Canadä

Sea ice rheology and numerical solvers for sea ice dynamics

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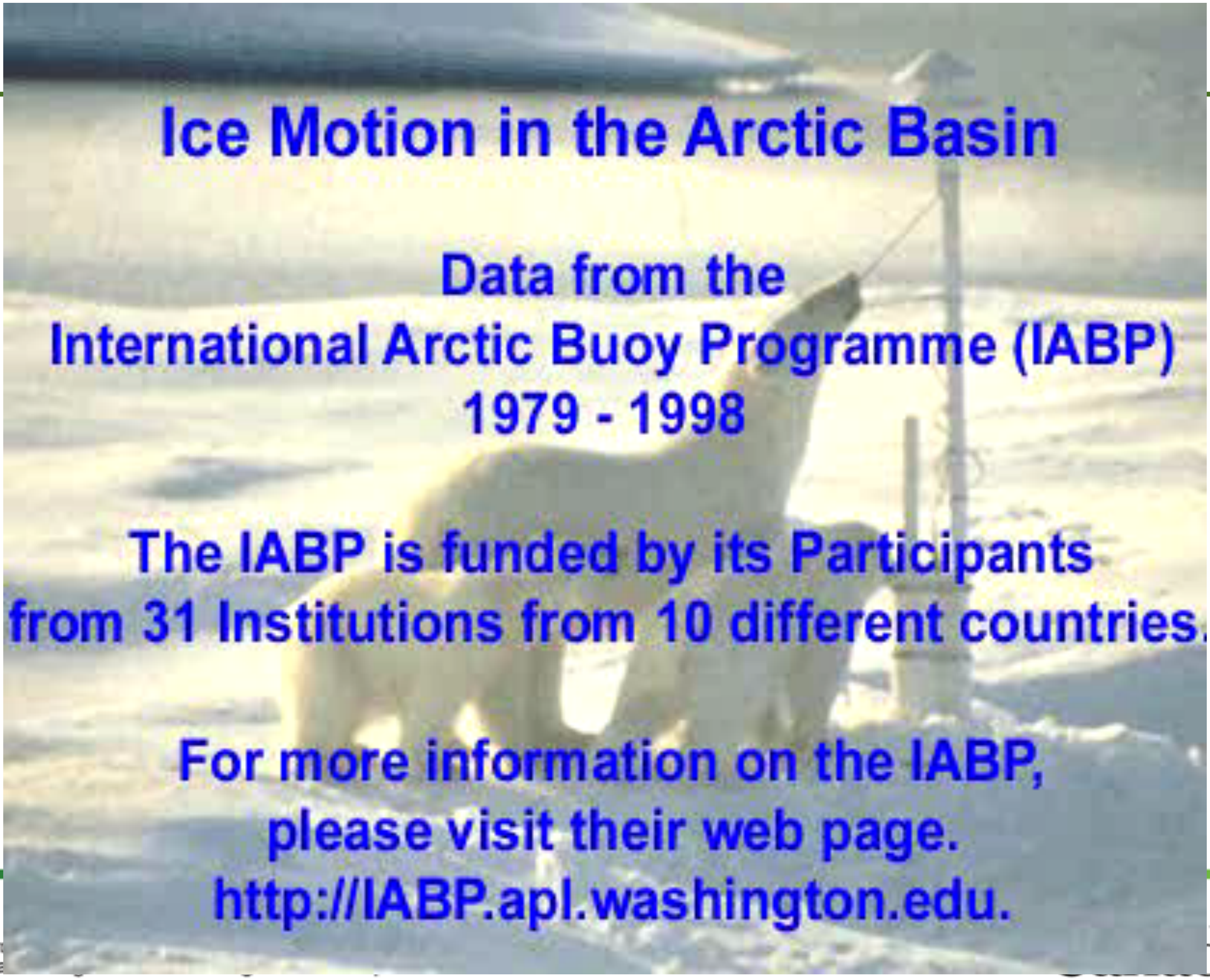
Outline

- Introduction on sea ice dynamics
- The importance of sea ice rheology
- The viscous-plastic (VP) rheology
- Current numerical solvers for sea ice dynamics
- Critics of the VP model and new approaches
- Conclusions



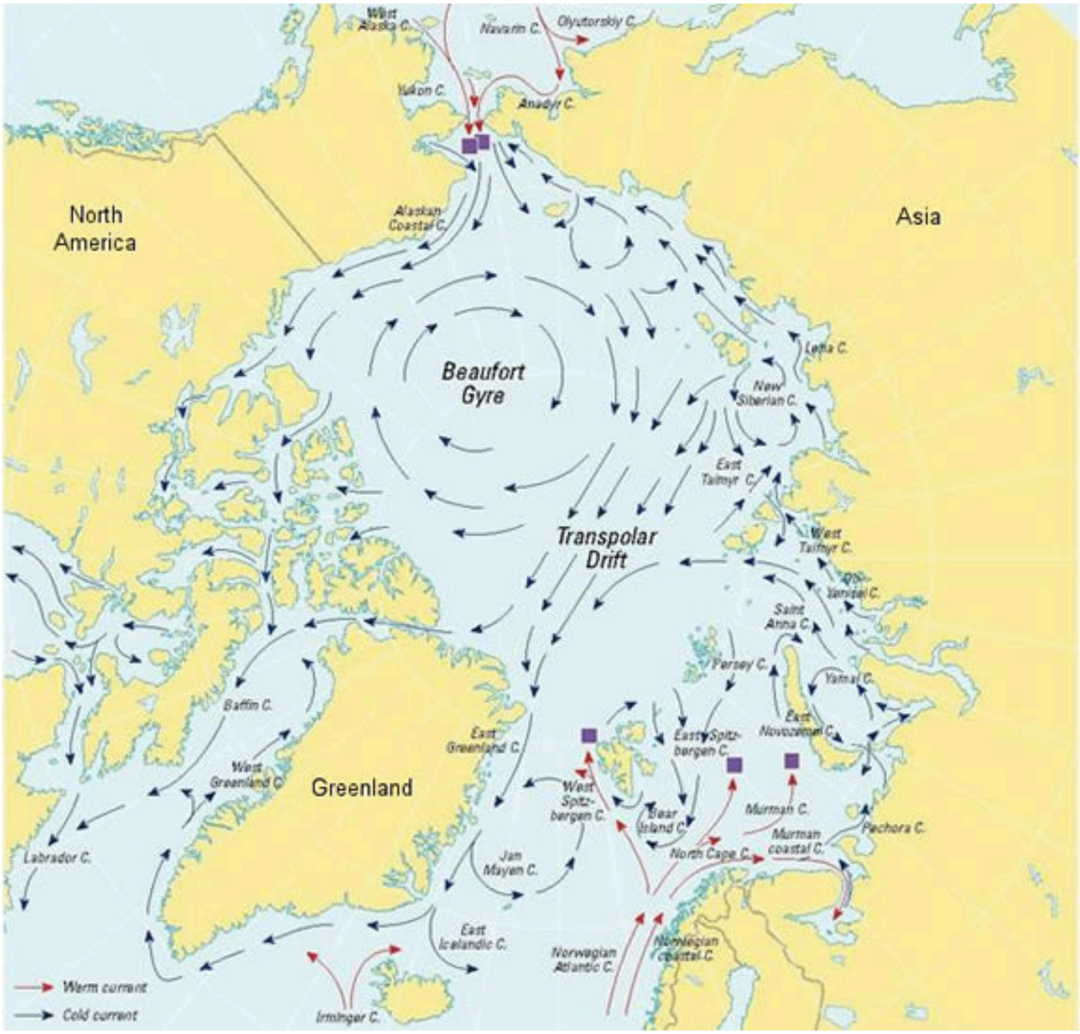
Ice Motion in the Arctic Basin

Data from the
International Arctic Buoy Programme (IABP)
1979 - 1998

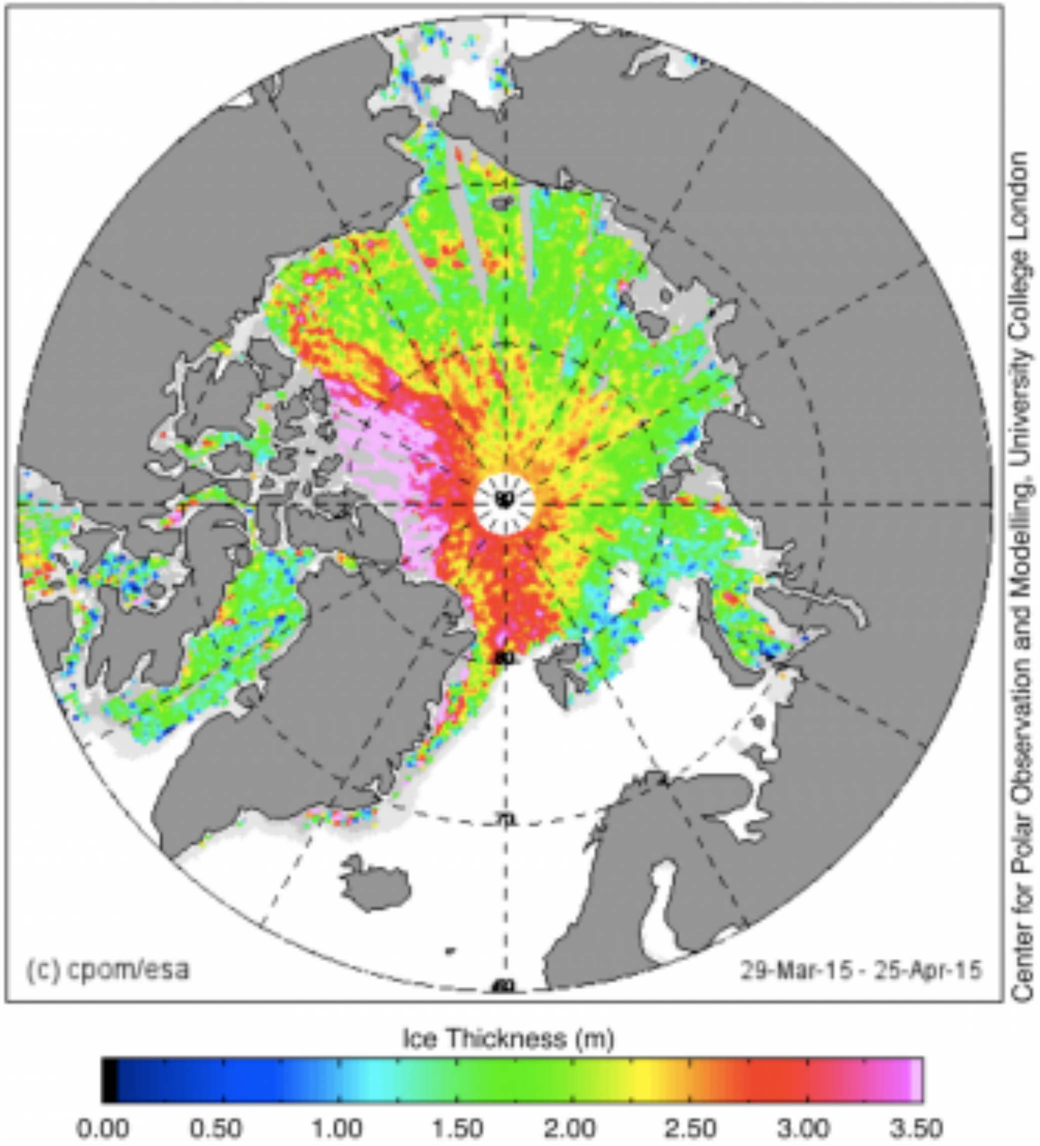
The IABP is funded by its Participants from 31 Institutions from 10 different countries.

