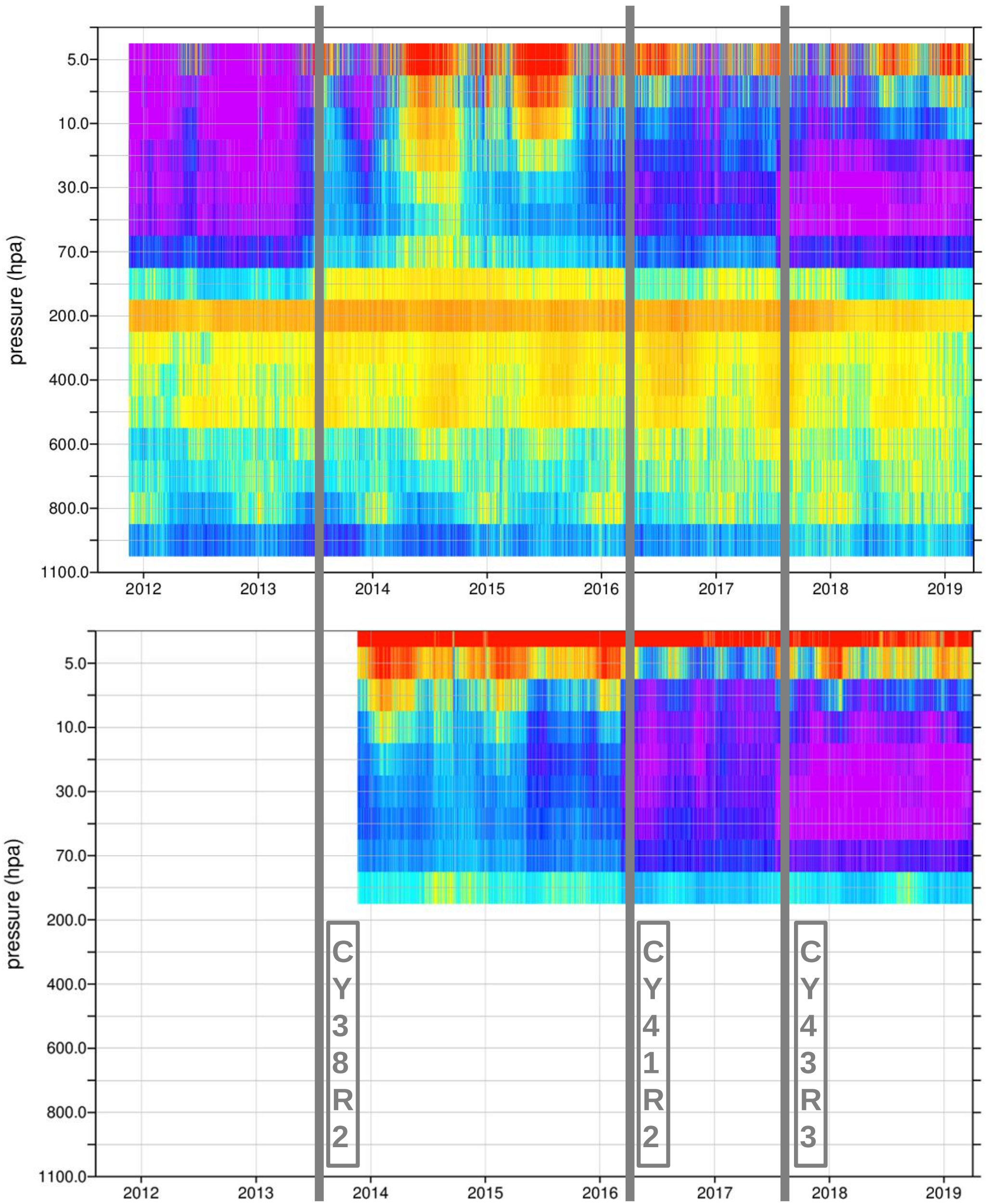
Estimation of model biases and the importance of scale separation

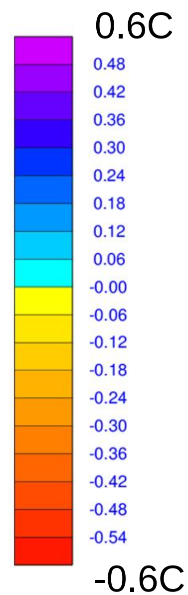
Patrick Laloyaux

EUROPEAN CENTRE FOR MEDIUM RANGE WEATHER FORECASTS

Temperature bias in operation



The short-term model bias is estimated by comparing the 12-hour first-guess trajectory with radiosondes and GPS-RO



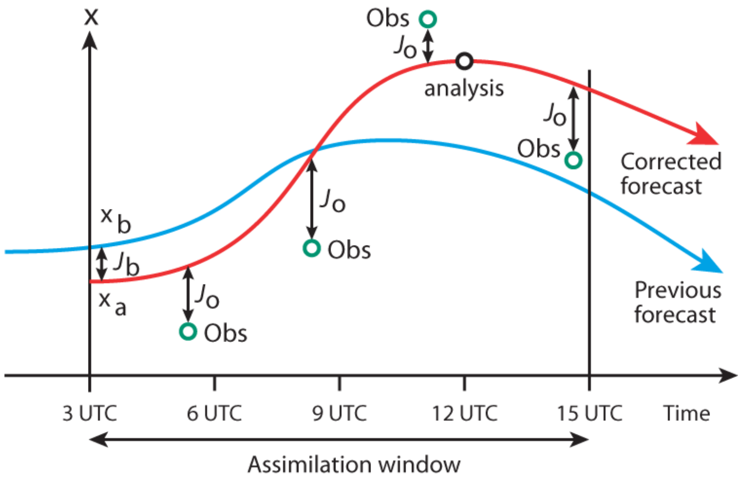
Similar signal with the two types of observations:

→ bias reduced with new vertical resolution (L137 in CY38R2)

→ bias increased with new horizontal resolution (Tco1279 in CY41R2)

→ bias increased with new radiative scheme (CY43R3)

4D-Var theory (strong-constraint 4D-Var)



$$J(\mathbf{x}_0, \mathbf{x}_1, ..., \mathbf{x}_N) = \frac{1}{2}\left(\mathbf{x}_0 - \mathbf{x}_0^b\right)^{\mathrm{T}} \mathbf{B}^{-1} \left(\mathbf{x}_0 - \mathbf{x}_0^b\right) + \frac{1}{2}\sum_{k=0}^{N} \left(\mathcal{H}_k(\mathbf{x}_k) - \mathbf{y}_k\right)^{\mathrm{T}} \mathbf{R}_k^{-1} \left(\mathcal{H}_k(\mathbf{x}_k) - \mathbf{y}_k\right)$$

If the model is assumed to be perfect (strong-constraint)

$$\mathbf{x}_k = \mathcal{M}_{k,k-1}(\mathbf{x}_{k-1}) \qquad \text{for} \qquad k = 1, \ldots, N$$

Cost function depends only on the state at the beginning of the assimilation window

