The Ocean Model ICON-O as Part of the Earth System Model ICON-ESM

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Outline

1. Overview of ICON Modelling System
2. Numerics (emphasis on ICON-O)
3. A Range of ICON-Simulations Across Scales

ICON Modelling System - Overview

Components

- Atmosphere: **ICON-A**
- Ocean: **ICON-O**
- Sea-Ice (part of ICON-O)
- Ocean Biogeochemistry **HAMOCC**
- Land: **JSBACH**
- Coupler: YAC (**Y**et **A**nother **C**oupler)
- Atmospheric Chemistry: **ICON-ART**

ICON-ESM

Combining all components above

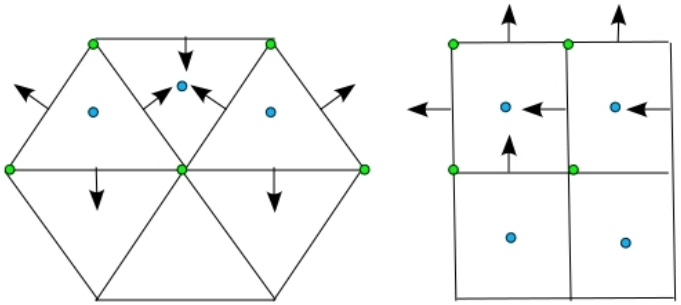
Partners

- German Weather Service (DWD)
- German Climate Computing Center (DKRZ)
- Karlsruhe Institute of Technology (KIT)

ICON Modelling System - Grid & Staggering

Basic Idea

- All ICON components use same horizontal grid & staggering but potentially different vertical coordinate
- Horizontal grid based on subdivision of icosahedron
- Icosahedron allows nearly uniform tessellation of the sphere without pole singularity
- C-Staggering on triangular grid



ICON Modelling System - Atmosphere

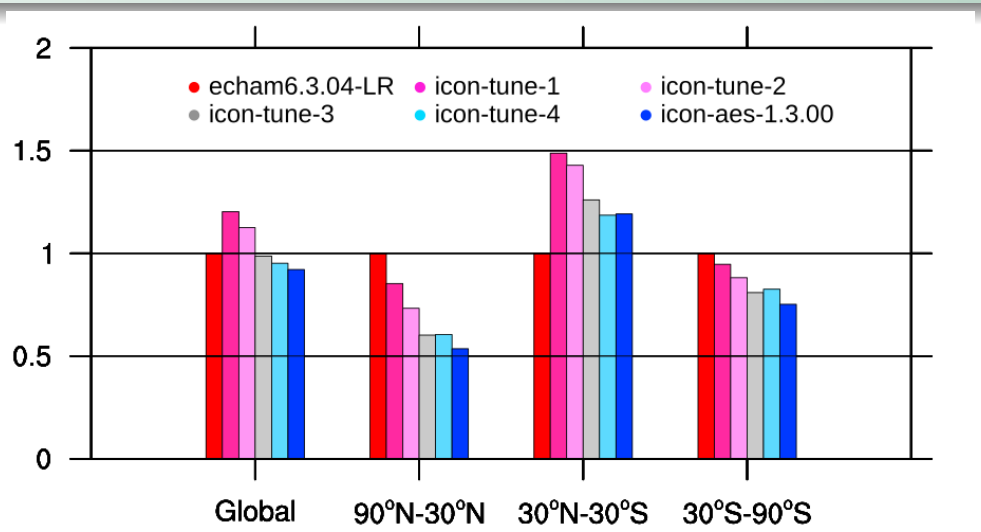
ICON-A

- **Model Equations:** compressible Navier-Stokes Equations
- **Prognostic variables:** 3D velocity field, density & virtual potential temperature
- Velocity equation in 2D vector invariant form
- Tracer equations in flux form

ICON-A

- **Vertical coordinate:** terrain-following hybrid sigma coordinate
- **Time stepping:** two-time level predictor-corrector scheme, explicit, except for vertical sound waves. Time splitting between dynamical core and tracer advection, parametrizations

ICON-A: Comparison with ECHAM



Reichler-Kim Scores

- Standardized annual mean climatological errors of 10y AMIP experiment of ECHAM and ICON-A
- Normalized such that ECHAM has bias of values 1
- A value larger/smaller than 1 indicates larger/smaller bias than ECHAM 6.3

M. Giorgetta et al. ICON-A, the Atmosphere Component of the ICON Earth System Model: I. Model Description, JAMES, 2017

Outline

1. Overview of ICON Modelling System
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3. A Range of ICON-Simulations Across Scales
    1. tell what is coming

The Ocean Primitive Equations

State vector: $\{v, h, \theta, S\}$,

$$\textbf{Horiz. Velocity} : \partial_t v + (f+\omega)\mathbf{k} \times v + \frac{\nabla |v|^2}{2} + w\partial_z v + \frac{g}{\rho_0}\nabla(p+p^s) - Lv = 0$$

$$\textbf{Hydrostatic} : \partial_z p = -\rho g$$

$$\textbf{Incompress.} : div\, v + \partial_z w = 0$$

$$\textbf{Free Surface} : \partial_t h + div(\int_B^h v\, dz) = P - E$$

$$\textbf{Tracer}(\mathbf{C} = \theta, \mathbf{S}) : \partial_t C + div(Cv) = L_C$$

$+$ Equation of state + Boundary conditions

Design Principle of ICON-O

Design Principle of ICON-O

Global ocean dynamics is modelled as a flow in discrete Hilbert space.

Requirements

- Discrete model equations formulated in **weak/variational form**. This is the path to global conservation laws.
- **Differential operators** are compatible with discrete weak form.
- **Reconstructions** for C-Staggered grid are compatible with discrete weak form.

Mimetic Discrete Differential Operators - Integral Form

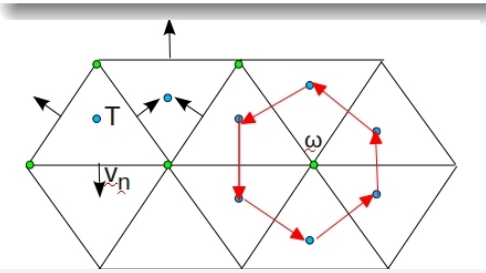
- **div**, **curl** discretized via Gauss' & Stokes Theorem and midpoint rule

$$\mathbf{div} v_{K,k} := \frac{1}{|K|\Delta z_{K,k}} \sum_{e \in \partial K} v_{e,k} |e| n_{e,K} \Delta z_{e,k}$$

- Define $\mathbf{grad_n}$ by

$$\langle f, \mathbf{grad_n} g \rangle_{\mathcal{H}_{\mathcal{E}_k}} = -\langle \mathbf{div} f, g \rangle_{\mathcal{H}_{\mathcal{C}_k}}$$

  Discrete version of integration-by-parts by definition.
- It holds that $\mathbf{curl grad_n} = 0$.



ICON-O: Hilbert-Space-Compatible Reconstructions

Consider reconstruction operators from edges to cells/vertices

$$\mathcal{P}_{\mathcal{G}} : \mathcal{H}_{\mathcal{E}_k} \to \mathbf{H}_{\mathcal{C}_k}, \qquad \hat{\mathcal{P}}_{\mathcal{G}} : \mathcal{H}_{\mathcal{E}_k} \to \mathbf{H}_{\mathcal{V}_k}$$

$$\mathcal{P}_{\mathcal{G}}^T : \mathbf{H}_{\mathcal{C}_k} \to \mathcal{H}_{\mathcal{E}_k} \qquad \hat{\mathcal{P}}_{\mathcal{G}}^\dagger : \mathbf{H}_{\mathcal{V}_k} \to \mathcal{H}_{\mathcal{E}_k}$$

Hilbert-Space-Compatible Reconstructions

The pair $(\mathcal{P}_{\mathcal{G}}, \hat{\mathcal{P}}_{\mathcal{G}})$ is Hilbert-Space compatible reconstruction, if

- $\mathcal{P}_{\mathcal{G}}, \hat{\mathcal{P}}_{\mathcal{G}}$ reproduces constants
- $\mathcal{P}_{\mathcal{G}}^T$ and $\hat{\mathcal{P}}_{\mathcal{G}}^\dagger$ provide unique and at least first order accurate fluxes at primal and dual edges.
- Null space of $\mathcal{M}_{\mathcal{G}} := \mathcal{P}_{\mathcal{G}}^T \mathcal{P}_{\mathcal{G}}$ coincides with space of divergence noise.
- $\mathcal{M}_{\mathcal{G}}$ is strictly positive-definite on image space $\mathcal{M}_{\mathcal{G}}(\mathcal{H}_{\mathcal{E}_k})$.
- $\hat{\mathcal{M}}_{\mathcal{G}} := \hat{\mathcal{P}}_{\mathcal{G}}^\dagger \hat{\mathcal{P}}_{\mathcal{G}}$ is skew-symmetric.

Example of Hilbert-Space-Compatible Reconstructions

Perot Reconstruction

$$P : \mathcal{H}_{\mathcal{E}_k} \to \mathbf{H}_{\mathcal{C}_k}$$

$$v \mapsto P v_{K,k} := \frac{1}{|K| \triangle z_{K,k}} \sum_{e \in \partial K} v_{e,k} |e| \Delta z_{e,k} (\vec{x}_e - \vec{x}_K),$$

The transposed operator is for $F \in \mathbf{H}_{\mathcal{C}_k}$ given by

$$P^T F_{K|L} := \frac{1}{|K|L^{\perp}|} \left\{ \vec{F}_K d_{e,K} + \vec{F}_L d_{e,L} \right\} \cdot \mathbf{n}_{K|L}.$$

Simulations

Perot reconstruction is used in current experimental configuration.