For more information on the IABP, please visit their web page.
http://IABP.apl.washington.edu.

Main features of Arctic sea ice drift



Arctic Sea Ice Thickness



Sea ice dynamics

- Strongly impact the local and geophysical distributions of sea ice thickness.
- It is not only velocity that matters but its spatial derivatives (i.e. deformations)
- The formulation of rheology is crucial to properly simulate sea ice dynamics.

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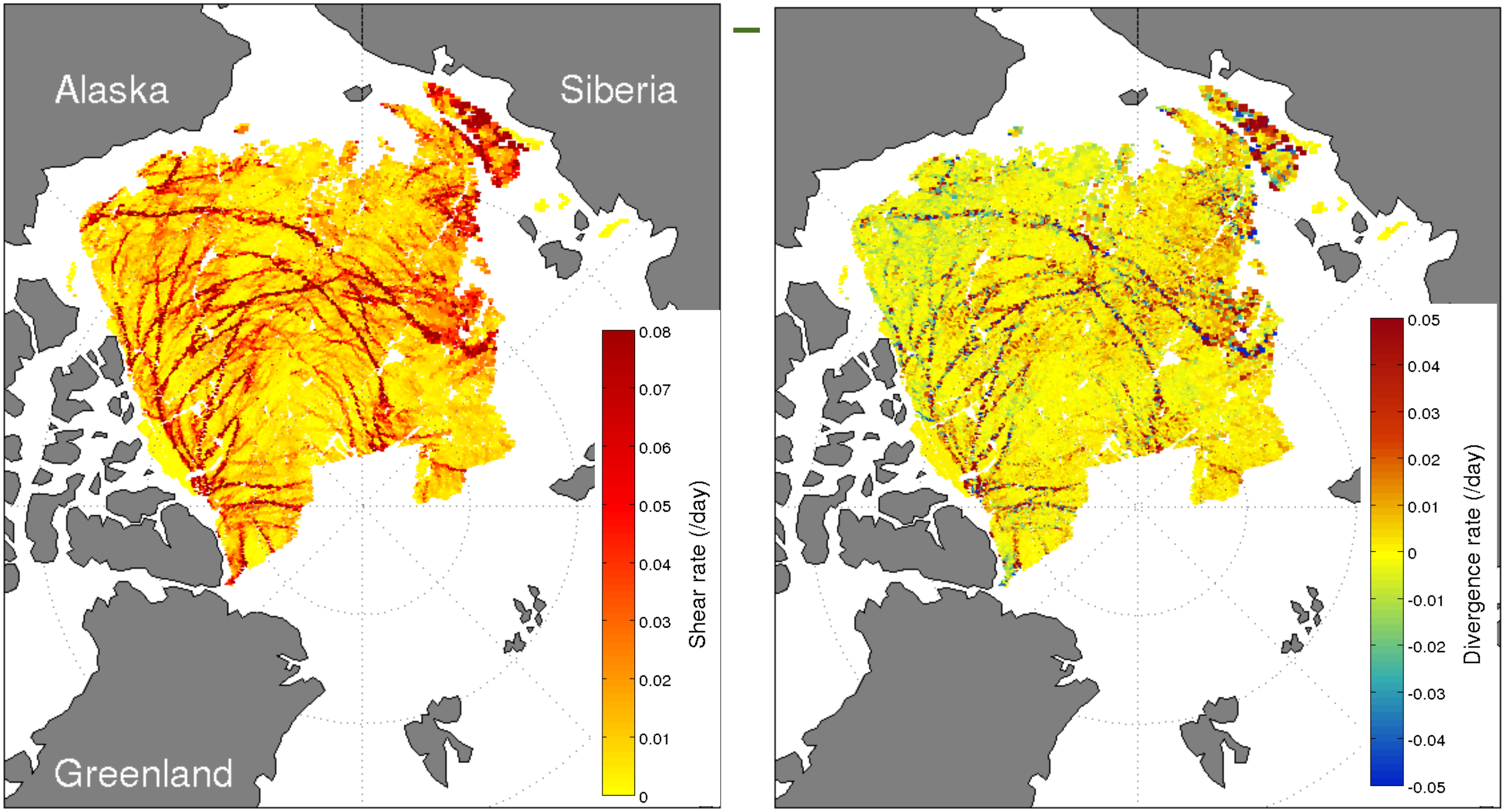
What is rheology?

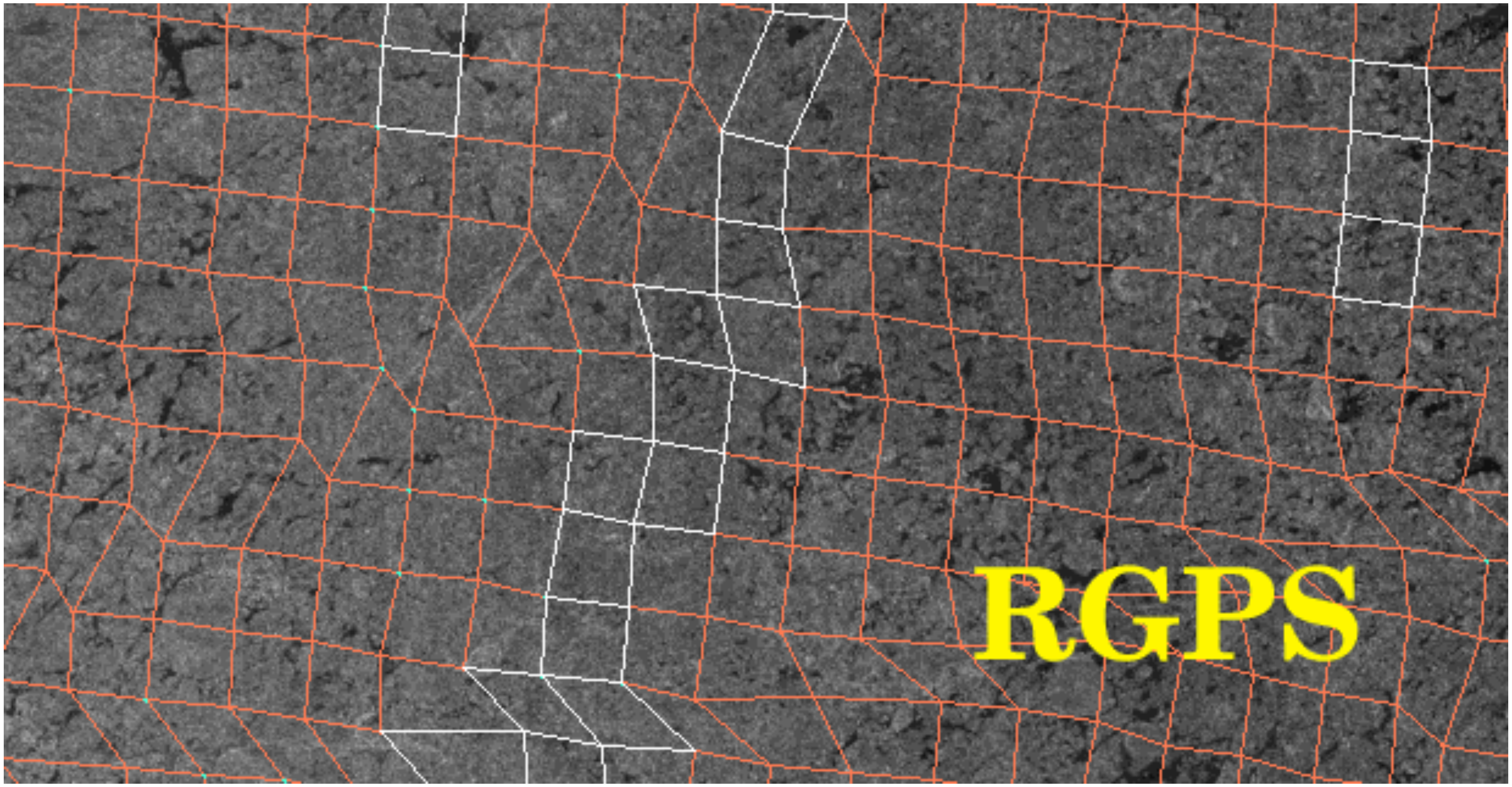
Rheology is the relationship between applied stresses, material properties and the resulting deformations.

