Deutscher Wetterdienst
Wetter und Klima aus einer Hand

Towards More Unified Treatment of Turbulence, Shallow Convection, and Clouds: A Third-Order Scheme with Prognostic Scalar Variances

Ekaterina Machulskaya and Dmitrii Mironov

German Weather Service, Offenbach am Main, Germany

(ekaterina.machulskaya@dwd.de, dmitrii.mironov@dwd.de)

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Outline

- Introduction
- Key features of a unified turbulence-shallow convection scheme
  - *non-local* nature of convective motions: prognostic scalar variances
  - *skewed* nature of convective motions: coupling with statistical cloud scheme
- Regularization of stability functions
- Future challenges

Introduction

$$\frac{d\bar{f}}{dt} \equiv \frac{\partial \bar{f}}{\partial t} + \bar{u}_i \frac{\partial \bar{f}}{\partial x_i} = -\frac{\partial}{\partial x_i}\overline{u_i' f'} + \bar{S} + \cdots$$

Sub-grid scale processes are traditionally described by means of various approaches:

$$\frac{\partial}{\partial x_i}\overline{u_i' f'} = \left[\frac{\partial}{\partial x_i}\overline{u_i' f'}\right]_{conv} + \left[\frac{\partial}{\partial x_i}\overline{u_i' f'}\right]_{turb} + \left[\frac{\partial}{\partial x_i}\overline{u_i' f'}\right]_{gwd} + \left[\frac{\partial}{\partial x_i}\overline{u_i' f'}\right]_{sso}$$

$$\left[\frac{\partial}{\partial x_i}\overline{u_i' f'}\right]_{conv} = \left[\frac{\partial}{\partial x_i}\overline{u_i' f'}\right]_{deep} + \left[\frac{\partial}{\partial x_i}\overline{u_i' f'}\right]_{shallow}$$

- This separation is sometimes artificial
- It is a very complicated task to bring different parts to correctly interact with each other
- **A unified description of turbulence and boundary-layer convection**

based on the equations for statistical moments is desirable and seems to be feasible

Towards a Unified Description of Turbulence and Shallow Convection

Key features of a unified scheme

- account for *non-local* nature of convective motions:

  large eddies → counter-gradient vertical heat and moisture transport

- account for *skewed* nature of convective motions:

  latent heat release within clouds of small area coverage (large skewness) induces intensive turbulence

