Gaussian process regression is used to emulate expensive models when performing parameter estimation

Machine learning surrogate models for parameter tuning: The Lorenz 96 as a test case

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The Lorenz96 model

Introduced by Edward Lorenz in a **ECMWF workshop** on predictability, the Lorenz 96 model (hereinafter referred to as L96) is still one of the most used toy models in geoscience.

Following notation from Schneider et al. 2017, The Lorenz-96 model consists of K **slow** variables $X_k(k = 1, \ldots, K)$, each of which is **coupled** to J **fast** variables $Y_{j,k}(j = 1, \ldots, J)$:

$$\frac{dX_k}{dt} = \underbrace{-X_{k-1}(X_{k-2} - X_{k+1})}_{\text{Advection}} \underbrace{-X_k}_{\text{Diffusion}} \underbrace{+F}_{Forcing} \underbrace{-hc\bar{Y}_k}_{Coupling} \tag{1}$$

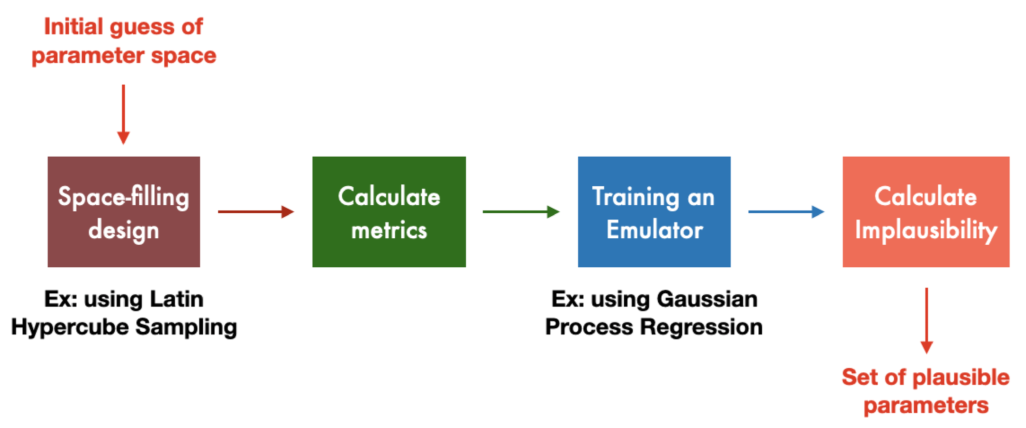
$$\frac{1}{c}\frac{dY_{j,k}}{dt} = \underbrace{-bY_{j+1,k}(Y_{j+2,k} - Y_{j-1,k})}_{\text{Advection}} \underbrace{-Y_{j,k}}_{\text{Diffusion}} \underbrace{+\frac{h}{J}X_k}_{\text{Coupling}} \tag{2}$$

where $\bar{Y}_k = \frac{1}{J}\sum_{j=1}^{J} Y_{j,k}$

The goal of the present ongoing work is to use observations about this system and consider the **History Matching technique** as a means to tune the four parameters $\theta = \{F, h, c, b\}$.

History Matching

The basis of History Matching is to use observed data to **rule-out** any parameter settings which are "**implausible**". Since climate simulation models are often too slow to individually check every possible parameter setting, this is usually done with the help of an emulator. Here a machine learning algorithm, namely, Gaussian Process Regression is used for the emulating step.



Experiments on the Lorenz96

If we denote the time average of a function $\phi(t)$ over the time interval $[t_0, t_0 + T]$ by

$$\langle \phi \rangle_T = \frac{1}{T}\int_{t_0}^{t_0+T} \phi(t)\mathrm{d}t,$$

we seek then to minimise the following **observational error**

$$J_0(\theta) = \frac{1}{2}\left\| \langle \boldsymbol{f}(X,Y)\rangle_T - \langle \boldsymbol{f}(\tilde{X},\tilde{Y})\rangle_\infty \right\|_\Sigma^2 \tag{3}$$

