An observation error correlation model for wind data.

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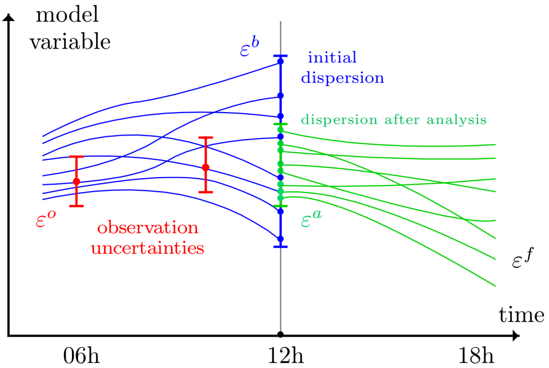
Ensemble data assimilation



The forecast error depends on the analysis error : $\varepsilon^f = \boldsymbol{M}\varepsilon^a + \varepsilon^m$.
The analysis error depends on the observation error : $\varepsilon^a = (\boldsymbol{I} - \boldsymbol{K}\boldsymbol{H})\varepsilon^b + \boldsymbol{K}\varepsilon^o$, where $\varepsilon^o = \boldsymbol{R}^{1/2}\boldsymbol{w}$ contains perturbations.

(e.g. Houtekamer et al. (1996), Fisher (2003), Berre et al. (2006))

Generation of the perturbed observations

General methodology :

- To generate new observations for the ensemble, we add a correlated noise ε_i such that $\boldsymbol{y}_i = \boldsymbol{y} + \varepsilon_i$.
- Given $\boldsymbol{w}$ a gaussian white noise, we set $\varepsilon_i = \boldsymbol{R}^{1/2}\boldsymbol{w}$.
- Therefore, ε_i is itself a realisation of $\mathcal{N}(0, \boldsymbol{R})$.

When $\boldsymbol{y}$ contains u and v :

- $\boldsymbol{R}$ will contain $u-u$ correlations, $v-v$ correlations and **cross-correlations**

$$\boldsymbol{R} = \begin{pmatrix} \boldsymbol{R}_{uu} & \boldsymbol{R}_{uv} \\ \boldsymbol{R}_{vu} & \boldsymbol{R}_{vv} \end{pmatrix}.$$

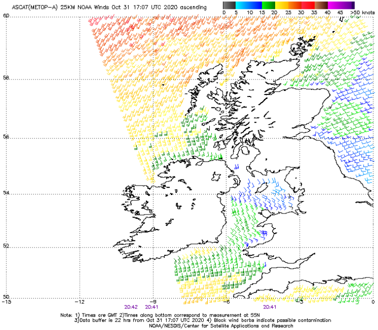
Scalar observations VS **vector observations**

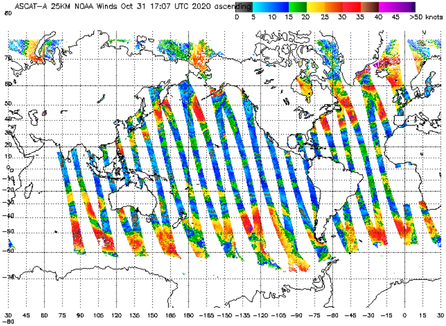
- Work has been done to account for **spatial** correlations in **scalar** observations such as **satellite data** (Brankart et al. (2009), Ruggiero et al. (2016), Michel (2018), Guillet et al. (2019)).
- The modelling of correlation operators for **vector** observations such as AMV or SCAT is a new topic (the only possible exception being Isaksen and Radnóti (2010)).

- Existing work mostly revolves around the **estimation** of wind errors correlations in AMV (Bormann et al. (2002)) / scatterometer data (Cotton (2016)).
- They show that wind errors may contain **cross-correlations** (i.e. between u and v)

Question : How to represent $\boldsymbol{R}$ and $\boldsymbol{R}^{1/2}$ for these types of observations in order to generate **wind pertubations for the EDA** ?

NOAA wind vectors (25km) retrived from ASCAT - METOP A on Sunday, november 1st.





Outline

From wind to potentials

One way to account for **cross-correlations** is to transform the winds into **velocity potential** and **streamfunction** (assuming these new control variables are uncorrelated).

This method was used in Schlatter (1974) with the additional condition $div(\boldsymbol{u}) = 0$ to model "geostrophic" covariances in $\boldsymbol{B}$.

