Increase in N → decrease in λ_z
Increase in U → increase in λ_z

N², climatology for July



Birner (GRL, 2006)

U, climatology for July



Randel (NCAR TM,1992)

Question: Which horizontal scales contribute to the global mean cooling?

- At higher horizontal resolution, more energy in the **divergent** part of the spectrum → more of the **gravity wave** spectrum is resolved.