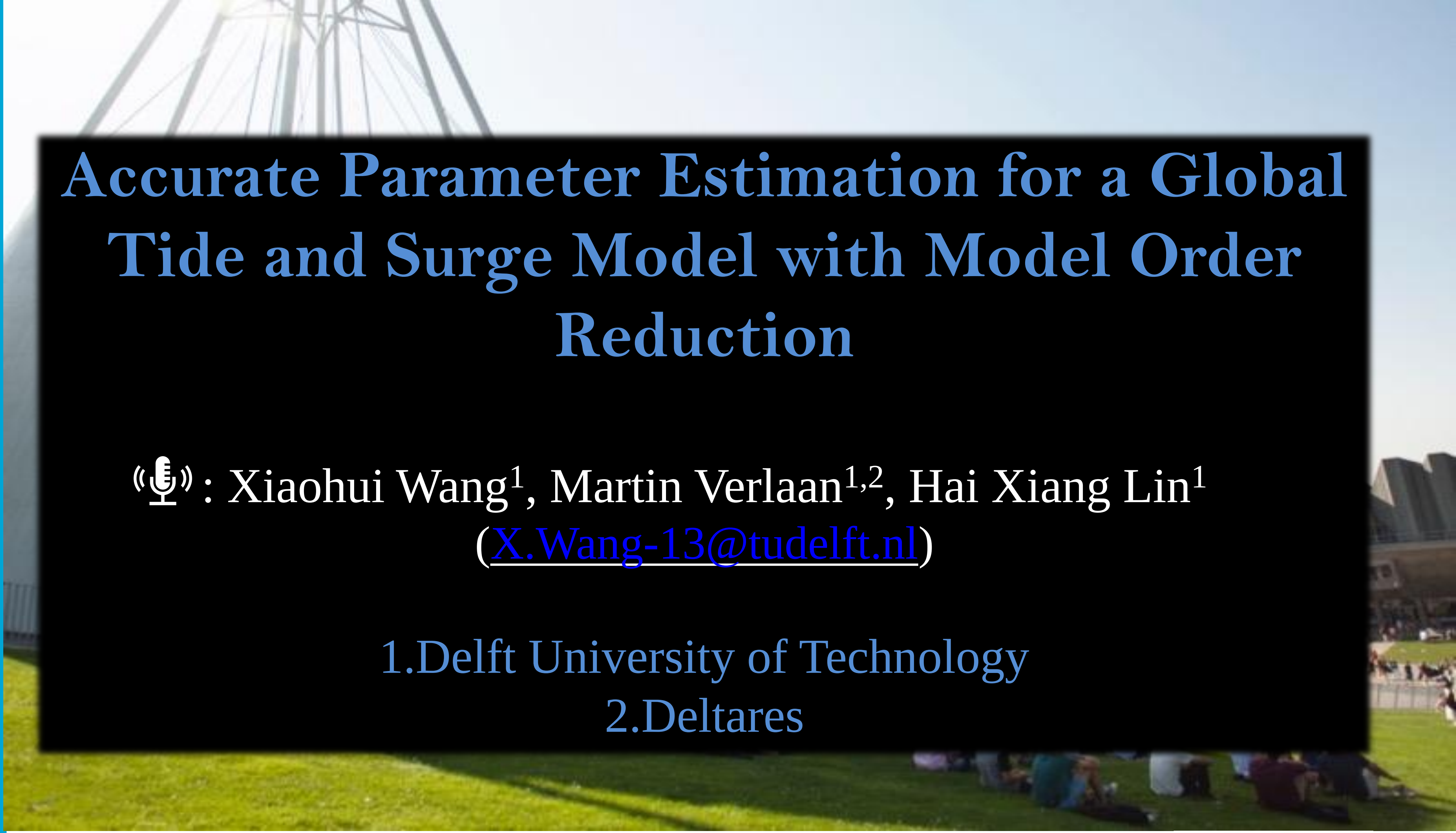

Accurate Parameter Estimation for a Global Tide and Surge Model with Model Order Reduction

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Outline

- ➢ Research Motivation
- ➢ Global Tide and Surge Model
- ➢ Parameter Estimation Scheme
- ➢ Numerical Experiments and Results
- ➢ Conclusions

