The impact of using reconditioned correlated observation error covariance matrices in the Met Office 1D-Var system

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Motivation

- Including observation error correlation (OEC) information in numerical weather predication (NWP) improves high resolution forecasts and allows the use of high-density satellite data.
- The minimum eigenvalue of the correlated OEC matrix was shown to be important in determining convergence speed of the minimisation of a variational data assimilation problem (Tabeart et al, 2018a).
- Reconditioning methods are often used to include OEC information in a more computationally efficient way (Tabeart et al, 2018b).

Aim: Study the use of the ridge regression method in the Met Office 1D-Var system - this reduces the condition number of a matrix by increasing all its eigenvalues.

Definition: Ridge regression method of reconditioning

Let $\mathbf{R}$ be an OEC matrix, and let κ_{max} be a user-defined target condition number. We define $\mathbf{R}_{RR}$ as the reconditioned OEC matrix, $\mathbf{R}_{RR} = \mathbf{R} + \delta\mathbf{I}$, such that $\kappa(\mathbf{R}_{RR}) = \kappa_{max}$. The ridge regression reconditioning constant, δ, is given by $\delta = (\lambda_{max}(\mathbf{R}) - \lambda_{min}(\mathbf{R})\kappa_{max})/(\kappa_{max} - 1)$.

Description of Met Office 1D-Var system

Case study: IASI (Infrared Atmospheric Sounding Interferometer) in Met Office 1D-Var system, 16th June 2016 0000Z. 97330 common observations across all experiments.

Roles of 1D-Var in the Met Office system

1. Quality control: reject observations if associated 1D-Var minimisation requires > 10 iterations.
2. Estimation of variables: Skin temperature (ST), cloud fraction (CF) and cloud top pressure (CTP) are not included in 4D-Var state vector, so estimates from 1D-Var retrieval are used.

Two groups of experiments

- Diagonal choices: E_{diag} (current operational choice of OEC matrix), E_{infl} (inflated diagonal matrix, previous choice of OEC matrix)
- Correlated choices: E_{est} (the estimated OEC matrix), E_{1500}, E_{1000}, E_{500}, E_{67} (reconditioned versions of E_{est}, where the subscript denotes the choice of κ_{max} for each experiment).

Table 1: Experiments, and minimum eigenvalue and condition number of corresponding OEC matrix.

Experiment name	E_{diag}	E_{est}	E_{1500}	E_{1000}	E_{500}	E_{67}	E_{infl}
$\lambda_{min}(\mathbf{R})$	0.025	0.00362	0.00482	0.007244	0.0145	0.1010	0.0625
$\kappa(\mathbf{R})$	9.263	2730	1500	1000	500	67	64

Impact on Met Office 1D-Var Routine

Convergence of 1D-Var improves with reconditioning

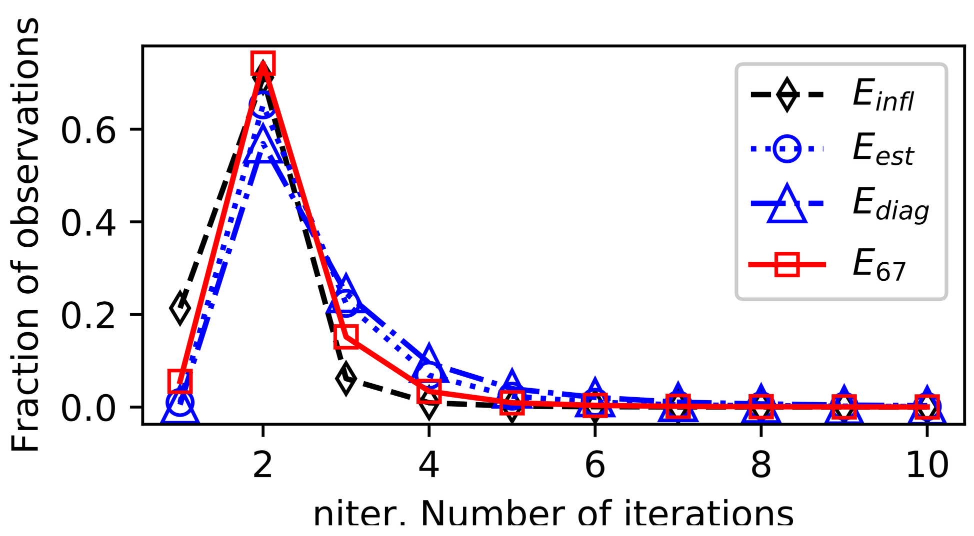

Figure 1: Number of iterations required for convergence of the minimization of the 1D-Var cost function as a fraction of 97330 (total number of observations common to all choices of $\mathbf{R}$).