The Mellor-Yamada hierarchy of second-order models

Considering turbulence anisotropy…

…

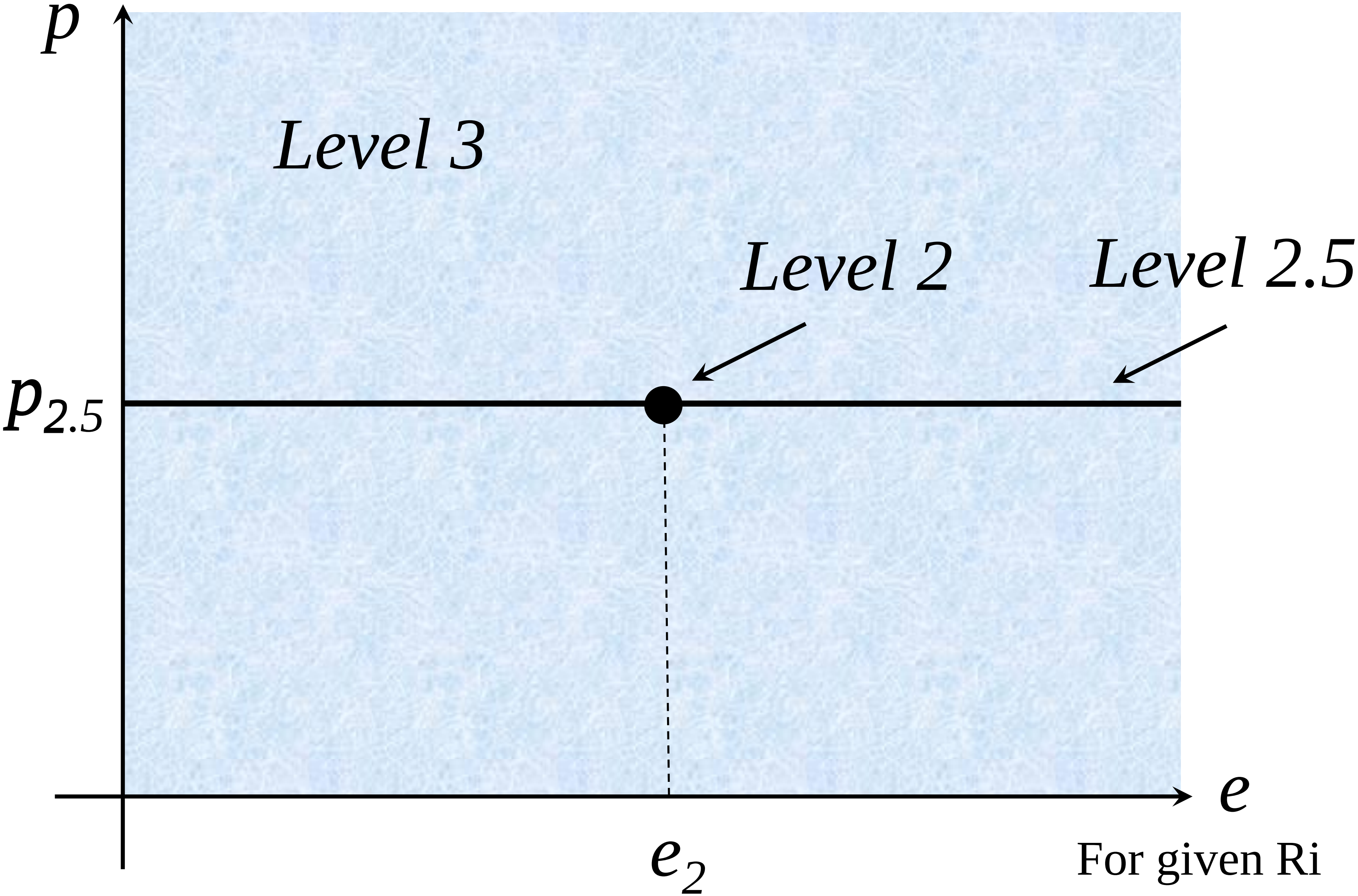
Level 2: all the second moments are computed algebraically (equilibrium equations, stationarity and homogeneity are assumed)

Level 3: TKE and scalar variances are computed prognostically (non-stationarity) with due regard for the transport terms (non-homogeneity); all other second moments – algebraically

Level 4: all the second moments are computed prognostically

„Artificial“ compromise – **Level 2.5**: TKE – prognostically, all other second moments – algebraically

The Mellor-Yamada hierarchy for second-order models



TKE-Scalar Variance Closure Scheme

Prognostic equations for $\overline{u_i'^2}$ (kinetic energy of SGS motions) and for $\overline{\theta_l'^2}$, $\overline{q_t'^2}$, $\overline{\theta_l' q_t'}$ (potential energy of SGS motions) including third-order transport.

Convection/stable stratification =
Potential Energy ↔ Kinetic Energy.
No reason to prefer one form of energy over the other!

The scalar-variance equation

$$\frac{1}{2}\frac{\partial \overline{\theta'^2}}{\partial t} = -\overline{w'\theta'}\frac{\partial \overline{\theta}}{\partial z} - \frac{1}{2}\frac{\partial}{\partial z}\overline{w'\theta'^2} - \varepsilon_\theta$$

non-stationarity — production — non-homogeneity — dissipation

Production = Dissipation (implicit in all models that carry the TKE equations only)

→ no way to get counter-gradient scalar fluxes

TKE-Scalar Variance Closure Scheme

Algebraic (equilibrium) formulations for scalar fluxes, Reynolds-stress components

$$\overline{u'u'} \equiv \overline{u'_1u'_1} = -\tau c_1\left(\overline{u'_iu'_k}\frac{\partial \overline{U}_j}{\partial x_k} + \overline{u'_iu'_k}\frac{\partial \overline{U}_k}{\partial x_j}\right) - \tau c_2 \beta \overline{w'\theta'} + c_3 e$$

$$\overline{u'v'} \equiv \overline{u'_1u'_2} = \ldots$$

$$\ldots$$

$$\overline{w'w'} \equiv \overline{u'_3u'_3} = -\tau c_1\left(\overline{u'_iu'_k}\frac{\partial \overline{U}_j}{\partial x_k} + \overline{u'_iu'_k}\frac{\partial \overline{U}_k}{\partial x_j}\right) - \tau c_4 \beta \overline{w'\theta'} + c_3 e$$