Processes/features related to sea ice rheology: **deformations**



***RGPS observations, 24-30 Mar. 2007, ~10km scale**, Source: Nasa JPL, Ron Kwok*

Processes/features related to sea ice rheology: leads

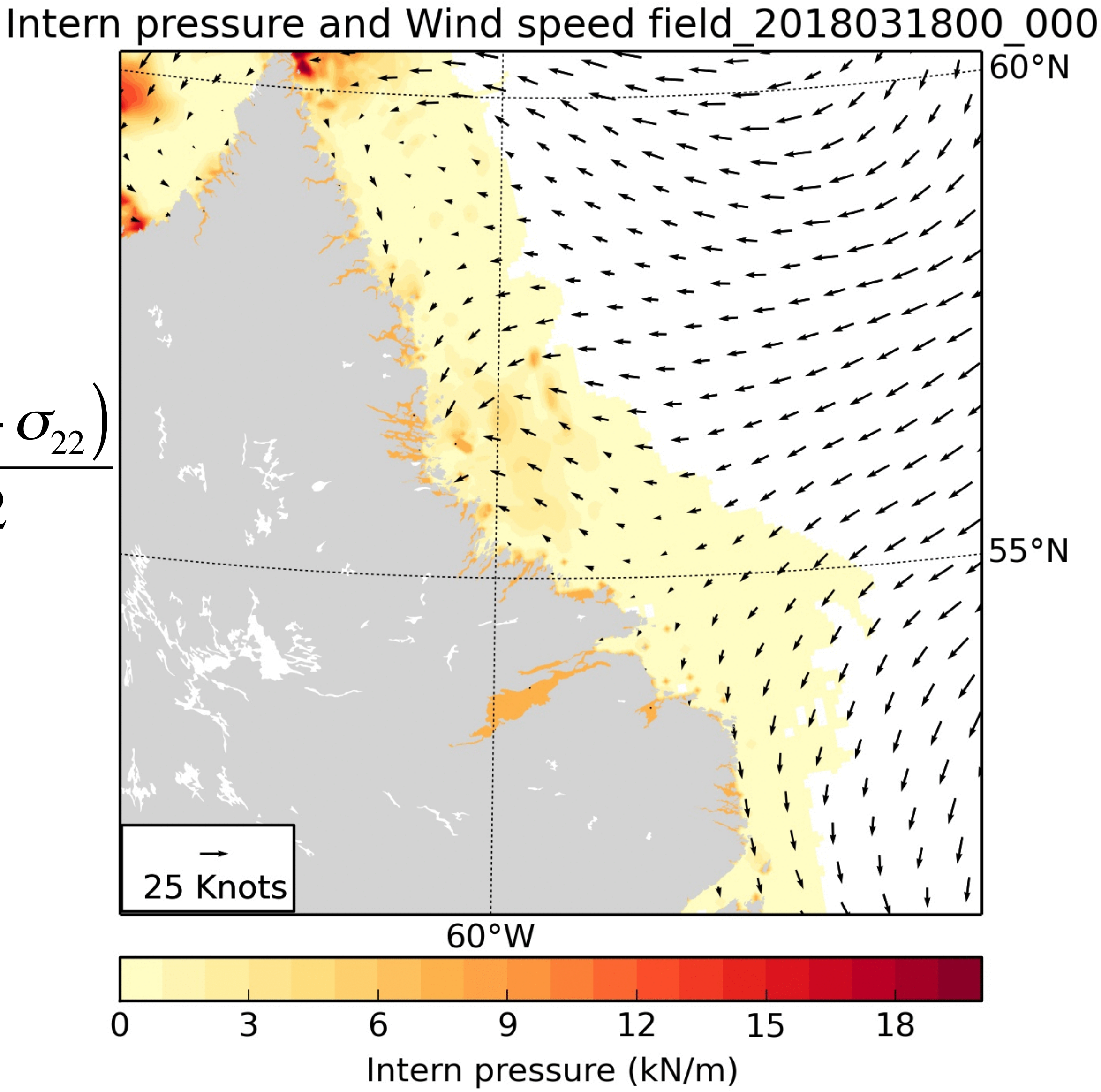


Processes/features related to sea ice rheology:
pressure ridges

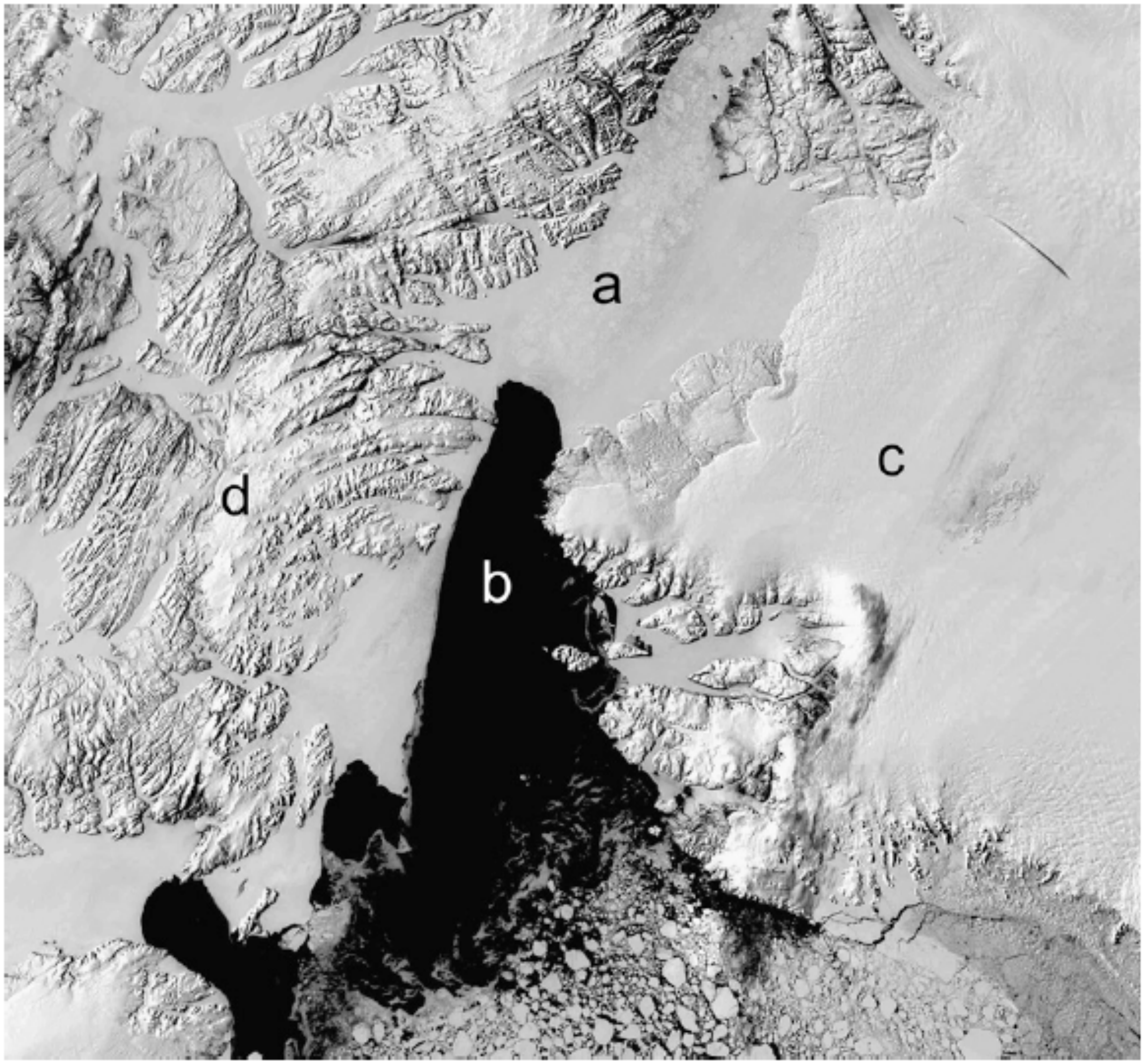
Processes/features related to sea ice rheology:

ice pressure

$$p = -\frac{(\sigma_{11} + \sigma_{22})}{2}$$



Processes/features related to sea ice rheology:
ice arches



Dumont et al. 2009

More on the importance of sea ice deformations

- affect the thickness distribution through the formation of ridges and leads
- heat flux through new leads is 1-2 orders of mag higher than over thick ice (Maykut, 1978)
- 25-40% of new ice formation occurs in leads (Kwok, 2006)
- Ridges affect the air-ice and ocean-ice drags

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The sea ice momentum equation

$$m\frac{Du}{Dt} = -mf\hat{k} \times u + \tau_a - \tau_w - \tau_b - mg\nabla H_d + \boxed{\nabla \cdot \sigma}$$

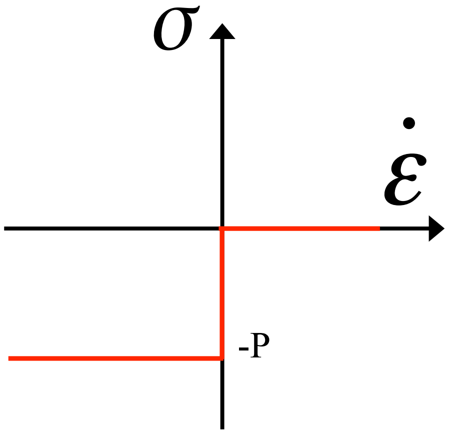
- 4 terms are nonlinear
- it is the rheology term that makes the equation so difficult to solve

