Although weather and climate predictions improve over time, models remain imperfect. Predictions might be improved if models are combined dynamically to produce solutions that are unique to the combined system. This supermodel is potentially closer to observations than the standard Multi Model Ensemble (MME) method, in which the different models are only combined after the simulations. Errors in the supermodel dynamics can be corrected at an earlier stage and since the models in a supermodel are synchronized we do not suffer from variance reduction and smoothing.

Key ingredients of supermodeling

- **Synchronization:** allows the supermodel to produce improved predictions as compared to the standard MME and to form a consensus solution which remains closest to the observed evolution.
- **Training:** to learn the connection strengths between the models within the supermodel.
- **Ensemble of different models:** the models need to be able to compensate for each others shortcomings.

Weighted supermodeling

Consider two imperfect models with parametric error, with $s$ denoting the supermodel solution. States are combined with frequency $\Delta t$, indicated by the Kronecker $\delta$.

$$
\begin{align*}
S_1 &= \delta_{i,j}(F(x_1,p_1) + (1 - \delta_{i,j})(F(x_2,p_2))) \\
S_2 &= \delta_{i,j}(F(x_1,p_1) + (1 - \delta_{i,j})(F(x_2,p_2))) \\
\end{align*}
$$

Aim: learning the optimal weights $W$.

Training of the Lorenz 63 system

The supermodeling approach is tested on a low-order dynamical system, the famous Lorenz 63 system. For certain parameter values a chaotic attractor appears, with the shape of a butterfly. The differential equations of the system contain system parameters $a, b, c$. The state space is described by coordinates $x, y, z$.

Observations are generated by the model with standard parameter values. Two imperfect models with perturbed parameter values are used.

![Figure 1: a supermodel: an interacting ensemble of imperfect models](image)

### Table 1: Parameter values of the imperfect and perfect models.

<table>
<thead>
<tr>
<th>Model</th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12.5</td>
<td>19</td>
<td>3.3</td>
</tr>
<tr>
<td>2</td>
<td>7.5</td>
<td>35</td>
<td>1.9</td>
</tr>
<tr>
<td>Truth</td>
<td>10</td>
<td>28</td>
<td>8/3</td>
</tr>
</tbody>
</table>

![Figure 4: After training the supermodel, the resulting attractor (blue) is much more similar to the perfect attractor (green)](image)

### References


### An ocean-connected super Earth System Model

Next step in supermodeling: connect 3 state-of-the-art Earth System Models (ESMs):
- Models NorESM, CESM, MPIESM
- Every month the models exchange Sea Surface Temperature (SST)
- The weighted average of the different model SST's is used as pseudo-observation
- The pseudo-observations are assimilated back into each model with Ensemble Optimal Interpolation.

![Figure 7: Annual mean precipitation climatology in the Tropical Pacific (GPCP: satellite-based rainfall estimates).](image)

Experimental setup, different ways of combining models:
- A posteriori averaging of the models (Standard), the models are only combined after the run, as in a standard MME
- Supermodel with equal weights (EW)
- Supermodel with spatially and monthly varying trained weights (TW). Weights are trained on the basis of offline simulations.

### Discussion and conclusion

Experiments with the Lorenz 63 supermodel and the SPEEDO supermodel showed a significant improvement in predictions and projections compared to individual models. We moved forward with combining state-of-the-art climate models that do not only have different parameters, but are also different in structure. Training the weights was performed offline in this experiment, so the next step is to use online methods. Also, the models are now only combined in the ocean component, so to further improve results the atmospheres will be combined as well.