$$\overline{u'\theta'} = -\tau c_5\left(S_{ij} + W_{ij}\right)\overline{u'_j\theta'} - \tau c_6 \overline{u'_iu'_k}\frac{\partial \overline{\theta}}{\partial x_k}$$

$$\ldots$$

$$\overline{w'\theta'} = -\tau c_5\left(S_{ij} + W_{ij}\right)\overline{u'_j\theta'} - \tau c_6 \overline{u'_iu'_k}\frac{\partial \overline{\theta}}{\partial x_k} - \tau c_7 \beta \overline{\theta'^2}$$

Algebraic relations in TKESV scheme (Level 3 system after Mellor&Yamada) form a linear algebraic system

External parameters:
e (=TKE), S^2, N^2,
scalar variances

The system was solved analytically →

TKE-Scalar Variance Closure Scheme

In the Level 3 system, the components of scalar fluxes are not down-gradient:

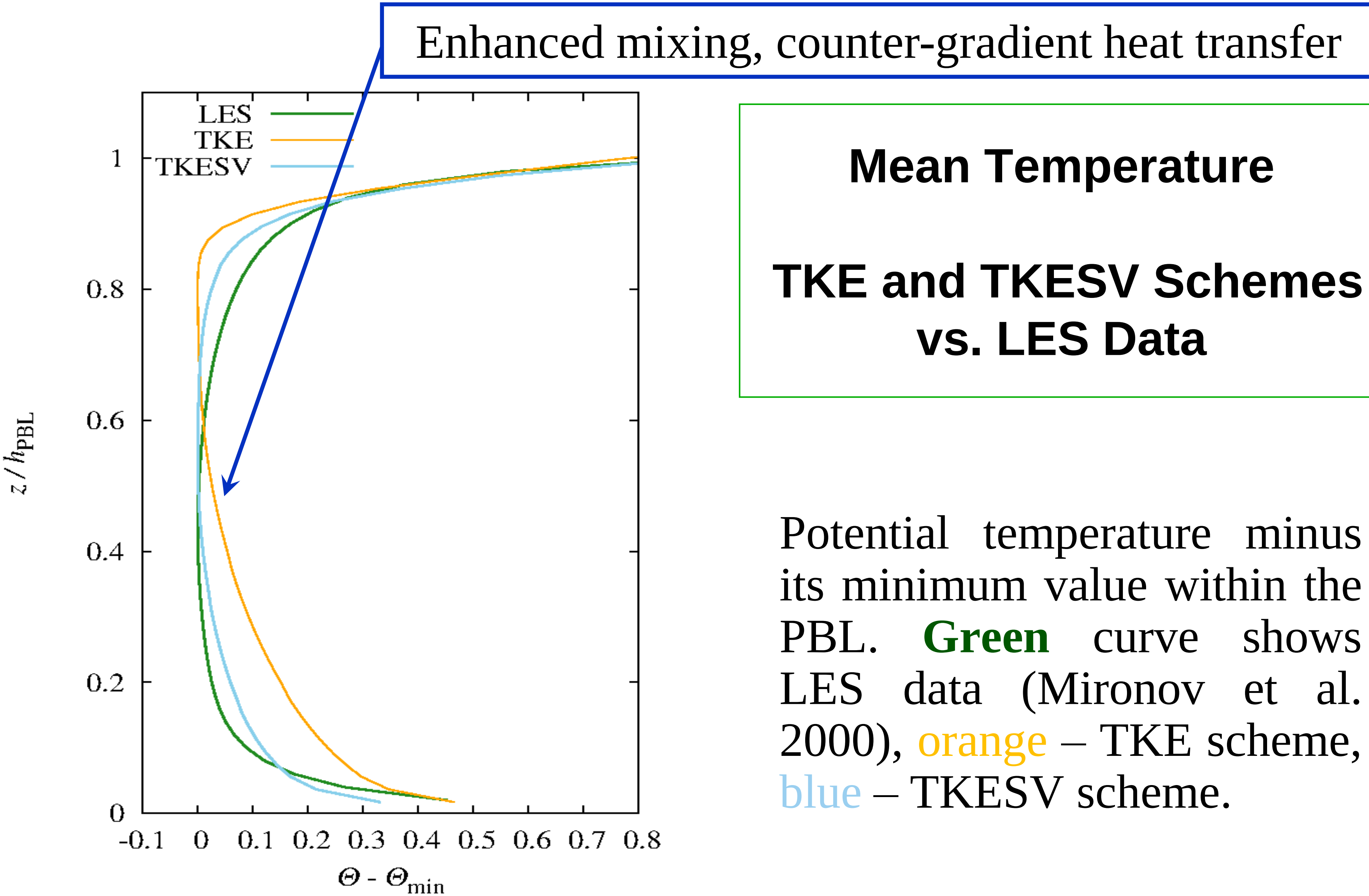
$$\overline{w'\theta'} = -F^3_{H1}K_H\frac{\partial\overline{\theta}}{\partial z} + F_{H2}(\tau^2S^2, Ri)\tau\beta\overline{\theta'^2}$$

