Sea ice rheology and numerical solvers for sea ice dynamics

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Outline

• Introduction on sea ice dynamics
• The importance of sea ice rheology
• The viscous-plastic (VP) rheology
• Current numerical solvers for sea ice dynamics
• Critics of the VP model and new approaches
• Conclusions
Ice Motion in the Arctic Basin

Data from the
International Arctic Buoy Programme (IABP)
1979 - 1998

The IABP is funded by its Participants from 31 Institutions from 10 different countries.

For more information on the IABP, please visit their web page.
Main features of Arctic sea ice drift
Sea ice dynamics

- Strongly impact the local and geophysical distributions of sea ice thickness.
- It is not only velocity that matters but its spatial derivatives (i.e. deformations)
- The formulation of rheology is crucial to properly simulate sea ice dynamics.
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What is rheology?

Rheology is the relationship between applied stresses, material properties and the resulting deformations.
Processes/features related to sea ice rheology: deformations
Processes/features related to sea ice rheology:

leads
Processes/features related to sea ice rheology:

**pressure ridges**
Processes/features related to sea ice rheology: ice pressure

\[ p = -\frac{\left(\sigma_{11} + \sigma_{22}\right)}{2} \]
Processes/features related to sea ice rheology: ice arches

Dumont et al. 2009
More on the importance of sea ice deformations

• affect the thickness distribution through the formation of ridges and leads

• heat flux through new leads is 1-2 orders of mag higher than over thick ice (Maykut, 1978)

• 25-40% of new ice formation occurs in leads (Kwok, 2006)

• Ridges affect the air-ice and ocean-ice drags
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The sea ice momentum equation

\[ m \frac{Du}{Dt} = -m \hat{k} \times u + \tau_a - \tau_w - \tau_b - mg \nabla H_d + \nabla \cdot \sigma \]

- 4 terms are nonlinear
- it is the rheology term that makes the equation so difficult to solve
1D momentum equation

\[ \rho h \frac{\partial u}{\partial t} + C_w(u)u - \frac{\partial \sigma(u)}{\partial x} = \tau_a \]

where

\[ \sigma = \zeta \varepsilon - \frac{P}{2} = \zeta \frac{\partial u}{\partial x} - \frac{P}{2} \]
1D VP rheology

\[ \sigma = \zeta \dot{\varepsilon} - \frac{P}{2} \]

\[ \zeta = \frac{P}{2 | \dot{\varepsilon} |} \]

\[ \sigma = \zeta \dot{\varepsilon} - \frac{P}{2} \]

\[ \zeta = \min \left( \frac{P}{2 | \dot{\varepsilon} |}, \zeta_{\text{max}} \right) \]
Viscous-plastic formulation

\[
\sigma_{ij} = 2\eta \dot{\varepsilon}_{ij} + \left[\zeta - \eta\right] \dot{\varepsilon}_{kk} \delta_{ij} - P \delta_{ij} / 2 \\
\zeta = P / 2\Delta, \quad \eta = \zeta \varepsilon^{-2}, \quad \Delta = \sqrt{f(\varepsilon_{ij}^{\dot{2}})}
\]

Hibler, 1979
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Time discretization (implicit approach)

\[ \rho h \frac{\partial u}{\partial t} + C_w(u)u - \frac{\partial \sigma(u)}{\partial x} = \tau_a \]

We want to solve the eqs at time levels \( n = 1, 2, 3, \ldots \) :

\[
\begin{align*}
0 & \quad \Delta t & \quad 2\Delta t & \quad 3\Delta t & \quad \ldots \\
\rho h \left( \frac{u^n - u^{n-1}}{\Delta t} \right) + C_w(u^n)u^n - \frac{\partial \sigma(u^n)}{\partial x} = \tau_a^n
\end{align*}
\]
The system of nonlinear equations

\[ \rho h \frac{u^n}{\Delta t} + C_w(u^n)u^n - \frac{\partial}{\partial x} \xi(u^n) \frac{\partial u^n}{\partial x} = \tau^n_a + \rho h \frac{u^{n-1}}{\Delta t} - \frac{1}{2} \frac{\partial P}{\partial x} \]

\[ A(u^n)u^n \quad b \]

\[ A(u)u = b \]