Effect on 1D-Var retrievals: changes to temperature retrievals are small, changes to humidity retrievals are larger

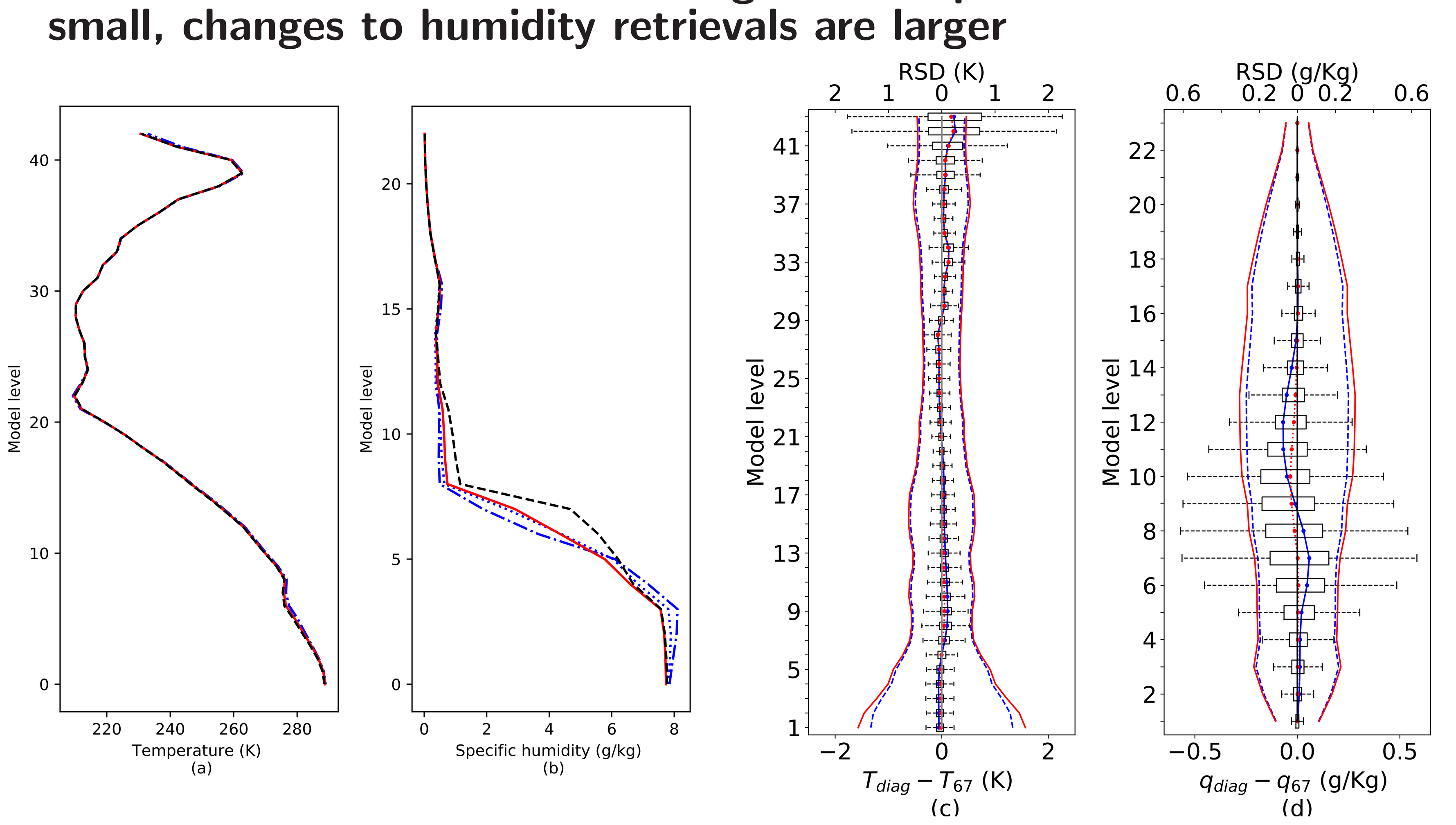

Figure 2: Example retrieved profiles of temperature (a) and specific humidity (b), and differences in retrievals between E_{diag} and E_{67} for temperature (c) and specific humidity (d) for 97330 observations.

Impact on variables that influence 4D-Var routine

Effect on quality control procedure: more reconditioning = more observations pass quality control

Table 2: Number of accepted observations for each experiment, E_{exp}. For E_{diag} the number of accepted observations is 100686.

Experiment	E_{est}	E_{1500}	E_{1000}	E_{500}	E_{67}	E_{infl}
No. of accepted obs (T)	100655	100795	101002	101341	102333	102859
No of obs accepted by both E_{diag} and E_{exp}	99039	99175	99352	99656	100382	100679

Agrees with the reduction in the number of iterations with decreasing κ_{max} seen in Figure 1.