1D momentum equation

$$\rho h \frac{\partial u}{\partial t} + C_w(u)u - \frac{\partial \sigma(u)}{\partial x} = \tau_a$$

where $$\sigma = \zeta \dot{\varepsilon} - \frac{P}{2} = \zeta \frac{\partial u}{\partial x} - \frac{P}{2}$$

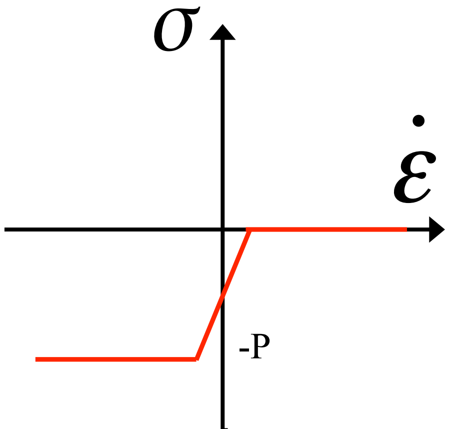
1D VP rheology



plastic

$$\sigma = \zeta\,\dot{\varepsilon} - \frac{P}{2}$$

$$\zeta = \frac{P}{2\,|\,\dot{\varepsilon}\,|}$$

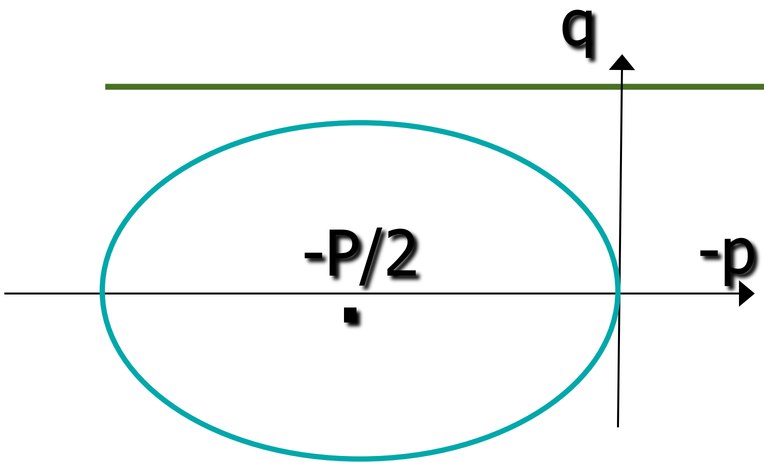


viscous-plastic

$$\sigma = \zeta\,\dot{\varepsilon} - \frac{P}{2}$$

$$\zeta = \min\left(\frac{P}{2\,|\,\dot{\varepsilon}\,|}, \zeta_{\max}\right)$$

Viscous-plastic formulation



$$\sigma_{ij} = 2\eta\,\dot{\varepsilon}_{ij} + [\zeta - \eta]\,\dot{\varepsilon}_{kk}\,\delta_{ij} - P\delta_{ij}/2 \qquad i, j = 1,2$$

$$\zeta = P/2\Delta,\ \ \eta = \zeta e^{-2} \qquad \Delta = \sqrt{f(\dot{\varepsilon}_{ij}^{\,2})}$$

Hibler, 1979

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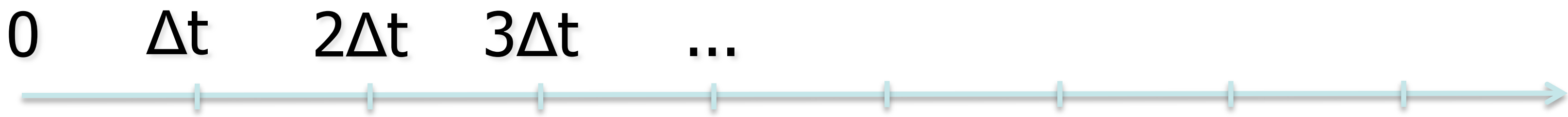
Time discretization (implicit approach)

$$\rho h \frac{\partial u}{\partial t} + C_w(u)u - \frac{\partial \sigma(u)}{\partial x} = \tau_a$$

We want to solve the eqs at time levels n= 1, 2, 3,... :



$$\rho h \left(\frac{u^n - u^{n-1}}{\Delta t} \right) + C_w(u^n)u^n - \frac{\partial \sigma(u^n)}{\partial x} = \tau_a^n$$

The system of nonlinear equations

$$\underbrace{\rho h \frac{u^n}{\Delta t} + C_w(u^n)u^n - \frac{\partial}{\partial x}\zeta(u^n)\frac{\partial u^n}{\partial x}}_{\mathbf{A}(\mathbf{u}^n)\mathbf{u}^n} = \underbrace{\tau_a^n + \rho h \frac{u^{n-1}}{\Delta t} - \frac{1}{2}\frac{\partial P}{\partial x}}_{\mathbf{b}}$$

$$\Downarrow$$

$$\mathbf{A}(\mathbf{u})\mathbf{u} = \mathbf{b}$$

$$\mathbf{F}(\mathbf{u}) = \mathbf{A}(\mathbf{u})\mathbf{u} - \mathbf{b}$$ (the residual)

Implicit solvers

Picard

```
do k=1, $k_{max}$
    'solve' $\mathbf{A}(\mathbf{u}^{k-1})\mathbf{u}^{k} = \mathbf{b}$
    stop if $\left\|\mathbf{F}(\mathbf{u}^{k})\right\| < \gamma_{nl}\left\|\mathbf{F}(\mathbf{u}^{0})\right\|$
enddo
```

e.g. Zhang and Hibler 1997

Newton

```
do k=1, $k_{max}$
    'solve' $\mathbf{J}(\mathbf{u}^{k-1})\delta\mathbf{u} = -\mathbf{F}(\mathbf{u}^{k-1})$
    $\mathbf{u}^{k} = \mathbf{u}^{k-1} + \delta\mathbf{u}$
    stop if $\left\|\mathbf{F}(\mathbf{u}^{k})\right\| < \gamma_{nl}\left\|\mathbf{F}(\mathbf{u}^{0})\right\|$
enddo
```

e.g. Lemieux et al. 2012,
Mehlmann and Richter 2017

Pros and cons of current implicit solvers

measure of numerical convergence

implicit approach (no stability issue)

slow to super-linear convergence

issues with parallelization

robustness

Anderson acceleration

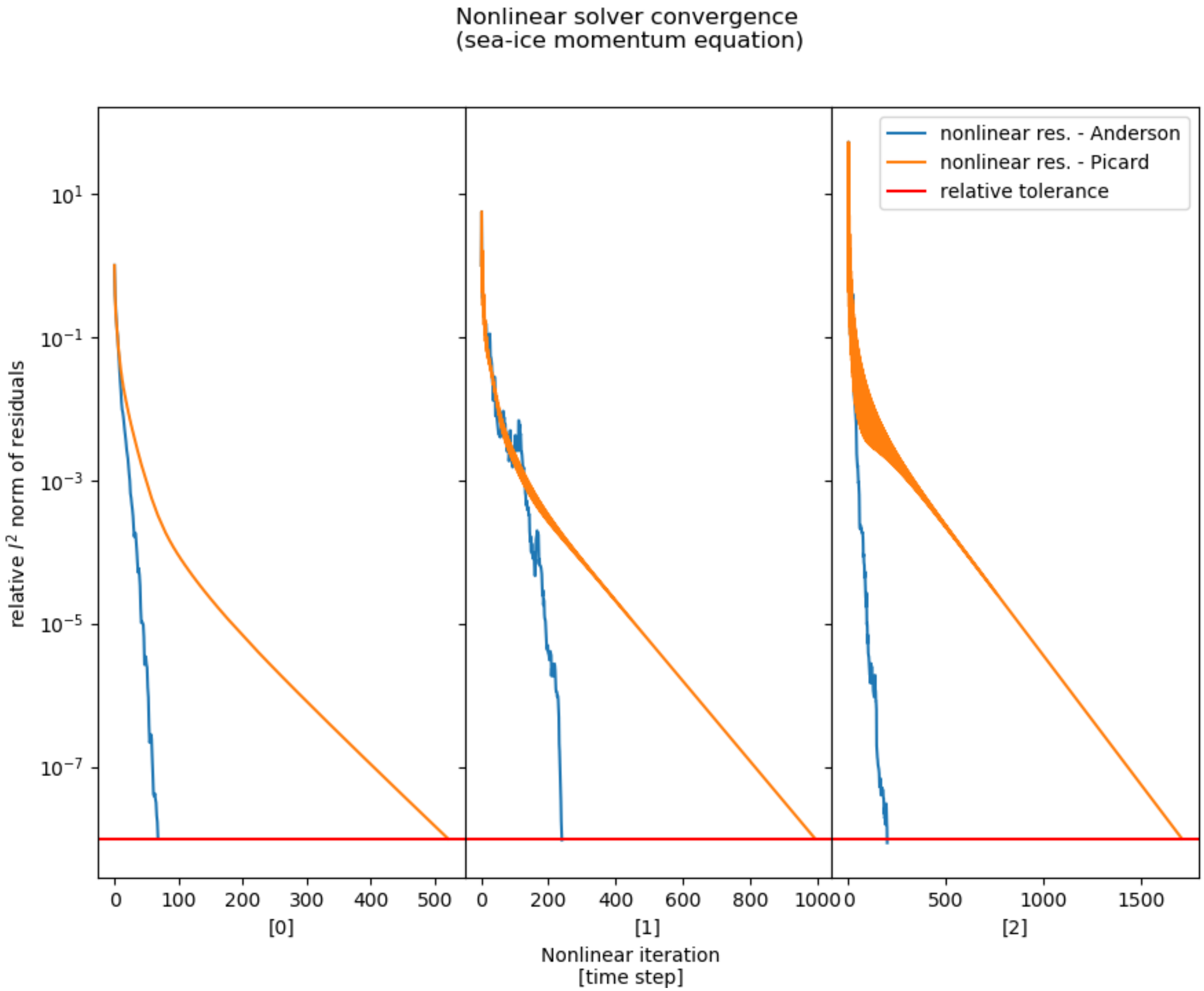
Anderson, 1965

$$\mathbf{A}(\mathbf{u})\mathbf{u}=\mathbf{b} \Rightarrow \mathbf{u}=\mathbf{G}(\mathbf{u}) \Rightarrow \mathbf{u}^{k+1}=\mathbf{A}^{-1}(\mathbf{u}^{k})\mathbf{b}$$

```
do k=1, $k_{max}$
    $\mathbf{f}^{k} = G(\mathbf{u}^{k}) - \mathbf{u}^{k}$
    min $\left\| \alpha_0 \mathbf{f}^{k-m} \ldots + \alpha_{m-1} \mathbf{f}^{k-1} + \alpha_m \mathbf{f}^{k} \right\|$
    $\mathbf{u}^{k+1} = \alpha_0 \mathbf{u}^{k-m} \ldots + \alpha_{m-1} \mathbf{u}^{k-1} + \alpha_m \mathbf{u}^{k}$
enddo
```

Anderson acceleration combines a few (m) iterates by minimizing the residual.