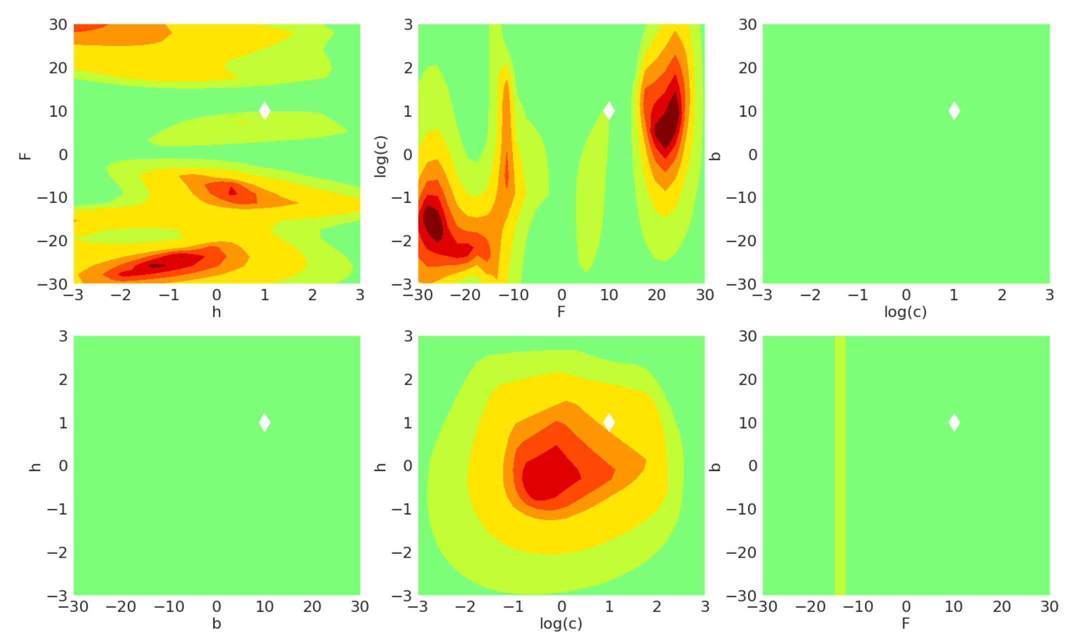
where $\boldsymbol{f}(X,Y) = [X, \bar{Y}, X^2, X\bar{Y}, \bar{Y}^2]$ and $\langle \boldsymbol{f}(\tilde{X},\tilde{Y})\rangle_\infty$ are the true moments calculated from a long control simulation with the true parameters. Σ is a covariance error assumed here for simplicity to be diagonal as in Schneider et al. (2017).

We start by calculating the metrics for 40 sets of parameters given by the **Latin Hypercube Sampling**. This yields 40 training points for the **Gaussian Process Regressor** based emulator. Thanks to the cheap cost of the emulator we calculate the **implausibility** for a high number of points in the parameter space and draw **NROY (Not Ruled Out Yet)** maps. The implausibility is as defined in Williamsson et al. (2017):

$$(z - E(M(x)))^T \,\mathrm{Var}[z - E(M(x))]^{-1}(z - E(M(x)))$$

, where $E(M(x))$ is the expectation for the model M obtained from the emulator. A set of parameters P is ruled out if $I(P) > 3$.

The figure below show the minimum implausibility plots for the expanded set of parameters tested thanks to the emulator. Red coloured regions depict combinations of parameters that led to high implausibility and are therefore ruled out of further tuning. The white point is our ground truth set of parameters.



Future work

Investigating the case of partial and noisy observations then applying the technique to ocean numerical simulations.

References

- Schneider et al. 2017: Earth system modeling 2.0: A blueprint for models that learn from observations and targeted high-resolution simulations. Geophysical Research Letters.
- Williamson et al. 2017: Tuning without over-tuning: parametric uncertainty quantification for the nemo ocean model. Geoscientific Model Development.

Acknowledgments

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