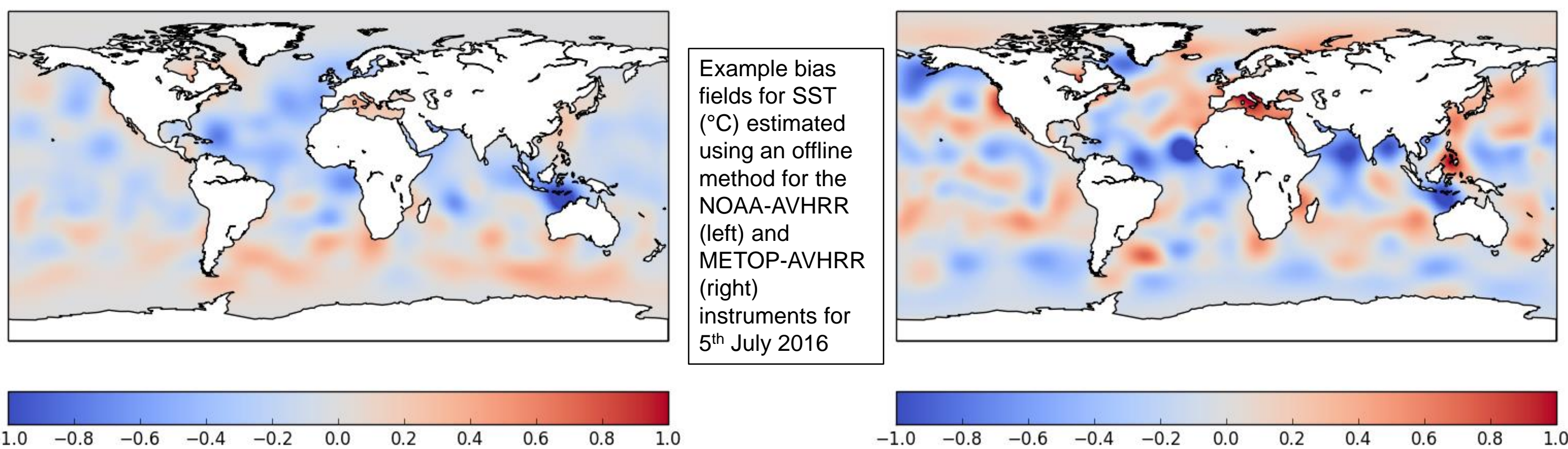


Variational bias correction of satellite oceanographic measurements incorporating observations of the bias

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1. Introduction

As generally formulated, data assimilation systems assume that observations are unbiased. In reality this is not the case and satellite observations of Sea Surface Temperature (SST), Sea Surface Salinity (SSS) and Sea Level Anomaly (SLA) are expected to contain systematic error. There are many sources of this bias, with atmospheric affects on the radiative signal a big issue for SST and SSS, while biases in the mean dynamic topography impact SLA measurements.



To account for these biases, the Met Office has developed a variational technique for bias correction. **A key component of this system is the use of 'observations-of-bias' to constrain the bias calculation (used for SST and SSS biases, not used for SLA).**

Operationally the variational method is used to correct:

- Biases in satellite SST data on a per satellite basis with reference data coming from surface drifters and some infrared SST data which are considered unbiased.
- A slowly varying bias correction for SLA data to account for errors in the MDT.
- Biases in satellite SSS data from SMOS/Aquarius/SMAP on a per satellite basis (this is not yet implemented operationally; Martin et al. 2019).

For Further details please see [While et al \(2019\)](#)

2. Variational Bias Correction Method

In our variational bias correction methodology we modify the standard variational cost function to become:

$$J = 0.5(\mathbf{x} - \mathbf{x}^f)^T \hat{\mathbf{B}}^{-1} (\mathbf{x} - \mathbf{x}^f) + 0.5(\mathbf{y} - (\mathbf{H}_y \mathbf{x} + \mathbf{H}_b \mathbf{b}))^T \hat{\mathbf{R}}^{-1} (\mathbf{y} - (\mathbf{H}_y \mathbf{x} + \mathbf{H}_b \mathbf{b})) + 0.5(\mathbf{b} - \mathbf{b}^f)^T \hat{\mathbf{O}}^{-1} (\mathbf{b} - \mathbf{b}^f) + 0.5(\mathbf{z} - \mathbf{H}_z \mathbf{b})^T \hat{\mathbf{L}}^{-1} (\mathbf{z} - \mathbf{H}_z \mathbf{b}).$$