Nonlinear convergence: Picard and Anderson



The (explicit) EVP solver

```
do s=1, $N_{sub}$
    $$\frac{\sigma^s - \sigma^{s-1}}{\Delta t_e} + \frac{\sigma^s}{\alpha T} = \frac{\zeta}{T}\frac{\partial u^{s-1}}{\partial x} - \frac{P}{2\alpha T}$$
    $$\rho h\left(\frac{u^s - u^{s-1}}{\Delta t_e}\right) = -C_w(u^{s-1})u^{s-1} + \frac{\partial \sigma^s}{\partial x} + \tau_a^n$$
enddo
```

Hunke, 2001

Pros and cons of current explicit solvers

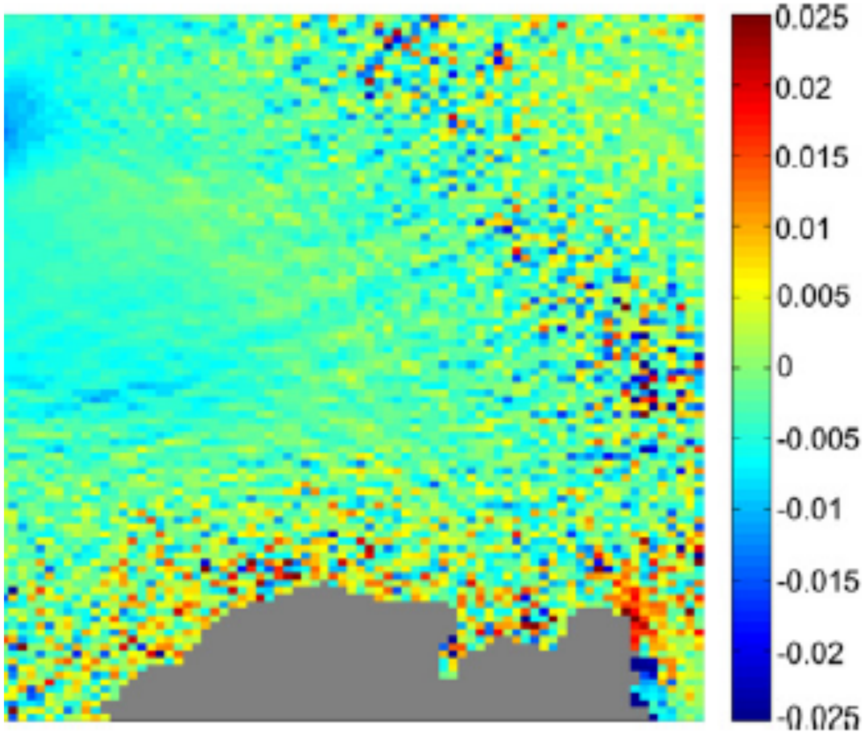