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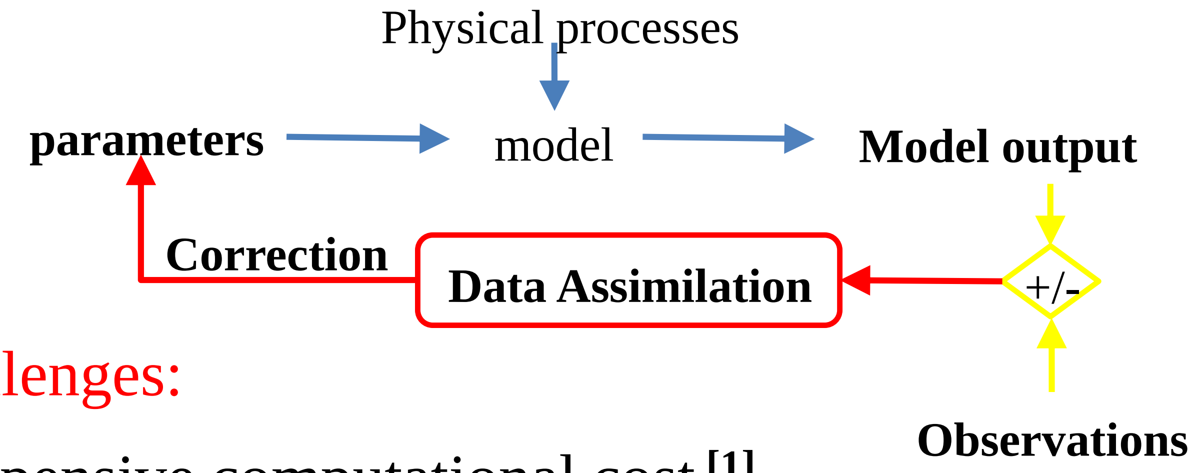
Research Motivation

Why to do **parameter estimation**?

- The **requirement** of the high accurate forecast of tide and surge
  - ✓ Global climate changes are increasing the risk of storm surges, flooding.
  - ✓ Accurate forecasts can substantially help evaluate the risk
- **Numerical tide models** can provide water level forecast, e.g. **GTSM**
- Model error remains, e.g. resolution, physical process, **uncertain parameters**.
- Some **measurements** (tide gauge & satellite altimetry) can be obtained.

Research Motivation



Physical processes

parameters → model → **Model output**

Correction **Data Assimilation** +/- **Observations**

Challenges:

- Expensive computational cost [1]
  - ✓ A large number of model simulations have to be run to find the optimal parameters.
- Huge memory requirement
  - ✓ Two weeks simulation time length leads calibration performance over-fit the observations used.
  - ✓ The long time-series results in huge memory requirement.
  - ✓ **Model order reduction** in time patterns.

[1] Wang, X., Verlaan, M., Apecechea, M. I., & Lin, H. X. (2021). Computation-efficient parameter estimation for a high-resolution global tide and surge model. *Journal of Geophysical Research: Oceans*, 126, e2020JC016917. https://doi.org/10.1029/2020JC016917

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Global Tide and Surge Model (v3.0)

- Delft3D Flexible Mesh (**unstructured mesh**)
- A **combined tide and surge** model

Tides

$$\frac{\partial \mathrm{u}}{\partial t}+\frac{1}{\boxed{h}}(\nabla\cdot(h\mathrm{uu}-\mathrm{u}\nabla\cdot(h\mathrm{u}))=-\mathrm{g}\nabla(\xi-\xi_{EQ}-\xi_{SAL})+\nabla\cdot(\nu(\nabla\mathrm{u}\nabla\mathrm{u}^{T}))+\frac{\tau+\tau_{IT}}{h}$$

$$\tau=-\frac{g}{\boxed{C_D^2}}\parallel \mathrm{u}\parallel \mathrm{u}$$

$$\tau_{IT}=-\boxed{C_{IT}}N(\nabla h\mathrm{u})\nabla h$$

Surges

Wind condition

Air pressure condition

Bathymetry

Bottom friction

Internal tides friction

Model	GTSM_coarse	GTSM_fine
Mesh	~2 million	~5 million
resolution	50km in deep ocean, 5km in coastal area	25km in deep ocean, 2.5km in coastal area, 1.25km in European
Computational cost (45 days, 200 cores)	25 min	70 min