Where:

\mathbf{x} :-	state vector	\mathbf{x}^f :-	background (forecast) vector
\mathbf{b} :-	state bias vector	\mathbf{b}^f :-	background (forecast) bias vector
\mathbf{y}	observation vector	\mathbf{z} :-	observations of bias vector
$\hat{\mathbf{B}}$:-	prescribed background error covariance	$\hat{\mathbf{R}}$:-	prescribed observation error covariance
$\hat{\mathbf{O}}$:-	prescribed bias error covariance	$\hat{\mathbf{L}}$:-	prescribed 'obs of bias' error covariance
$\mathbf{H}_y, \mathbf{H}_b, \mathbf{H}_z$:-	observation operators		

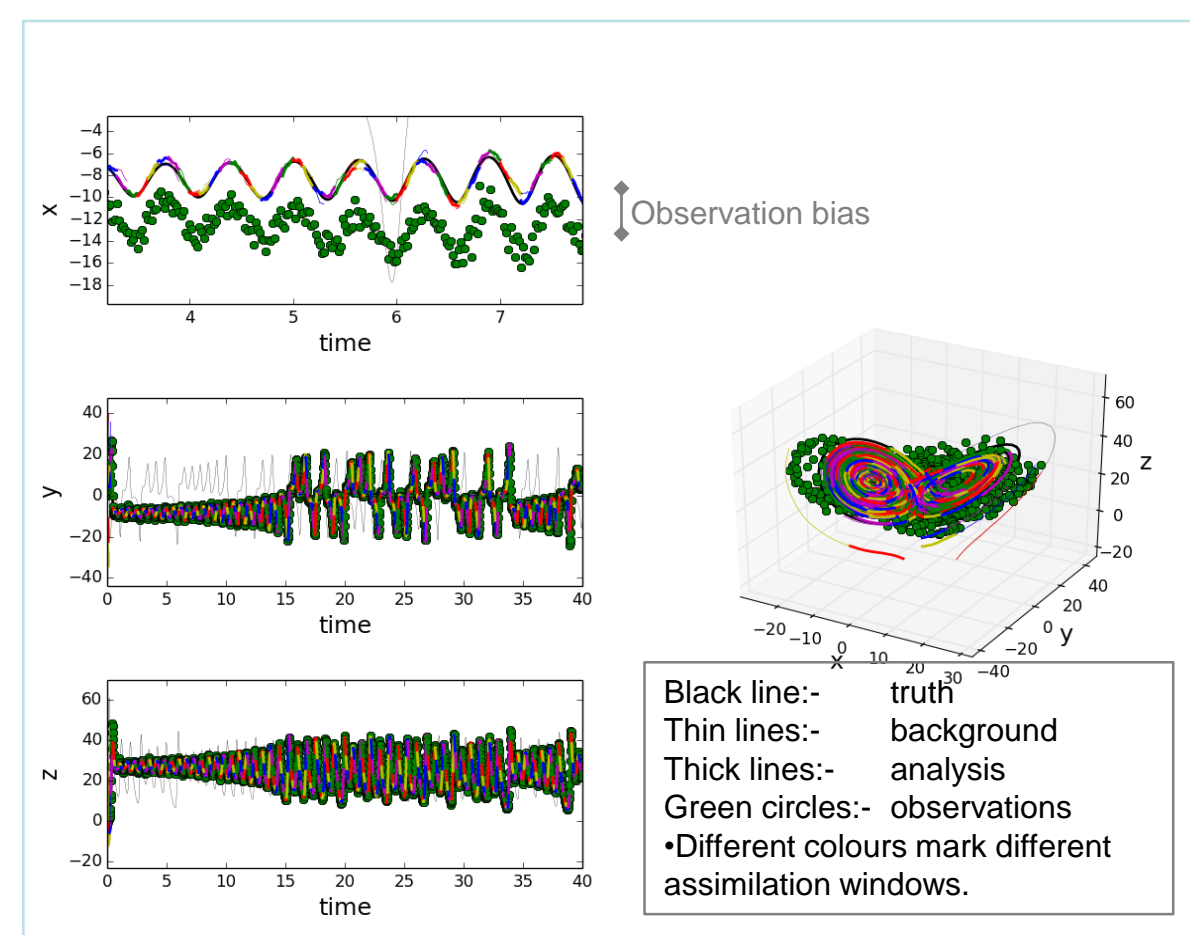
In deriving this equation we assume that model bias has already been accounted for and is negligible.