F_M and F_H are functions of τ^2S^2 and τ^2N^2, or τ^2S^2 and $Ri = N^2/S^2$ ($\tau=l/e^{1/2}$ is the turbulence time scale) – "stability functions"

They are merely the notation used to represent the solution of system of linear equations for scalar-flux and Reynolds-stress components in a compact form.

The stability functions of the level 3 system and the level 2.5 system differ (different set of arguments, different functional form – "other stability functions").

Single-column testing: Dry Convective PBL



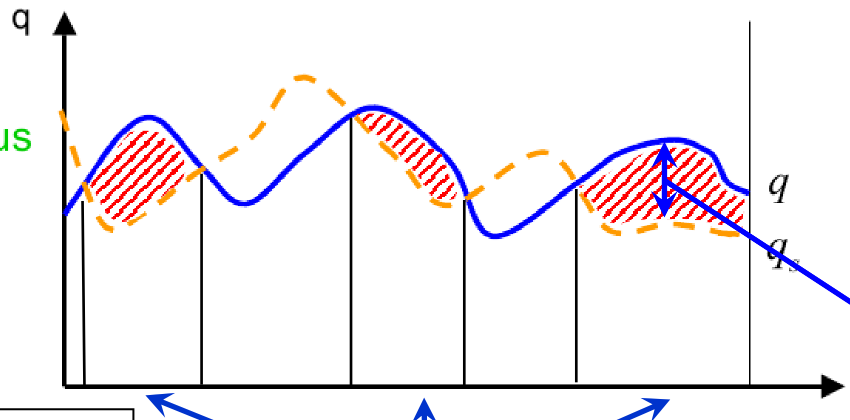
Mean Temperature

TKE and TKESV Schemes vs. LES Data

Potential temperature minus its minimum value within the PBL. **Green** curve shows LES data (Mironov et al. 2000), orange – TKE scheme, blue – TKESV scheme.

Coupling with Statistical Cloud Scheme

Heterogeneous distribution of T and q



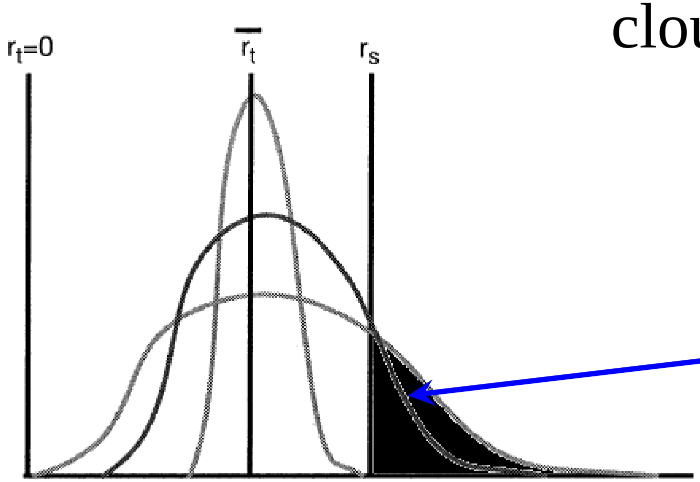
after Tompkins (2002)

clouds

SGS fluctuations of ***q*** and q_s (due to SGS fluctuations of *T*) result in fractional cloud cover

Saturation deficit/excess

$$s = q_t - q_s(T_l)$$



Assumed PDF approach

PDF form is assumed

Parameters should be determined

→ cloud cover, cloud condensate = integrals over supersaturated part of PDF

For shallow cumuli regime (highly localized clouds) the skewness is very important!