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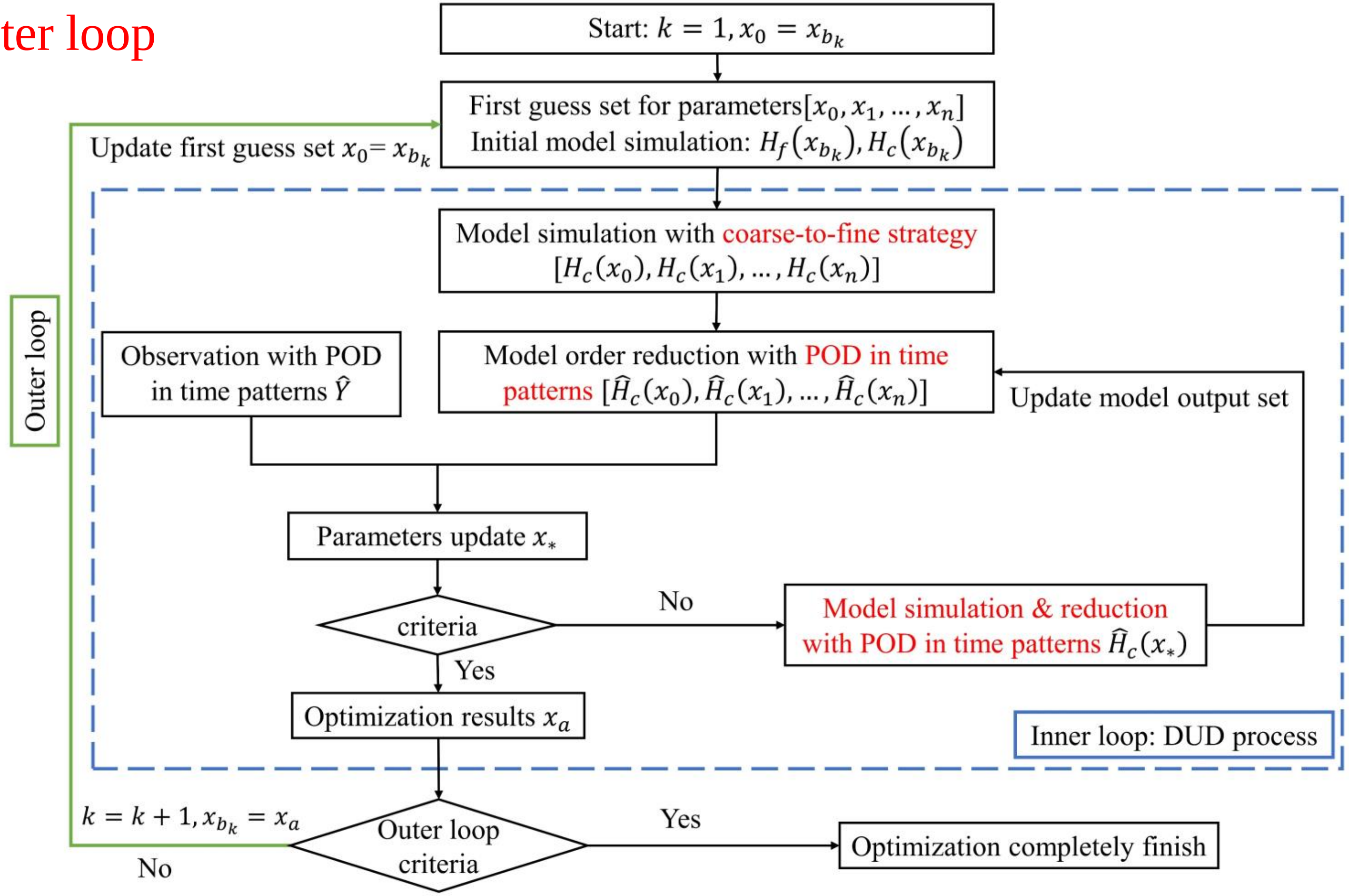
Parameter Estimation Scheme

- OpenDA software: a generic data-assimilation toolbox
- Basic Algorithm: A derivative-free calibration algorithm Does not use derivative (DUD)
- New developments:
  - ☐ Coarse to fine strategy (Computational time reduction): $H_f(x) = H_f(x_b) + (H_c(x) - H_c(x_b))$
  - ☐ Model order reduction (Memory requirement reduction)
  - ☐ Inner-outer loop



Start: $k = 1, x_0 = x_{b_k}$

First guess set for parameters $[x_0, x_1, \dots, x_n]$
Initial model simulation: $H_f(x_{b_k}), H_c(x_{b_k})$

Update first guess set $x_0 = x_{b_k}$

Outer loop

Model simulation with coarse-to-fine strategy
$[H_c(x_0), H_c(x_1), \dots, H_c(x_n)]$

Observation with POD in time patterns $\hat{Y}$

Model order reduction with POD in time patterns $[\hat{H}_c(x_0), \hat{H}_c(x_1), \dots, \hat{H}_c(x_n)]$

Update model output set

Parameters update x_*

criteria

No

Model simulation & reduction with POD in time patterns $\hat{H}_c(x_*)$

Yes

Optimization results x_a

Inner loop: DUD process

$k = k + 1, x_{b_k} = x_a$

No

Outer loop criteria

Yes

Optimization completely finish

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Parameter Estimation Scheme

- POD (Proper Orthogonal Decomposition) in time patterns

  - ☐ reducing the model order by **identifying several modes with the most energies** from a high-dimension system and uses these modes as a **lower-dimension subspace approximation.**

$$H_{N_t,N_s}(x) = [h^1(x), h^2(x), \cdots h^{N_s}(x)] \in \mathbb{R}^{N_t \times N_s}, N_t \gg N_s \qquad (1)$$

$$\left\| H_{N_t,N_s}(x) - K H_{N_t,N_s}(x) \right\|_2^2 = \sum_{i=1}^{N_s} \left\| h^i(x) - K h^i(x) \right\|_2 \qquad (2)$$

$$K = U_{N_p} U_{N_p}^T, U_{N_p} \in \mathbb{R}^{N_t \times N_p}, N_p \ll N_t \qquad (3)$$

  - ☐ **Truncated SVD:**

$$H_{N_t,N_s}(x) = U \Sigma V^T, U = [u_1, u_2, \dots, u_{N_t}] \in \mathbb{R}^{N_t \times N_t} \qquad (4)$$

$$\widehat{H}_{N_p,N_s}(x) = U_{N_p}^T H_{N_t,N_s}(x) \in \mathbb{R}^{N_p \times N_s} \qquad (5)$$

