Towards a more stable formulation for the Non-hydrostatic "constant-coefficient" ICI dynamical core of AROME system

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Outline

1. GENERALITIES ABOUT ICI TIME SCHEMES
2. RECIPES FOR STABLE CC-ICI TIME DISCRETISATION OF THE FULLY-COMPRESSIBLE SYSTEM
3. DESIGN OF AN ALTERNATE FORMULATION FOR A MORE STABLE CC-ICI SCHEME

Ideal time discretisation for NWP

Consider $\mathcal{X}$ a vector of prognostic variables governed by the non-linear dynamical system :

$$\frac{\partial \mathcal{X}}{\partial t} = \mathcal{M}\left(\mathcal{X}\right)$$

- Unconditionally stable time-scheme :

$$\frac{\mathcal{X}^{+} - \mathcal{X}^{0}}{\Delta t} = \frac{\mathcal{M}\left(\mathcal{X}^{+}\right) + \mathcal{M}\left(\mathcal{X}^{0}\right)}{2}$$

$\Rightarrow \mathcal{X}^{+}$ is solution of a non-linear differential equation :

$$\mathcal{F}\left(\mathcal{X}^{+}\right) := \left[\mathcal{X}^{+} - (\Delta t/2)\mathcal{M}\left(\mathcal{X}^{+}\right)\right] - \left[\mathcal{X}^{0} + (\Delta t/2)\mathcal{M}\left(\mathcal{X}^{0}\right)\right] = 0$$

* Superscripts $(+, 0)$ denote values at times $(t + \Delta t, t)$

Iterative-Centered-Implicit (ICI) time schemes

Newton method to the rescue :

$$\frac{\partial \mathcal{X}}{\partial t} = \underbrace{(\mathcal{M} - \mathcal{L})(\mathcal{X})}_{\textit{NL-residual}} + \underbrace{\mathcal{L}\mathcal{X}}_{\textit{Linear}}$$

- For $i \in [0, N_{\text{siter}}]$

$$\frac{\mathcal{X}^{+(i)} - \mathcal{X}^0}{\Delta t} = \frac{(\mathcal{M} - \mathcal{L})\left(\mathcal{X}^{+(i-1)}\right) + (\mathcal{M} - \mathcal{L})\left(\mathcal{X}^0\right)}{2} + \frac{\mathcal{L}\mathcal{X}^{+(i)} + \mathcal{L}\mathcal{X}^0}{2}$$

$\Rightarrow$

$$\mathcal{X}^{+(i)} = \mathcal{X}^{+(i-1)} - [\mathcal{I} - (\Delta t/2)\mathcal{L}]^{-1} . \mathcal{F}\left(\mathcal{X}^{+(i-1)}\right)$$

1. **Newton-Raphson approach** : $\mathcal{L} = (\partial\mathcal{M}/\partial\mathcal{X})$ is the true jacobian operator of $\mathcal{M}$ at $\mathcal{X}^{+(i-1)}$ $\Rightarrow$ $\mathcal{L}$ must be updated at each iteration.
2. **Quasi-Newton approach** : $\mathcal{L} = \mathcal{L}^*$ is taken stationary during all iterations. It can be chosen "more or less arbitrarily".

* $\mathcal{I}$ denotes identity operator in $\mathcal{X}$- space.

ICI scheme viewed as a correction :

- Starting from $\mathcal{X}^{+(-1)} = \mathcal{X}^0$,
- Iterations proceed for $i \in [0, N_{\text{siter}}]$ as

$$\mathcal{X}^{+(i)} = \overbrace{\left[\mathcal{X}^0 + \frac{\Delta t}{2} \left\{\mathcal{M}\left(\mathcal{X}^0\right) + \mathcal{M}\left(\mathcal{X}^{+(i-1)}\right)\right\}\right]}^{\widetilde{\mathcal{X}}^{+(i)}} + \underbrace{\frac{\Delta t}{2}\left[\mathcal{L}\mathcal{X}^{+(i)} - \mathcal{L}\mathcal{X}^{+(i-1)}\right]}_{\delta\mathcal{X}_{\text{ICI}}^{+(i)}}$$