P.K., L. Linardakis, A conservative discretization of the shallow-water equations on triangular grids, J. Comp. Phys. 2018

Filter of Divergence Noise on Triangular C-grid

Filter Divergence Noise

ICON-O contains a mathematical operator/matrix $\mathcal{M}_{\mathcal{G}} := \mathcal{P}_{\mathcal{G}}^T \mathcal{P}_{\mathcal{G}}$ such that

- $$\begin{aligned} \gamma \text{ is a noise field} \quad &\Rightarrow \quad \mathcal{M}_{\mathcal{G}}\gamma = 0 \\ \mathcal{M}_{\mathcal{G}}\gamma = 0 \quad &\Rightarrow \quad \gamma \text{ is a noise field .} \end{aligned}$$
- Nullspace of $\mathcal{M}_{\mathcal{G}}$ = Space of Divergence Noise

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Decomposition of Velocity Space

$$\text{Velocity Space} = \text{ Nullspace of } \mathcal{M}_\mathcal{G} \oplus \text{Physical Space}$$

$$\mathcal{H}_{\mathcal{E}_k} = \mathit{NULL}_{\mathcal{M}_\mathcal{G}} \oplus \mathcal{M}_\mathcal{G}(\mathcal{H}_{\mathcal{E}_k})$$

P.K., L. Linardakis, A conservative discretization of the shallow-water equations on triangular grids, J. Comp. Phys. 2018

Choice of Energy Norm

Kinetic Energy

$$E_K^{kin} := \frac{\mathcal{P}_{\mathcal{G}} \mathbf{v}_K \cdot \mathcal{P}_{\mathcal{G}} \mathbf{v}_K}{2}, \quad E_K^{kin} \text{ scalar at cell } K$$

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Energy Equation & Time Derivative

The form of the kinetic energy implies for the time derivative

$$\int \underbrace{\frac{\partial \mathbf{v}}{\partial t} \cdot \mathbf{v}\, dx}_{=\frac{1}{2}\frac{d}{dt}||v||_{L^2}^2} \rightsquigarrow \left\langle \frac{d}{dt} \underbrace{\mathcal{P}_\mathcal{G} v, \mathcal{P}_\mathcal{G} v}_{=\frac{1}{2}\frac{d}{dt}||\mathcal{P}_\mathcal{G} v||_{\mathbf{H}_{\mathcal{C}_k}}} \right\rangle_{\mathbf{H}_{\mathcal{C}_k}} = \left\langle \frac{d}{dt} \mathcal{P}_\mathcal{G}^T \mathcal{P}_\mathcal{G} v, v \right\rangle_{\mathcal{H}_{\mathcal{E}_k}}.$$

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Choice of Kinetic Energy and Consequences

$$\textbf{Velocity}: \left\langle \frac{d}{dt}\mathcal{P}_{\mathcal{G}}^{T}\mathcal{P}_{\mathcal{G}}v, \phi \right\rangle_{\mathcal{H}_{\mathcal{E}_k}} + \left\langle \hat{\mathcal{P}}_{\mathcal{G}}^{T}[(f+\omega)\hat{\mathcal{P}}_{\mathcal{G}}v], \phi \right\rangle_{\mathcal{H}_{\mathcal{E}_k}}$$

$$+ \left\langle \mathcal{P}_{\mathcal{G}}^{T}\mathcal{Q}_{\mathcal{G}}(w\mathbf{D_z}\mathcal{P}_{\mathcal{G}}v), \phi \right\rangle_{\mathcal{H}_{\mathcal{E}_k}} + \left\langle \mathcal{P}_{\mathcal{G}}^{T}\mathcal{P}_{\mathcal{G}}\mathbf{grad_n}[\frac{|\mathcal{P}_{\mathcal{G}}v|^2_{\mathbb{R}^3}}{2}], \phi \right\rangle_{\mathcal{H}_{\mathcal{E}_k}}$$

$$+ \left\langle \mathcal{P}_{\mathcal{G}}^{T}\mathcal{P}_{\mathcal{G}}\mathbf{grad_n}(g\eta + p_{hyd}), \phi \right\rangle_{\mathcal{H}_{\mathcal{E}_k}} - \left\langle Lv, \phi \right\rangle_{\mathcal{H}_{\mathcal{E}_k}} = 0$$

$$\textbf{Incompress.}: \mathbf{div}(\mathcal{P}_{\mathcal{G}}^{T}\mathcal{P}_{\mathcal{G}}v) + \mathbf{D_z}w = 0$$

$$\textbf{Free Surface}: \left\langle \frac{\partial \eta}{\partial t}, \psi \right\rangle_{\mathcal{H}_{\mathcal{C}_k}} + \left\langle \mathbf{div}[\sum_{k=N_z}^{k=N_{top}} \mathcal{P}_{\mathcal{G}}^{T}\mathcal{P}_{\mathcal{G}}v_{K_k}\Delta z_{K_k}], \psi \right\rangle_{\mathcal{H}_{\mathcal{C}_k}} = 0$$

$$\textbf{Tracer}: \left\langle \frac{\partial C}{\partial t}, \psi \right\rangle_{\mathcal{H}_{\mathcal{C}_k}} - \left\langle \mathbf{div}\mathcal{F}[\mathcal{P}_{\mathcal{G}}^{T}(C\mathcal{P}_{\mathcal{G}}v)], \psi \right\rangle_{\mathcal{H}_{\mathcal{C}_k}} + \left\langle LC, \psi \right\rangle_{\mathcal{H}_{\mathcal{C}_k}} = 0$$

Choice of Kinetic Energy and Consequences

$$\textbf{Velocity}: \left\langle \frac{d}{dt}\mathcal{M}_{\mathcal{G}} v, \phi \right\rangle_{\mathcal{H}_{\mathcal{E}_k}} + \left\langle \hat{\mathcal{P}}_{\mathcal{G}}^T [(f+\omega)\hat{\mathcal{P}}_{\mathcal{G}} v], \phi \right\rangle_{\mathcal{H}_{\mathcal{E}_k}}$$

$$+ \left\langle \mathcal{P}_{\mathcal{G}}^T \mathcal{Q}_{\mathcal{G}}(w\mathbf{D_z}\mathcal{P}_{\mathcal{G}} v), \phi \right\rangle_{\mathcal{H}_{\mathcal{E}_k}} + \left\langle \mathcal{P}_{\mathcal{G}}^T \mathcal{P}_{\mathcal{G}} \mathbf{grad_n}[\frac{|\mathcal{P}_{\mathcal{G}} v|^2_{\mathbb{R}^3}}{2}], \phi \right\rangle_{\mathcal{H}_{\mathcal{E}_k}}$$

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Choice of Kinetic Energy and Consequences

$$\textbf{Velocity}: \left\langle \frac{d}{dt}\mathcal{M}_{\mathcal{G}} v, \phi \right\rangle_{\mathcal{H}_{\mathcal{E}_k}} + \left\langle \hat{\mathcal{P}}_{\mathcal{G}}^T [(f+\omega)\hat{\mathcal{P}}_{\mathcal{G}} v], \phi \right\rangle_{\mathcal{H}_{\mathcal{E}_k}}$$

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P. K. Formulation of an Unstructured Grid Model for Global Ocean Dynamics (J. Comp. Phys. 339 (2017))

Conservation: Volume & Tracer Concentration

- If $(\mathcal{P}_{\mathcal{G}}, \hat{\mathcal{P}}_{\mathcal{G}})$ is an admissible reconstruction then the semi-discretized ocean equations conserve volume and tracer, more precisely they satisfy

$$\frac{\mathbf{d}\mathcal{V}}{\mathbf{dt}} = \mathbf{0} \quad \text{and} \quad \frac{\mathbf{d}\mathcal{T}}{\mathbf{dt}} = \mathbf{0},$$

Volume and tracer budgets are given by

$$\mathcal{V}(t) := \sum_{k=1}^{n_z} \sum_{K \in \mathcal{C}_k} |K| \Delta z_{k,K}, \; \mathcal{T}(t) := \sum_{k=1}^{n_z} \sum_{K \in \mathcal{C}_k} C_{k,K} |K| \Delta z_{k,K},$$

- The semi-discretized ocean equations conserve tracer variance, provided the tracer diffusivity vanishes

$$\left\langle \frac{d}{dt}(\Delta z T^2) \right\rangle_{\mathcal{H}_{\mathcal{C}}} = 0$$

Conservation: Total Energy

Theorem: Energy Conservation

If $(\mathcal{P}_\mathcal{G}, \hat{\mathcal{P}}_\mathcal{G})$ is an admissible reconstruction and external forcing and viscous dissipation are absent then the global energy is conserved

$$\frac{d}{dt}E = 0,$$

where the global total energy is defined as

$$E(t) := \sum_{k\in\mathcal{Z}} \sum_{K\in\mathcal{C}_k} \left(E^{kin}_{K,k}(t) + E^{pot}_{K,k}(t) \right)$$

with kinetic and potential energy per cell given by

$$E^{kin}_{K,k}(t) := \frac{|\mathcal{P}_\mathcal{G} v_{K,k}(t)|^2}{2} |K| \, \Delta z_{K,k},$$
$$E^{pot}_{K,k}(t) := \mathcal{Q}_\mathcal{G}(w_{K,k}(t)) \, \rho_{K,k}(t) |K| \Delta z_{K,k}.$$