The Hodge-Helmholtz decomposition leads to :

$$\boldsymbol{u} = \mathrm{rot}(\psi \boldsymbol{e}_z) - \mathrm{grad}(\chi).$$

$$\boldsymbol{u} = \begin{pmatrix} -\mathrm{grad} & \mathrm{rot} \end{pmatrix} \begin{pmatrix} \chi \\ \psi \end{pmatrix} = \boldsymbol{S} \begin{pmatrix} \chi \\ \psi \end{pmatrix}.$$

(Here, we used the identity : $\mathrm{rot}(-\psi \boldsymbol{e}_z) = \boldsymbol{e}_z \times \mathrm{grad}(\psi)$).

The wind correlation operator

The wind correlation operator is formulated as $\boldsymbol{R} = \boldsymbol{R}^{1/2}\boldsymbol{R}^{\mathrm{T}/2}$ where

$$\boldsymbol{R}^{1/2} = \boldsymbol{S}\boldsymbol{C}^{1/2}.$$

The operator $\boldsymbol{S}$ transforms (χ, ψ) into (u, v) and is written :

$$\boldsymbol{S} = \begin{pmatrix} -\mathrm{grad} & \mathrm{rot} \end{pmatrix}.$$

The matrix $\boldsymbol{C}^{1/2}$ is block diagonal :

$$\boldsymbol{C}^{1/2} = \begin{pmatrix} \boldsymbol{C}_{\chi\chi}^{1/2} & 0 \\ 0 & \boldsymbol{C}_{\psi\psi}^{1/2} \end{pmatrix}.$$

where each component is itself built from a diffusion operator $\boldsymbol{C}_{\star\star}^{1/2} = \gamma^{1/2}(1 - \ell^2\Delta)^{-m/2}$.

Finite element discretization of the diffusion equation

Using the finite element method, the continuous equation

$$(1 - \ell^2 \Delta) u_{k+1} = u_k$$

becomes

$$(\boldsymbol{M} + \boldsymbol{K}) \boldsymbol{\alpha}_{k+1} = \boldsymbol{M} \boldsymbol{\alpha}_k,$$

where $0 \leq k < m$.

The mass matrix $\boldsymbol{M}$ and the stiffness matrix $\boldsymbol{K}$ are very sparse and can be factored using a Cholesky algorithm.

Therefore, provided m is even :

$$\boldsymbol{R}^{1/2} = [(\boldsymbol{M} + \boldsymbol{K})^{-1} \boldsymbol{M}]^{m/2} \boldsymbol{M}^{-1/2}.$$

Finite element discretization of the 1st order derivatives

First, we write

$$\boldsymbol{S} = \begin{pmatrix} -\text{grad} & \text{rot} \end{pmatrix} = \begin{pmatrix} -\partial_x & \partial_y \\ -\partial_y & \partial_x \end{pmatrix}.$$

The finite element discretization yields

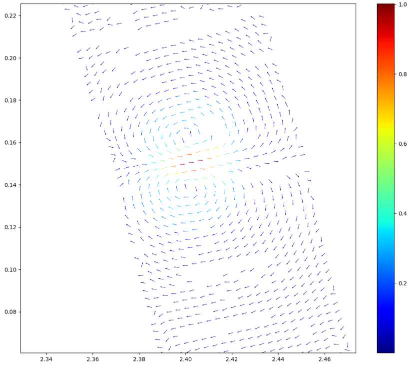
$$\boldsymbol{S} = \begin{pmatrix} -\boldsymbol{D}_x & \boldsymbol{D}_y \\ -\boldsymbol{D}_y & \boldsymbol{D}_x \end{pmatrix}.$$

Again, the matrices $\boldsymbol{D}_x$ and $\boldsymbol{D}_y$ are sparse.

Their transpose can be computed via $\boldsymbol{D}_x^{\mathrm{T}} = -\boldsymbol{D}_x$ and $\boldsymbol{D}_y^{\mathrm{T}} = -\boldsymbol{D}_y$.