- **Step 1** : Computation of provisional explicit guess, $\widetilde{\mathcal{X}}^{+(i)}$.
- **Step 2** : Computation of explicit part of ICI correction $\mathcal{X}^{\bullet} = \widetilde{\mathcal{X}}^{+(i)} - \frac{\Delta t}{2}\mathcal{L}\mathcal{X}^{+(i-1)}$.
- **Step 3** : Solving the implicit linear problem, $[\mathcal{I} - \frac{\Delta t}{2}\mathcal{L}]\mathcal{X}^{+(i)} = \mathcal{X}^{\bullet}$

Constant-coefficient Iterative-Centered-Implicit (CC-ICI) schemes

Constant-coefficient assumption :

$\mathcal{L}$ is chosen as a linear counterpart of $\mathcal{M}$ whose coefficients are taken constant in time and along horizontal directions $\Rightarrow \mathcal{L} = \mathcal{L}^*$

- **Strength** : Efficient inversion of the 'constant-coefficient' implicit problem $\left[I - \frac{\Delta t}{2}\,\mathcal{L}^*\right] \mathcal{X}^+ = \mathcal{X}^\bullet$ $\Rightarrow$ Allow the use of direct spectral solvers, or the use of very robust and efficient iterative solvers compared to non-constant ICI schemes, [Hussain *et al.* (2019), Degrauwe *et al.* (2020)].
- **Weakness** : Stability strongly depends on the magnitudes of the non-linear residual terms $(\mathcal{M} - \mathcal{L}^*)$, [Bénard *et al.* (2003,2005)]

Suitability of CC-ICI schemes for NWP models :

- IFS/ARPEGE-NH version global spectral models and AROME regional spectral model, both in mass-based coordinate [Bénard *et al.* (2010)]
- GEM NH global Canadian models : Log-hydrostatic pressure coordinate [Girard *et al.* (2014)], recently z-based coord. version by [Hussain *et al.* (2019)].

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Recipes for stable CC-ICI scheme

What has been learnt from decades of research on the design of stable CC-ICI time-discretisation of fully-compressible (EE) system :

(I1) Minimize the amount of explicitly treated non-linear residual quantities related to the 3D divergence term, $\mathbb{D}_3$. [Bénard *et al.* (2005)].

(I2) A clever choice of the associated linear operator $\mathcal{L}^*$. The most stable choice for $\mathcal{L}^*$ does not necessarily correspond to the proper linearisation of $\mathcal{M}$ around a given basic-state $\mathcal{X}^*$, [Bénard (2004)].

(I3) Ensure a consistent space- and time- treatment of vertical boundary imposition between $\mathcal{M}$ and $\mathcal{L}^*$, [Inherited for SI scheme, Caya and Laprise (1999)].

Non-rotating, adiabatic Euler equations cast in terrain-following mass-based η-coordinate :

$$\frac{\mathrm{D}\mathbf{v}}{\mathrm{D}t} + [RT\nabla(\ln\pi + \hat{q}) + \nabla\phi] + \frac{\nabla\phi}{m}\partial_\eta\left[\pi\left(e^{\hat{q}} - 1\right)\right] = 0$$

$$\frac{\mathrm{D}w}{\mathrm{D}t} - \frac{g}{m}\partial_\eta\left[\pi\left(e^{\hat{q}} - 1\right)\right] = 0$$

$$\frac{\mathrm{D}T}{\mathrm{D}t} + \frac{RT}{C_v}\mathbb{D}_3 = 0$$

$$\frac{\mathrm{D}\hat{q}}{\mathrm{D}t} + \frac{\dot{\pi}}{\pi} + \frac{C_p}{C_v}\mathbb{D}_3 = 0$$

$$\frac{\partial q_s}{\partial t} + \frac{1}{\pi_s}\int_0^1 \nabla\cdot(m\mathbf{v})\,\mathrm{d}\eta = 0$$

$$\frac{\mathrm{D}}{\mathrm{D}t} \equiv \partial_t + \mathbf{v}\cdot\nabla + \dot{\eta}\partial_\eta.$$

- Vertical coord. definition :

$$\pi = A(\eta) + B(\eta)\pi_s, \quad \eta \in [0,1]$$
$$m = \partial_\eta\pi$$
$$\partial_\eta\phi = -m/\rho, \qquad [\phi = gz]$$

- Prognostic variables :
  - $\mathbf{v}$: Cartesian horizontal velocity vector, $[\dot{\mathbf{x}}]$
  - w : Cartesian Vertical velocity, $[\dot{z}]$
  - T : temperature,
  - $\hat{q}$: NH pressure depature $[\ln(p/\pi)]$.
  - q_s : Log of hydrostatic surf pressure $[\ln(\pi_s)]$.