  - ☐ **Observation:**

$$\hat{Y} = U_{N_p}^T Y \in \mathbb{R}^{N_p \times N_s} \qquad (6)$$

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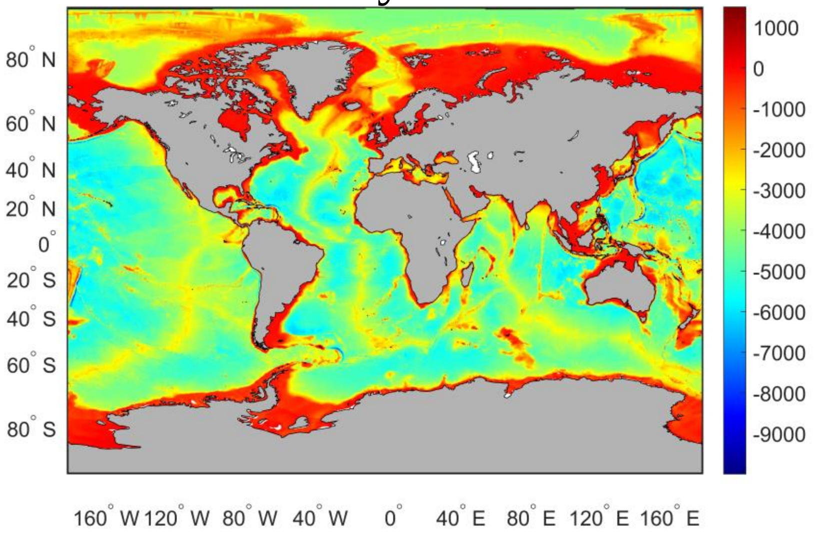
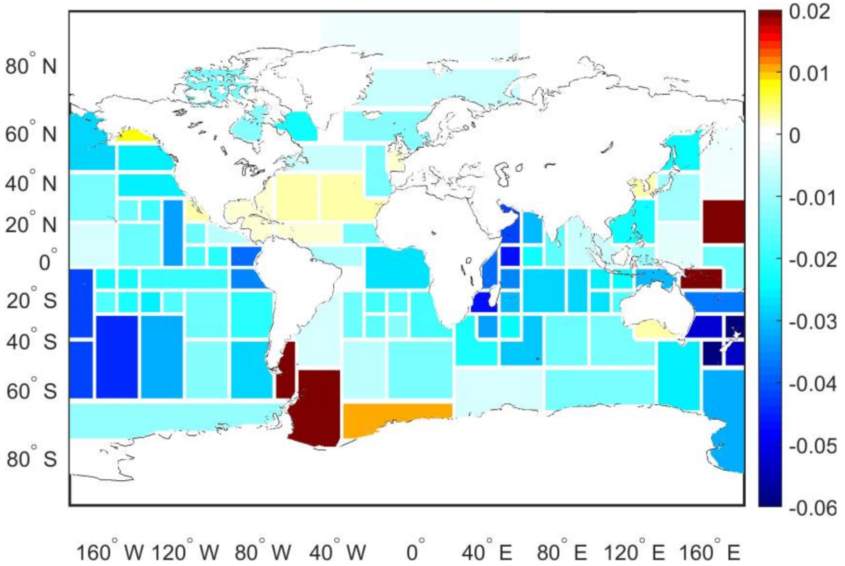
Numerical Experiments and Results

- **Experiment Set-up**
  - **Observation Network**
    - 1973 time-series from FES2014 dataset
  - **Parameter:** Bathymetry (110 subdomains)
    - Uncertainty: 5%
    - Sensitivity test





Name	Simulation time	Time Steps	Outer loop	POD	Truncation size	Data size Before POD	Data size after POD
EX1	1-14 Jan. 2014	2017	No	No	N/A	3.32Gb	N/A
EX2	1-14 Jan. 2014	2017	No	Yes	200	3.32Gb	0.33Gb
EX3	1-31 Jan. 2014	4465	Yes	Yes	200	7.35Gb	0.33Gb