Changes to retrieved values for ST, CF, CTP: most changes are small. As κ_{max} decreases, differences between retrievals for E_{diag} and E_{exp} reduce

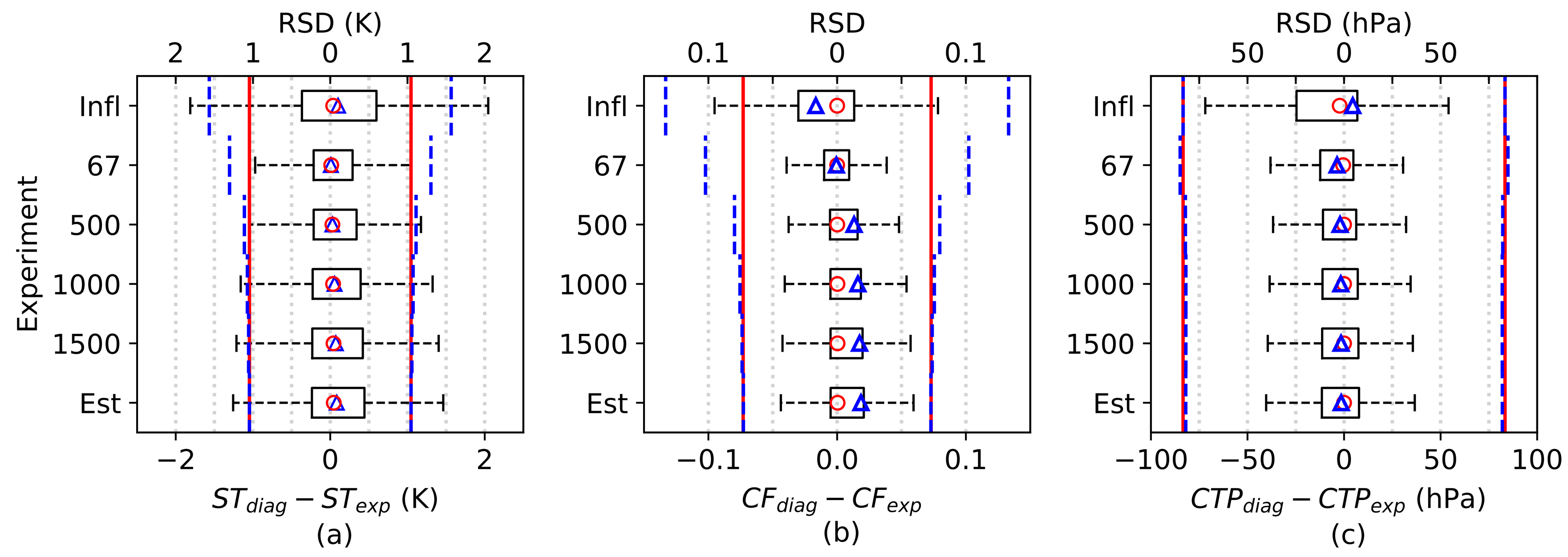

Figure 3: Box plot of differences between retrieved variables for $E_{diag} - E_{exp}$. Outliers not shown.

Notice the qualitative difference between the plots for correlated choices of $\mathbf{R}$ and E_{infl}; although convergence is fastest for E_{infl}, the retrieved values for ST, CF and CTP are all very different from E_{diag}.

Changes to ST, CF, CTP: There are lots of outliers, but the number of 'extreme' outliers is very small.

Table 3: Percentage of observations with retrievals that are outliers for CF, CTP and ST. The maximum absolute difference for CF is 1 and for CTP is 913.25hPa.

	E_{est}	E_{1500}	E_{1000}	E_{500}	E_{67}	E_{infl}
% outliers (CF)	23.9	24.0	24.2	24.6	25.3	21.4
% outliers (CTP)	22.8	22.8	23.0	22.9	21.4	18.8
% outliers (ST)	15.1	15.3	15.6	16.3	17.6	15.9
Maximum difference (ST (K))	21.67	21.12	21.14	22.38	21.03	26.83
Minimum difference (ST (K))	-33.52	-33.01	-32.14	-29.76	-23.82	-20.88

Table 4: Percentage of extreme outliers for CF, CTP and ST for each experiment. This corresponds to absolute differences greater than 25% of the maximum differences introduced in Table 3.

	E_{est}	E_{1500}	E_{1000}	E_{500}	E_{67}	E_{infl}
% extreme outliers ($\lvert CF\rvert > 0.25$)	4.9	4.7	4.4	3.9	3.2	7.5
% extreme outliers ($\lvert CTP\rvert > 225hPa$)	3.3	3.3	3.3	3.3	2.7	4.4
% extreme outliers ($\lvert ST\rvert > 5K$)	1.6	1.5	1.5	1.4	1.4	3.6

It is likely that these large differences are caused by changes to cloud for different choices of OEC. However, further work is needed to understand the origin and consequences of these extreme differences fully.

Conclusions

- Introducing correlated observation error covariance (OEC) matrices improves convergence compared to the current operational OEC matrix. Increasing the amount of reconditioning further improves convergence.
- Increasing the amount of reconditioning increases the number of observations that pass the quality control step.
- Changing the OEC matrix results in mostly small changes to retrieved ST, CF and CTP values. However, a small number ($< 5\%$) of retrieved values are changed by very large amounts.
- As the minimum eigenvalue of the OEC matrix is increased, the difference between the control and experimental retrieved values reduces.

References

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