The Two-Energies Turbulence Scheme and Length Scale Formulations

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1, 2TE+APDF scheme

2, Budget-Based Turbulence Length Scale Diagnostics

3, TKE scheme in IFS

2TE+APDF scheme

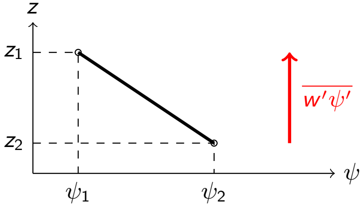
- Separate modelling of turbulence and clouds in Atmospheric Boundary Layer (ABL) causes inconsistencies.
- Unified parameterization of turbulence and clouds should improve the representation of interactions and transition of processes.
- A parameterization based on **two prognostic turbulence energies** and the **Assumed Probability Density Function** (APDF) approach for modelling both turbulence and clouds is a possible solution.

ECMWF

Local down-gradient turbulent diffusion

$$\overline{u'w'} = -\mathbf{K_M}\frac{\partial u}{\partial z}, \quad \overline{v'w'} = -\mathbf{K_M}\frac{\partial v}{\partial z},$$
$$\overline{\theta_l'w'} = -\mathbf{K_H}\frac{\partial \theta_l}{\partial z}, \quad \overline{q_t'w'} = -\mathbf{K_H}\frac{\partial q_t}{\partial z},$$

$\mathbf{K_M}$ and $\mathbf{K_H}$ - turbulent diffusion coefficients for momentum and heat/moisture



ECMWF

2TE+APDF scheme

Turbulent diffusion coefficients in TKE scheme

$$\mathbf{K_M} = \frac{\nu^4}{C_\epsilon}\chi_3(Ri_f^*)\sqrt{e_k}L, \quad \mathbf{K_H} = C_3\frac{\nu^4}{C_\epsilon}\phi_3(Ri_f^*)\sqrt{e_k}L$$

ECMWF

2TE+APDF scheme

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- TKE - measure of turb. intensity

2TE+APDF scheme

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- TKE - measure of turb. intensity
- length scale - scale of the problem

2TE+APDF scheme

Turbulent diffusion coefficients in TKE scheme

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- TKE - measure of turb. intensity
- length scale - scale of the problem
- stability functions - influence of stratification

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Turbulent diffusion coefficients in TKE scheme

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- TKE - measure of turb. intensity
- length scale - scale of the problem
- stability functions - influence of stratification
- closure constants

ν - free parameter, C_3 - inverse Prandtl number at neutrality, Ri_f^* - stability parameter in the form of flux Richardson number:
$Ri_f \equiv (\frac{g}{\theta_v}\overline{\theta_v' w'})/(\overline{u'w'}\frac{\partial u}{\partial z} + \overline{v'w'}\frac{\partial v}{\partial z})$

Prognostic TKE equation

$$\frac{d\mathbf{e_k}}{dt} = \frac{\partial}{\partial z}\left(K_{e_k}\frac{\partial \mathbf{e_k}}{\partial z}\right) + I + II - \epsilon_k,$$

$$\mathbf{e_k} \equiv \frac{\overline{u'u'} + \overline{v'v'} + \overline{w'w'}}{2} \quad \text{-Turbulence Kinetic Energy (TKE)},$$

$$I \equiv -\overline{u'w'}\frac{\partial u}{\partial z} - \overline{v'w'}\frac{\partial v}{\partial z} \quad \text{-Shear term},$$

$$II \equiv \frac{g}{\theta_v}\overline{\theta_v' w'} = E_{q_t}\overline{q_t' w'} + E_{\theta_l}\overline{\theta_l' w'} \quad \text{-Buoyancy term}$$

$$\epsilon_k \equiv \frac{2\,\mathbf{e_k}}{\tau_k} \quad \text{-Dissipation term}$$

K_{e_k} - turb. exchange coefficients for e_k; τ_k and τ_s - are dissipation time scales; E_{q_t} and E_{θ_l} are cloud-dependent weights.