ICON-O: Extension to Generalized Vertical Coordinates

$$\textbf{Velocity}: \Big\langle \frac{d}{dt}\mathcal{M}_{\mathcal{G}} v, \phi \Big\rangle_{\mathcal{H}_{\mathcal{E}_k}} + \Big\langle \hat{\mathcal{M}}_{\mathcal{G}}[(f+\omega), v], \phi \Big\rangle_{\mathcal{H}_{\mathcal{E}_k}}$$
$$+ \Big\langle \mathcal{P}_{\mathcal{G}}{}^T \mathcal{Q}_{\mathcal{G}}(\dot{s}\mathbf{D_z}\mathcal{P}_{\mathcal{G}} v), \phi \Big\rangle_{\mathcal{H}_{\mathcal{E}_k}} + \Big\langle \mathcal{M}_{\mathcal{G}}\mathbf{grad_n}[\frac{|\mathcal{P}_{\mathcal{G}} v|^2_{\mathbb{R}^3}}{2}], \phi \Big\rangle_{\mathcal{H}_{\mathcal{E}_k}}$$
$$+ \Big\langle \mathcal{M}_{\mathcal{G}}\mathbf{grad_n} p_{hyd} + g\eta, \phi \Big\rangle_{\mathcal{H}_{\mathcal{E}_k}} + \Big\langle \mathcal{M}_{\mathcal{G}}(g\rho)\mathcal{P}_{\mathcal{G}}{}^T \mathcal{Q}_{\mathcal{G}}\mathbf{grad_n} z, \phi \Big\rangle_{\mathcal{H}_{\mathcal{E}}}$$
$$- \Big\langle Lv, \phi \Big\rangle_{\mathcal{H}_{\mathcal{E}_k}} = 0,$$

$$\textbf{Continuity}: d_t z_{,s}\Delta s + \mathbf{div}(\mathcal{M}_{\mathcal{G}}[z_{,s}\Delta s, v]) + \mathbf{D}_{\bar{s}}(\widetilde{z_{,s}\Delta s}\dot{s}) = 0,$$

$$\textbf{Surface}: \Big\langle \frac{\partial \eta}{\partial t}, \psi \Big\rangle_{\mathcal{H}_{\mathcal{C}}} + \sum_{k=1}^{k_{bot}} \Big\langle \mathbf{div}[\mathcal{M}_{\mathcal{G}}[z_{,s}\Delta s_k, v_k], \psi \Big\rangle_{\mathcal{H}_{\mathcal{C}_k}} = 0,$$

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Conservation Properties: Generalized Vertical Coordinates

Lagrangian vs Eulerian treatment of vertical direction in continuity eq.

- Eulerian: use continuity eq. diagnostically
- Lagrangian: use continuity eq. prognostically, requires remapping

Corollary: Conservation Properties - Eulerian Approach

The conservation properties of ICON-O with vertical z-coordinate translate to the case of generalized vertical coordinates if the vertical direction in the continuity eq. is computed in an Eulerian way.

V. Singh, P. K. A Structure-Preserving Discretization of the Ocean Primitive Equations in Generalized Vertical Coordinates (in preparation)

ICON-O: New Sea-Ice Dynamics

Governing Equations

Momentum

$$\underbrace{m}_{\textit{sea ice mass}} \partial_t v = \underbrace{f_c(v)}_{\textit{Coriolis force}} + \underbrace{\tau(v)}_{\textit{atmos/ocean stress}} + \underbrace{div(\sigma)}_{\textit{internal stress}}$$

Rheology

$$\sigma := \frac{1}{2} \underbrace{\eta(v)}_{\textit{viscosity}} \left(\nabla v + \nabla^T v\right) + \frac{3}{4}\eta(v) tr\left(\nabla v + \nabla^T v\right) I + \underbrace{\frac{1}{2} P(h, A) I}_{\textit{icestrength}}, \qquad \text{with } \eta(v) := \frac{P(h, A)}{\Delta v)}$$

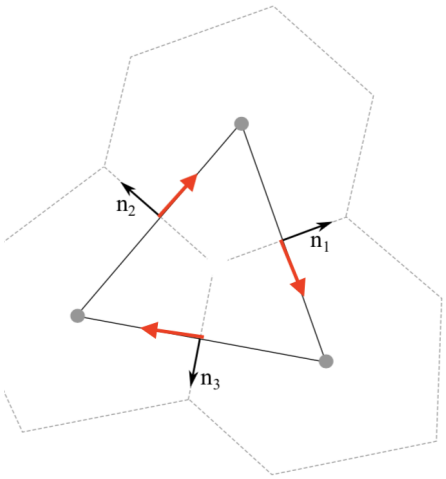
Challenge

Compute strain rate tensor $(\nabla v + \nabla^T v)$ if only normal velocity component is available.

ICON-O: New Sea-Ice Dynamics

Solution

- Enlarge velocity space: from C-grid (Raviart-Thomas FE) to full vector at edges (Crouzeix-Raviart FE)
- **But:** CR element does **not satisfy Korn's inequality**: $c||\nabla v|| \leq ||\nabla v + \nabla^T v||$, with c independent of grid size. This prohibits convergence.
- $\Rightarrow$ Additional stabilization is required.

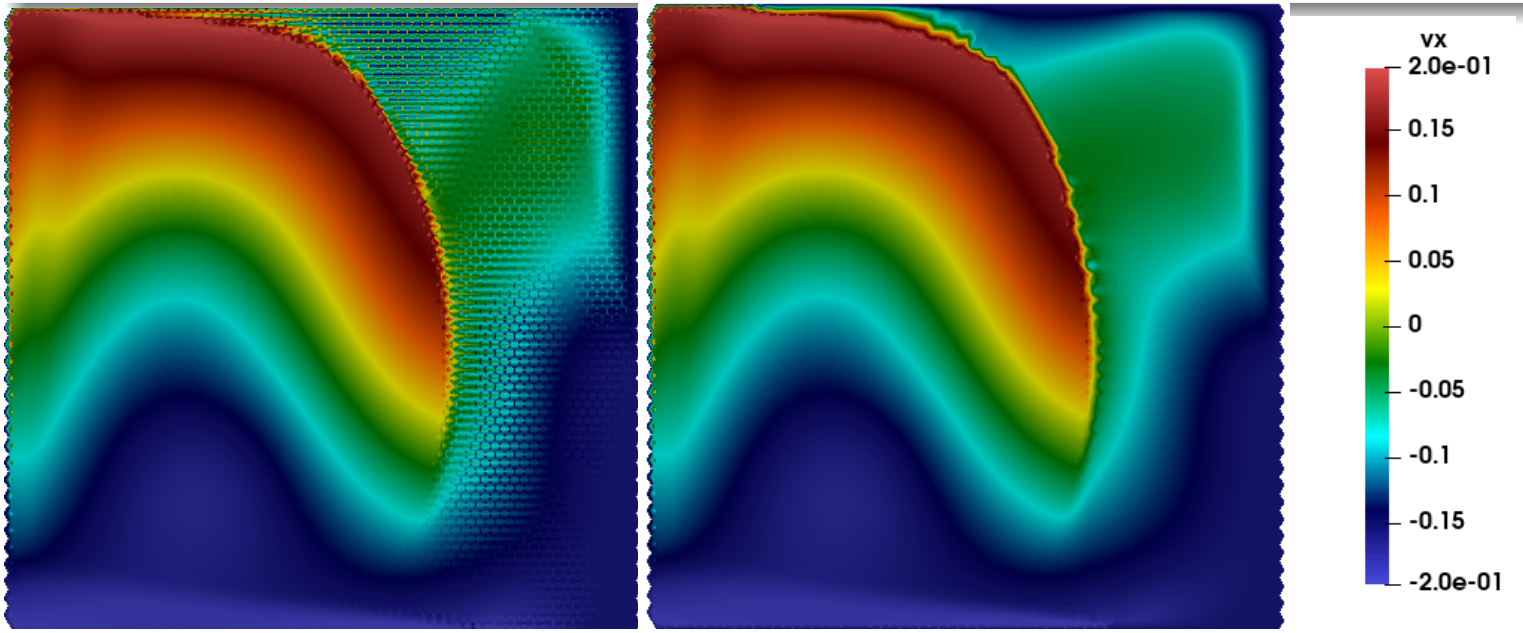


C.Mehlmann, P. K. Sea-Ice dynamics on triangular grids (in review)

ICON-O: New Sea-Ice Dynamics

Idealized Test with CR Element

- Simulation of ice dynamics on rectangular box with prescribed winds and ocean currents
- Shown sea-ice velocities (x-component) after 1 month
- **Left:** Without stabilization **Right:** With stabilization



New Sea-ice dynamics within EVP solver currently being implemented in ICON-O.

Outline

1. Overview of ICON Modelling System
2. Numerics (emphasis on ICON-O)
3. A Range of ICON-Simulations Across Scales

Options to configure ICON-O

Model Details

- **Vertical coordinate:** z-coordinate, z^*
- **Transport algorithm:** flux-corrected transport
- **Time stepping:** semi-implicit Adams-Bashford
- Fractional Land-Sea mask, removal of land-points in ocean domain
- **Parametrizations**
  - Isoneutral diffusion & eddy-induced advection of Gent-McWilliams
  - Vertical mixing: TKE, KPP
  - Internal wave parametrization: IDEMIX
  - Langmuir parametrization
  - Velocity dissipation: Biharmonic, Leith
- MPI & OpenMP parallel

A Range of ICON-Simulations Across Scales

Resolution	Duration	Coupled	Goal
$160km$	10ky	yes	paleoclimate dynamics
$40km$	1ky	yes	climate change
$10km$	100y	no	ocean variability
$5km$	1y	yes	air-sea coupling
$12km - 600m$	1y	no	oceanic energy cascades

