Dynamical core of the Russian global NWP model: current state and further development

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1. Dynamical core of the SLAV operational model - recent developments

1.1 Brief description
- hydrostatic semi-Lagrangian semi-implicit core of own development,
- vorticity-divergence formulation, unstagerrated grid,
- details in (Tolsykh et al, GMD 2017),
- variable resolution in latitude,
- current operational version with ~22 km horizontal resolution and 51 levels,
- new version with ~10 km horizontal resolution and 104 levels.

1.2 Improving parallel efficiency for new SLAV version (3600x1946x104 grid)

Percentage of time used in different parts of SL-AV model code while using 3888 cores at Cray XC40 (left) and 3888 cores at T-Platforms V6000 (right) after optimizations.

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Advection</th>
<th>Fast waves</th>
</tr>
</thead>
<tbody>
<tr>
<td>SI-SL</td>
<td>not retarded</td>
<td>fast retarded</td>
</tr>
<tr>
<td>SI-EUL</td>
<td>retarded(&lt;&lt;spatial discretization)</td>
<td>retarded</td>
</tr>
<tr>
<td>EUL-EXP</td>
<td>Retarded(&lt;&lt;spatial discretization)</td>
<td>not retarded</td>
</tr>
<tr>
<td>SL-EXP</td>
<td>not retarded</td>
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</tr>
</tbody>
</table>

Time integration schemes:
- SI-SL – semi-implicit semi-Lagrangian
- SI-EUL – semi-implicit Eulerian
- EUL-EXP – exponential Eulerian
- SL-EXP – semi-Lagrangian exponential

2. Development of dynamical core for new-generation global NWP model

2.1 Features
- compressible non-hydrostatic equation set in height-based terrain-following coordinates
- gnomonic cubed-sphere grid with nested regions of higher resolution
- reduced lat-lon grid is also a possible option
- Possible integration strategies: semi-implicit semi-Lagrangian, exponential (SL), Runge-Kutta IMEX with Eulerian advection (all under testing in shallow-water and vertical slice Euler-eqs models).

2.2 Exponential time-integration schemes

Based on:
\[ \frac{\partial \mathbf{v}}{\partial t} = \mathbf{L} \mathbf{v} + \mathbf{a} \exp(\lambda A t) \mathbf{v}^0 \]
Couppling with semi-Lagrangian (SL) advection – exclude advection operator from \( L \Rightarrow \) cheaper iterative exp-term integration

2.2.1 Second order exponential-Rosenbrock scheme for non-linear system

Computing \( \exp(\lambda A t) \) at the midpoints (unstructured Lagrangian grid) is difficult \( \Rightarrow \) 3 stage scheme:
1) SI transport D1->A by the first half of trajectory
2) Calculation of exp-like function at arrival point A
3) SI transport by the second half of trajectory D2->A

2.2.2 Phase velocities of processes with different time-integration strategies

Galewski (Tellus, 2004) case, phase I: inertia-gravity wave propagation

Divergence after 5hr of integration. The position of initial flow disturbance is shown by red star. Dashed circle indicates the distance of mean phase velocity multiplied by the time. SI-SL solution is characterized by the spurious numerical dispersion of inertia-gravity waves. EXP-SL solution is similar to the solution of explicit scheme.

2.2.3 Shallow water experiments with different integration methods.

Galewski (Tellus, 2004) case, phase II: barotropic instability development

Vorocity after 6 days of integration. EXP-SL solution agrees well with the SISL solution as well as lat of published results from other SW models.

3. Horizontal discretization at the reduced latitude-longitude grid with Arakawa C staggering

3.1 C-staggered reduced lat-lon grid design

Reduced grid construction:
\[ N^0 = -2N_y + N_x (1 - \cos \phi_x \cos \phi_y - \cos \phi_y) \]
Here \( \epsilon \in [0, 1] \) is the grid reduction rate.

3.2 Horizontal approximation at C-staggered reduced lat-lon grid

Interpolation approach is used. Notations for the grid function values at fixed latitude
\[ \mathbf{f}^0 = (\mathbf{f}_x, \mathbf{f}_y, \mathbf{f}_z) \]
Interpolation design:
\[ \mathbf{f}^0 = \mathbf{f}^0 + \mathbf{f}^1 = \mathbf{f}^0 + \mathbf{f}^1 \]
Restriction:
\[ (\mathbf{r}^0 f_y, \mathbf{r}^0 f_x, \mathbf{r}^0 f_z) \]
Conservation properties
\[ \sum_{y=1}^{N_y} f_y(x) = 1, \sum_{x=1}^{N_x} f_x(y) = 1 \]
Comparison of \( \mathbf{f}^0 \) for the different schemes.

3.3 Results of numerical experiments

Three schemes:
- Energy mass conservative ‘En-cons’
- Only mass conservative ‘Mass-cons’
- No conservation properties ‘Non-cons’

3.3.1 Gravity waves dispersion

Spherical harmonic \( Y^m_n \) average eigenvalue
\[ \lambda = -2\pi n / N_y, \sum_{y=1}^{N_y} (\overline{\Delta f(t)(\Delta f(t))^T})_{y,y} \]
computed for the grids with \( N_x = 16 \)

3.2.2 Steady state solution

Linear shallow-water equations on a rotating sphere. Geostrophically balanced flow in the pole-rotated spherical coordinate system. Grids with \( N_n = 128 \). Crank-Nicolson scheme with the iterative treatment of the Coriolis force terms. Helmholtz problem is solved using geometric multigrid preconditioned bicgstab solver.

4. Conclusions

1. SI advection is difficult for massively-parallel implementation, however, there are ways to reduce its cost somehow.
2. SI-exponential time-stepping and staggered reduced grid are attractive alternatives for mainstream computational methods. Further research is interesting.