easy to understand, easy to implement

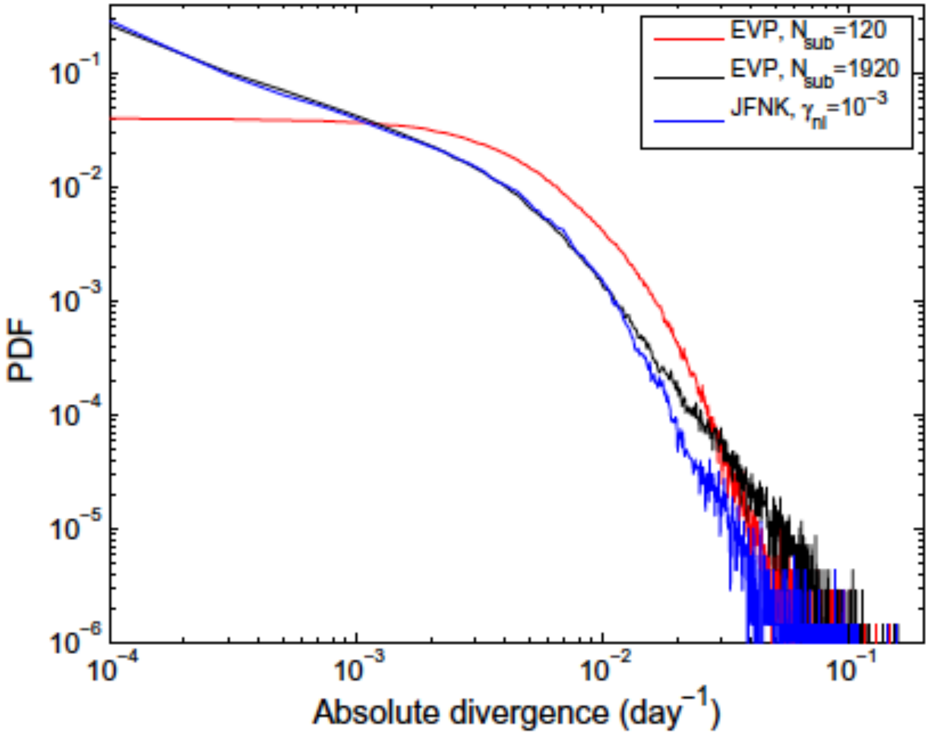
well suited for parallelization

no measure of numerical convergence

noise in numerical solutions

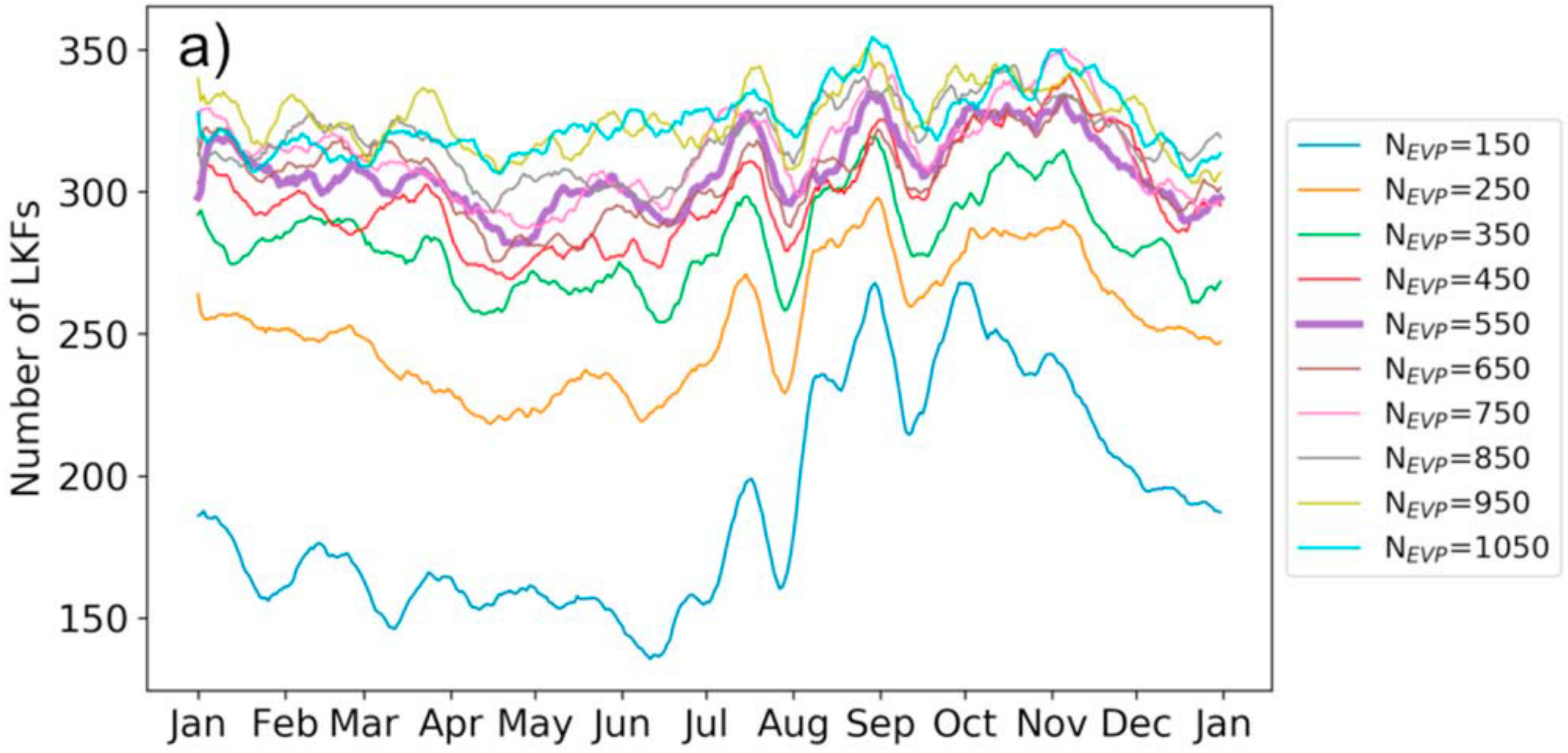
Simulated deformations with the EVP





Lemieux et al. 2012

Simulated deformations with the EVP



Koldunov et al. 2019

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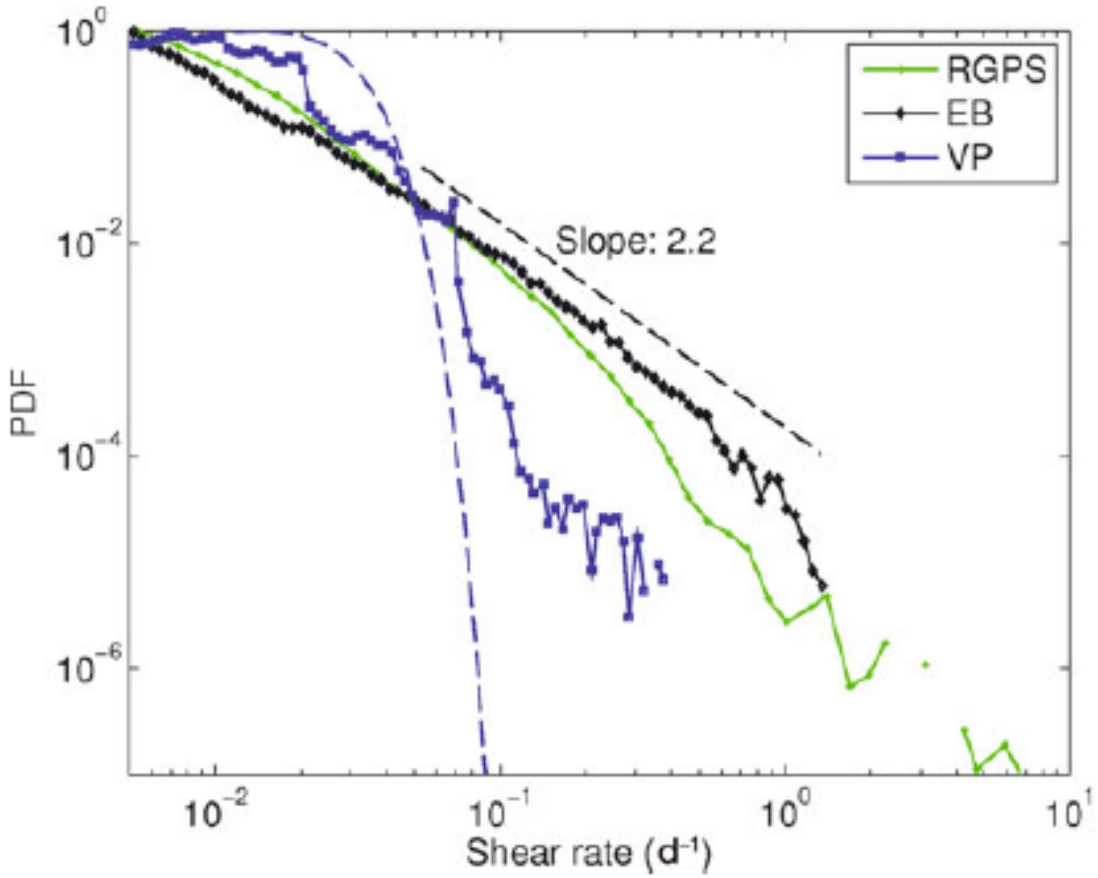
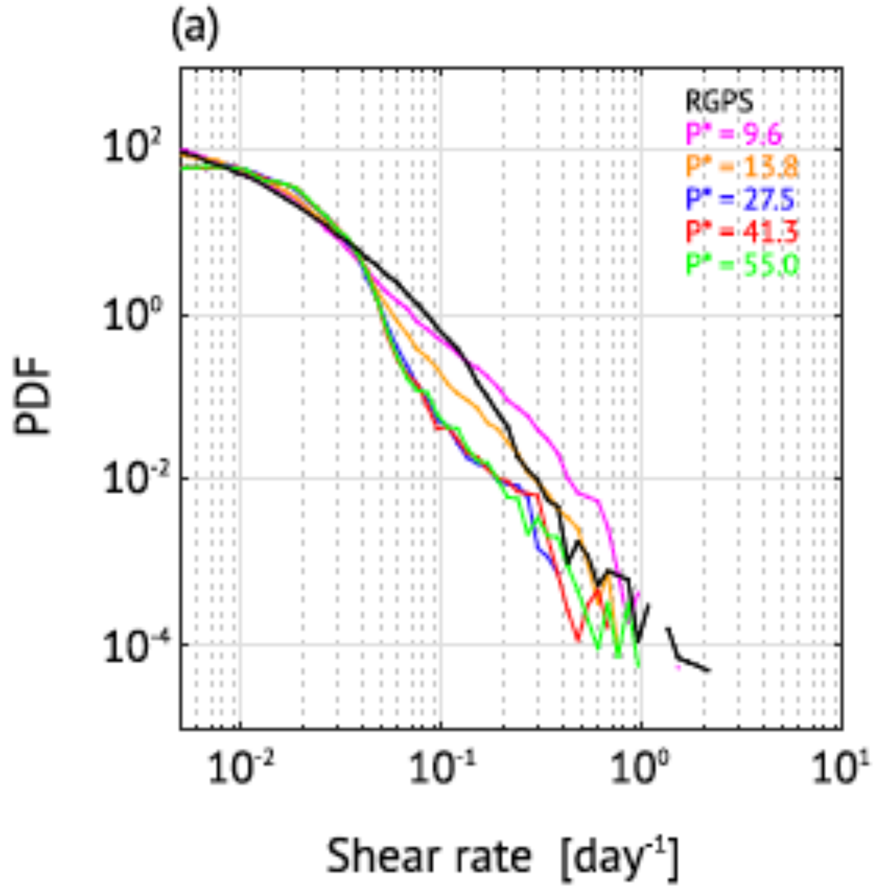
Observed and simulated deformations