- Vertical BCs :

$$\dot{\eta}_s = \dot{\eta}_T = 0, \qquad \text{[Material BC]}$$
$$\hat{q}_T = 0, \qquad \text{[Elastic top BC]}$$
$$w_s = \mathbf{v}_s\cdot\nabla z_s, \qquad \text{[Rigid bottom BC]}$$

Current ingredients of the CC-ICI scheme of AROME

(I1) 3D Divergence expression in terrain-following η-coordinate :

$$\mathbb{D}_3 = \nabla \cdot \mathbf{v} + g \frac{\partial_\eta w}{\partial_\eta \phi} - \frac{\partial_\eta \overline{\mathbf{v}}^{w}}{\partial_\eta \phi} \cdot \nabla \phi$$

To avoid any NL residual terms in $\mathbb{D}_3$, a pseudo-vertical divergence variable, termed as d_4, is defined in such a way that

$$\mathbb{D}_3 = \nabla \cdot \mathbf{v} + d_4$$

with

$$d_4 = g \frac{\partial_\eta w}{\partial_\eta \phi} - \frac{\partial_\eta \overline{\mathbf{v}}^{w}}{\partial_\eta \phi} \cdot \nabla \phi$$

• d_4 is used as a new prognostic variable for the vertical momentum budget.

(I2) $\mathcal{L}^* \Rightarrow$ Linearisation of $\mathcal{M}$ around an isothermal T^*, quiescent, hydrostatic, horizontally homogeneous reference state ($\pi_s^* = cst$, and $z_s = 0$). Use of a colder constant temperature ($T_a^* < T^*$) in the linear Eq. of d_4 for stability reason :

$$\frac{\partial \mathbf{v}}{\partial t} + [R \nabla \mathcal{G}^* (T - T^* \hat{q}) + R T^* \nabla (\hat{q} + q_s)] = 0$$

$$\frac{\partial d_4}{\partial t} + \frac{g^2}{R T_a^*} \frac{\pi^*}{m^*} \partial_\eta \left[\frac{1}{m^*} \partial_\eta (\pi^* \hat{q}) \right] = 0$$

$$\frac{\partial T}{\partial t} + \frac{R T^*}{C_v} [\nabla \cdot \mathbf{v} + d_4] = 0$$

$$\frac{\partial \hat{q}}{\partial t} + \frac{C_p}{C_v} \left[\nabla \cdot \mathbf{v} - \frac{C_v}{C_p} \mathcal{S}^* (\nabla \cdot \mathbf{v}) + d_4 \right] = 0$$

$$\frac{\partial q_s}{\partial t} + \mathcal{N}^* (\nabla \cdot \mathbf{v}) = 0$$

* $\overline{(\;)}^{w}$: Interpolation operator from **v**- to w- space.

Current ingredients of the CC-ICI scheme of AROME

Potential conflict between ingredients **[I1]** and **[I3]** : BBC treatment consistency ?

• Rigid BBC specification for d_4 NL prognostic equation takes the form of a Neumann boundary condition →

$$\frac{g}{m}\partial_\eta\left[\pi(e^{\hat{q}}-1)\right]\Big|_s = \dot{w}_s = \dot{\overline{[\mathbf{v_s}\cdot\nabla\mathbf{z_s}]}}$$

On contrary, in absence of orography BBC treatment in the linear equation of d_4 consists in

$$\frac{g}{m^*}\partial_\eta\left[(\pi^*\hat{q})\right]\Big|_s = 0$$

⇒ For consistency between $\mathcal{M}$ and $\mathcal{L}^*$ BBC treatments, the Neumann condition in $\mathcal{M}$ should be perfectly compensated by some of other source terms present in the NL equation of d_4. This can be difficult to achieve in space-and time- discretized context.

• This issue is alleviated when using w NL prognostic equation because rigid BBC is now a Dirichlet condition and w-equation is much simpler ⇒ Compensation easier to fulfil in space- and time- discrete context. But considering w as prognostic variable will violate **[I1]**.