Minimising J provides both an analysis of \mathbf{x} and an analysis of the bias \mathbf{b} .

A key feature of the bias correction system is the presence of 'observations-of-bias' \mathbf{z} . These help to constrain the bias analysis. In practice observations-of-bias are the difference between matchups of co-located biased and (assumed) unbiased observations. We assume that most satellite data are biased, but that in-situ data and data from some high quality IR satellite instruments are unbiased.

To prevent double counting, observations used to calculate the matchups in \mathbf{z} are not used as direct observations in \mathbf{y}

3. Tests with Lorenz 63 system

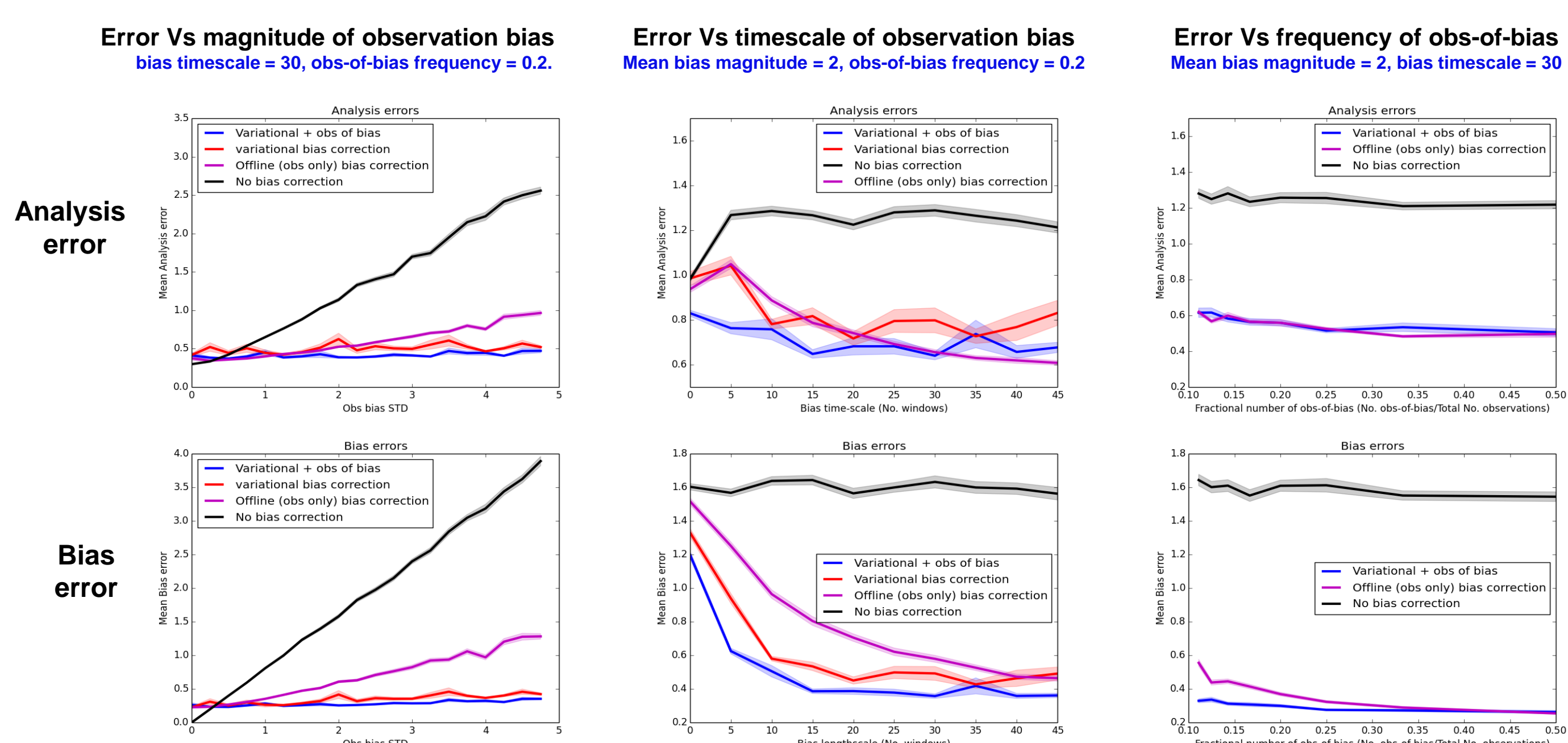


We have tested our variational bias correction system in a 3DVar context using the Lorenz 63 model.