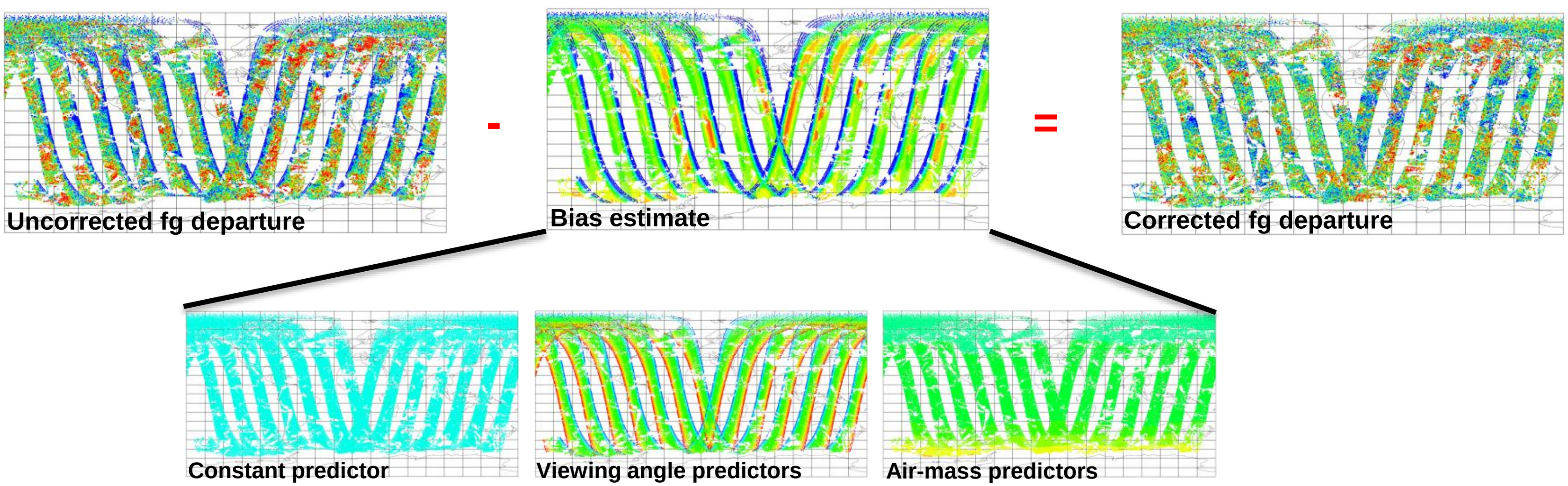
$$J(\mathbf{x}_0) = \frac{1}{2}\left(\mathbf{x}_0 - \mathbf{x}_0^b\right)^{\mathrm{T}} \mathbf{B}^{-1} \left(\mathbf{x}_0 - \mathbf{x}_0^b\right) + \frac{1}{2}\sum_{k=0}^{N} \left(\mathcal{H}_k \mathcal{M}_{k,0}(\mathbf{x}_0) - \mathbf{y}_k\right)^{\mathrm{T}} \mathbf{R}_k^{-1} \left(\mathcal{H}_k \mathcal{M}_{k,0}(\mathbf{x}_0) - \mathbf{y}_k\right)$$

4D-Var assumes **random zero-mean errors** in **observations** and in the **model**

4D-Var theory (strong-constraint 4D-Var)

VarBC has been designed to remove biases from instruments and radiative transfer models (estimating the systematic differences between the observations and model inside 4D-Var)

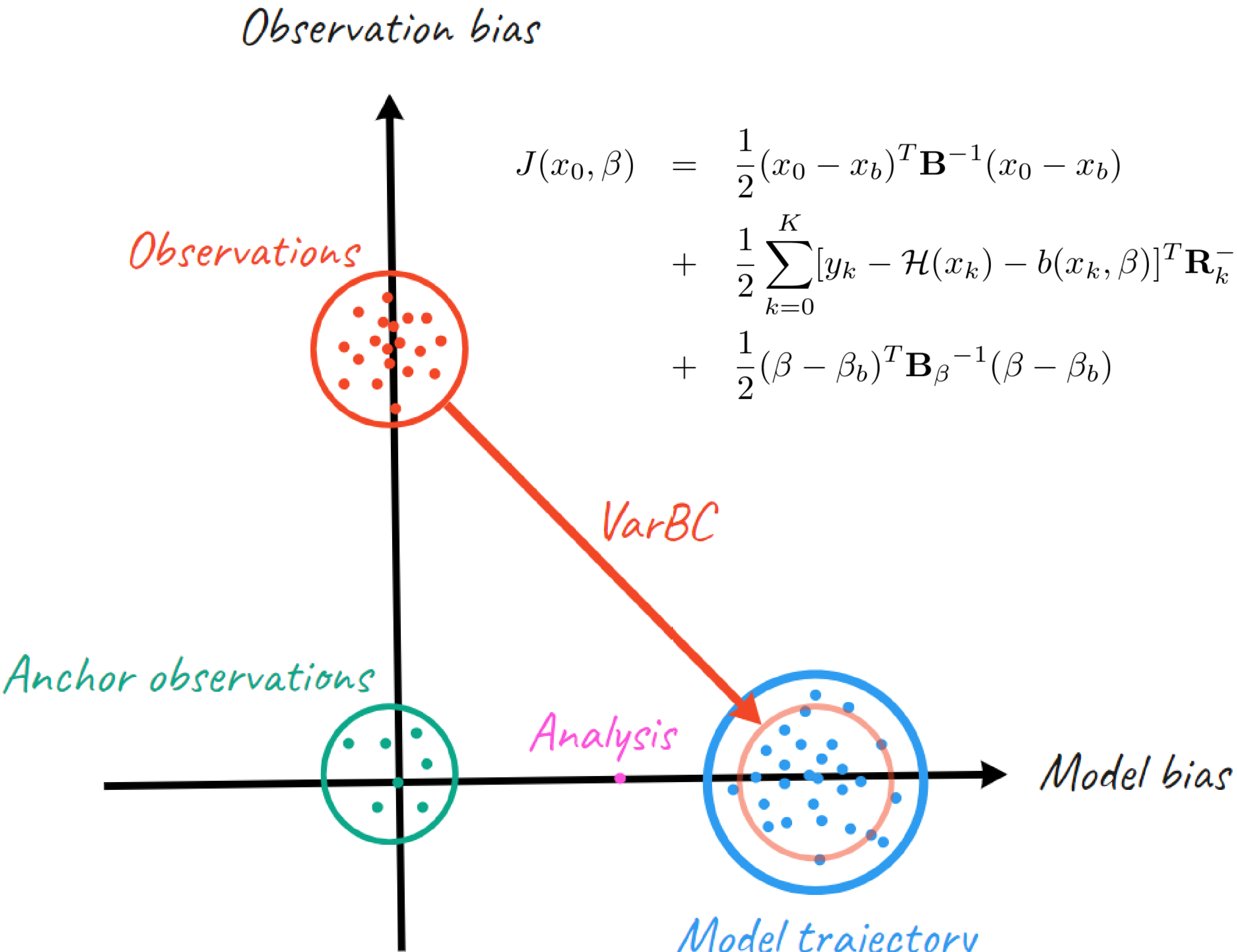
$$
\begin{aligned}
J(x_0,\beta) &= \frac{1}{2}(x_0 - x_b)^T \mathbf{B}^{-1}(x_0 - x_b) \\
&+ \frac{1}{2}\sum_{k=0}^{K}[y_k - \mathcal{H}(x_k) - b(x_k,\beta)]^T \mathbf{R}_k^{-1}[y_k - \mathcal{H}(x_k) - b(x_k,\beta)] \\
&+ \frac{1}{2}(\beta - \beta_b)^T \mathbf{B}_\beta{}^{-1}(\beta - \beta_b)
\end{aligned}
$$