• Current solution in AROME : use of w equation in NL explicit part of the model, then transform from explicit-guess of w into a explicit guess for d_4, finally deal with d_4 in implicit part. Dirichlet BBC

$$w_s = \mathbf{v}_s\cdot\nabla zs$$

is specified during the explicit guess transformation in a consistent manner.

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Motivations :

- Current CC-ICI scheme of AROME has proven its robustness to deal with kilometric scales and moderate slope of orography. However, there is evidence that it may suffer from some drawbacks when hectometric scales and steep orography are considered.
- Try to derive a more robust CC-ICI scheme for the mass-based EE system that allow high-resolutions and attractive time-steps for NWP.

Some room for improvements :

1. Consider a new vertical momentum prognostic variable less sensitive to the destabilising effect of steep orography slopes.
2. Design of a bespoke linear ICI operator that somehow contains a key information about orography slope.

Design of a more stable formulation for CC-ICI scheme

A bespoke vertical momentum prognostic variable with a homogeneous rigid BBC

- Consider the variable :

$$\mathbb{W} = w - \overline{\mathbf{v}}^{\mathrm{w}} \cdot \nabla \left[\mathtt{S} z_s \right]$$

$\mathtt{S} = \mathtt{S}(\eta)$ is a prescribed monotonic vertical function satisfying $\mathtt{S}(0) = 0$ at the top, and $\mathtt{S}(1) = 1$ at bottom. $\mathtt{S}$ is mainly used to avoid "jet streams of $\mathbb{W}$" close to model top. Thus, $\mathbb{W}$ behaves as : w at top, and as $\dot{\eta}$ at the bottom. $\Rightarrow$ Rigid BBC amounts to

$$\mathbb{W}_s = 0.$$

- $\mathtt{S}$ is defined as :

$$\mathtt{S}(\eta) = \left[\frac{B(\eta)\, \pi_s^{ref}}{B(\eta)\, \pi_s^{ref} + A(\eta)} \right], \quad \eta \in]0, 1],$$

This definition ensures that $\mathtt{S}\nabla zs$ fits ∇z for an stationary isothermal hydrostatic atmosphere.

- Noting

$$\mathsf{Y} = -\mathtt{S}\, \overline{\mathbf{v}}^{\mathrm{w}} \cdot \nabla z_s$$

Prognostic Eq. of $\mathbb{W}$ read as :

$$\frac{\mathrm{D}\mathbb{W}}{\mathrm{D}t} = \frac{g}{m} \partial_\eta \left[\pi \left(e^{\hat{q}} - 1 \right) \right] + \dot{\mathsf{Y}}$$

where $\dot{\mathsf{Y}} = \mathrm{D}\mathsf{Y}/\mathrm{D}t$ is evaluated according to the current transport scheme (Eul. or SL). in order to avoid the appearance of erroneous responses due to inconsistencies in the treatment of metrics terms [Klemp *et al.* (2003)].

Adding ingredients : [**I1**] and [**I3**]

(I1) No NL residual terms in $\mathbb{D}_3 \Rightarrow$ use of d_4 as prognostic variable, however reformulation in terms of $\mathbb{W}$ leads to a new definition for d_4 referred for now on as d_5 and given by :

$$d_5 = g\frac{\partial_\eta \mathbb{W}}{\partial_\eta \phi} + \mathrm{X}_5,$$

with

$$\mathrm{X}_5 = \mathbf{v}\cdot\frac{\partial_\eta \nabla\phi}{\partial_\eta \phi} + \frac{\partial_\eta}{\partial_\eta \phi}\left[\overline{\mathbf{v}}^{\mathrm{w}}\cdot\nabla(\mathrm{S}\phi_s - \phi)\right],$$

(I3) BBC space and time treatment consistency is automatically guarantee with $\mathbb{W}$ and d_5, since they are respectively subjected to zero BBC:

$$\mathbb{W}_s = 0,$$
$$\dot{\mathbb{W}}_s = 0$$

in both NL and linear model. $\Rightarrow$ Now we can adopt either d_5- or $\mathbb{W}$- prognostic equation in $\mathcal{M}$ with guarantee of no BBC inconsistency issue. In case of $\mathbb{W}$-option, since d_5 is necessarily used as prognostic variable for stability reason [*cf.* **I1**], then transformations from $\mathbb{W}$ to d_5 and *vice versa* are required.