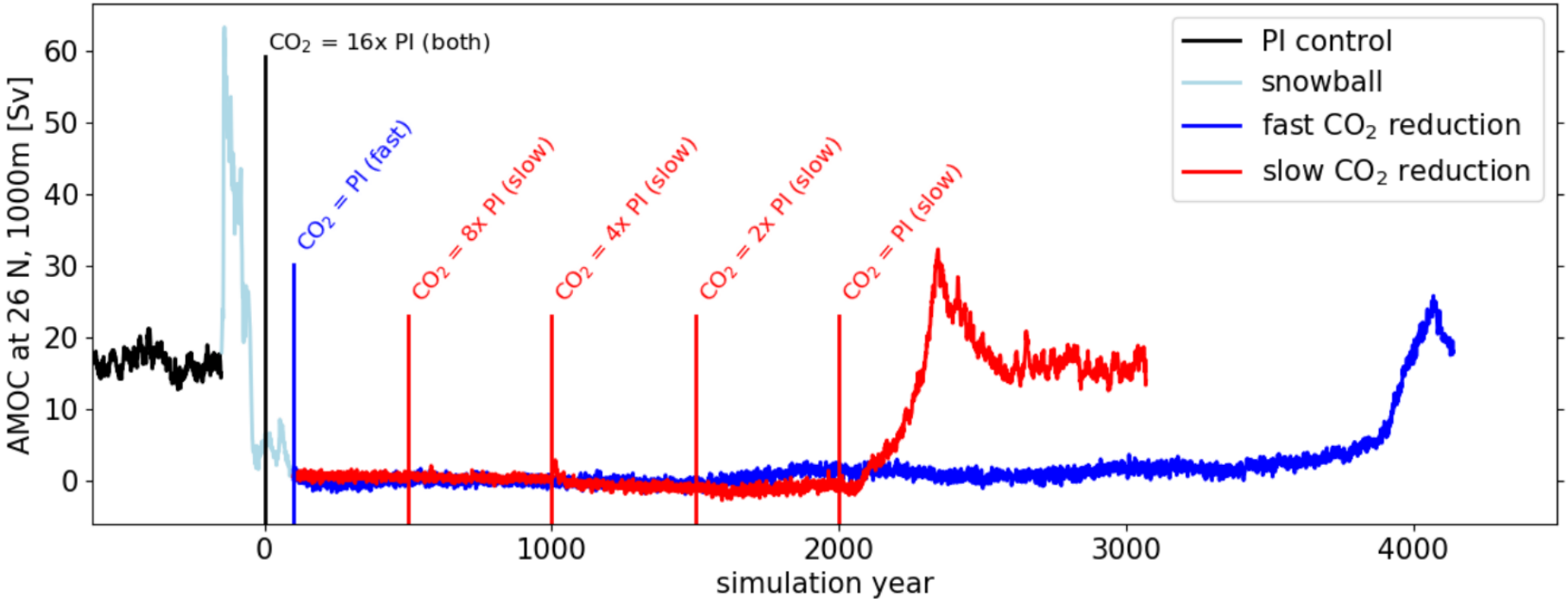
Snowball Earth Hypothesis

- Earth was nearly ice covered $\sim$ 640 My ago
- Snowball terminated by accumulation of CO_2 concentrations
- **Question:** How does ocean circulation look after snowball Earth ?

Experiment with ICON-ESM

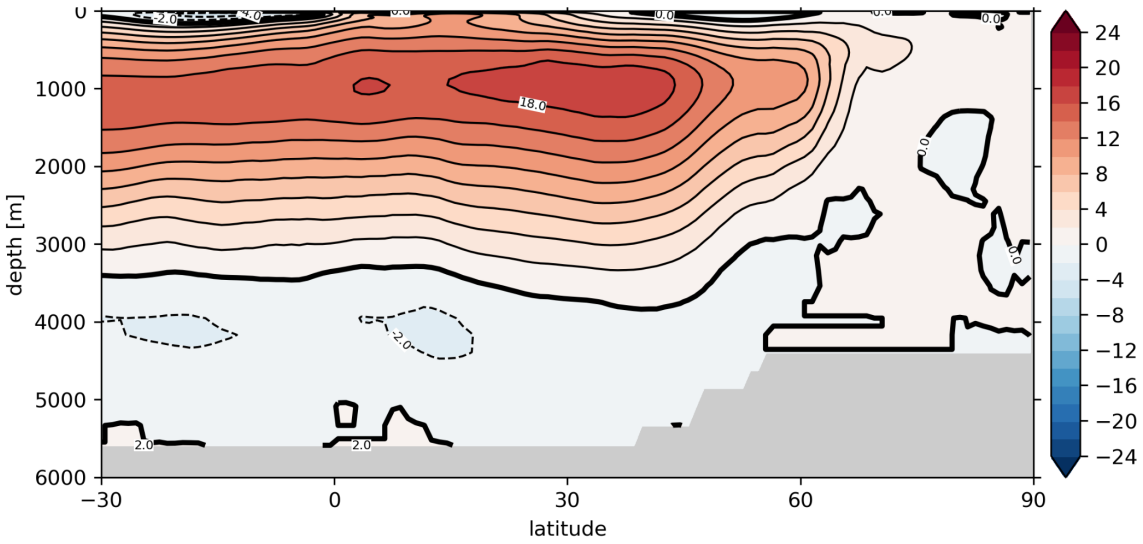
1. Create snowball Earth by reduction of incoming solar radiation
2. Melt snowball by increasing CO_2 for 100 years
3. Reduce CO_2 again
   1. Fast: set CO_2 to pre-industrial value after 100y
   2. Slow: start from 16x pre-industrial CO_2 and half every 500y until pi-value

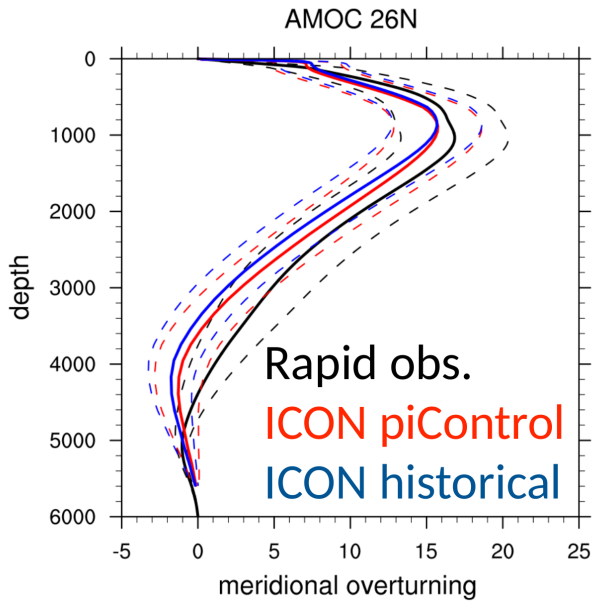
Paleoclimate Dynamics with ICON-ESM



Response of overturning at 26 N to different CO_2 reduction pathways

Climate Change at Long Time Scales: ICON-ESM at 40km

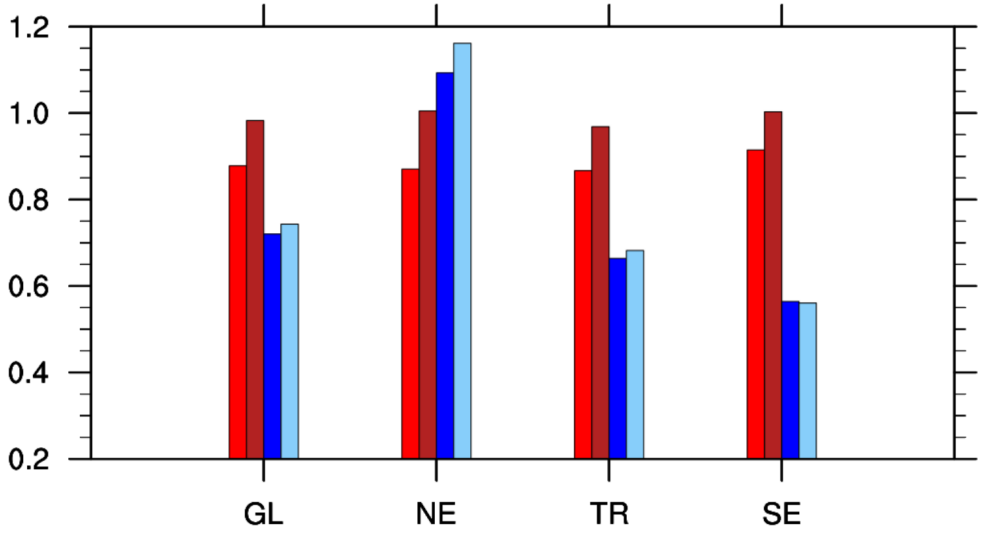




pi control simulation

ICON-ESM produces strength and structure of Atlantic Meridional Overturning Circulation

Climate Change at Long Time Scales: ICON-ESM at 40km

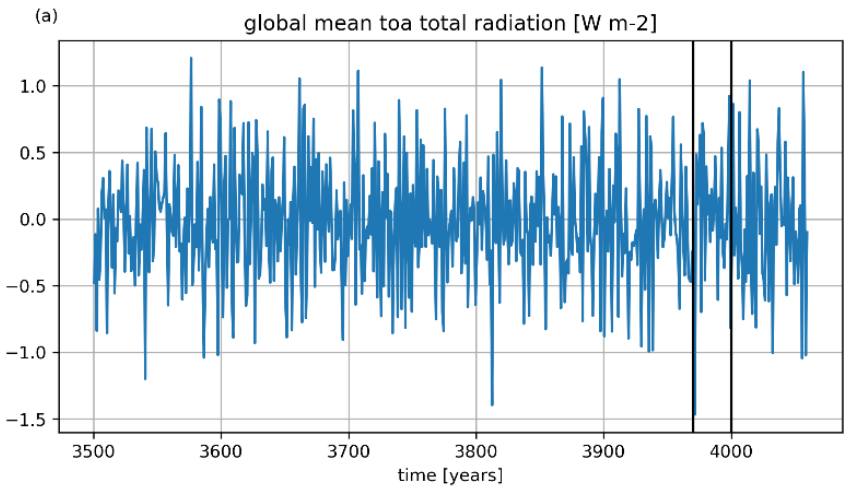


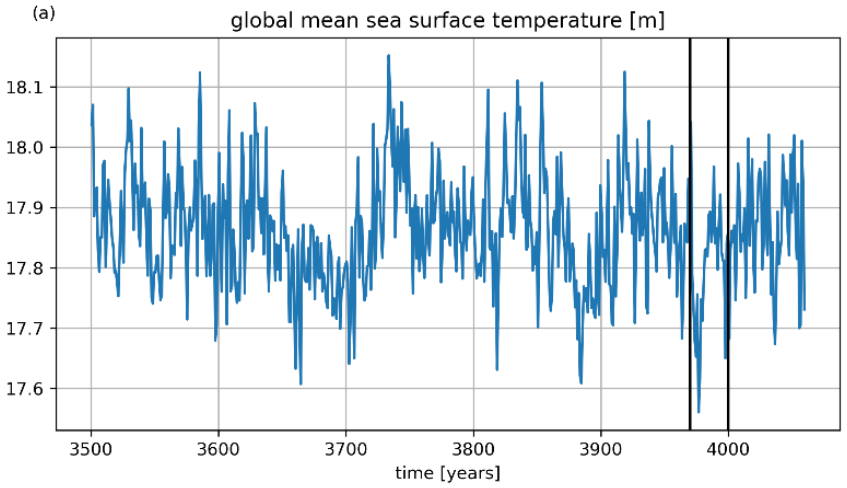
- Red: MPI-ESM
- Blue: ICON-ESM

Reichler-Kim Scores

- Standardized annual mean climatological errors of pi-control experiment of MPI-ESM and ICON-ESM
- Normalized such that MPI-ESM has bias of values 1
- A value larger/smaller than 1 indicates larger/smaller bias than MPI-ESM