Noisy observations were assimilated into all 3 axes, but only observations in the x-axis are biased.

The observation bias was not constant, but instead slowly varied on a user determined time-scale.

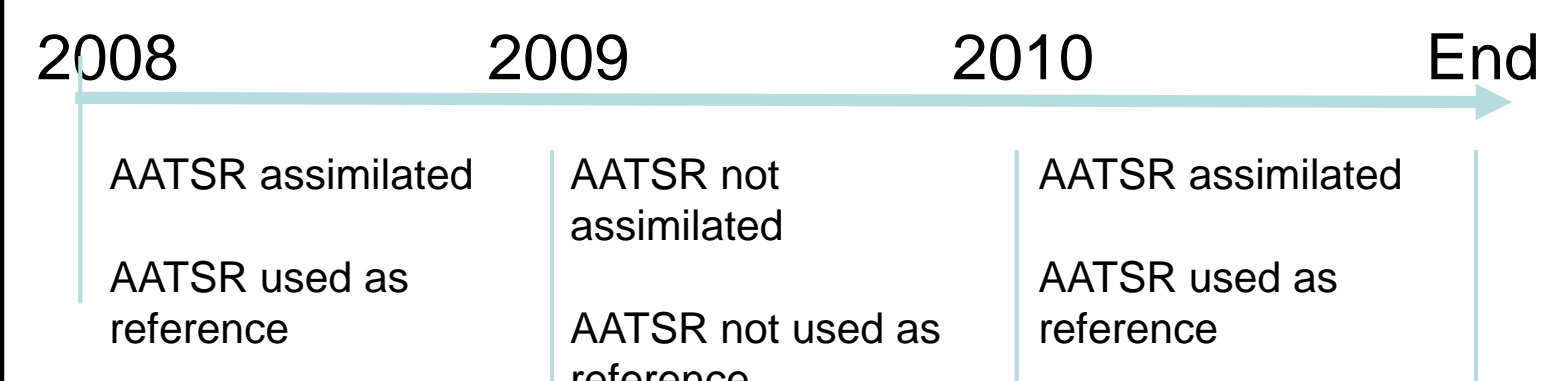
For each data point in the experiments, an ensemble of 100 model runs with perturbed parameters and initial state were used to calculate the statistics.



From the plots above we can see that:

- Without bias correction (black line) there is a near linear increase in the errors as the bias increases.
- Errors also increase linearly, though more slowly, when bias correcting using just the observations (purple line).
- Variational methods (blue and red lines) are near agnostic to the amount of bias.
- All bias correction methods degrade results at very small biases, but improve results at larger biases.
- Bias correction with observations-of-bias produces the best results, especially when observations-of-bias are frequent.
- Not shown, but the above results can be shown to agree with theoretical predictions for a simple linear system (see While et al; 2019)