Design of a more stable formulation for CC-ICI scheme

Construction of the NL equation for d_5 :

- Prognostic Eq. for d_5 can be formed applying Lagrangian $\dot{[\,]} = \mathrm{D}/\mathrm{D}t$ to its definition :

$$\dot{d}_5 = g\left[\frac{\partial_\eta \dot{\mathbb{W}}}{\partial_\eta \phi} - \frac{\partial_\eta \overline{\mathbf{v}}^{\,\mathrm{w}}}{\partial_\eta \phi}\cdot\nabla\mathbb{W}\right] + (\mathrm{X}_5 - d_5)\, d_5 + \dot{\mathrm{X}}_5$$

with

$$\dot{\mathbb{W}} = \dot{w} - \nabla(\mathrm{S} z_s)\cdot\overline{[\dot{\mathbf{v}}]}^{\,\mathrm{w}} - \overline{\mathbf{v}}^{\,\mathrm{w}}\cdot\dot{\overline{[\nabla(\mathrm{S}z_s)]}}$$

where $\dot{w}$ and $\dot{\mathbf{v}}$ are replaced by their respective NL Lagrangian source terms of w and $\mathbf{v}$. For simplicity it is assumed that

$$[\dot{\overline{\mathbf{v}}}^{\mathrm{w}}] \equiv \overline{[\dot{\mathbf{v}}]}^{\,\mathrm{w}}$$

- Hence, after some basic algebra, it yields

$$\begin{aligned}\dot{d}_5 = g^2\frac{\partial_\eta}{\partial_\eta\phi}\Bigg[& \frac{1}{m}\partial_\eta\left[\pi(e^{\hat{q}}-1)\right] \\ & +\mathrm{S}(\nabla z_s)^2\,\overline{\left\{\frac{1}{m}\overline{\mathrm{S}\,\partial_\eta\left[\pi(e^{\hat{q}}-1)\right]}^{\,\mathrm{v}}\right\}}^{\,\mathrm{w}} \\ & +\mathrm{S}\nabla z_s\cdot\overline{\left\{\frac{1}{m}\overline{\frac{\nabla(\phi-\mathrm{S}\phi_s)}{g}\partial_\eta\left[\pi(e^{\hat{q}}-1)\right]}^{\,\mathrm{v}}\right\}}^{\,\mathrm{w}}\Bigg] \\ & + \textit{Cross–Derivative terms} + \ldots\ldots\end{aligned}$$

with

$$(\nabla z_s)^2 = (\partial_x z_s)^2 + (\partial_y z_s)^2$$

$^*\overline{(\ )}^{\,\mathrm{v}}$: Interpolation operator from w- to $\mathbf{v}$- space.

Design of a more stable formulation for CC-ICI scheme

Ingredient [I2] : Choice of the linear equation for d_5

(I2) The linear Eq. for d_5 is made up of the same linear contribution as for d_4 + an additional contribution taking into account the effect of a "characteristic uniform slope" on the fast vertically propagating of waves in reference atmosphere ⇒

$$\frac{\partial d_5}{\partial t} = -\frac{g^2}{R T_a^*} \mathcal{L}_v^*(\hat{q})$$

where

$$\mathcal{L}_v^*(\hat{q}) = \frac{\pi^*}{m^*} \partial_\eta \left[\frac{1}{m^*} \partial_\eta \left(\pi^* \hat{q}\right) + \mathrm{S}\, \Lambda_s^{*2} \overline{\left\{ \frac{1}{m^*} \overline{\mathrm{S}\, \partial_\eta \left(\pi^* \hat{q}\right)}^{\mathrm{v}} \right\}}^{\mathrm{w}} \right]$$

• where Λ_s^* is a newly introduced constant parameter that can be chosen either as the mean or as the maximum effect of slopes over the domain ⇒

$$\Lambda_s^{*2} = \frac{\int_{\mathcal{S}} (\nabla z_s)^2 \mathrm{d}s}{\int_{\mathcal{S}} \mathrm{d}s}$$

or

$$\Lambda_s^{*2} = \max \left[(\nabla z_s)^2 \right]$$

both choices have shown similar gain in term of stability.