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The two-energies turbulence scheme (2TE)

$$\frac{d\mathbf{e_k}}{dt} = \frac{\partial}{\partial z}\left(K_{e_k}\frac{\partial \mathbf{e_k}}{\partial z}\right) + \boxed{I + II - \frac{2\,\mathbf{e_k}}{\tau_k}},$$

$$\frac{d\mathbf{e_s}}{dt} = \frac{\partial}{\partial z}\left(K_{e_s}\frac{\partial \mathbf{e_s}}{\partial z}\right) + \boxed{I - \frac{2\,\mathbf{e_s}}{\tau_s}},$$

$$\mathbf{e_s} \equiv e_k + \frac{E_{q_t}\overline{q_t'^2}}{2\frac{\partial q_t}{\partial z}} + \frac{E_{\theta_l}\overline{\theta_l'^2}}{2\frac{\partial \theta_l}{\partial z}},$$

$$Ri_f^{TE} = \frac{\mathbf{e_s} - \mathbf{e_k}}{\mathbf{e_s} + \mathbf{e_k}\left(\frac{C_4}{2\,C_3} - 1\right)}$$

K_{e_s} - turb. exchange coefficients for e_s; τ_s - dissipation time scale

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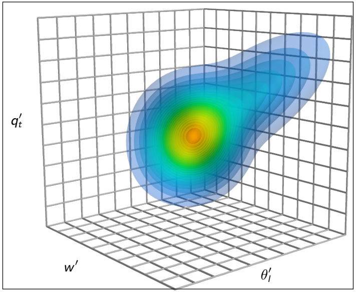
APDF method

Trivariate PDF



PDF shape given

C - Cloud fraction, θ_v - virtual potential temperature

APDF method

Input

Available

$\overline{w}$ $\overline{q_t}$ $\overline{\theta'_l}$

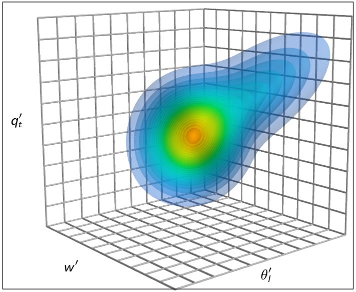
$\overline{w'\theta'_l}$ $\overline{w'q'_t}$ $\overline{w'^2}$

Additional

$\overline{\theta'^2_l}$ $\overline{q'_t\theta'_l}$

$\overline{q'^2_t}$ $\overline{\mathbf{w'^3}}$

Trivariate PDF



estimate PDF from model state

PDF shape given

C - Cloud fraction, θ_v - virtual potential temperature

ECMWF

APDF method

Input

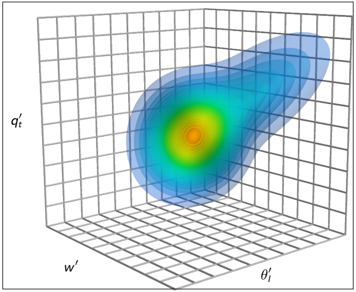
Available

$\overline{w}$	$\overline{q_t}$	$\overline{\theta_l'}$
$\overline{w'\theta_l'}$	$\overline{w'q_t'}$	$\overline{w'^2}$

Additional

$\overline{\theta_l'^2}$	$\overline{q_t'\theta_l'}$
$\overline{q_t'^2}$	$\overline{\mathbf{w'^3}}$

Trivariate PDF



Output

Cloud related variables

C

$\overline{w'\theta_v'}$

...