Outline

Typical response on unstructured meshes

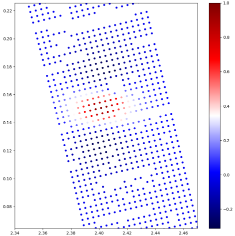


A plot of $\boldsymbol{R}\delta_k$ where k is the observation at the center of the domain. (Experiments using SCAT observations thinned at a 50km resolution. The length-scale is $\ell = 190km$)

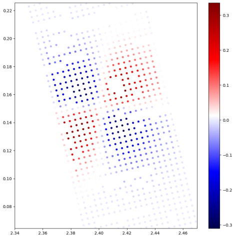
Results :

- Imposing $div(\boldsymbol{u}) = 0$ results in geostrophic balance.
- Missing data do not penalize the method.
- the amplitude (variance) has been normalized.

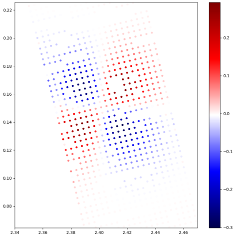
Cross correlations in $\boldsymbol{R}$



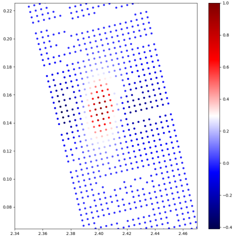
$u - u$ **correlations**



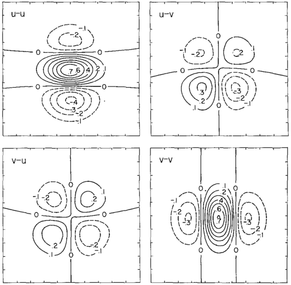
$u - v$ **correlations**



$v - u$ **correlations**



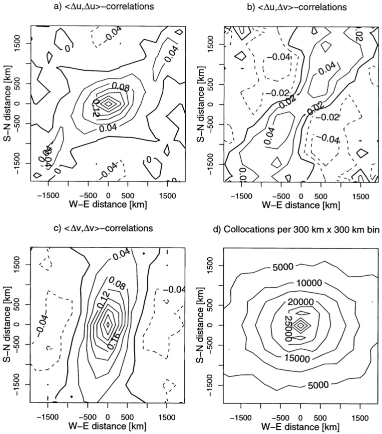
$v - v$ **correlations**



Reproduces the theoretical functions shapes from Schlatter (1974) in the matrix $\boldsymbol{B}$.

Geostrophy in practice

Diagnostics of AMV error correlations from Bormann (2003).



In practice, there is a slight **tilt** : non geostrophy ? χ and ψ in reality ?

Outline

Summary of the method

- We extended the method introduced in Guillet et al. (2019) to the case of **vector observations**.
- This time, we are interested in $\boldsymbol{R}^{1/2}$ to generate ensemble perturbations.
- To account for **cross-correlations** in the wind vectors, we worked on the **velocity potential** and **streamfunction**.
- The 1st order and 2nd order derivatives are all discretized using the **finite element method**.
- This allows treatment on **unstructured meshes**.
- We recover results from previous studies (Schlatter (1974) for $\boldsymbol{B}$ or Bormann (2003) for $\boldsymbol{R}$).

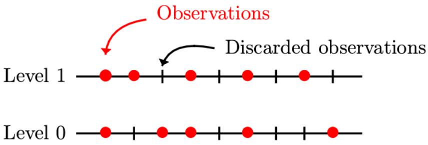
Future of this work

This work on vector observations is new and there is still work to do before implementation in a larger scale system :

- The **normalization** of the correlation operators is not computed analytically (needs tuning).
- Diagnostics for AMV data suggests the potentials χ and ψ might be (cor)related in practice.
- We did not adress the question of $\boldsymbol{R}^{-1}$: the existence of the inverse may depend on the boundary conditions on the domain.

Related topic : LEFE project on 3D correlations

- We know how to account for **horizontal** or **vertical** correlations.
- But the horizontal structure of observations can be different at each vertical level (quality control, thinning).



- We want to investigate 3D finite elements (e.g. for radar data) or a 2D+1D approach (for satellite data).
- **LEFE project for the period 2020-2023**.

Related topic (long term) : a software based on Atlas

- Atlas : a library for developing flexible next-generation NWP models.
- Provides mesh-generation capabilities from a wide catalogue of meshes.
- Mainly developped in C++, can be interfaced with Fortran.

- Our goal : provide a software for modelling / applying matrix $\boldsymbol{R}$ and its inverse based on Atlas.
- Work postponed due to quarantine etc. **Will 2021 be the year ? ?**

Thank you