4. Testing using a global ocean reanalysis

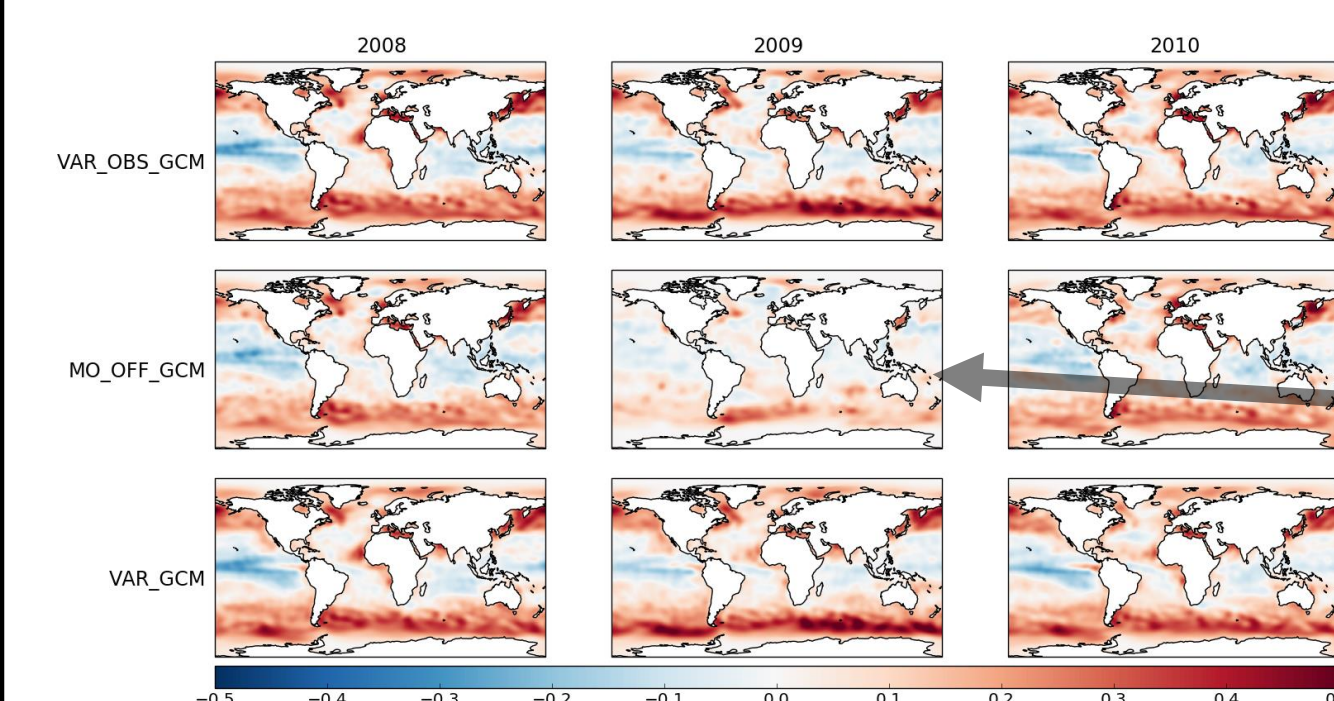


To test the bias correction scheme for SST we ran four 3 year experiments (2008-2010):

- NO_COR_GCM**:- No bias correction, all observations assimilated directly
- VAR_GCM**:- Pure variational bias correction of SST, no observations-of-bias..
- MO_OBS_GCM**:- Offline bias correction of SST using just the observations-of-bias. In this scheme the bias was estimated and removed from the observations before assimilation took place
- VAR_OBS_GCM**:- Variational bias correction of SST including observations-of-bias.

High quality AATSR data was used as a reference to generate the observations-of-bias, but was withheld during 2009.

All experiments were based on the Met Office's FOAM system using 3DVar assimilation (Blockley et al; 2014)

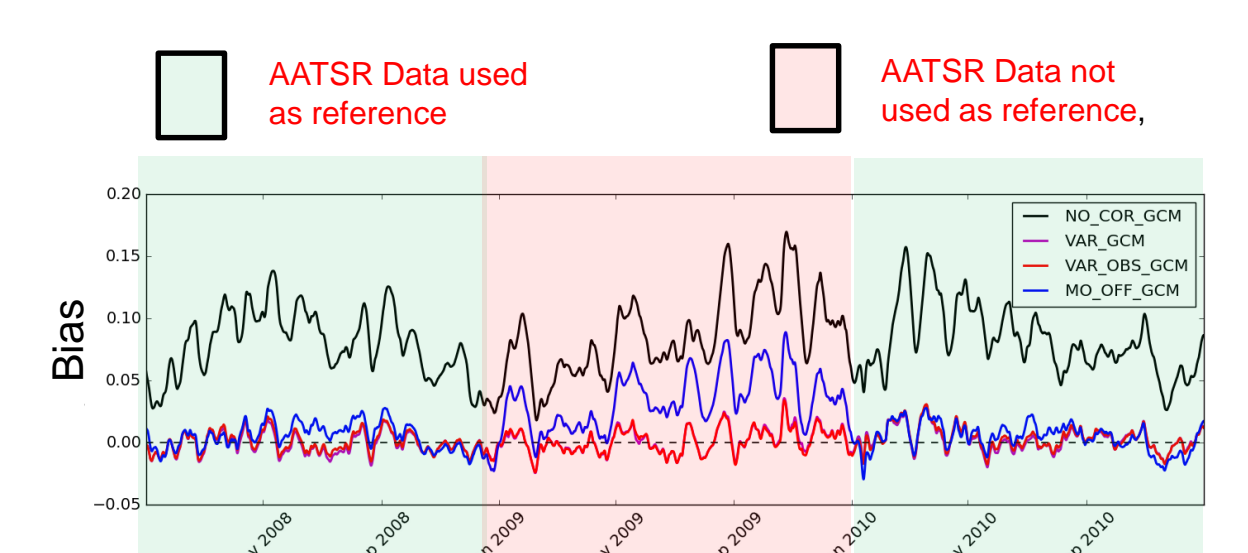


This plot shows the mean bias calculated for the microwave AMSR-E instrument.