Higher order Moments

$\overline{w'^2q_t'^2}$

$\overline{w'^3q_t'}$

$\overline{w'^4}$

...

estimate PDF from model state

PDF shape given

analytical integration of PDF

C - Cloud fraction, θ_v - virtual potential temperature

ECMWF

2TE+APDF (1)

- the buoyancy term, II, is computed via APDF
- The stability parameter is computed from local gradients (Ri_f^{GR}) and turbulence energies:

  $Ri_f^* = \mathbf{C_{Ri_f}}\, Ri_f^{\mathrm{GR}} + (1 - \mathbf{C_{Ri_f}})\, Ri_f^{\mathrm{TE}}$

- Turbulence exchange coefficient for TOMs:

$$K_{e_k} = K_{e_s} = \left(C_{e_k} \overline{w'^2} + C_{\theta_s} \frac{g}{\theta_0} \overline{w'\theta'_s} \tau_k \right) \tau_k,$$

$$\overline{w'^3} = -K_{e_k} \frac{\partial \overline{w'^2}}{\partial z}$$

C_{e_k}, C_{θ_s}, and C_{Ri_f} - closure constants, $Ri_f^{GR} \equiv Ri \frac{K_H}{K_M}$ - computed from conventional gradient Richardson number, $\overline{w'\theta'_s}$ turbulent flux of the entropy potential temperature (Marquet and Geleyn, 2014)

ECMWF

2TE+APDF (2)

- Canuto et al. (2007) - dry case:

$$\overline{w'^3} = -A_1 \frac{\partial \overline{w'^2}}{\partial z} - A_2 \frac{\partial \overline{w'\theta'}}{\partial z} - A_3 \frac{\partial \overline{\theta'^2}}{\partial z}$$

$$A_1 = \left(a_1 \overline{w'^2} + a_2 \frac{g}{\theta_0} \tau \overline{w'\theta'} \right) \tau$$

- simplification for moist 2TE+APDF:

$$A_2 = A_3 = 0, \; \overline{w'\theta'} = \overline{w'\theta'_s}$$

2TE+APDF (3)

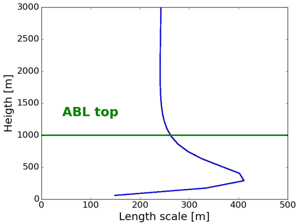
- Turbulence length scale:

$$L = \frac{(C_K\, C_\epsilon)^{\frac{1}{4}}}{C_K} \frac{\kappa z}{1 + \frac{\kappa z}{\lambda_m}\left[\frac{1+\exp\left(-a_m\sqrt{\frac{z}{H_{\rm abl}}}+b_m\right)}{\beta_m+\exp\left(-a_m\sqrt{\frac{z}{H_{\rm abl}}}+b_m\right)}\right]},$$

$$H_{\rm abl} = C_{ablh}\int_z L_{up} dz,$$

$$\Delta\theta_s > 0 : \beta_{\mathbf{m}} = 0, \qquad \Delta\theta_s \leq 0 : \beta_{\mathbf{m}} \neq 0$$

$$\Delta\theta_s = \theta_s(z = 1.5H_{\rm abl}) - \theta_s(z = 0)$$



$\kappa = 0.4$ is the von Kármán constant; $H_{\rm abl}$ is the ABL height; L_{up} is upward par of the non-local length scale (Bougeault and Lacarrere, 1989); a_m, b_m, β_m, and λ_m are shape constants; C_{ablh} and C_K are closure constants.

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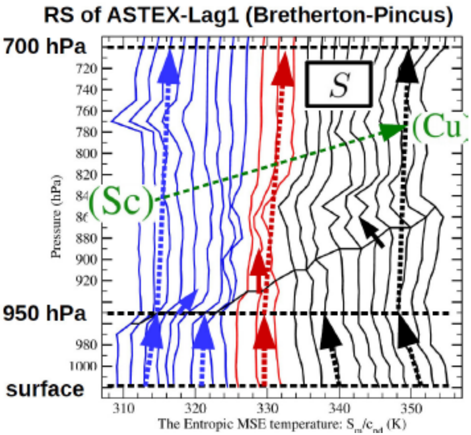
2TE+APDF (4)

- $\beta_m \sim$ entrainment
- $\Delta\theta_s \sim$ entrainment



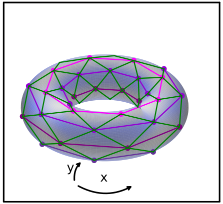
Marquet and Bechtold (2021)

ICON experiments

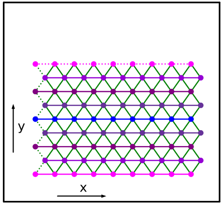
- Two modes:
  - **Single Column Mode (SCM)** : Torus grid (8x8), no dynamics
  - **Cloud Resolving Mode (CRM)** : Torus grid(100x100, 2.5km), with dynamics
- Two setups:
  - **NWP** : ICON operational turbulence and convection scheme
  - **2TE+APDF** : Two-energies scheme with APDF (without convection par.)