Gaussian distribution works badly.

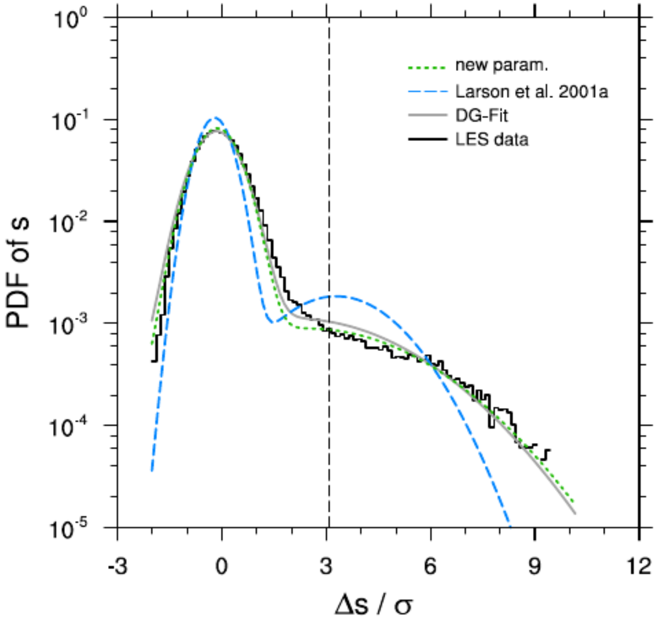
Coupling with Statistical Cloud Scheme

Double Gaussian distribution – very flexible, but expensive, if a joint PDF for θ_l, q_t and w is assumed

For cloud representation the PDF of $s = q_t - q_s(T_l)$ is sufficient

Still the DG PDF of s requires 5 input parameters – too many

A three-moment (mean, variance, and skewness) statistical SGS cloud scheme (Naumann et al., 2013); 5 parameters are reduced to 3 using LES findings



(Naumann et al., 2013)

– a good compromise between flexibility and computational costs

First moment $\bar{s}$ is provided by the grid-scale equations

Second moment $\overline{s'^2}$ is computed from $\overline{\theta_l'^2}$, $\overline{\theta_l' q_t'}$ and $\overline{q_t'^2}$ provided by TKESV

Third moment $\overline{s'^3}$ is computed through its own transport equation

Coupling with Statistical Cloud Scheme

Turbulence in clouds – the buoyancy flux $\frac{g}{T_0}\overline{w'\theta_v'}$, a very important source of TKE

$$\overline{w'\theta_v'} = \overline{w'(\theta[1+(R-1)q_t - Rq_l])'} = A\overline{w'\theta_l'} + B\overline{w'q_t'} + C\overline{w'q_l'}$$

