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Efficient Computation of Matrix-vector Products in Data Assimilation

DARE: Data Assimilation for the Resilient City

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Introduction

In parallel processing, the calculation of matrix-vector products for observations with spatially correlated errors over long lengthscales requires significant communications between CPUs. This may not be computationally feasible in an operational system. The aim of this work is to explore the use of a particular type of the Fast Multipole Method [1] to rapidly compute matrix-vector products.

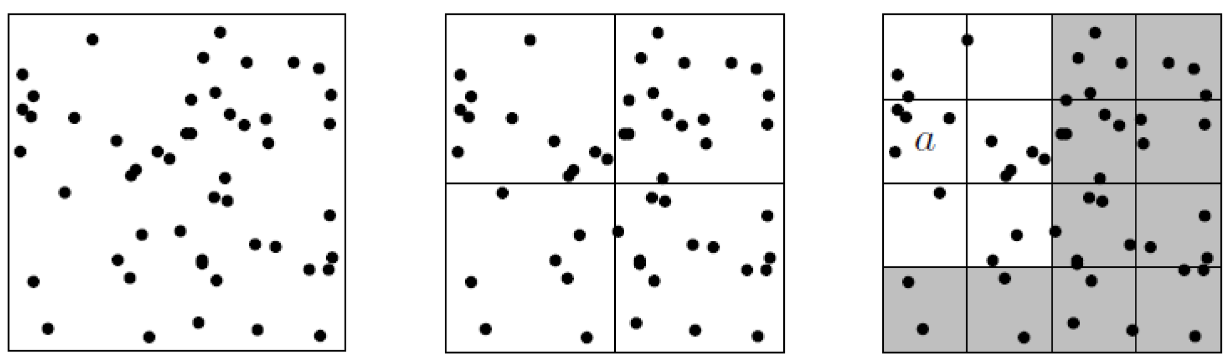
Method

What is it?

The SVD approach of the fast multiple method (SVD-FMM) is a numerical algorithm for efficiently computing matrix-vector products: $\mathbf{q} = \mathbf{A}\mathbf{d}$ [1]. It uses a singular value decomposition (SVD) and works for a wide choice of matrices.

How it works?

- The SVD-FMM starts with partitioning the observations into a boxtree – a hierarchical structure of nested boxes



- Then the near field of a box is made of itself and all its neighbours. The far field of a box is made of everything else. For example, the near field of the box a is all the white boxes, and the far field is all the grey boxes
- The near field calculations are done by the standard matrix-vector multiplication
- The far field calculations are done by using the singular values and vectors of the sub-matrices of $\mathbf{A}$ and the projections of the sub-vectors of $\mathbf{d}$ onto the bases given by different singular vectors

Why use it?

- It reduces the algorithmic complexity of the standard matrix-vector multiplication
- It requires less communication costs in parallel computing

Application to data assimilation

- We need to multiply the inverse of observation error covariance matrix with a vector, i. e., $\mathbf{q} = \mathbf{A}\mathbf{d}$ with $\mathbf{A} = \mathbf{R}^{-1}$
- The matrix-vector multiplication has a linear complexity if $\mathbf{A}$ is diagonal (uncorrelated observation errors)
- The matrix-vector multiplication has a quadratic complexity if $\mathbf{A}$ is dense (correlated observation errors)