Predictors are chosen to estimate observation biases (hopefully)

4D-Var theory (strong-constraint 4D-Var)

VARBC can potentially absorb model error into the observation correction
(this will reinforce the bias in the analysis)



$$
\begin{aligned}
J(x_0,\beta) &= \frac{1}{2}(x_0-x_b)^T\mathbf{B}^{-1}(x_0-x_b) \\
&+ \frac{1}{2}\sum_{k=0}^{K}[y_k-\mathcal{H}(x_k)-b(x_k,\beta)]^T\mathbf{R}_k^{-1}[y_k-\mathcal{H}(x_k)-b(x_k,\beta)] \\
&+ \frac{1}{2}(\beta-\beta_b)^T\mathbf{B}_\beta{}^{-1}(\beta-\beta_b)
\end{aligned}
$$

Developing the solution: weak-constraint 4D-Var

4D-Var theory (weak-constraint 4D-Var)

We assume that the model is not perfect, adding an error term η in the model equation

$$x_k = \mathcal{M}_k(x_{k-1}) + \eta \quad \text{for } k = 1, 2, \cdots, K$$

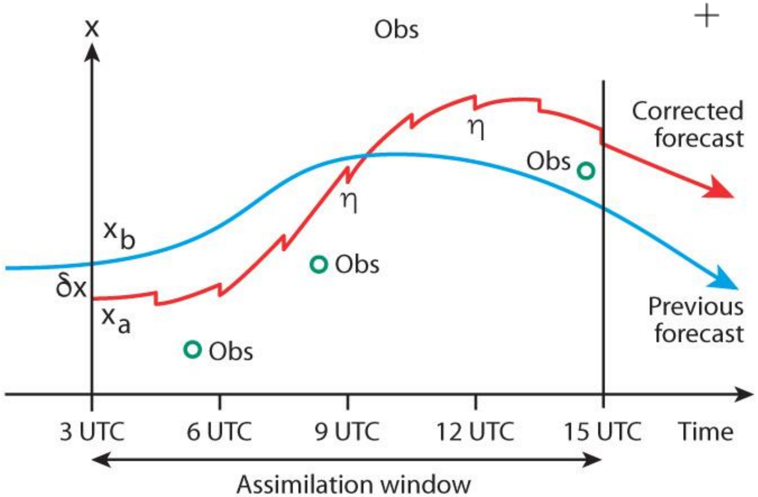
The model error estimate η contains 3 physical fields

- temperature
- vorticity
- divergence

Constant model error forcing over the assimilation window

$$\begin{aligned} J(x_0, \beta, \eta) &= \frac{1}{2}(x_0 - x_b)^T \mathbf{B}^{-1}(x_0 - x_b) \\ &+ \frac{1}{2}\sum_{k=0}^{K}[y_k - \mathcal{H}(x_k) - b(x_k, \beta)]^T \mathbf{R}_k^{-1}[y_k - \mathcal{H}(x_k) - b(x_k, \beta)] \\ &+ \frac{1}{2}(\beta - \beta_b)^T \mathbf{B}_\beta{}^{-1}(\beta - \beta_b) \\ &+ \frac{1}{2}(\eta - \eta_b)^T \mathbf{Q}^{-1}(\eta - \eta_b) \end{aligned}$$

Model state → x_0
Observation bias parameters → β
Model error → η



→ Introduce additional controls to target an unbiased analysis

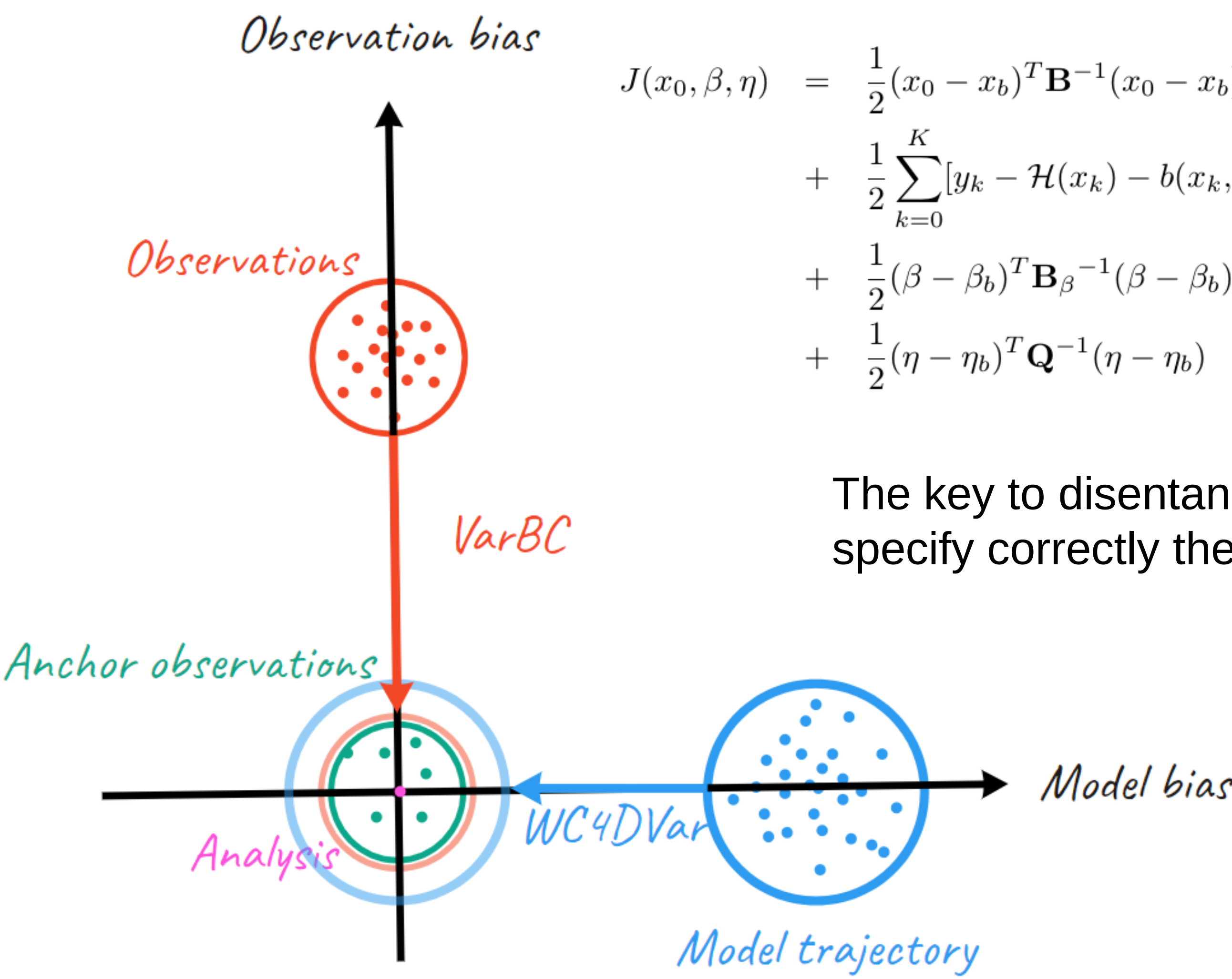
→ The model error covariance matrix Q contains the model error statistics (need to be estimated)

4D-Var theory (weak-constraint 4D-Var)