• Interpolation operators $\overline{(\,)}^{\mathrm{w}}$ and $\overline{(\,)}^{\mathrm{v}}$ are determined so that $\mathcal{L}_v^*$ akin to a negative definite vertical Laplacian operator.

ICI time discretisation procedure from $\mathbb{W}$ to d_5

- **Step 1** : Computation of d_5 explicit-guess :
  - **Step 1.1** : Computation of $\mathbb{W}$ explicit-guess :

$$\widetilde{\mathbb{W}}^{+(i)} = \left[\mathbb{W} + \overline{\mathbf{v}}^{\,\mathrm{w}} \cdot \nabla\left(\mathrm{S}z_s\right) + \frac{\Delta t}{2}\left(\mathcal{M}_{\mathrm{w}}\right)\right]^0 + \left[-\widetilde{[\overline{\mathbf{v}}^{\,\mathrm{w}}]}^{+(i)} \cdot \nabla\left(\mathrm{S}z_s\right) + \frac{\Delta t}{2}\left(\mathcal{M}_{\mathrm{w}}\right)^{+(i\text{-}1)}\right]$$

  - **Step 1.2** : Transformation from $\widetilde{\mathbb{W}}^{+(i)}$ to $\widetilde{d_5}^{+(i)}$:

$$\widetilde{d_5}^{+(i)} = g\,\frac{\partial_\eta \widetilde{\mathbb{W}}^{+(i)}}{\partial_\eta \widetilde{\phi}^{+(i)}} + \widetilde{[\mathrm{X}_5]}^{+(i)}$$

- **Step 2** : Computation of the explicit part of ICI correction:

$$d_5^{\bullet} = \widetilde{d_5}^{+(i)} - \frac{\Delta t}{2}\left[-\frac{g^2}{RT_a^*}\mathcal{L}_{\mathrm{v}}^*\left(\hat{q}^{+(i\text{-}1)}\right)\right]$$

Step 3 :Solving Linear implicit problem : $(\mathcal{I} - \frac{\Delta t}{2}\mathcal{L}^*)\mathcal{X} = \mathcal{X}^\bullet$

• Consider the linear implicit problem to be solved in vertically discrete space :

$$D + \frac{\Delta t}{2} R\nabla^2 \left[\mathbf{G}^* \left(T - T^* \hat{q}\right) + T^* \left(\hat{q} + q_s\right)\right] = \nabla \cdot \mathbf{v}^\bullet$$

$$d_5 + \frac{\Delta t}{2} \frac{g^2}{RT_a^*} \mathbf{L}_v{}^*(\hat{q}) = d_5^\bullet$$

$$T + \frac{\Delta t}{2} \frac{RT^*}{C_v} \left[D + d_5\right] = T^\bullet$$

$$\hat{q} + \frac{\Delta t}{2} \left[-\mathbf{S}^*(D) + (C_p/C_v)\left(D + d_5\right)\right] = \hat{q}^\bullet$$

$$q_s + \frac{\Delta t}{2} \mathbf{N}^* \left(D\right) = q_s^\bullet$$

• Current strategy for solving the linear implicit problem consists in eliminating in favour of (d_4) variables ⇒ some algebraical constraints between discrete vertical operators $\mathbf{G}^*$, $\mathbf{S}^*$, $\mathbf{N}^*$ and $\mathbf{L}_v{}^*$ must be satisfied [Budnova *et al.* (1995)]. However, the fulfilment of some of these constraints might no longer hold true for the newly proposed formulation of $\mathbf{L}_v{}^*$.

• A new reduction strategy that does not induce any constraints [⇒less restrictive] is proposed ⇒ Elimination in favour of horizontal divergence (D) variable along the lines of HPE model.