Climate Change at Long Time Scales: ICON-ESM at 40km

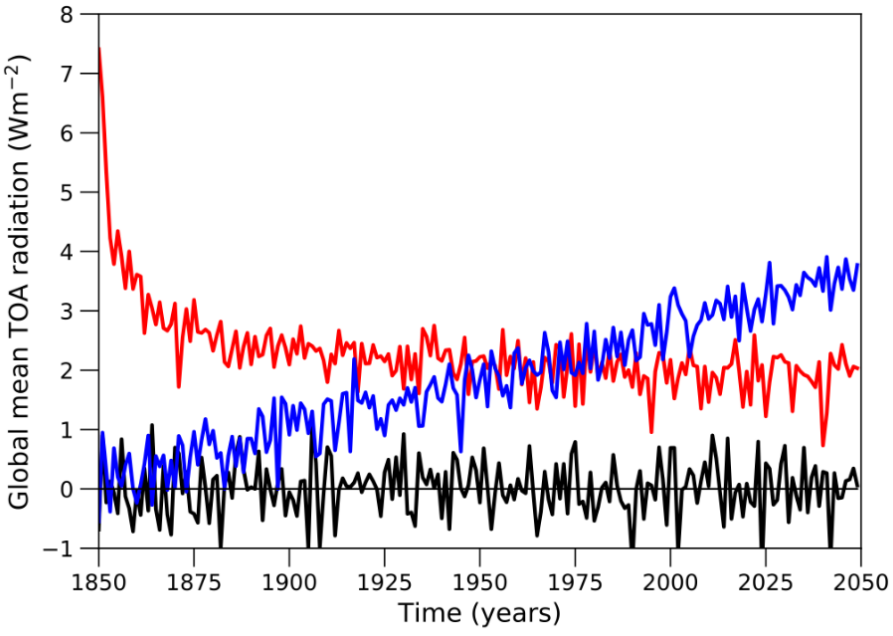


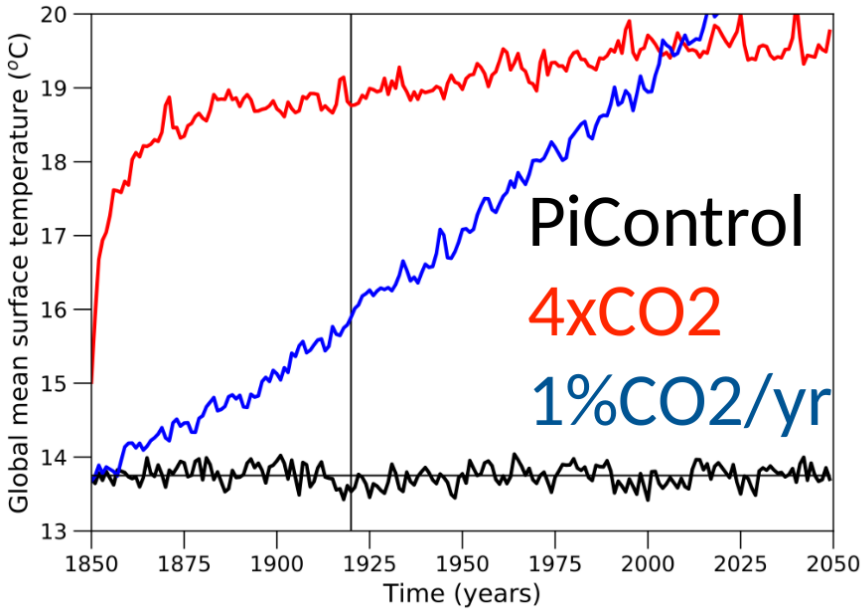


pi control simulation - no trend visible

- **Left:** global mean top of atmosphere radiation
- **Right:** global mean sea surface temperature, near pi-observation

Climate Change at Long Time Scales: ICON-ESM at 40km

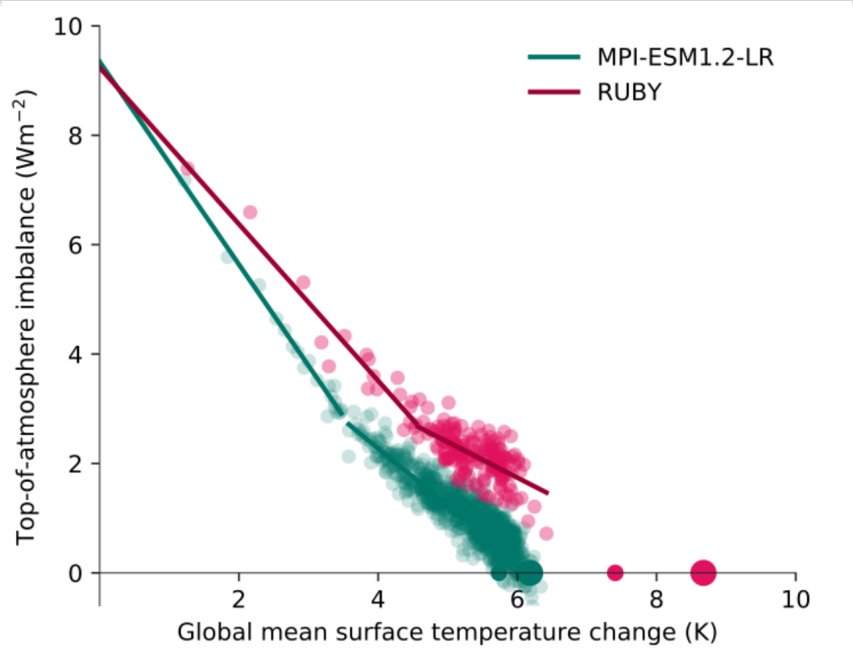




Response to CO_2-increase

- **Left:** global mean top of atmosphere radiation
- **Right:** global mean sea surface temperature

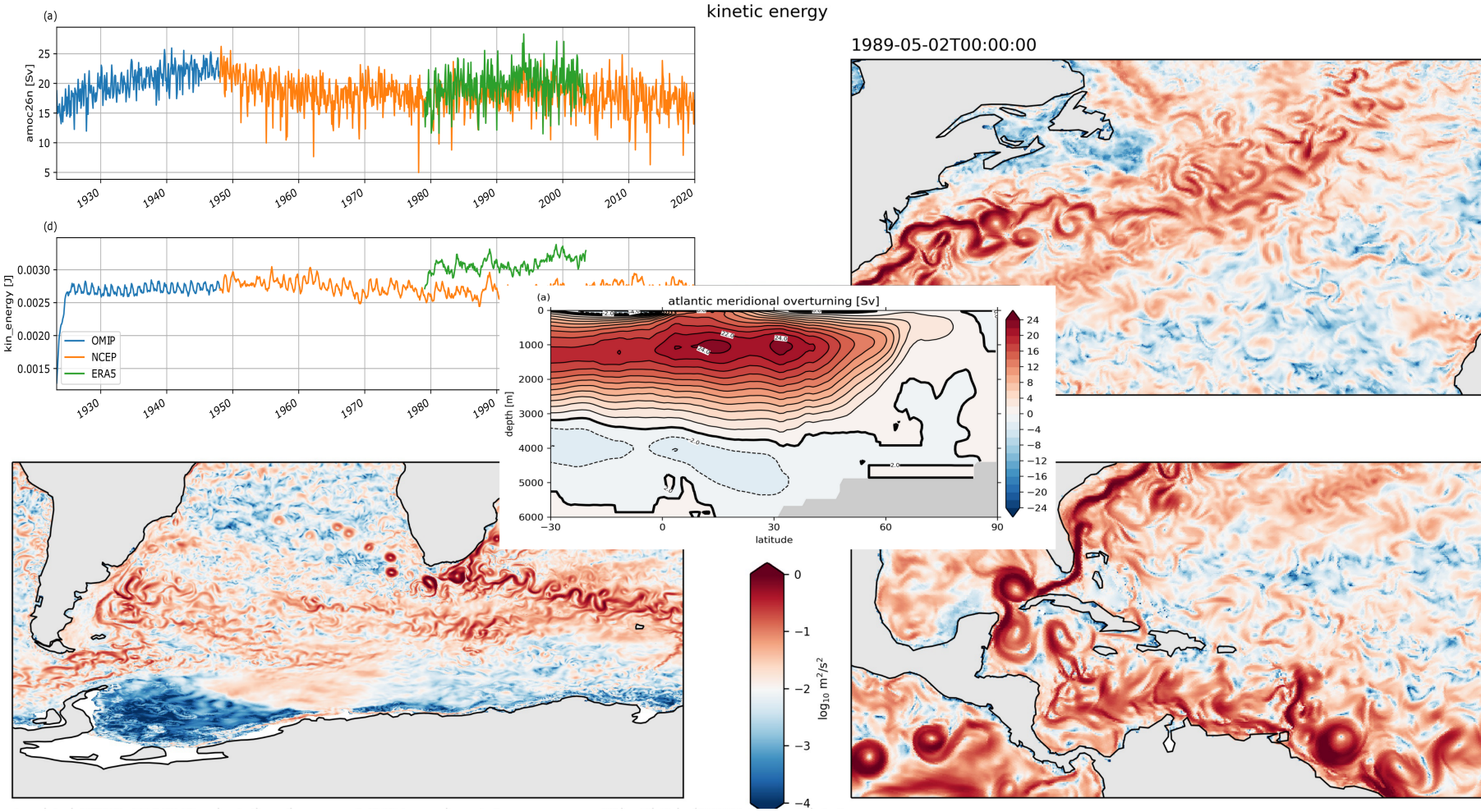
Climate Change at Long Time Scales: ICON-ESM at 40km