The different sources of biases are correctly attributed. This will produce an unbiased analysis

$$x_k = \mathcal{M}_k(x_{k-1}) + \eta \quad \text{for } k = 1, 2, \cdots, K$$

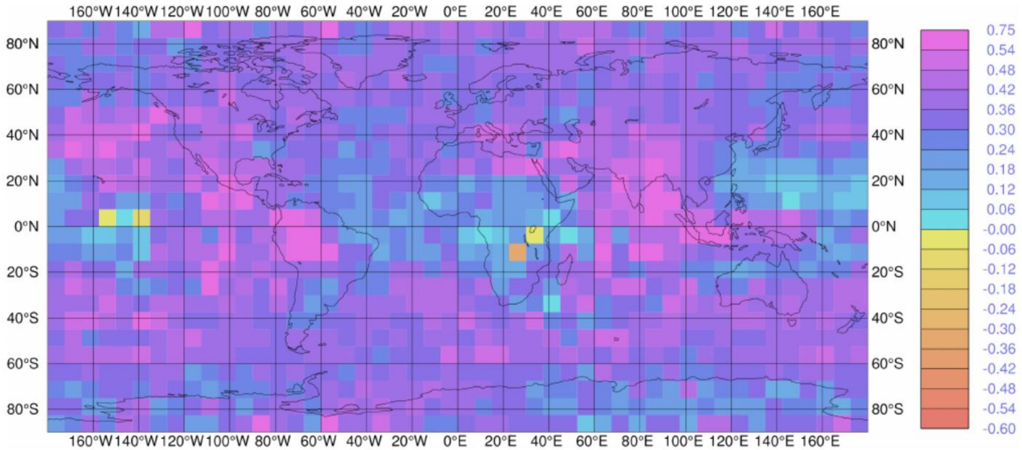
$$\begin{aligned} J(x_0, \beta, \eta) &= \frac{1}{2}(x_0 - x_b)^T \mathbf{B}^{-1}(x_0 - x_b) \\ &+ \frac{1}{2}\sum_{k=0}^{K}[y_k - \mathcal{H}(x_k) - b(x_k, \beta)]^T \mathbf{R}_k^{-1}[y_k - \mathcal{H}(x_k) - b(x_k, \beta)] \\ &+ \frac{1}{2}(\beta - \beta_b)^T \mathbf{B}_\beta{}^{-1}(\beta - \beta_b) \\ &+ \frac{1}{2}(\eta - \eta_b)^T \mathbf{Q}^{-1}(\eta - \eta_b) \end{aligned}$$

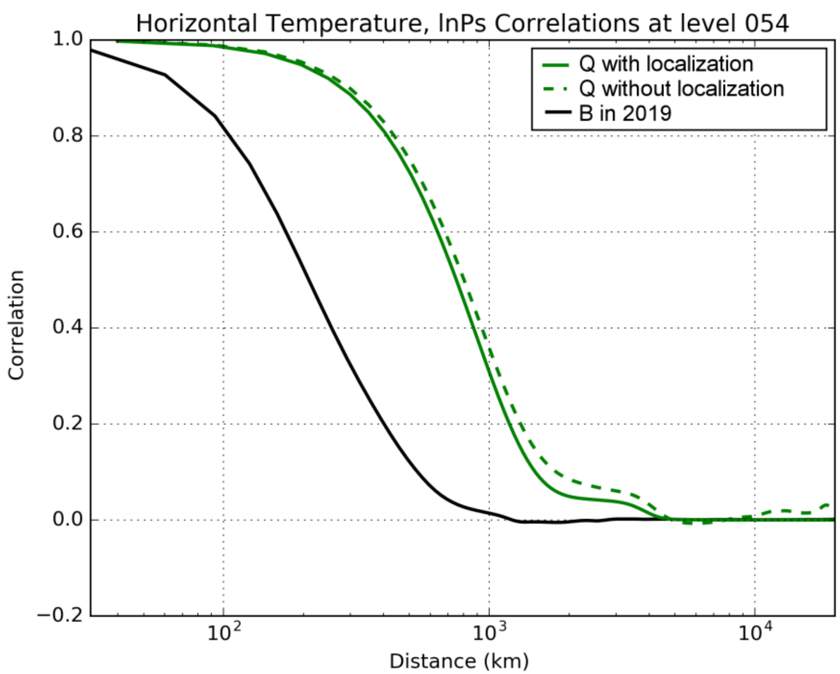


The key to disentangle the biases is to specify correctly the covariance matrices

Specification of model error covariance matrix Q

Difference between RO temperature retrievals and first-guess temperatures (70hPa)





Model space (scale separation)

- B corrects the background and contains small scales
- Q corrects the model bias and contains large scales

Observation space

- Good choice of predictors to model observation errors

4D-Var corrects small scale errors (background errors) by changing the initial condition and large scale errors (model errors) by changing the model forcing

When is weak-constraint 4D-Var expected to perform well?

Quarterly Journal of the
Royal Meteorological Society

RESEARCH ARTICLE

Exploring the potential and limitations of weak-constraint 4D-Var

P. Laloyaux, M. Bonavita, M. Chrust, S. Gürol

First published: 15 August 2020 | https://doi.org/10.1002/qj.3891

WC4DVAR can accurately estimate the model bias and the initial state when

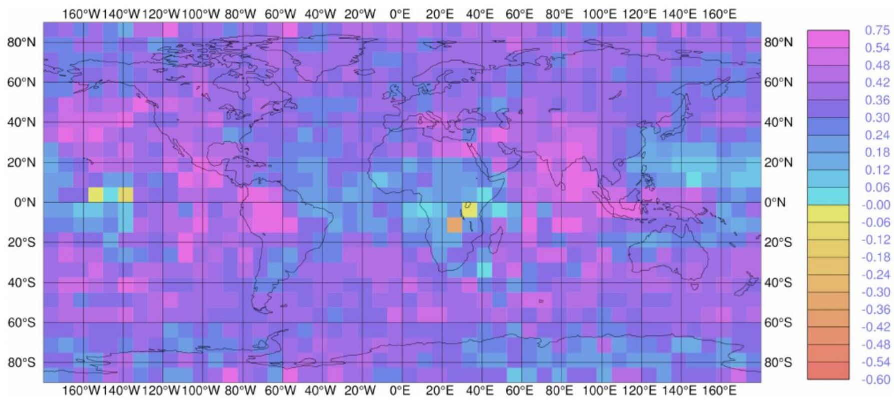
- background and model errors have different spatial scales
- the observing system is spatially homogeneous
- the observing system is unbiased