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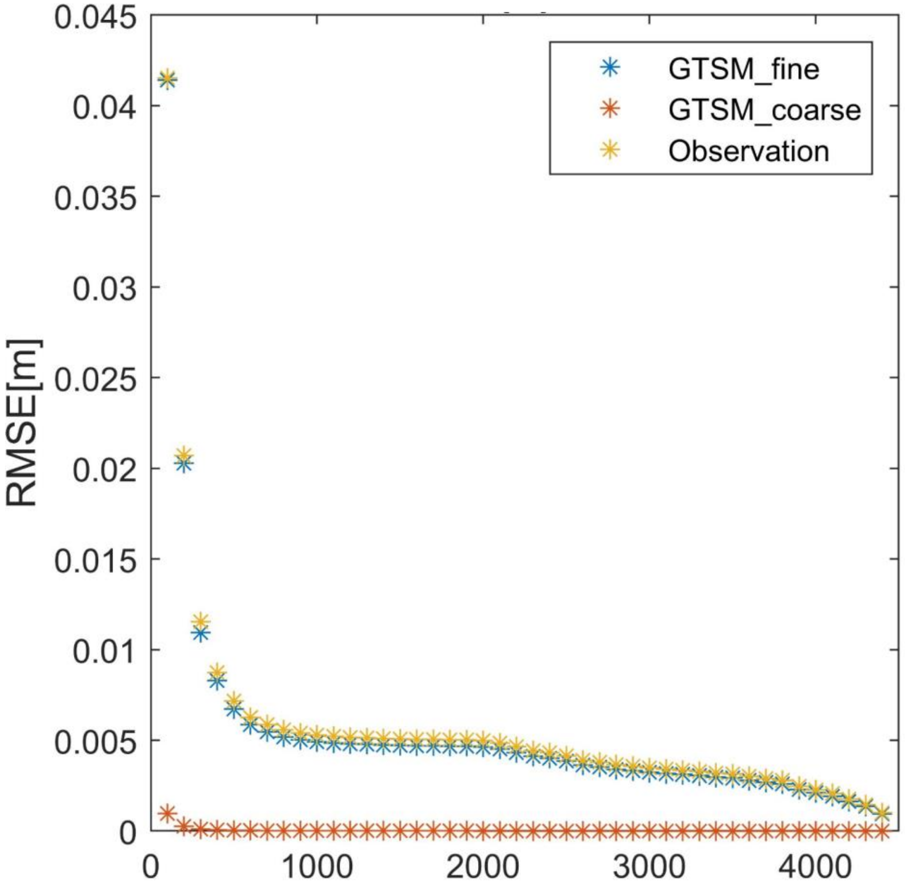
Numerical Experiments and Results

- **POD performance analysis**
  - ☐ Projection and reconstruction accuracy
    - Model simulation time: 1-31 Jan. 2014
    - Generate basis matrix U_{N_p} from coarse model output and reconstruct matrix with changed truncation size.
  - ☐ RMSE is decreased with the increasing truncation size.
  - ☐ Excellent accuracy of the reconstructed coarse model.
  - ☐ The reconstructed fine model and observations have similar performance when the truncation size varies.
  - ☐ 200 modes ensure the reconstructed observation error is smaller than the observation uncertainty (0.05m).



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Numerical Experiments and Results