Design of a more stable formulation for CC-ICI scheme

A new elimination procedure

Implicit problem is now reduced in favour of $(D, \hat{q})$

$$\left(\mathbf{I} - \tfrac{\Delta t^2}{4}\,\nabla^2 \mathbf{B}^*\right) D = D^{\bullet\bullet}$$

$$\mathbf{P}^* \hat{q} = \hat{q}^{\bullet\bullet} - \tfrac{\Delta t}{2}\,\left(\frac{C_p}{C_v}\mathbf{I} - \mathbf{S}^*\right) D$$

with

$$\mathbf{P}^* = \left(\mathbf{I} - \tfrac{\Delta t^2}{4}\frac{C_p}{C_v}\frac{g^2}{RT_a^*}\mathbf{L}_v^*\right), \qquad \text{(pentadiagonal FD vertical operator)}$$

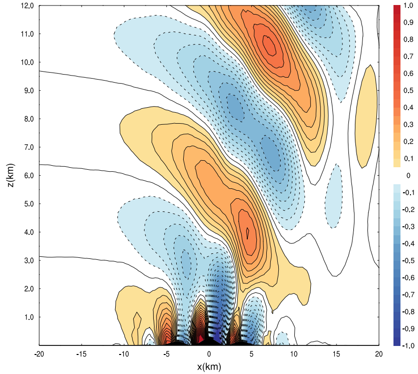
$$\mathbf{B}^* = RT^*\left(\frac{R}{C_p}\mathbf{G}^*\mathbf{S}^* + \mathbf{N}^*\right) + RT^*\left(\frac{C_p}{C_v}\mathbf{I} - \mathbf{G}^*\right)\mathbf{P}^{*\,-1}\left(\mathbf{I} - \frac{C_p}{C_v}\mathbf{S}^*\right)$$

Idealized AROME 2D experiments :

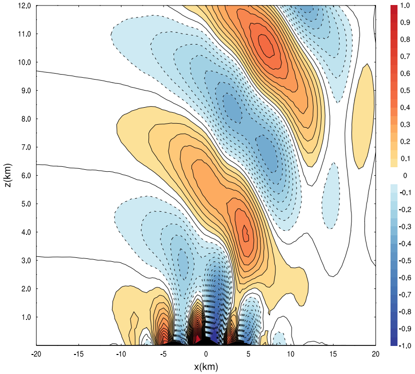
Schär orography test-case

- Vertical velocity solution for Schär-mountain of $h = 250\,\mathrm{m}$ after $6\,\mathrm{h}$ of integration with $N_{\mathrm{siter}} = 1$ (1 ICI iteration). Experimental settings : $\Delta x = 500\,\mathrm{m}$, $A(\eta) = 0$ and $B(\eta)$ is chosen in such a way that $\Delta z \approx 312\,\mathrm{m}$, $\Delta t = 32\,\mathrm{s}$, $T^* = 350K$, with $T_a^* = 100K$. Slope max $= 19^\circ \Rightarrow \Lambda_s^* = 0.11$.

(w, d_4)

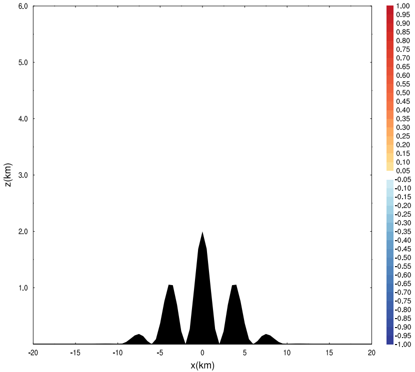


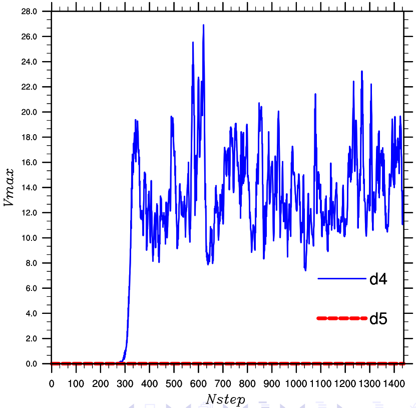
$(\mathbb{W}, d_5)$



- AROME 3D (375 m, L90) academic experiment of a "No-flow test-case" in presence of a Schär orography of $h = 2000\,\mathrm{m}$. Starting from an isothermal $T_0 = 288.15K$, hydrostatically balanced and resting atmosphere. After $6\,\mathrm{h}$ of integration with $N_{\mathrm{siter}} = 1$ (1 ICI iteration). $\Delta t = 15\,\mathrm{s}$, $T^* = 350K$, with $T^*_a = 100K$. Slope max $= 58^\circ \rightarrow \Lambda^*_s = 1.6$.