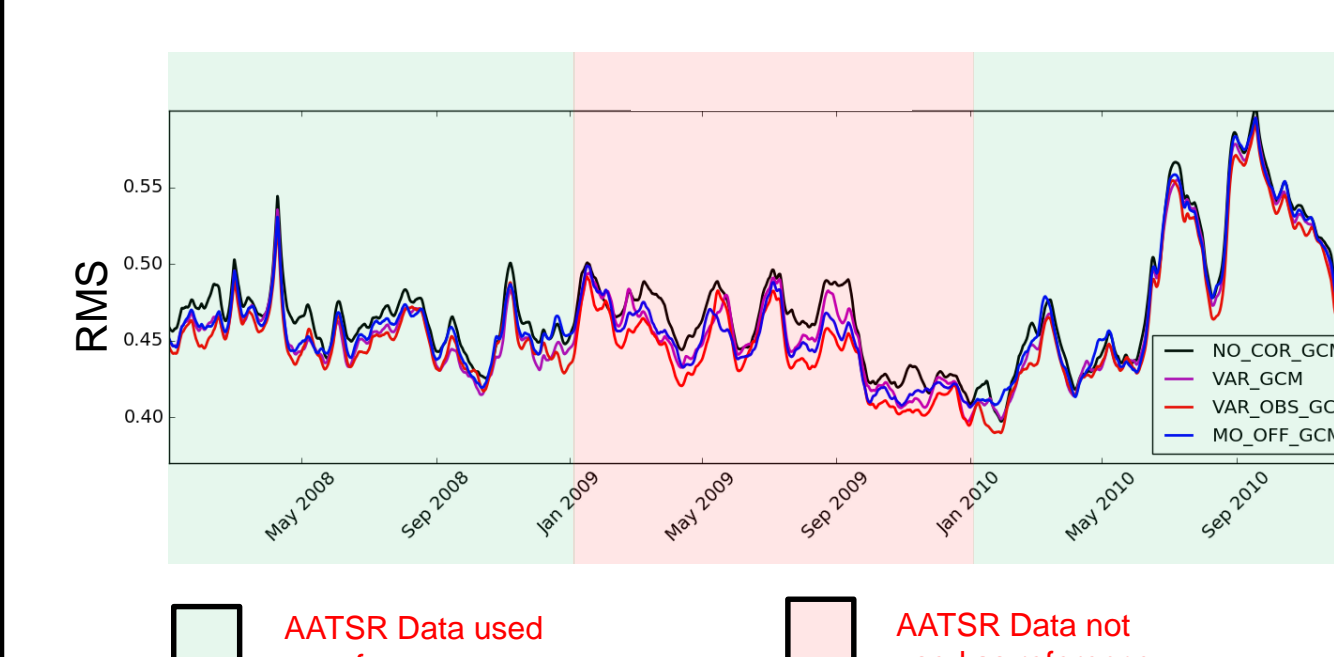
While the 2 variational experiments are roughly the same for all 3 years. There is a significant change in the middle year in the MO_OBS_GCM experiment.

Shown right is the global bias between 1 day forecasts of the model and AMSR-E observations.

All methods reduced the observation bias, but the offline method of MO_OBS_GCM struggled during the year without the AATSR reference.



Taken together the above plots show the variational methods outperforming an offline method when reference observations are sparse. Yet it is difficult to see much difference between the variational schemes with and without observations-of-bias.



However, the impact of the observations-of bias can be seen if we look at the RMS difference between 1 day forecasts and in-situ data (left).

For this metric the VAR_OBS_GCM experiment (which included observations-of-bias) shows slightly improved statistics compared to the other experiments.

References

- Blockley, E. W., Martin, M. J., McLaren, A. J., Ryan, A. G., Walters, J., Lea, D. J., ... Storkey, D. (2014). Recent development of the Met Office operational ocean forecasting system: an overview and assessment of the new Global FOAM forecasts. *Geoscientific Model Development*, 7(6), 2613–2638. <https://doi.org/10.5194/gmd-7-2613-2014>
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