- **POD performance analysis**
  - ❑ **Estimation performance in EX1 and EX2**

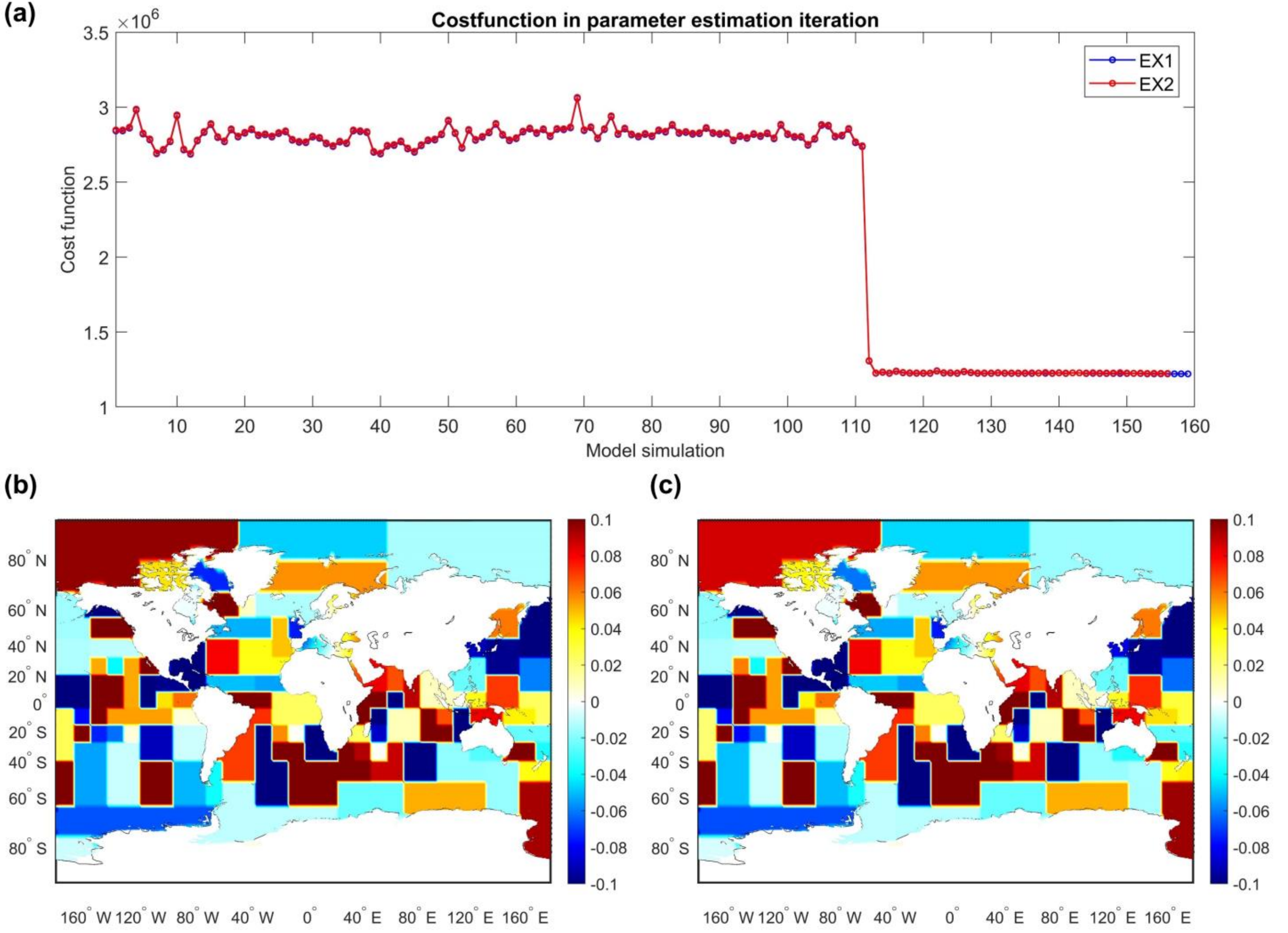

Figure 3: (a) Cost function in EX1 and EX2; (b) Bathymetry changes in EX1; (c) Bathymetry changes in EX2

Numerical Experiments and Results

- **Parameter estimation results (EX3): CPU time: 12days for 200 cores (57600h)**

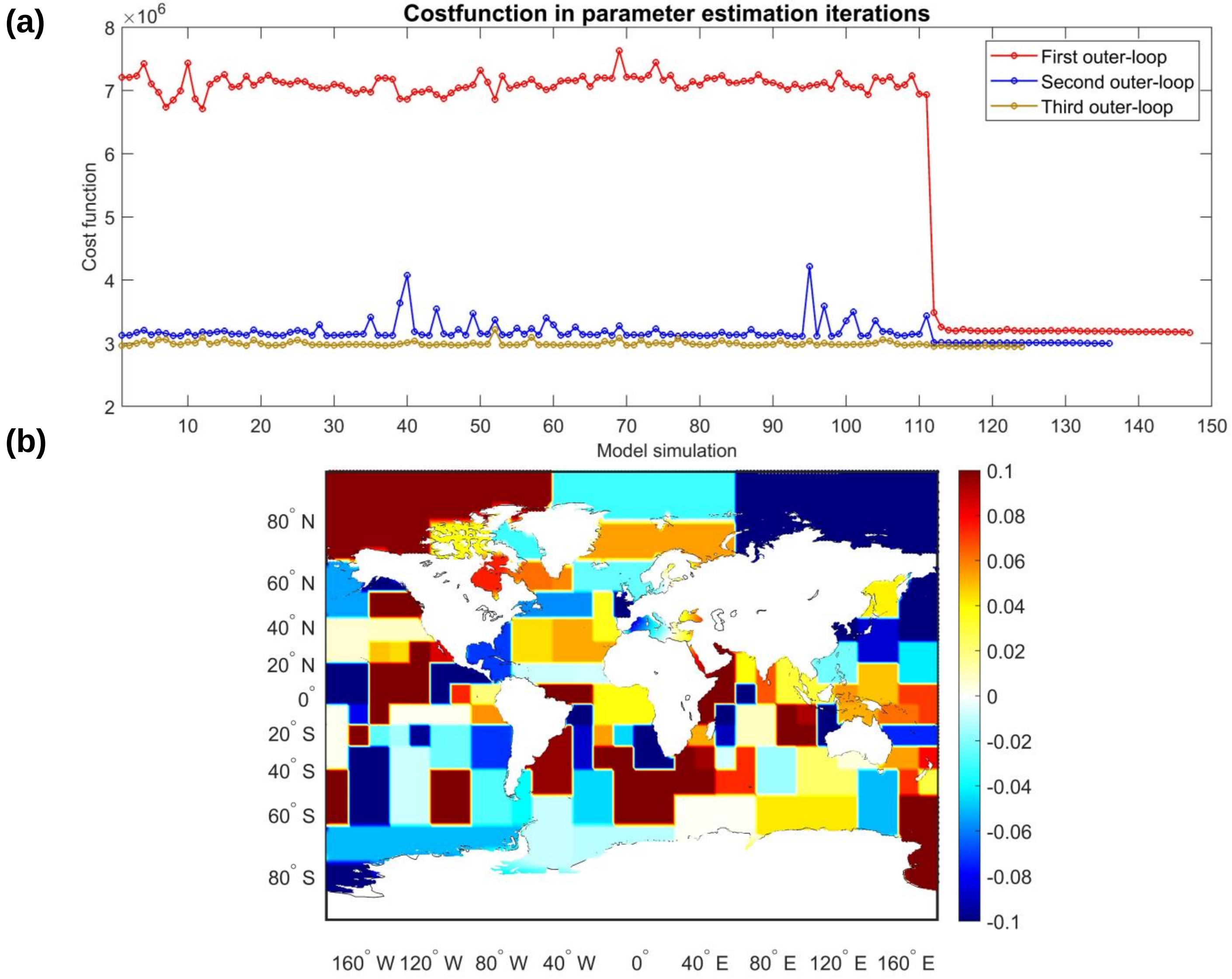

Figure: (a) Cost function for three outer loop iterations of EX3; (b) Final bathymetry changes in EX3;