Study is done with a quasi-geostrophic model

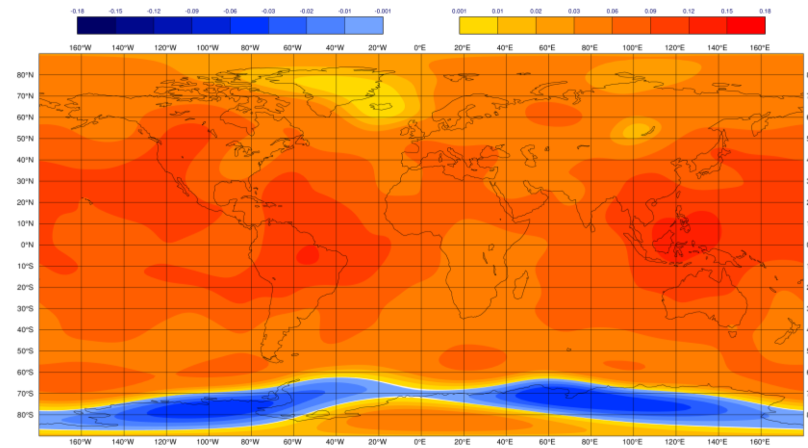
Results of weak-constraint 4D-Var

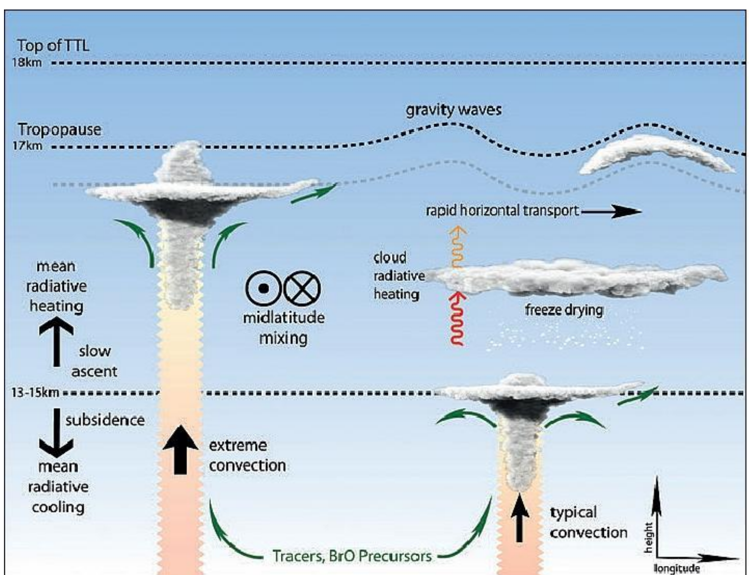
Weak constraint 4D-Var captures the model error structure

Model bias estimated from GPS-RO temperature retrievals



Model correction estimated by weak constraint 4D-Var

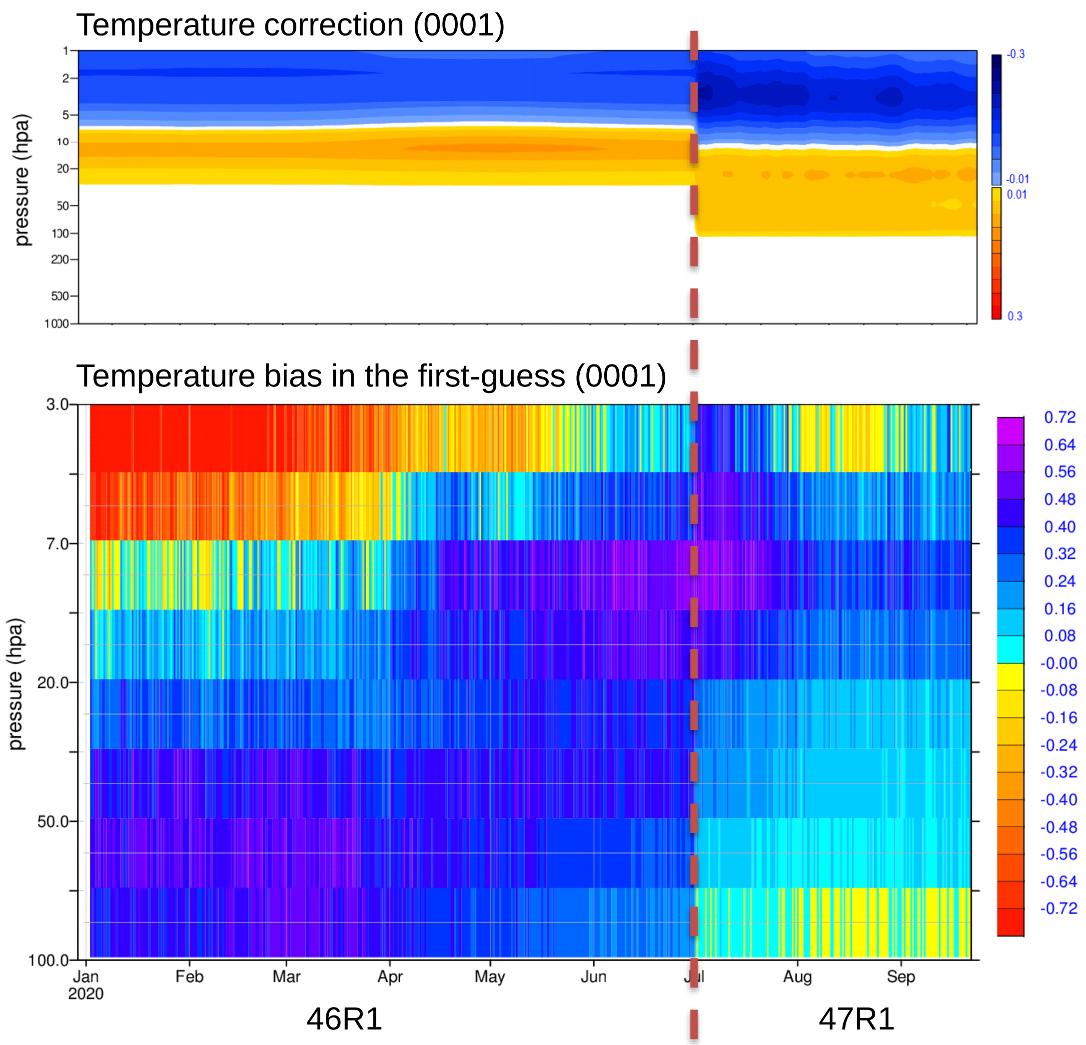




The cooling is due to discretization errors in the vertical advection, associated with inadequate representation of resolved gravity waves in the vertical direction

Weak constraint 4D-Var is in operation

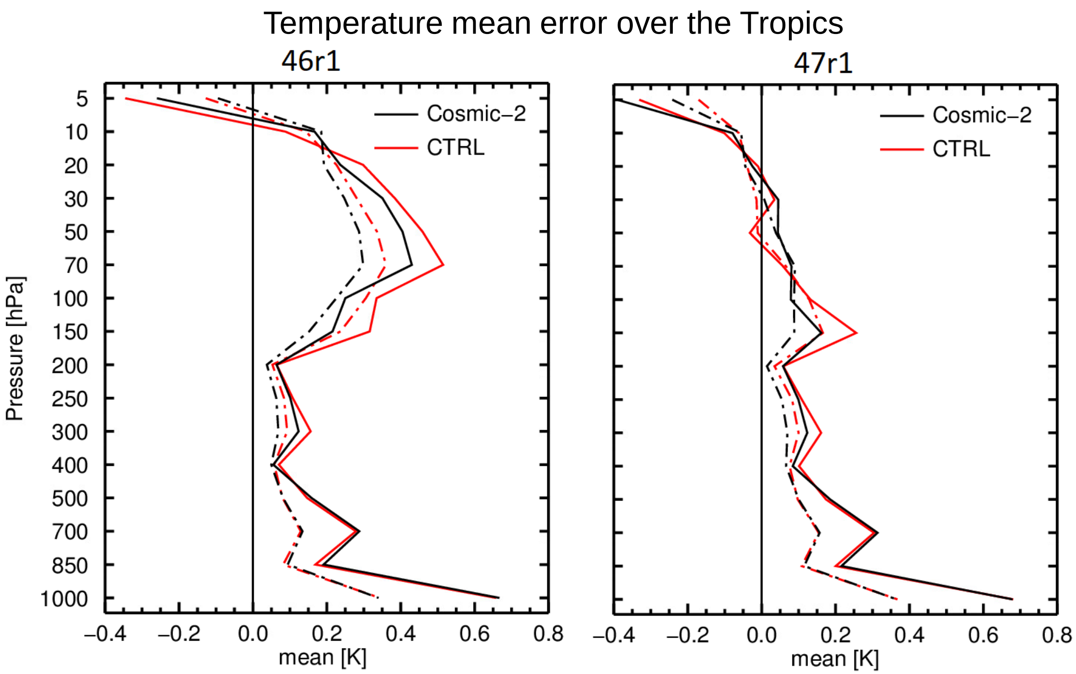
First time that weak-constraint 4D-Var works as expected in an operational NWP system