\[ F(u) = A(u)u - b \quad \text{(the residual)} \]
## Implicit solvers

<table>
<thead>
<tr>
<th>Picard</th>
<th>Newton</th>
</tr>
</thead>
<tbody>
<tr>
<td>do $k=1, k_{\text{max}}$</td>
<td>do $k=1, k_{\text{max}}$</td>
</tr>
<tr>
<td>‘solve’ $A(u^{k-1})u^k = b$</td>
<td>‘solve’ $J(u^{k-1})\delta u = -F(u^{k-1})$</td>
</tr>
<tr>
<td>stop if $|F(u^k)| &lt; \gamma_{\text{nl}}|F(u^0)|$</td>
<td>$u^k = u^{k-1} + \delta u$</td>
</tr>
<tr>
<td>enddo</td>
<td>stop if $|F(u^k)| &lt; \gamma_{\text{nl}}|F(u^0)|$</td>
</tr>
<tr>
<td>enddo</td>
<td>enddo</td>
</tr>
</tbody>
</table>

e.g. Zhang and Hibler 1997

e.g. Lemieux et al. 2012, Mehlmann and Richter 2017
Pros and cons of current implicit solvers

- Measure of numerical convergence
- Implicit approach (no stability issue)
- Slow to super-linear convergence
- Issues with parallelization
- Robustness
Anderson acceleration

\[ A(u)u = b \quad \Rightarrow \quad u = G(u) \quad \Rightarrow \quad u^{k+1} = A^{-1}(u^k)b \]

do \ k=1, k_{\text{max}}

\[ f^k = G(u^k) - u^k \]

\[ \min \left\| \alpha_0 f^{k-m} + \ldots + \alpha_{m-1} f^{k-1} + \alpha_m f^k \right\| \]

\[ u^{k+1} = \alpha_0 u^{k-m} + \ldots + \alpha_{m-1} u^{k-1} + \alpha_m u^k \]

endo

Anderson acceleration combines a few (m) iterates by minimizing the residual.
Nonlinear convergence: Picard and Anderson

Nonlinear solver convergence
(sea-ice momentum equation)
The (explicit) EVP solver

do s=1, N_{sub} \n\frac{\sigma^s - \sigma^{s-1}}{\Delta t_e} + \frac{\sigma^s}{\alpha T} = \frac{\zeta}{T} \frac{\partial u^{s-1}}{\partial x} - \frac{P}{2 \alpha T} \n\rho h \left( \frac{u^s - u^{s-1}}{\Delta t_e} \right) = -C_w (u^{s-1}) u^{s-1} + \frac{\partial \sigma^s}{\partial x} + \tau^n_a \nendo

Hunke, 2001
Pros and cons of current explicit solvers

easy to understand, easy to implement

well suited for parallelization

no measure of numerical convergence

noise in numerical solutions
Simulated deformations with the EVP

Lemieux et al. 2012
Simulated deformations with the EVP

Koldunov et al. 2019
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Observed and simulated deformations

Girard et al. 2011
Observed and simulated deformations

Bouchat and Tremblay 2017
Intersection angle between lines of deformations

Hutter and Losch 2019
Intersection angle between lines of deformations

Ringeisen et al. 2019
The elastic-anisotropic-plastic rheology

- Considers subgrid-scale anisotropy
- Notable changes to sea ice drift and geophysical distribution of thickness
- Same solver approach than EVP

Tsamados et al. 2013, Heorton et al. 2018
The Maxwell-elasto-brittle (MEB) rheology

neXtSIM (Rampal et al., 2015)
The MEB rheology

- rigid state of sea ice is elastic
- Mohr-Coulomb failure criterion
- use of a damage parameterization

Girard et al. 2011, Rampal et al. 2016, Dansereau et al., 2016
Ideal MEB simulations

Plante et al. 2020

The angle of fractures are not following granular theory

The damage is unstable in compressive failures
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Conclusions

• Solving the sea ice momentum equation is challenging

• Explicit and implicit solvers all have pros and cons.

• New (rheology) approaches also have numerical issues

• These numerical problems get more serious as $dx$ decreases and as more processes and coupling to other components are included.

• As $dx$ decreases, the continuum assumption breaks down....
Thank you!