$C = 0$

$(\overline{w'q_l'} = 0)$

$C = 1$

$(q_l = q_t - q_s(T_l)$ everywhere $\rightarrow q_l' = q_t' - q_s')$

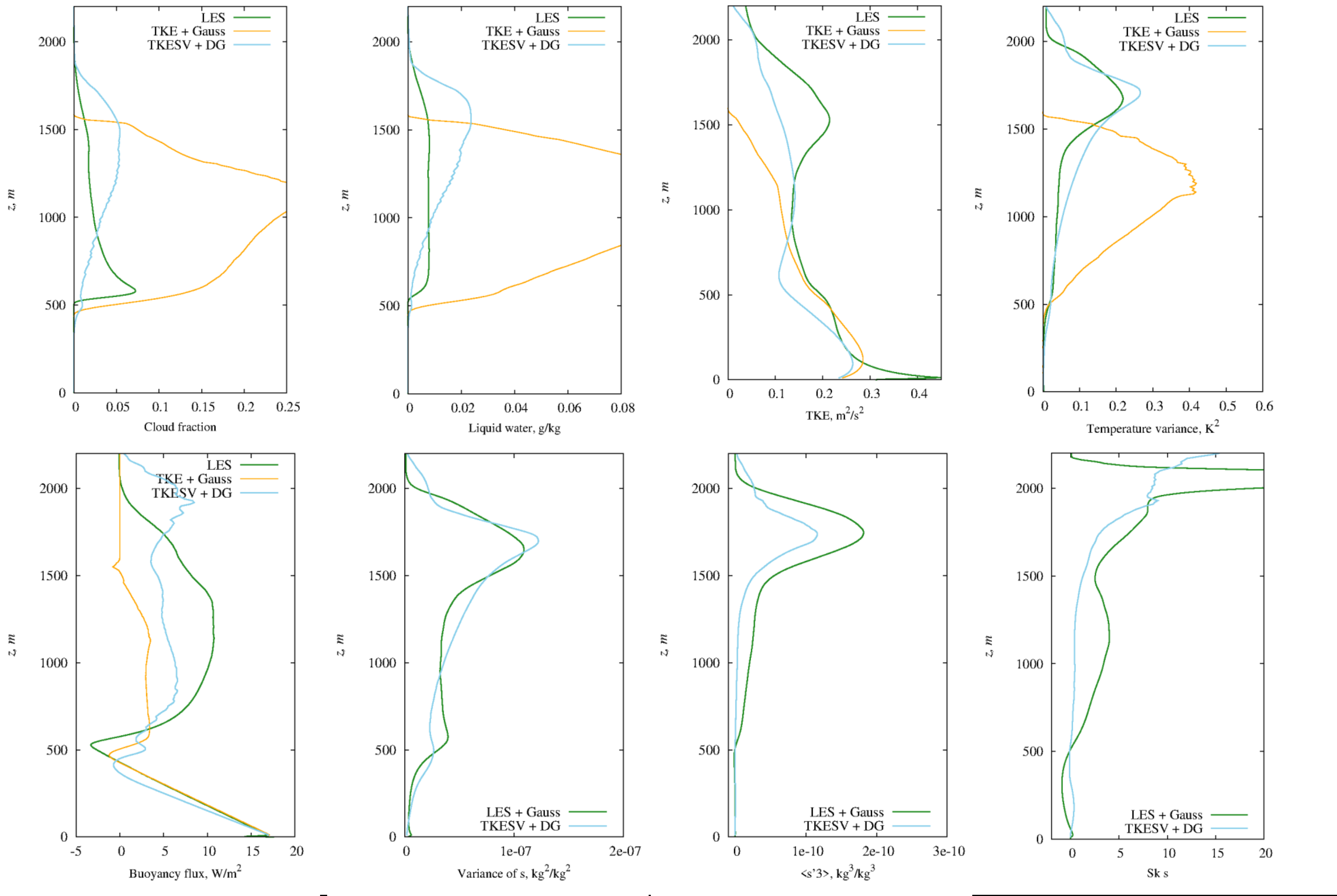
In between $\overline{w'q_l'}$ is unknown

The linear interpolation with C corresponds to a Gaussian PDF,
does not work in many situations, e.g. for cumulus type clouds
(C is small but $\overline{w'\theta_v'}$ is dominated by $\overline{w'\theta_v'}_{cloud}$)

Use the parameterization of Naumann et al. (2013) of $\overline{w'q_l'}$ as a function of s-skewness: $S = \frac{\overline{s'^3}}{\left(\overline{s'^2}\right)^{3/2}}$

The moments are provided by TKESV

Single-column testing: BOMEX



Regularization of the stability functions

Pathological behaviour of stability functions in non-stationary conditions

The problem is well-known and is recognized to be associated with the truncation of equations (neglecting of the terms that are responsible for inhomogeneity and non-stationarity).

The ways to handle it: either to regularize the solution (widely used, but too crude) or to regularize the equations (more mild and model-friendly).

Level 2.5: the problem is confined to growing turbulence (Helfand & Labraga, 1988)

→ for $\frac{e}{e_2} < 1$, re-insert into the algebraic equations the "transport" terms that would emulate what was neglected by the truncation.

Level 3: similarly for $\frac{e}{e_{2p}} < 1$ – regularized stability functions reveal no pathological behavior (Machulskaya & Mironov, 2020)

Future Challenges

- The base-line version of TKESV is implemented into the global 3d NWP ICON model, it was extensively tested, runs stably
- The advanced version (equation for $\overline{s'^3}$ + SGS clouds after Naumann et al. (2013)) is implemented and being tested
- Coupling with the microphysics (Schemann & Seifert, 2017)

Thanks for your kind attention!