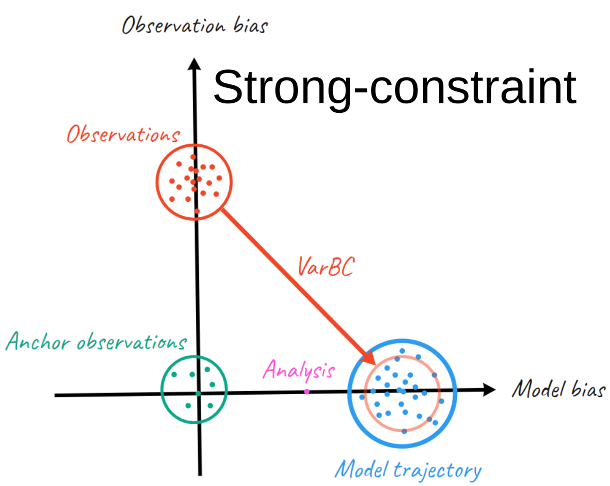


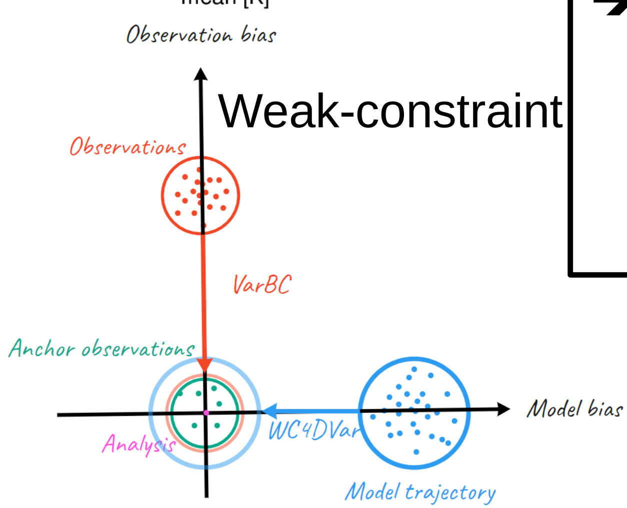
The last cycle (47R1) has implemented weak-constraint 4D-Var for the whole stratosphere (bias reduced up to 50%)

Do we need weak-constraint if we get more RO observations?

ECMWF started assimilating COSMIC-2 RO in March 2020



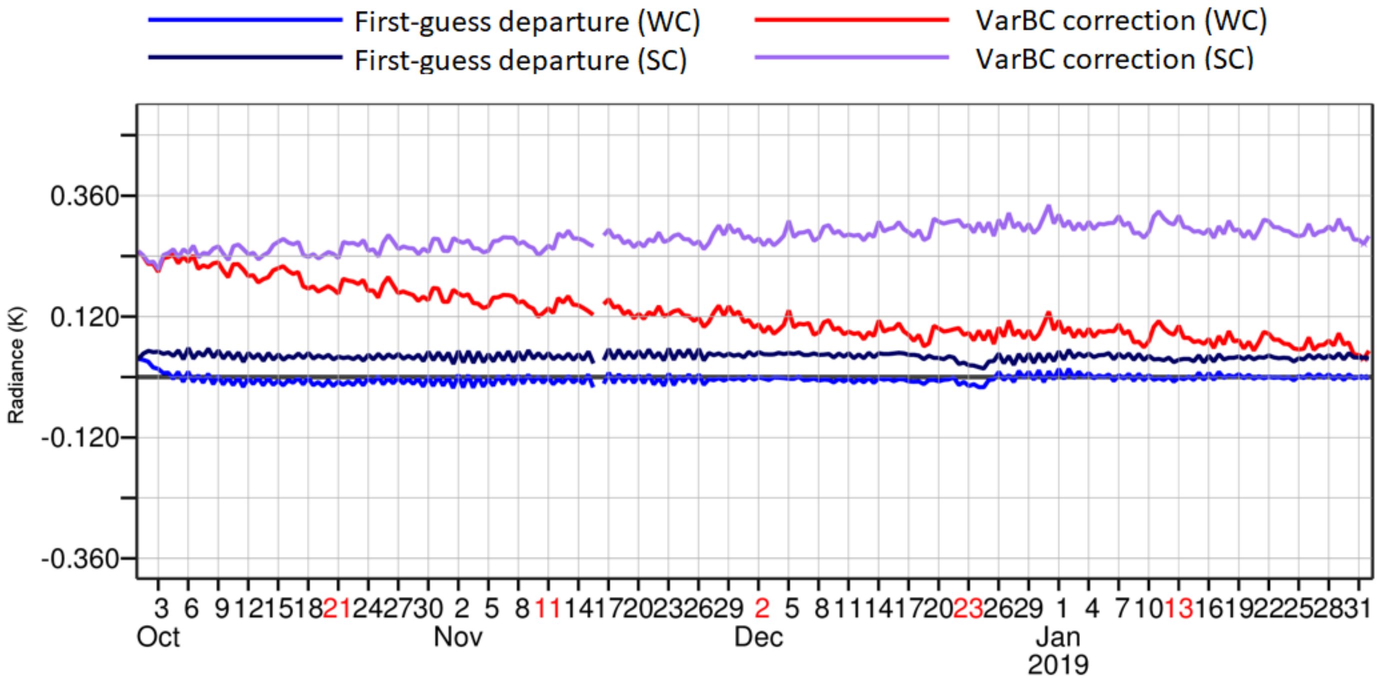




- ➔ More anchor observations is critical to reduce the mean and standard error in strong-constraint 4D-Var
- ➔ Largest impact of extra anchor observations is on the standard error in weak-constraint 4D-Var
- ➔ Research on data assimilation methodology is as important as the acquisition of more observations

Interactions with the observation bias correction (VarBC)