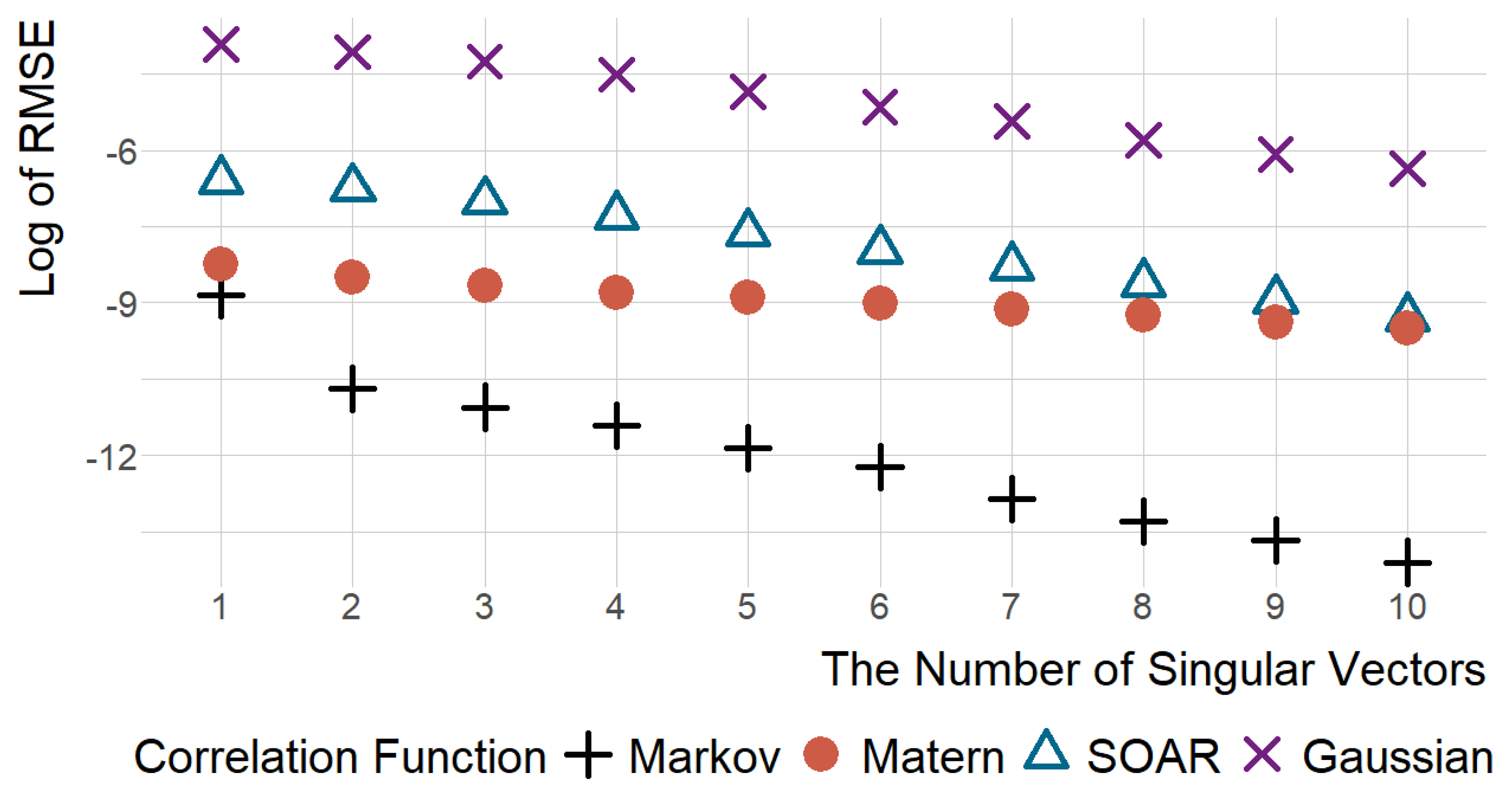
Numerical experiments

Design

We use a wide choice of observation error covariance matrices. The matrices are created using the Gaussian, Markov, Matern and second-order auto-regressive (SOAR) correlation functions. The elements of the innovation vector $\mathbf{d}$ are uniformly distributed random values. We also consider the covariance matrices with reconditioning.

Results

The accuracy of the SVD-FMM is shown as a function of the number of singular vectors for different correlation functions. The covariance matrices are reconditioned to have a condition number of 1000. The RMSE is defined as the root-mean-squared deviations between the elements of $\mathbf{q}$ given by the standard matrix-vector multiplication and SVD-FMM.



- The accuracy of the SVD-FMM increases as more singular vectors are used. However, an increase of the number of singular vectors will slow down the computation
- An optimal number needs to be determined by considering the trade-off between accuracy and efficiency
- Although the covariance matrices are reconditioned to have the same condition number, the RMSE still relies on the original condition numbers of each matrix. The larger the number the bigger the error
- The effects of correlation lengthscale, condition number, reconditioning method and missing observations have also been investigated but are not shown

References

1. Z. Gimbutas and V. Rokhlin, 'A generalized fast multipole method for nonoscillatory kernels', *SIAM Journal on Scientific Computing*, 2003, pp. 796-817
2. G. Hu and S. L. Dance, 'Efficient computation of matrix-vector products in data assimilation', *Manuscript*

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