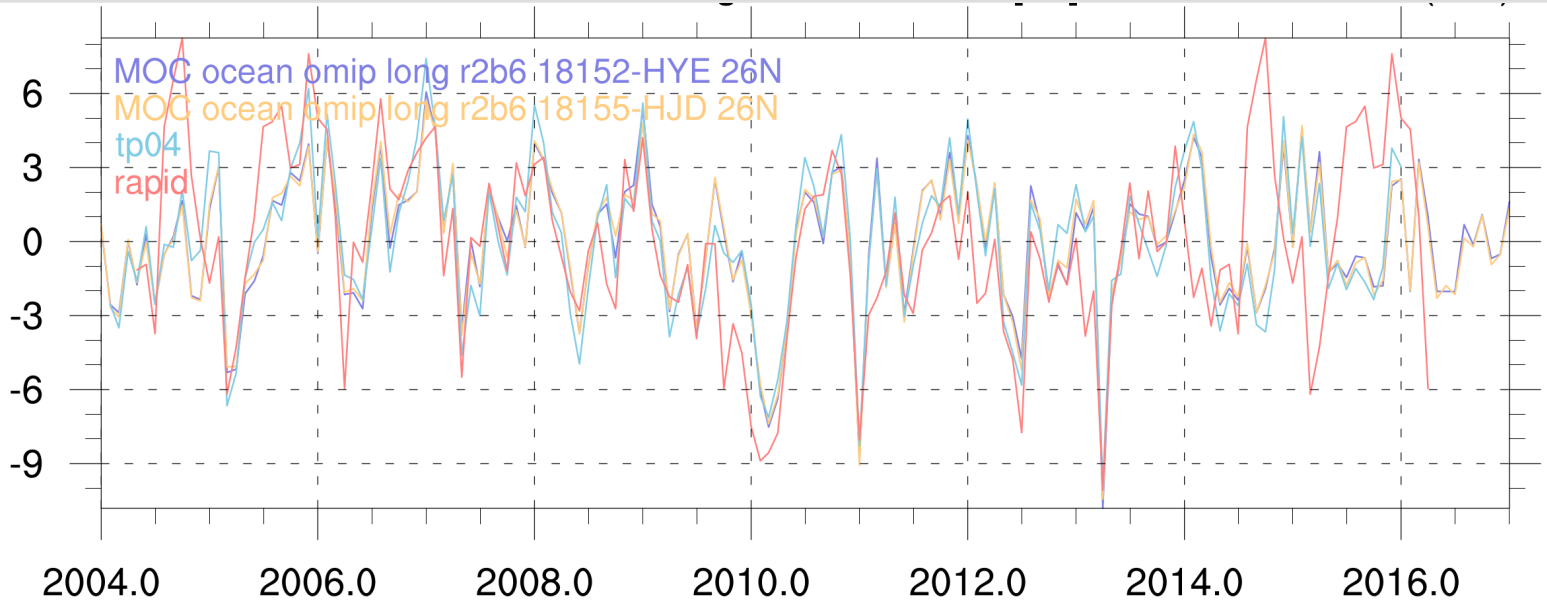
Climate sensitivity

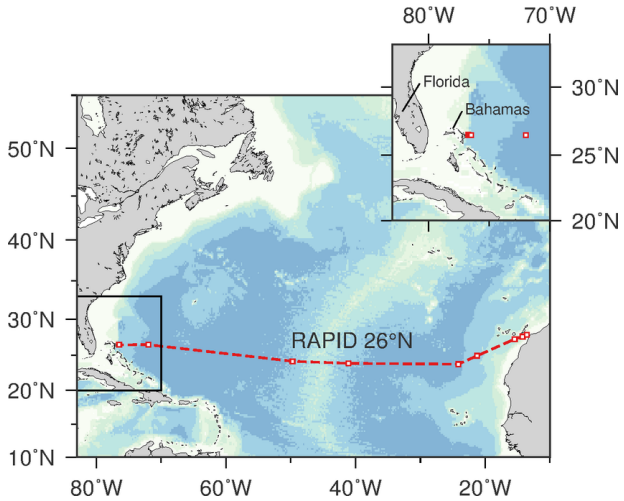
- Transient climate response: 2.1°
- Equilibrium climate sensitivity: 3.7°
- ICON-ESM and MPI-ESM have different climate sensitivities.