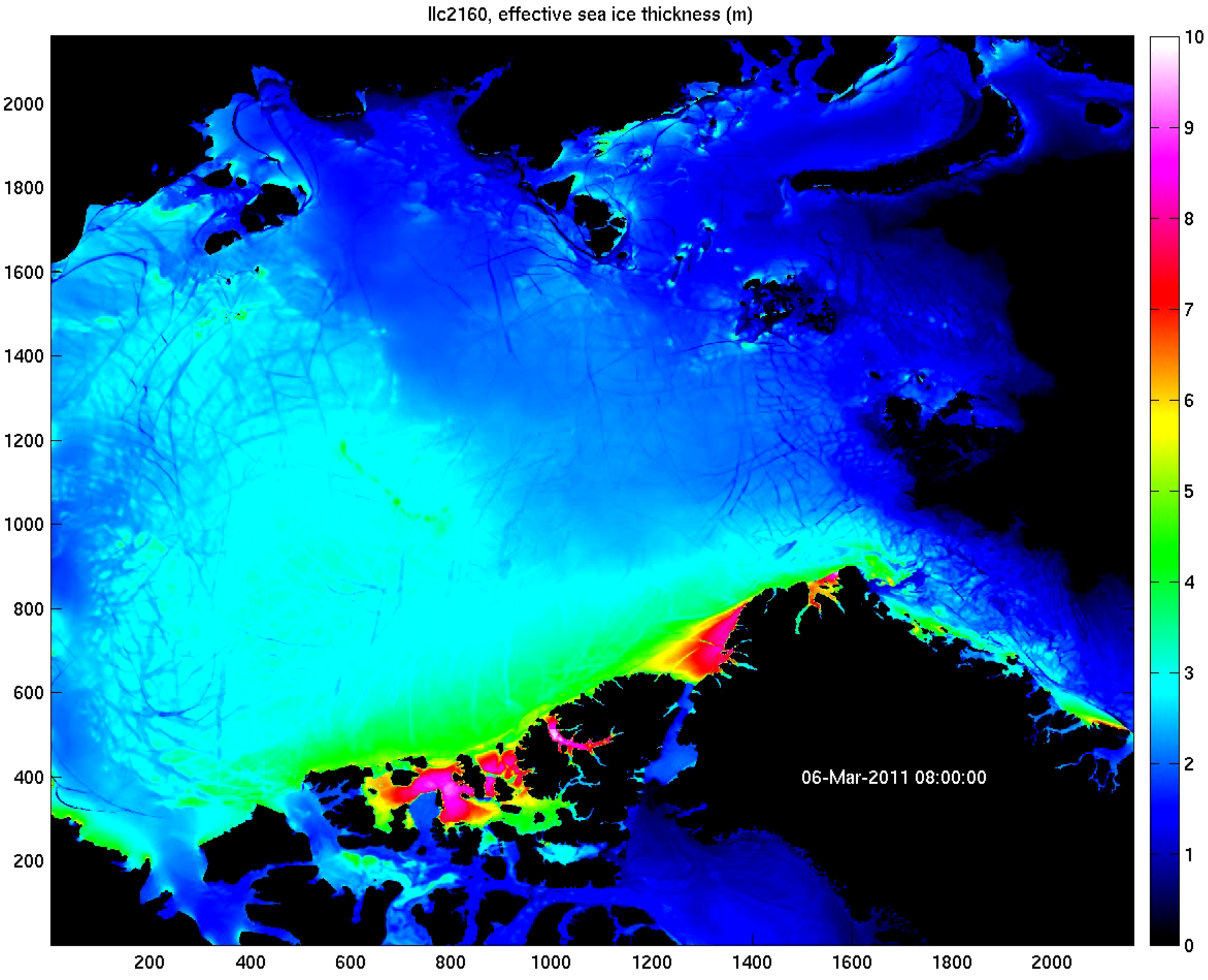


Girard et al. 2011

Observed and simulated deformations

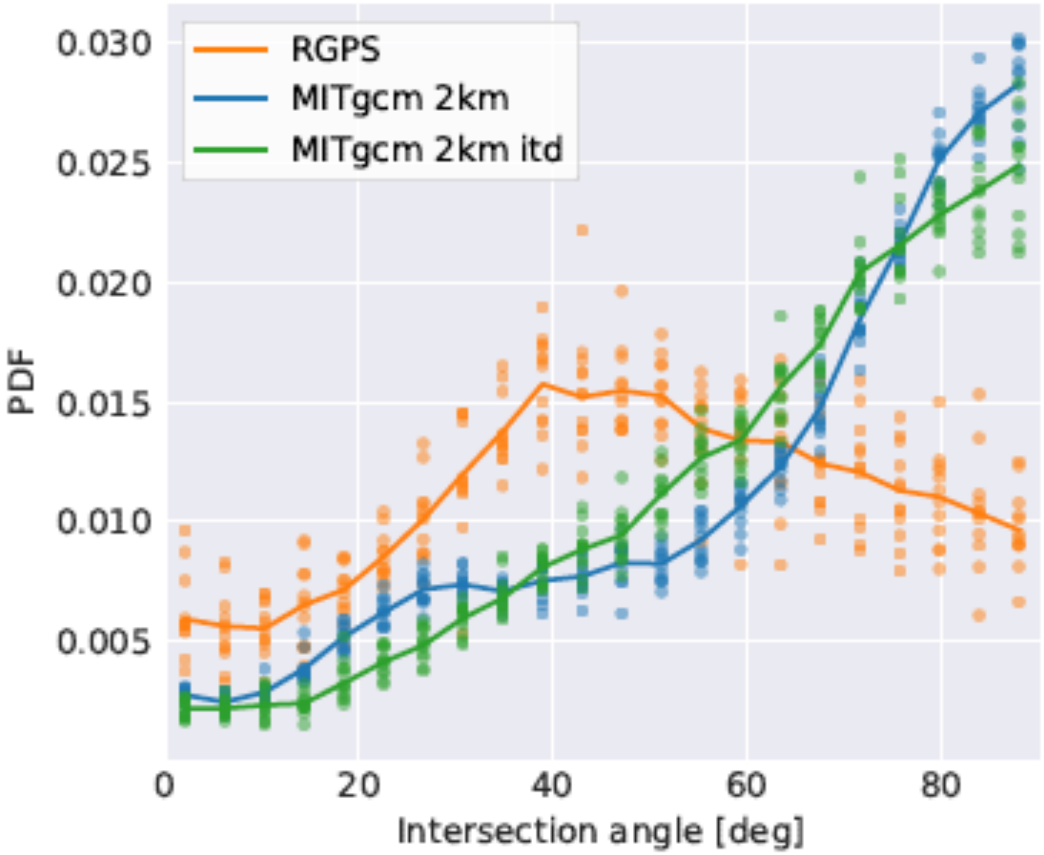


Bouchat and Tremblay 2017



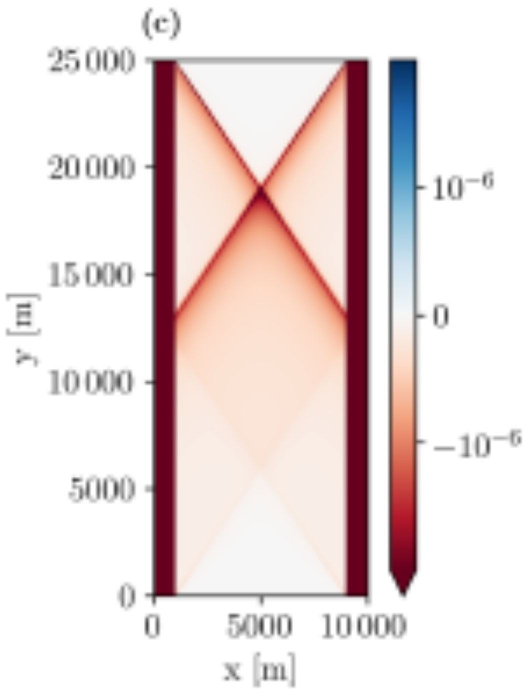
MITGCM-1km. Courtesy of Nils Hutter

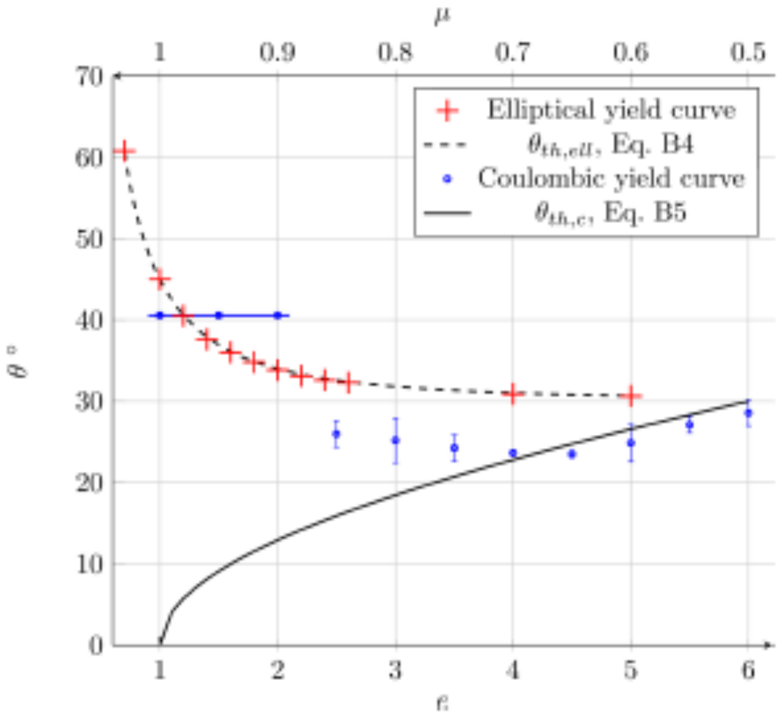
Intersection angle between lines of deformations



Hutter and Losch 2019

Intersection angle between lines of deformations

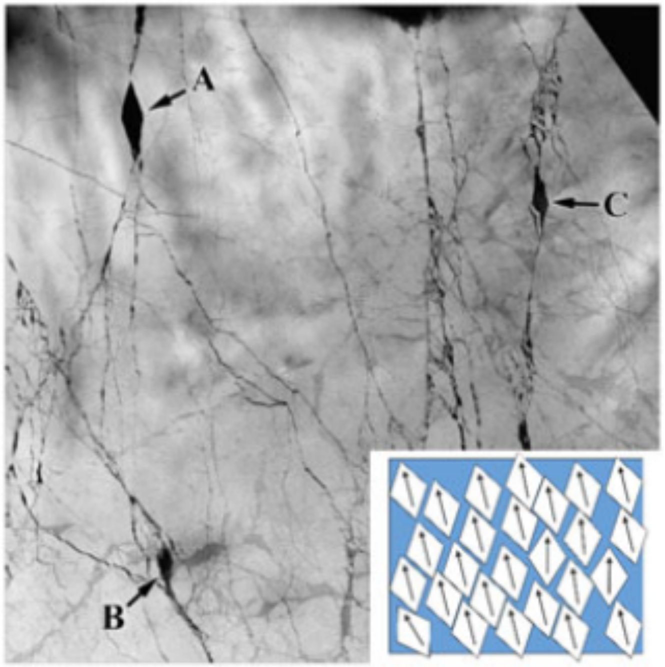




Ringeisen et al. 2019

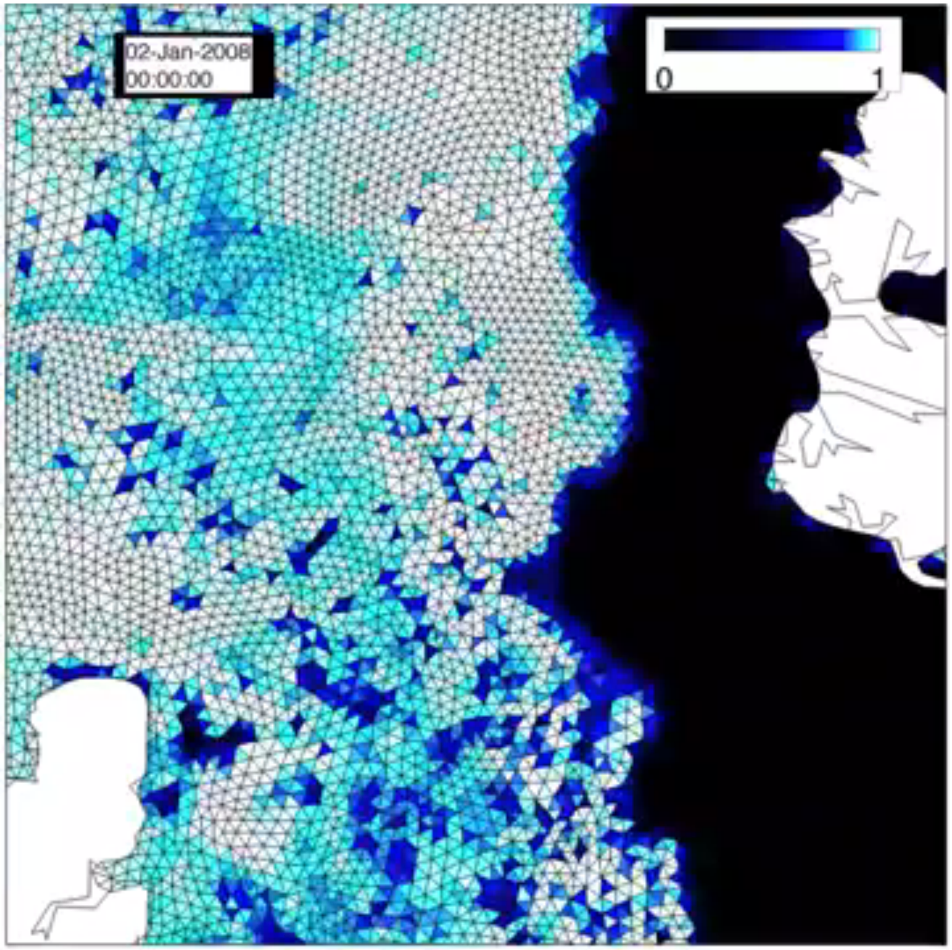
The elastic-anisotropic-plastic rheology