ICON SCM and ICON CRM-PER

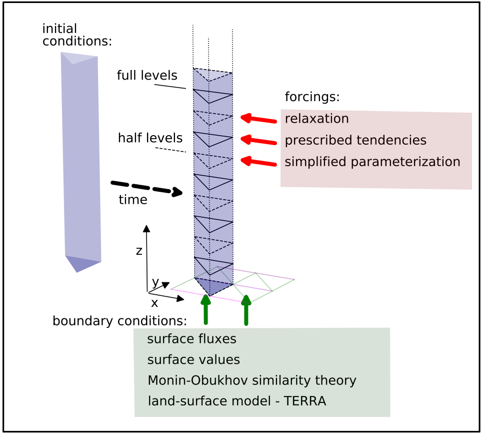
(a)



(b)



(c)



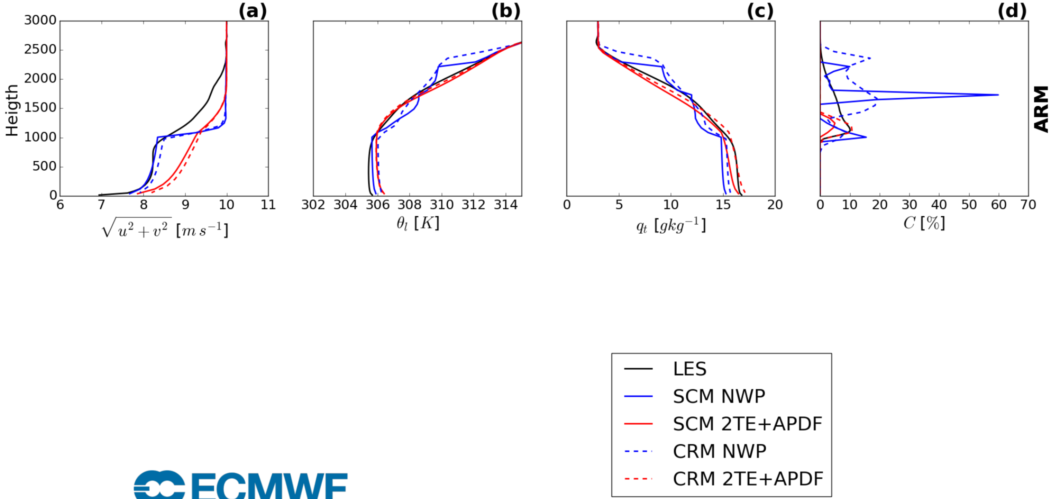
ICON experiments (2)

- **MicroHH (van Heerwaarden et al., 2017)** LES is used as reference.
- Four idealized cases:
  - **ARM**: Continental shallow,
  - **BOMEX**: Non-precipitating trade cumulus,
  - **DYCOMS-II**: Stratocumulus,
  - **GABLS(1)**: weakly stable stratification