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Numerical Experiments and Results

- **Parameter estimation results (EX3)**

RMSE	Time period	Initial	EX1	EX3
GTSM (coarse)	January 1-14	6.47	4.19	4.06
	January 15-31	7.14	5.20	4.41
GTSM (fine)	January 1-14	5.23	3.49	3.62
	January 15-31	5.84	4.33	3.66

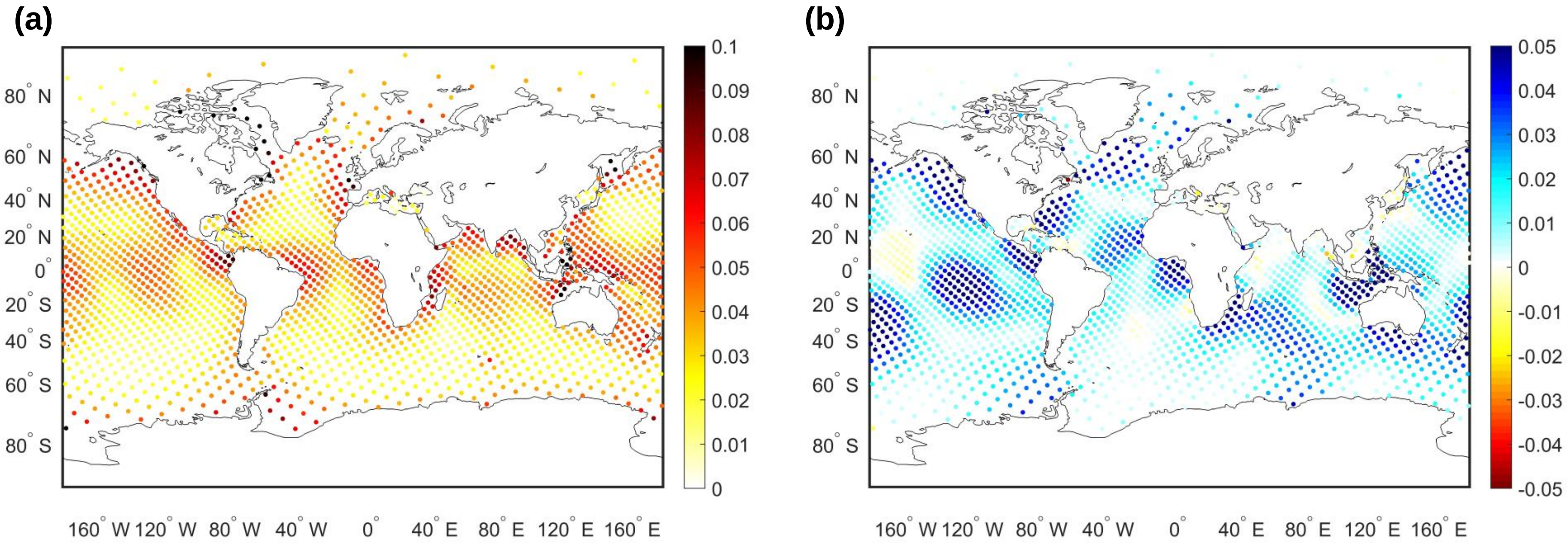

Figure: (a) RMSE between estimated fine grid GTSM in EX3 and FES2014 dataset in January 2014; (b) Difference of RMSE between initial model and estimated model, color blue shows improvement. [unit:m]

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Numerical Experiments and Results