- Considers subgrid-scale anisotropy
- notable changes to sea ice drift and geophysical distribution of thickness
- same solver approach than EVP



Tsamados et al. 2013, Heorton et al. 2018

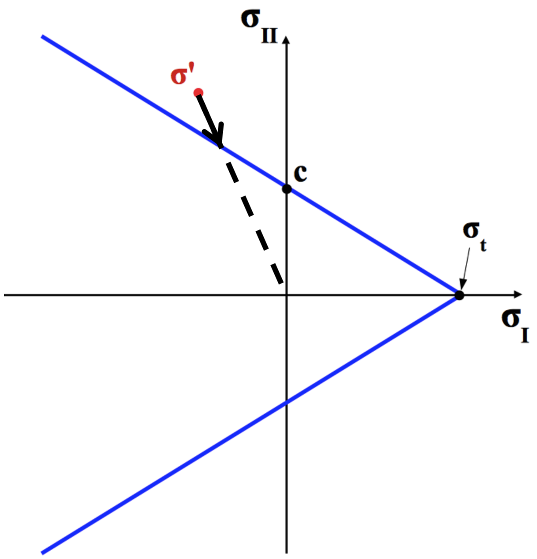
The Maxwell-elasto-brittle (MEB) rheology



neXtSIM (Rampal et al., 2015)

The MEB rheology

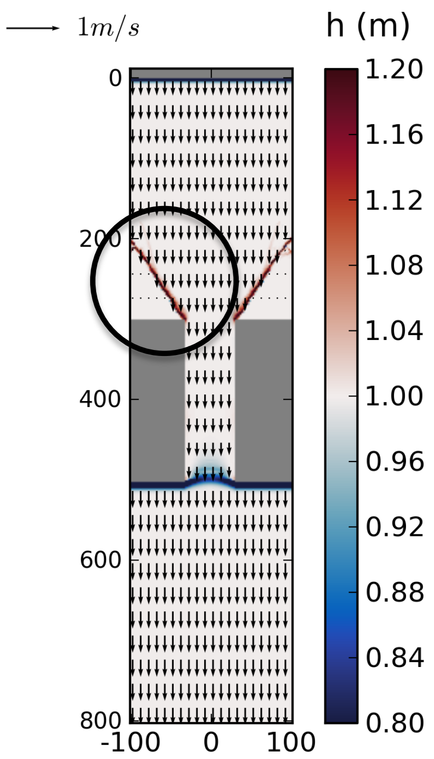
- rigid state of sea ice is elastic
- Mohr-Coulomb failure criterion
- use of a damage parameterization

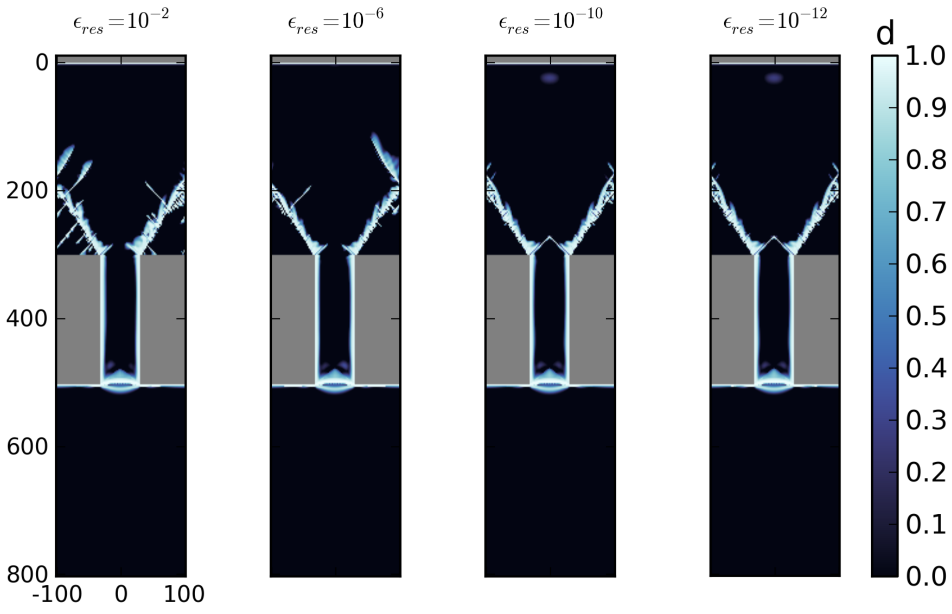


Girard et al. 2011, Rampal et al. 2016, Dansereau et al., 2016

Ideal MEB simulations

Plante et al. 2020





The angle of fractures are not following granular theory

The damage is unstable in compressive failures

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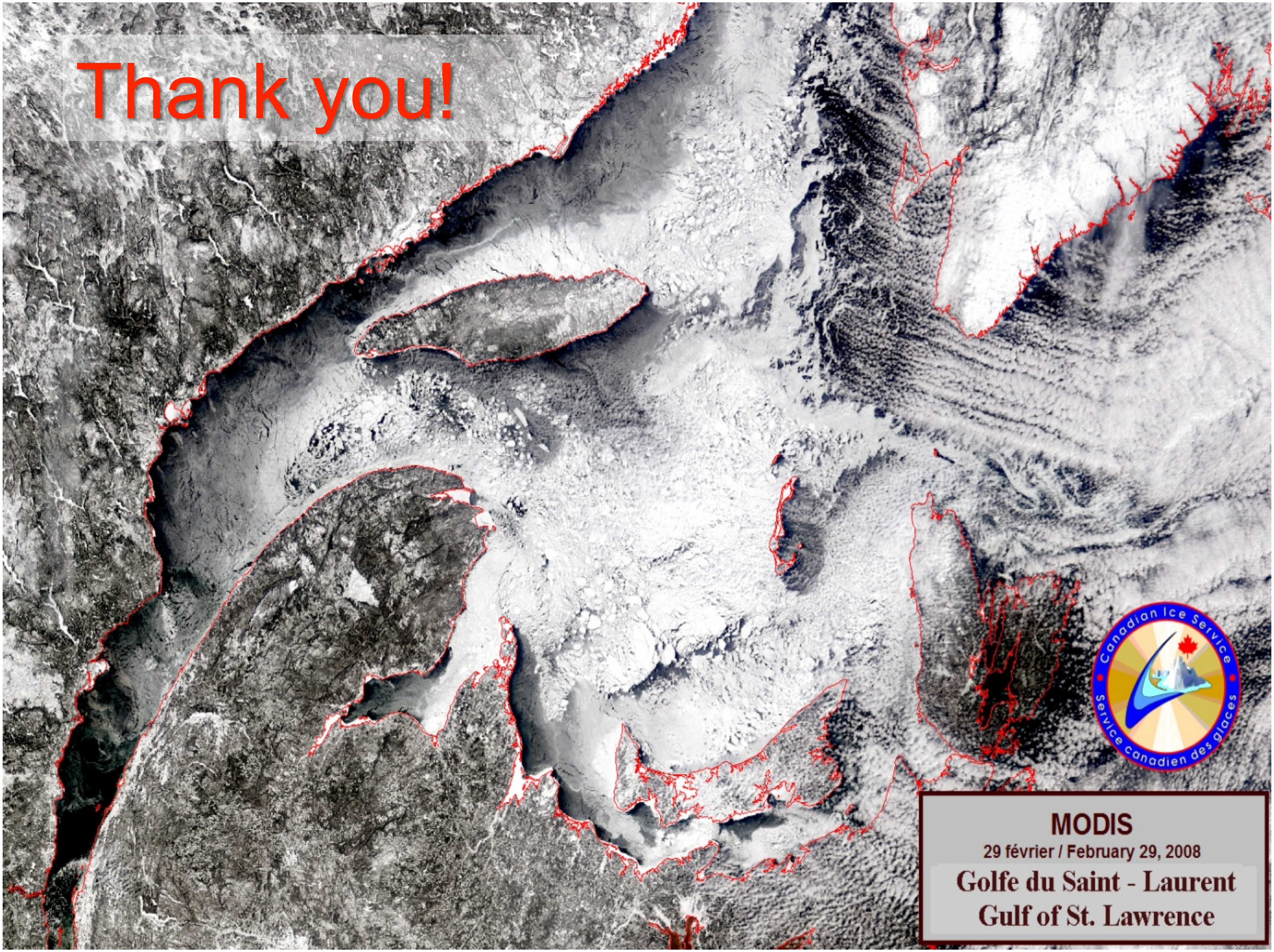
Conclusions

- Solving the sea ice momentum equation is challenging
- Explicit and implicit solvers all have pros and cons.
- New (rheology) approaches also have numerical issues
- These numerical problems get more serious as dx decreases and as more processes and coupling to other components are included.
- As dx decreases, the continuum assumption breaks down....

Thank you!



MODIS

29 février / February 29, 2008

Golfe du Saint - Laurent
Gulf of St. Lawrence