First-guess departure and observation bias correction in AMSU-A channel 10



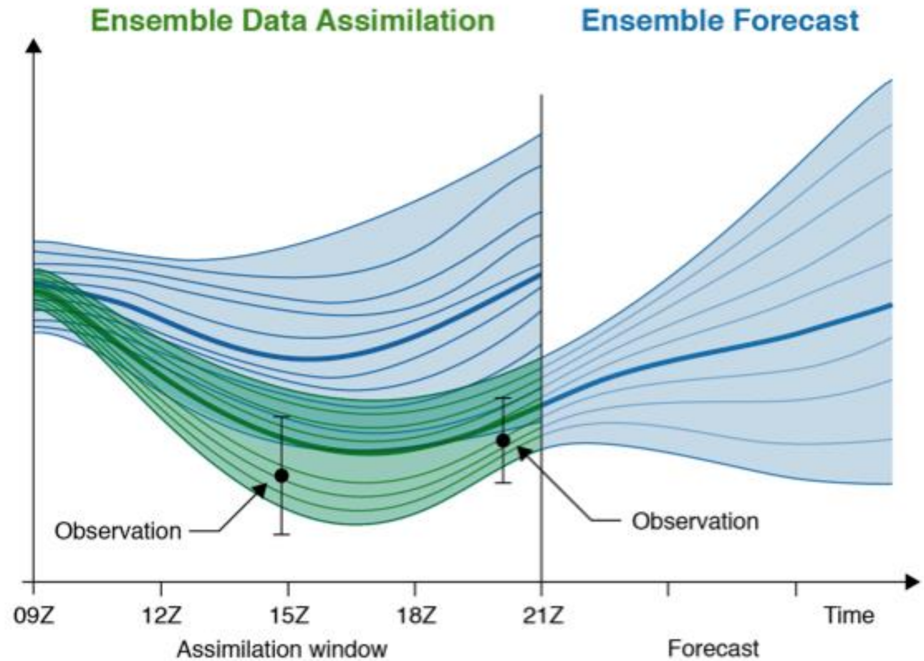
→ Observation bias correction is large in SC4DVAR (part of model bias is absorbed in VARBC). VarBC is much smaller in WC4DVAR

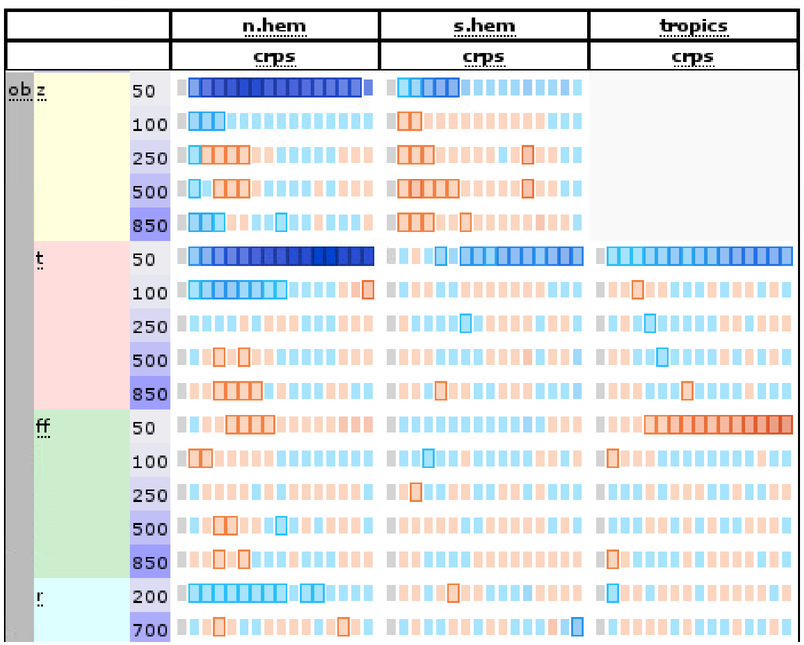
Tentative correction of medium-range forecasts

Weak-constraint 4D-Var in the stratosphere for

- HRES system
- EDA system

Model error estimate is prescribed in the ENS system (ensemble of 15-day forecasts initialized from HRES and EDA)



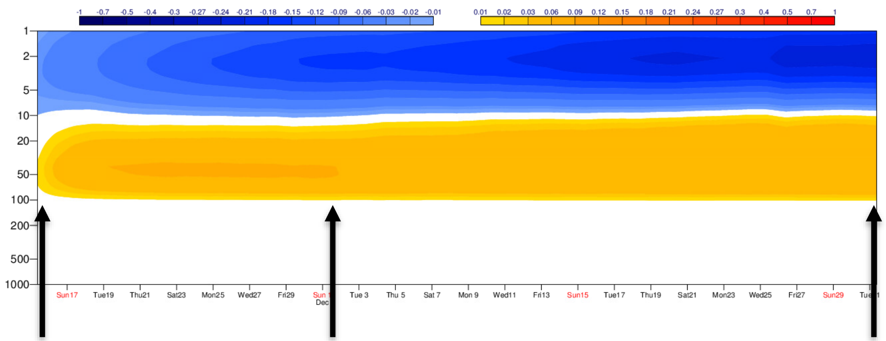


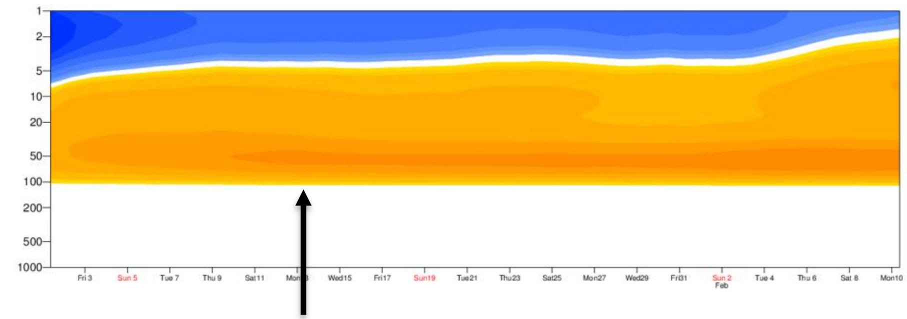
Between 20/07/2019 and 12/09/2019

➔ Validity of a constant error forcing over 15 days is questionable

➔ A flow-dependent correction is probably more appropriate

How fast does weak-constraint 4D-Var learn?





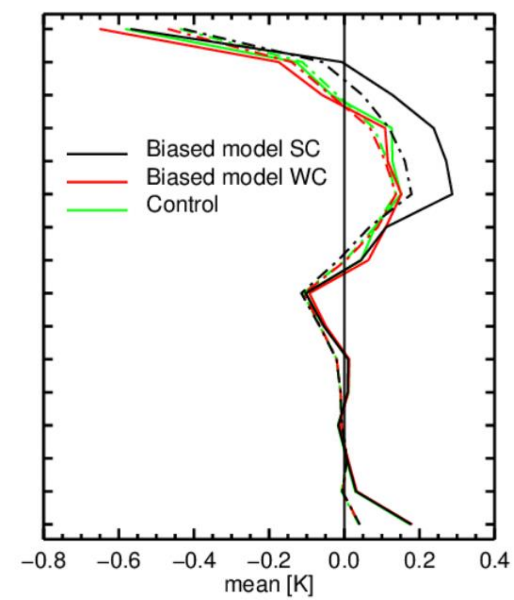
Weak-constraint 4D-Var is cold started (model error is zero)

After 2 weeks, the model error estimate is steady

Introduce a (bad) model change (vertical finite element replaced by vertical finite difference)

Weak-constraint 4D-Var learns the new bias

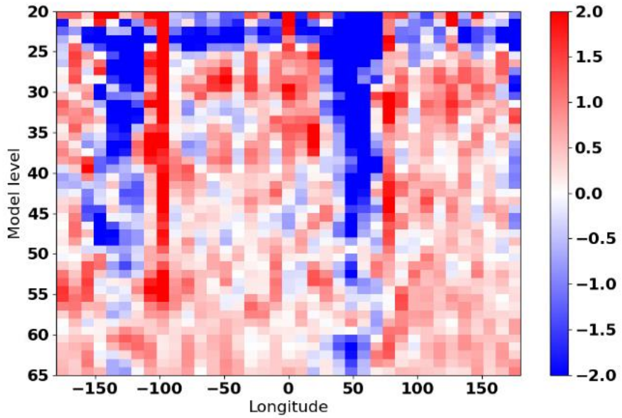
➔The fit to the observations is not degraded as with weak-constraint 4D-Var learns the new model error quickly