Ocean Variability - ICON-O at 10km

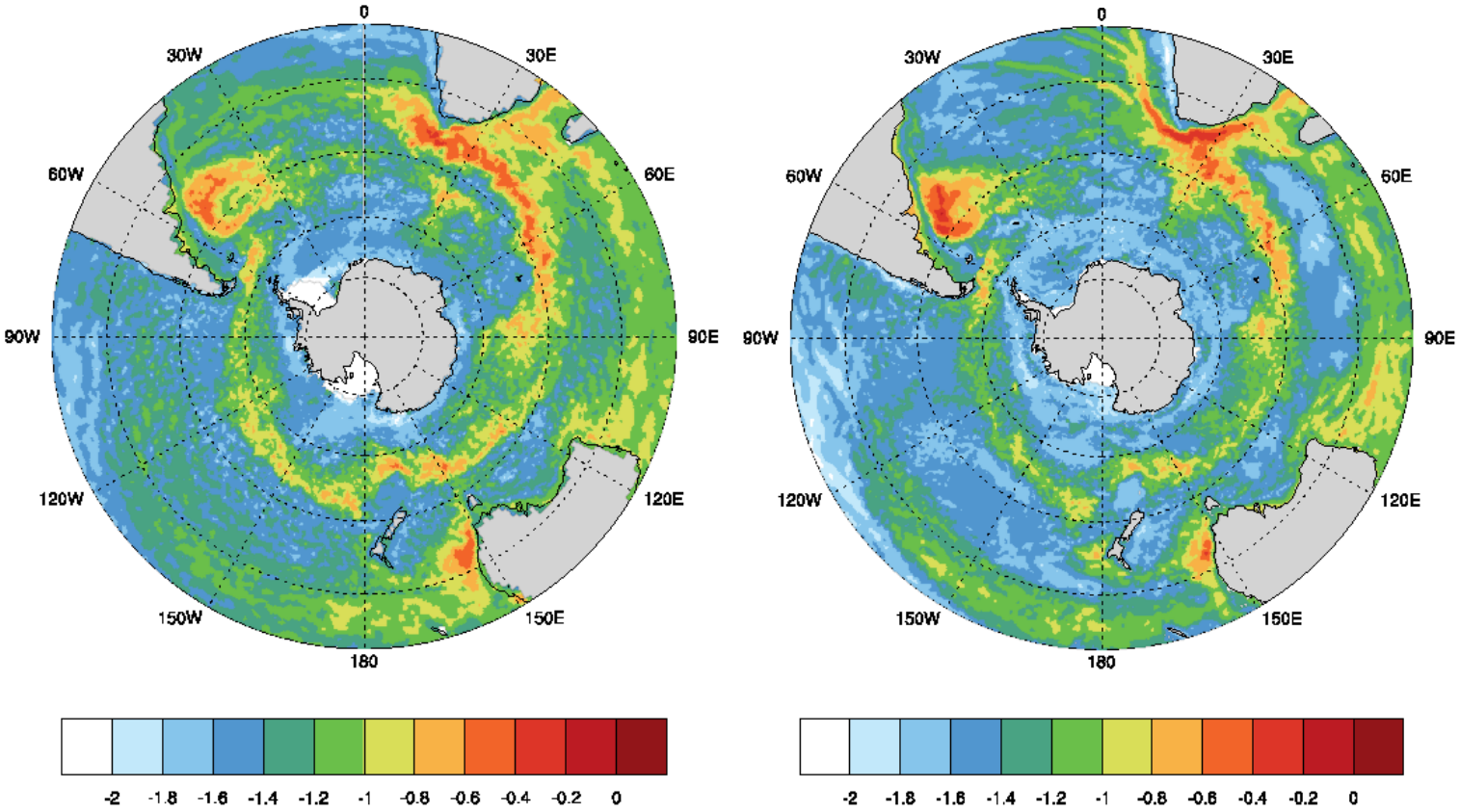


Ocean Variability - ICON-O at 10km: Comparison with Rapid Data





Ocean Variability - ICON-O at 10km: Comparison with Observations

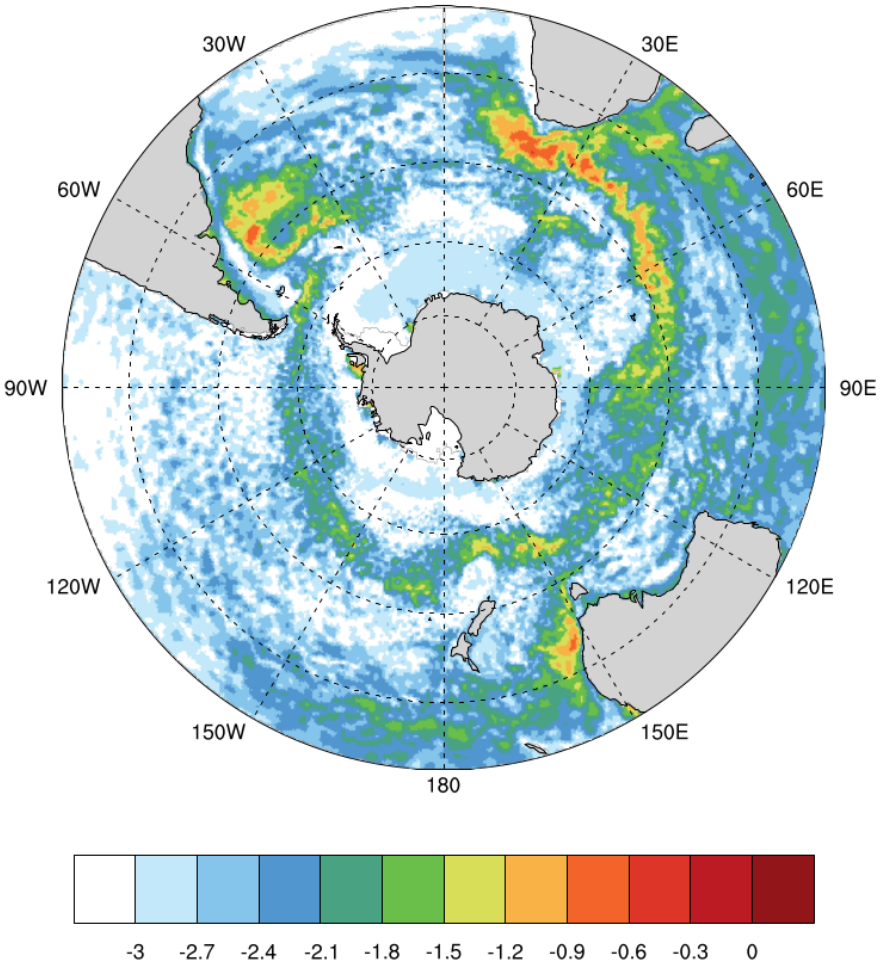


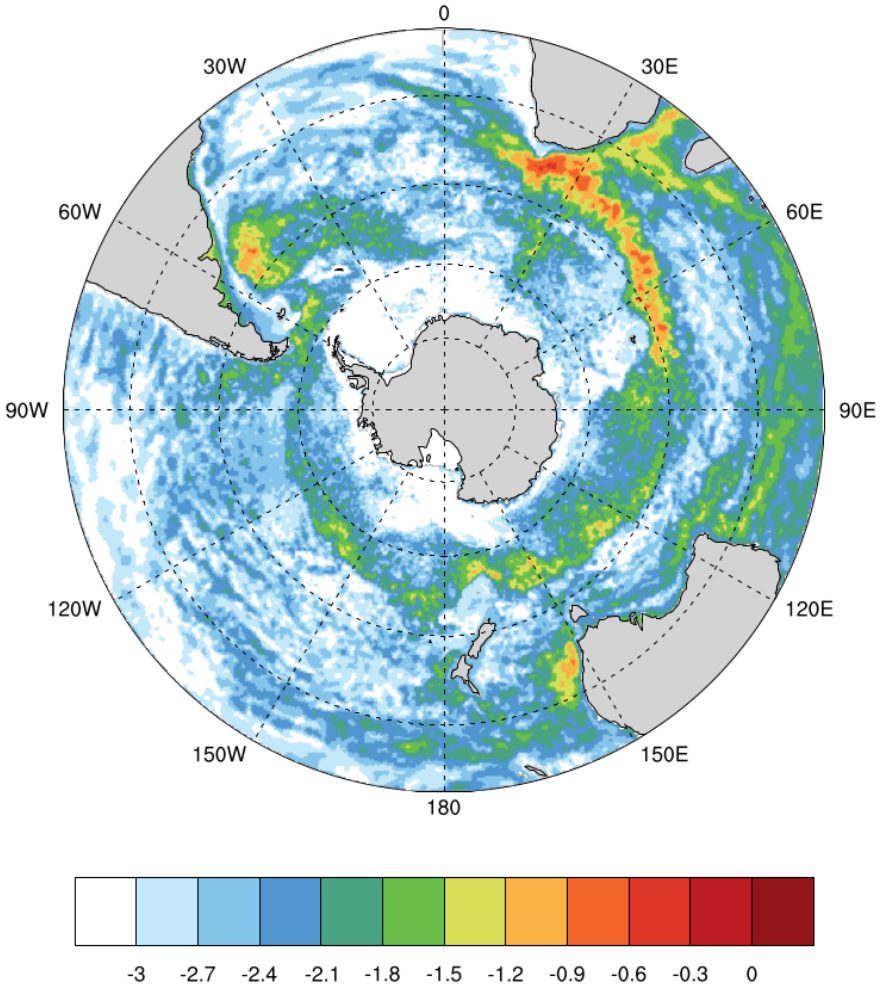
Standard deviation of sea-surface height

LEFT: Observations

RIGHT: ICON-O (10km horizontal resolution)