- **Model validation: Monthly Comparison with FES2014 time-series of 2014**

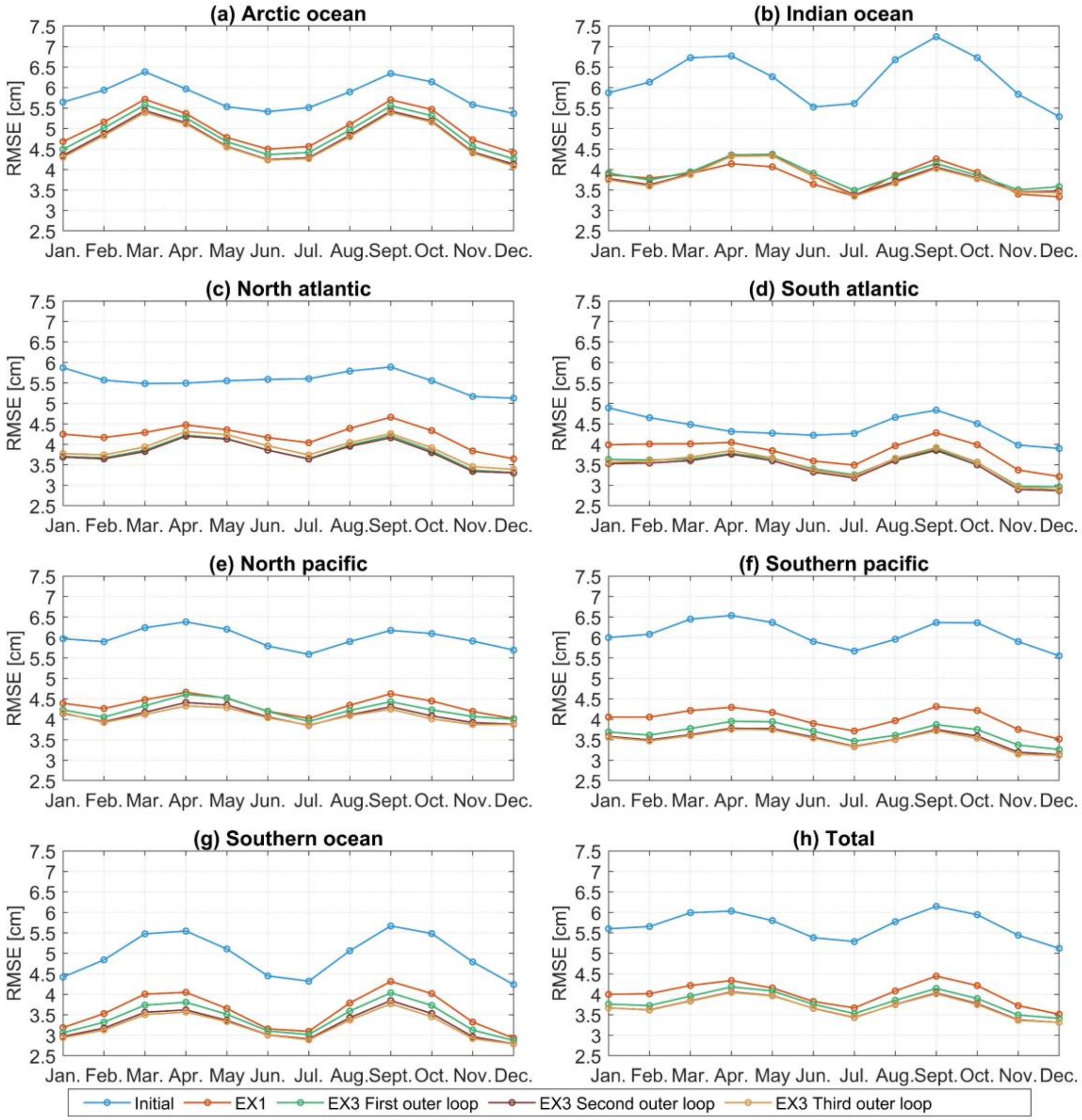

Figure: RMSE between model output and FES2014 dataset in 2014

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Numerical Experiments and Results

- **Model validation: Monthly Comparison with UHSLC time-series of 2014**

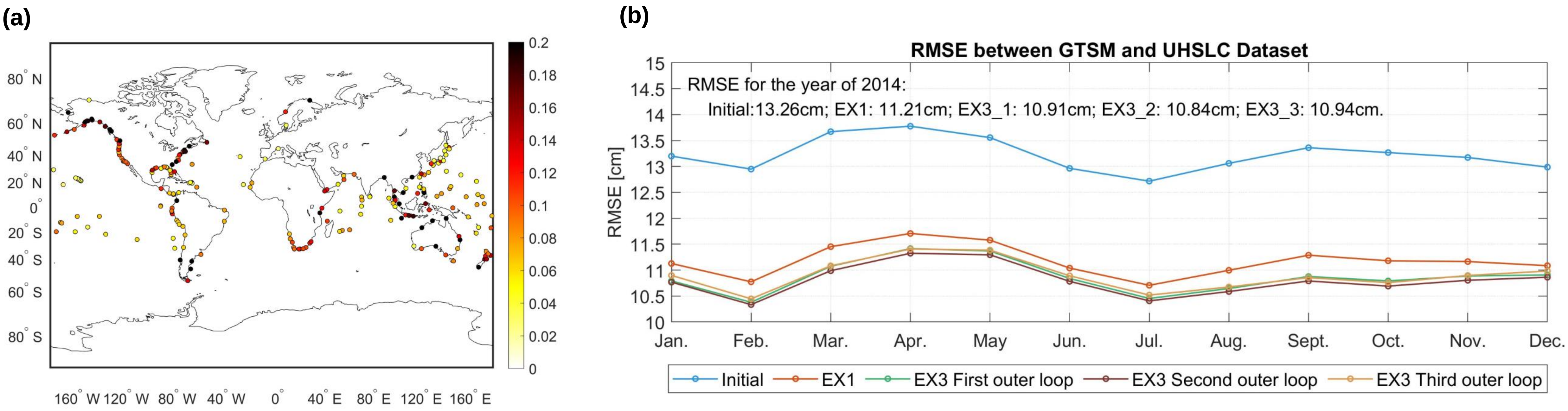

Figure: (a): RMSE between initial fine GTSM and UHSLC dataset in year 2014; (b): RMSE Difference between initial model and estimated model in EX3, color blue shows improvement.[unit:m]

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Conclusions

- Model order reduction can significantly reduce the memory requirement for parameter estimation procedure without estimation accuracy loss.
- Parameter estimation of GTSM benefits from long observation time series.
- An outer-loop can improve the calibration performance for non-linear models or approximate linearization.
- Future work will continue on the estimation of bottom friction for the fine model.

Thank You!

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