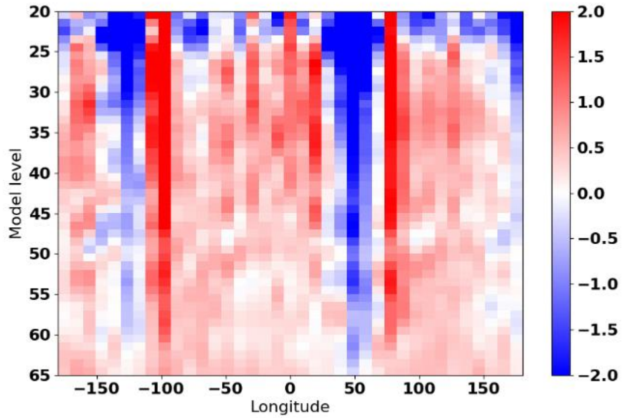
A pure Neural Network approach (in collaboration with NVIDIA)

Train Neural Networks on the NVIDIA high-performance GPU systems

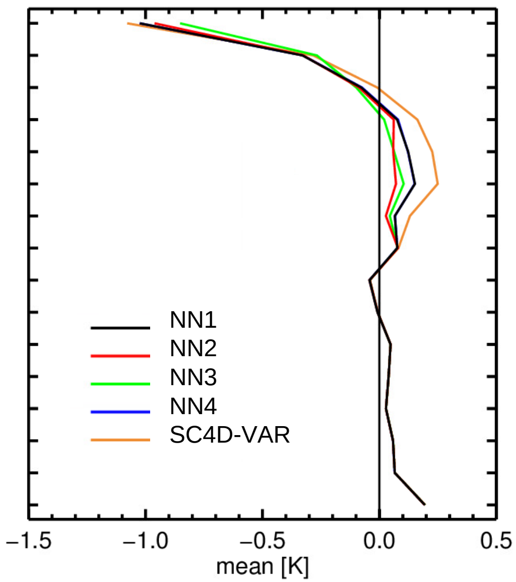
Background departure



Estimation from NN

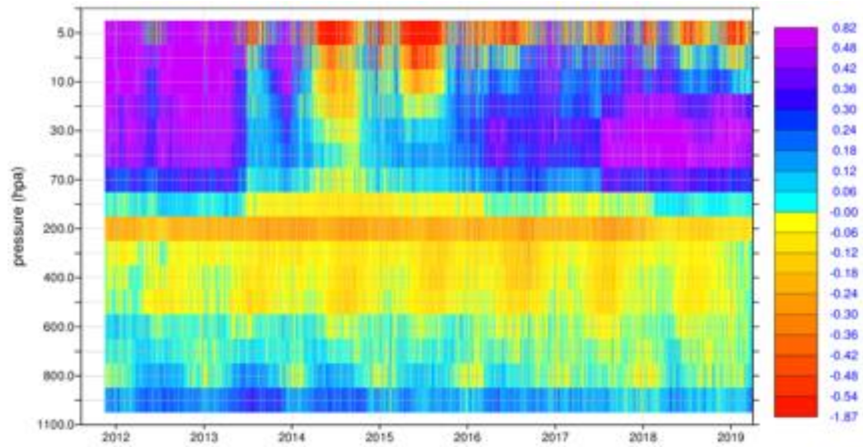


Temperature first-guess error (wrt radiosondes)

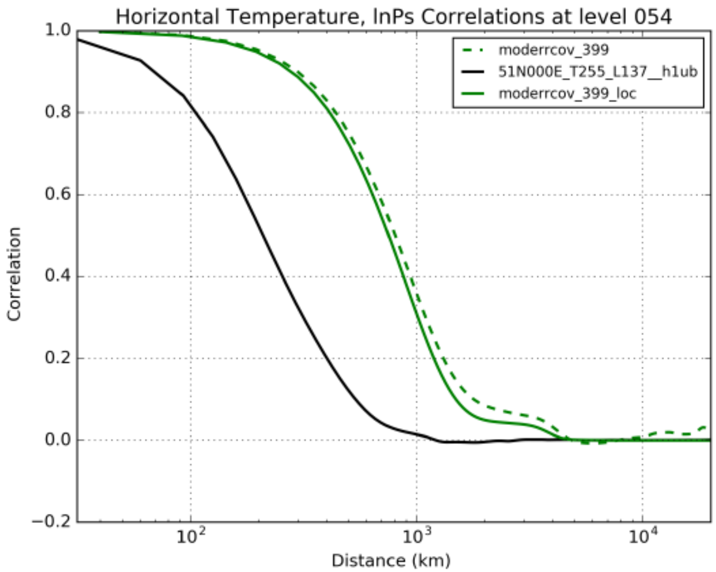


- ➔Database with ERA5 temperature background and RO background departure. 3D convolutional neural networks trained on this database
- ➔Reduction in the mean error when correction is applied in 4D-Var. Comparison with weak-constraint 4D-Var is ongoing
- ➔NN requires a large training dataset and needs to be retrained if the model is changing

Summary

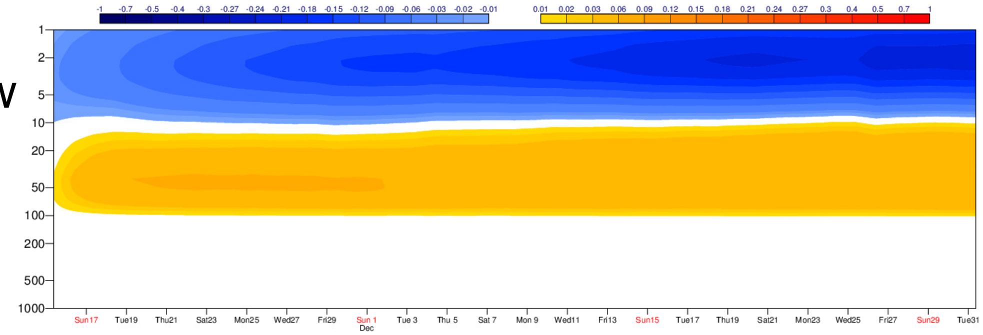


1 “The presence of bias can be detected by monitoring differences between observations and their model equivalents”



2 “Separation of different bias sources requires additional information, such as hypotheses about the error characteristics”

3 “The algorithm learns, after the first few analyses, that the model forecast consistently overestimates or underestimates the observations.”



D. Dee, Bias and data assimilation, 2006.

Summary

Quarterly Journal of the
Royal Meteorological Society

RMetS

RESEARCH ARTICLE

Towards an unbiased stratospheric analysis

P. Laloyaux, M. Bonavita, M. Dahoui, J. Farnan, S. Healy, E. Hólm, S. T. K. Lang

First published: 05 April 2020 | https://doi.org/10.1002/qj.3798 | Citations: 3

Future work

- Investigate how to extend weak-constraint 4D-Var in the troposphere
- Produce a fair comparison with a pure Neural Network approach
- Provide a better understanding on the impact for VarBC

Thank you!

Massimo Bonavita, Marcin Chrust, Mohamed Dahoui, Peter Dueben, Jacky Goddard, Selime Gürol, Sean Healy, Elias Holm, Simon Lang, Inna Polichtchouk and many others