Ocean Variability - ICON-O at 5km: Comparison with Observations



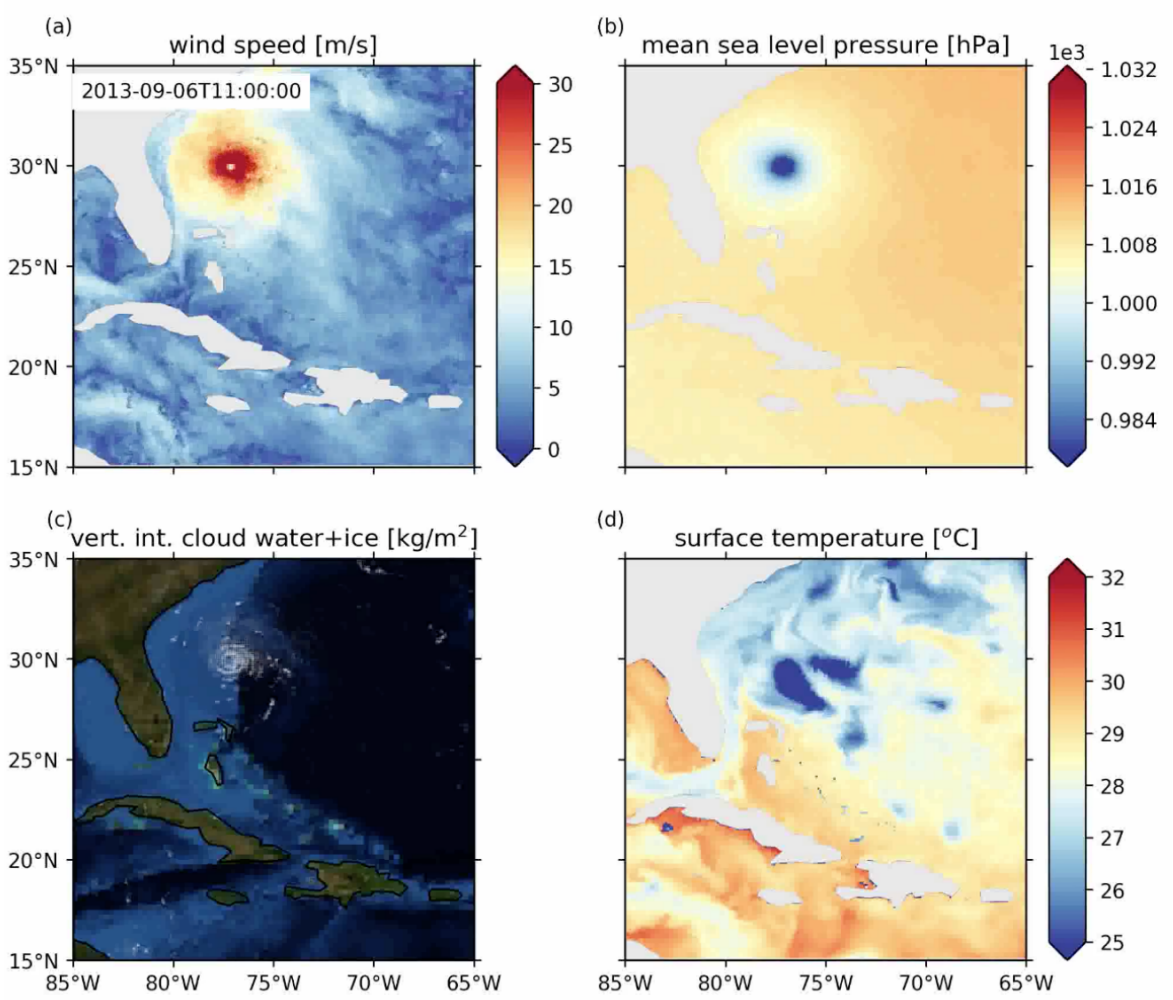


Standard deviation of sea-surface height

ICON-O at 5km - Global View

Start movie

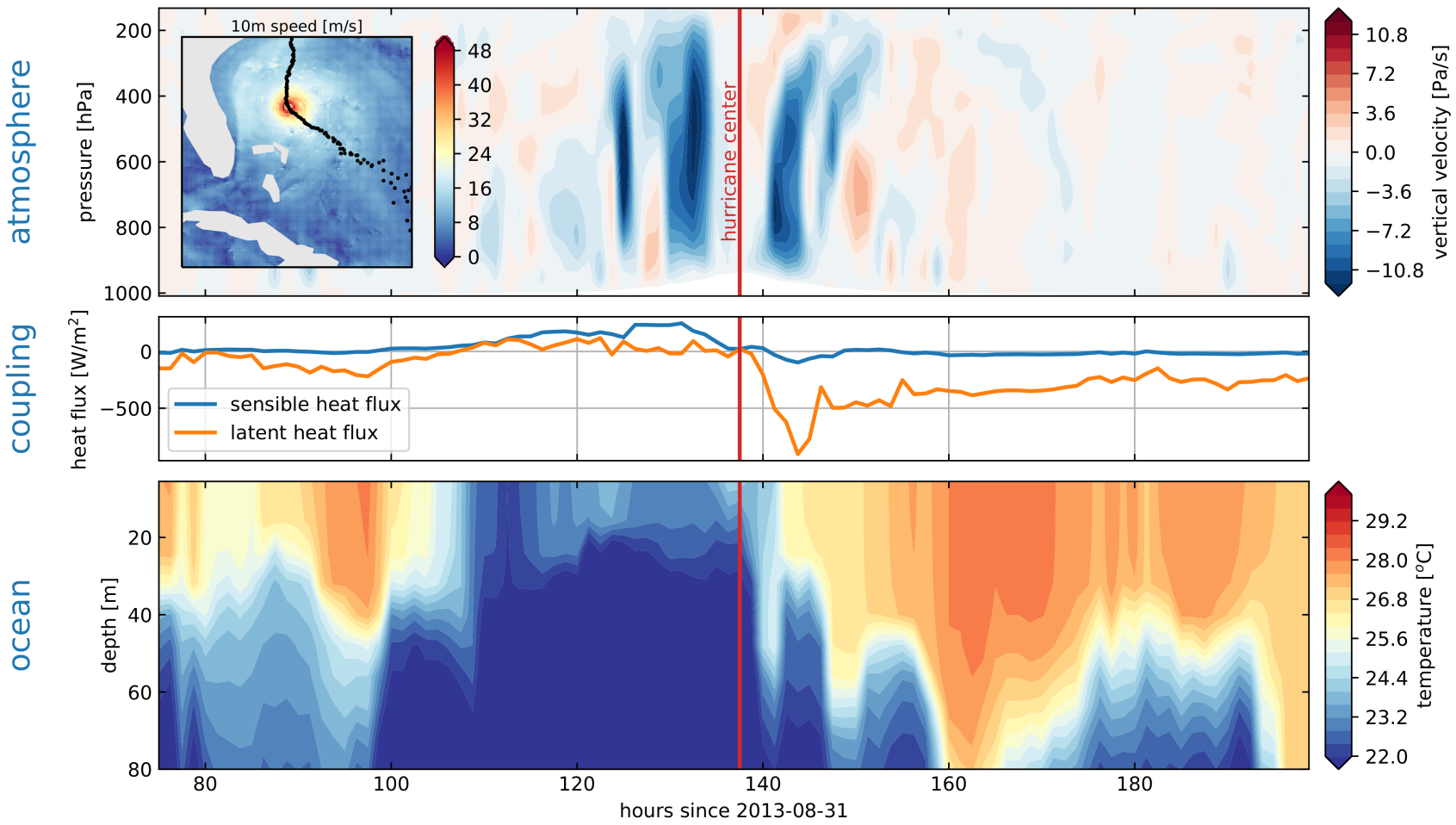
Air Sea Coupling - ICON-ESM at 5km



Coupled ICON-A|O simulation at 5km resolution

Hurricane signature in different fields

Air Sea Coupling - ICON-ESM at 5km

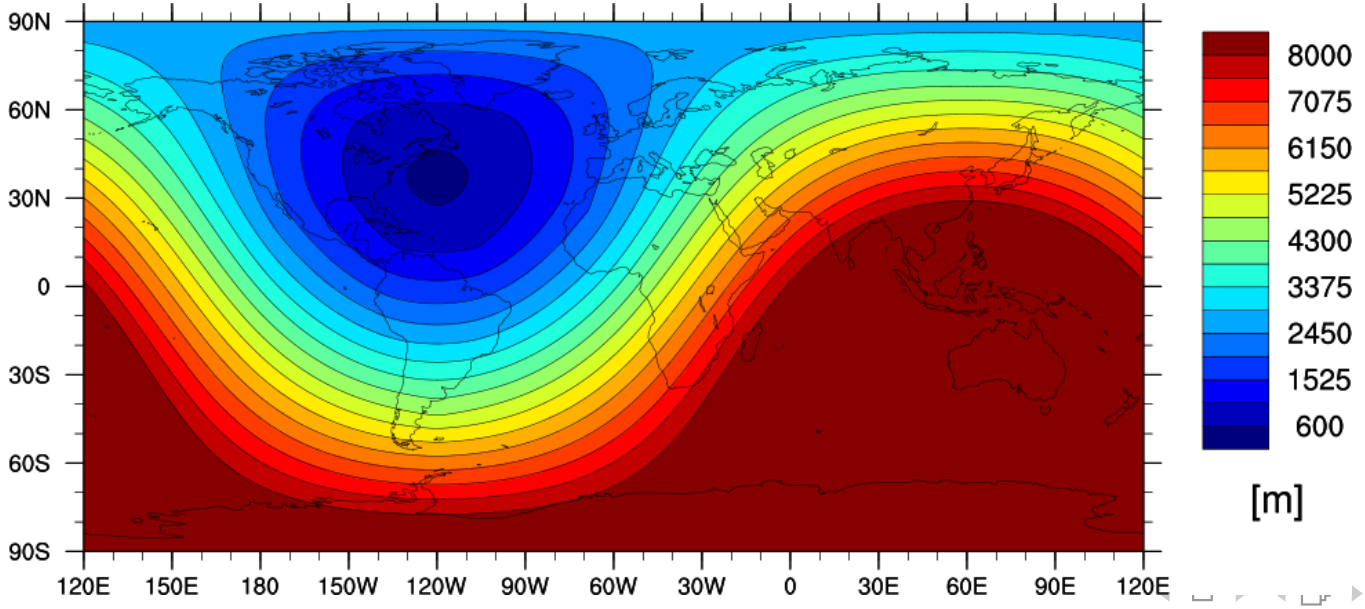


Coupled ICON-A|O simulation at 5km resolution

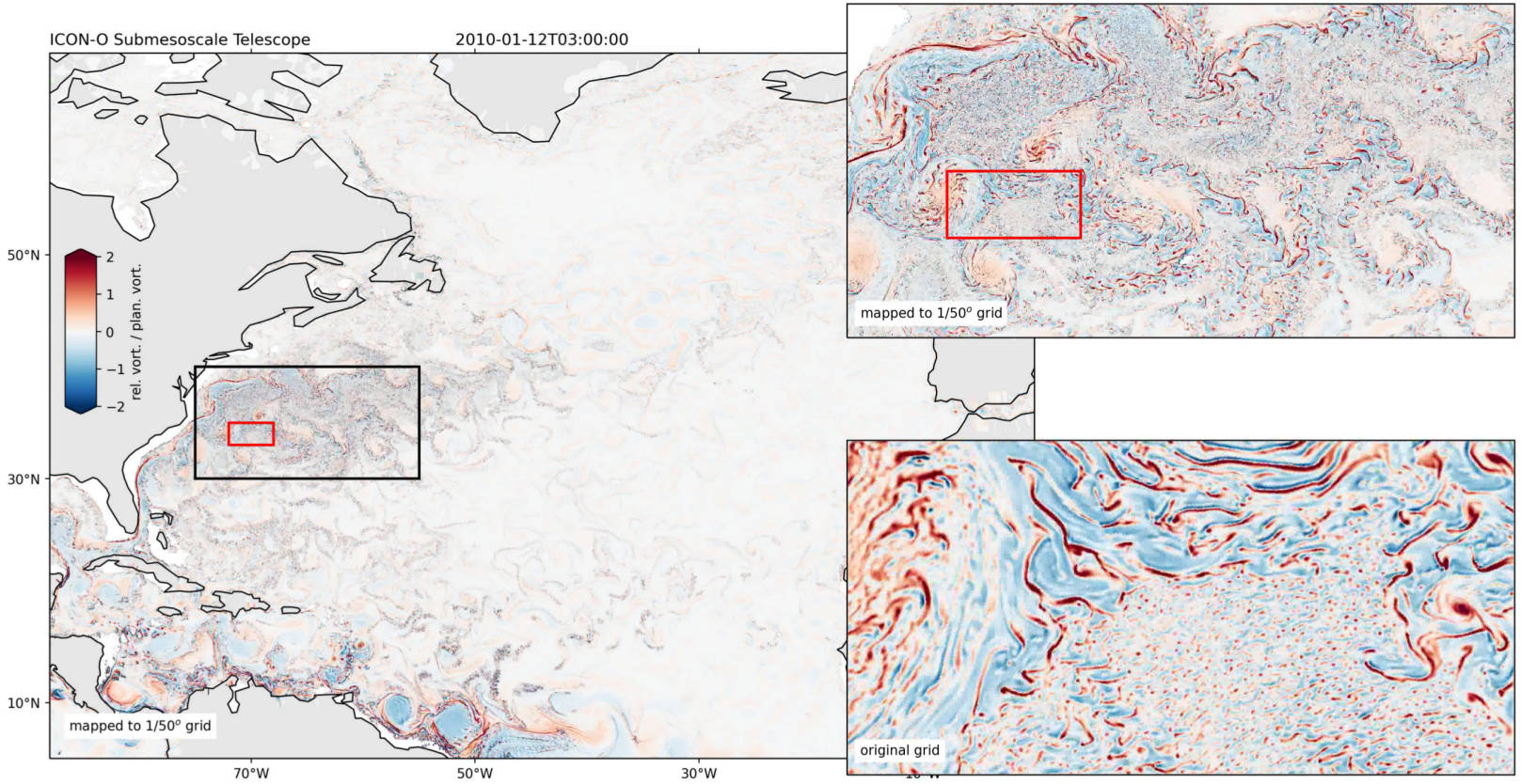
Energy Cascades: Submesoscale Turbulence via Computational Telescope

Create Computational Telescope

- Change locally grid resolution to achieve zooming effect
- Generate grid with a focal area in Gulf stream region
  - gridsize $< 600m$ in a circular area of 10° radius
  - gridsize $< 1000m$ in a circular area of 25° radius
  - and approaches outside a resolution of $10km$



Energy Cascades: Submesoscale Turbulence via Computational Telescope



Shown Local vorticity over planetary vorticity

Energy Cascades

Start movie

Thank you for your attention