$(\mathbb{W}, d_5)$





Real Case over the Alps

Stability of AROME (375 m) as function of (**SITRA**$=T_a^*$)

The experiments are run for one date 3.10.2015, for 24 hours. The calculation is considered stable if the run terminates. maximum slope is $57^\circ \rightarrow \Lambda_s^* = 1.55$

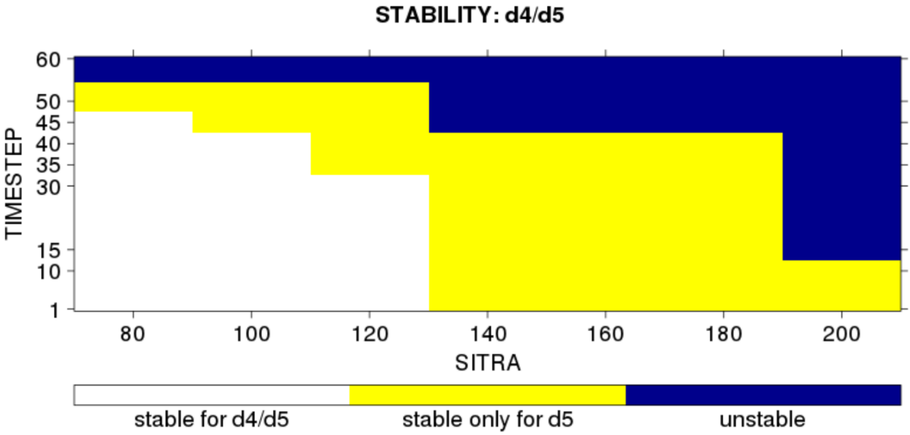
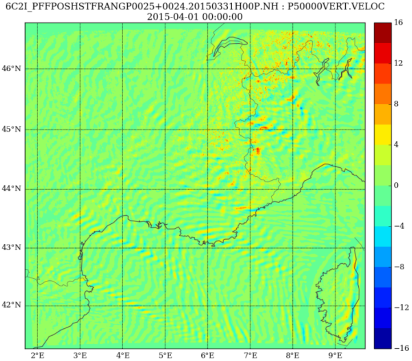


Table 2: Stability of experiments in 375m hor. resolution.

Real Case over the Alps

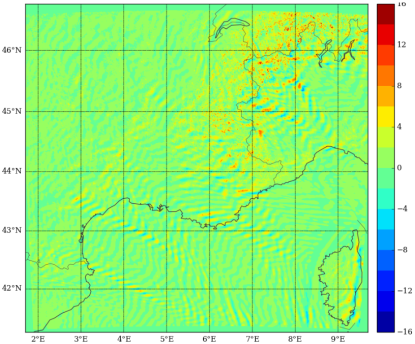
AROME (375 m, L90) 24H forecast with $\Delta t = 15\,s$ and $T_a^* = 100\,K$

(w, d_4)



$N_{\text{siter}} = 1$ (ICI outer iterations)
$+$ filtered orography $>$1000 m

$(\mathbb{W}, d_5)$



$N_{\text{siter}} = 1$ (ICI outer iterations)
no orography filtering

Conclusion

1. An alternate formulation using new vertical momentum prognostic variables termed as ($\mathbb{W}$, d_5) is proposed for the dynamical core of AROME .
2. These new variables systematically guarantee a consistent space-and-time treatment of BBC (without effort).
3. A more stable linear ICI operator together with a less restrictive strategy for solving implicit problem are also proposed.
4. These new ingredients put together substantially improve the robustness of the CC-ICI scheme. A potential gain of 30% in the horizontal resolution limit of AROME.

perspectives

1. Deep tests on real cases at 250 m of resolution over the Alps are presently examined.
2. Objective scores with AROME (1.3km) should be available soon.
3. Tests with IFS-ST NH at high-resolution are on progress.

Thanks for your attention !!!

The so-called vertical integral operators : $\mathcal{G}$, $\mathcal{S}$, and $\mathcal{N}$.

$$\mathcal{G}(X) = \int_{\eta}^{1} \frac{m}{\pi} X \mathrm{d}\eta'$$

$$\mathcal{S}(X) = \frac{1}{\pi} \int_{0}^{\eta} mX \mathrm{d}\eta'$$

$$\mathcal{N}(X) = \frac{1}{\pi_s} \int_{0}^{1} mX \mathrm{d}\eta'$$