ECMWF

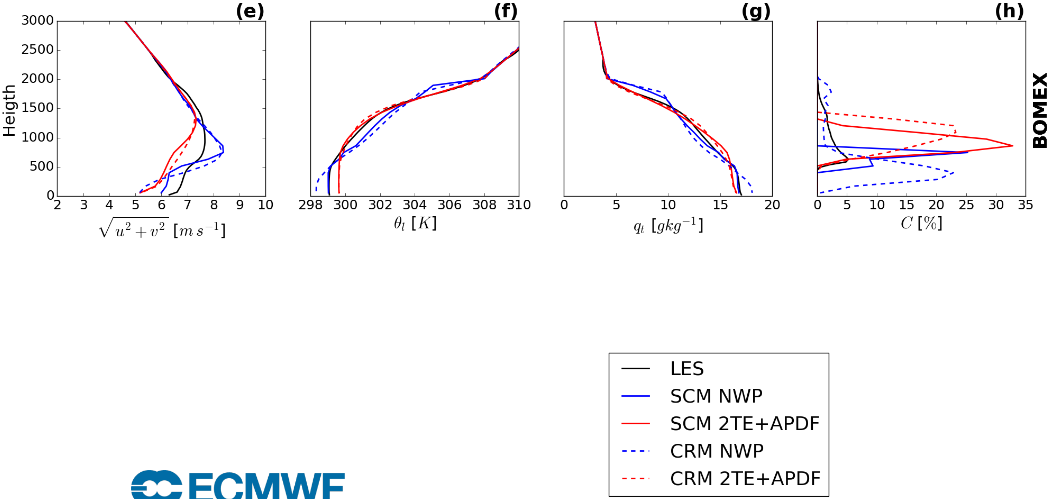
Vertical profiles after 8 hours if integration

ARM



Vertical profiles after 8 hours if integration

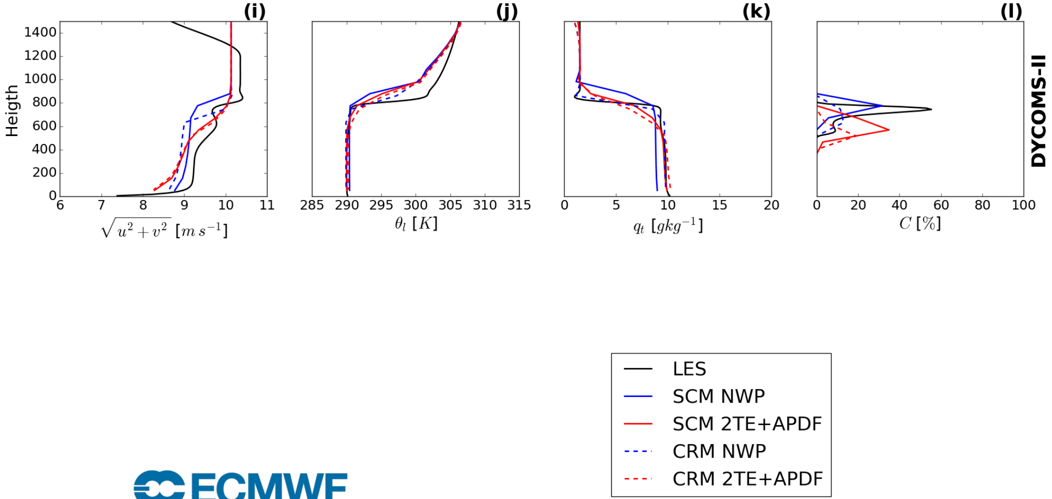
BOMEX



2TE+APDF scheme

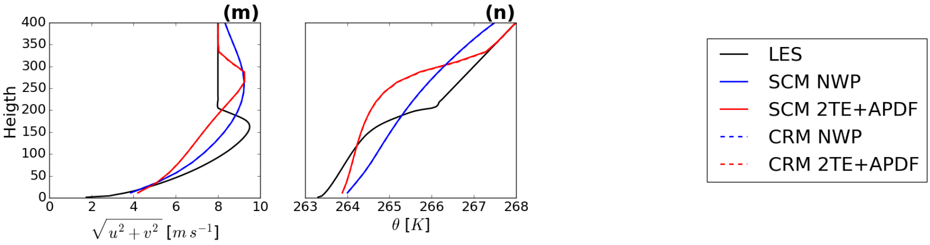
Vertical profiles after 8 hours if integration

DYCOMS-II



Vertical profiles after 8 hours if integration

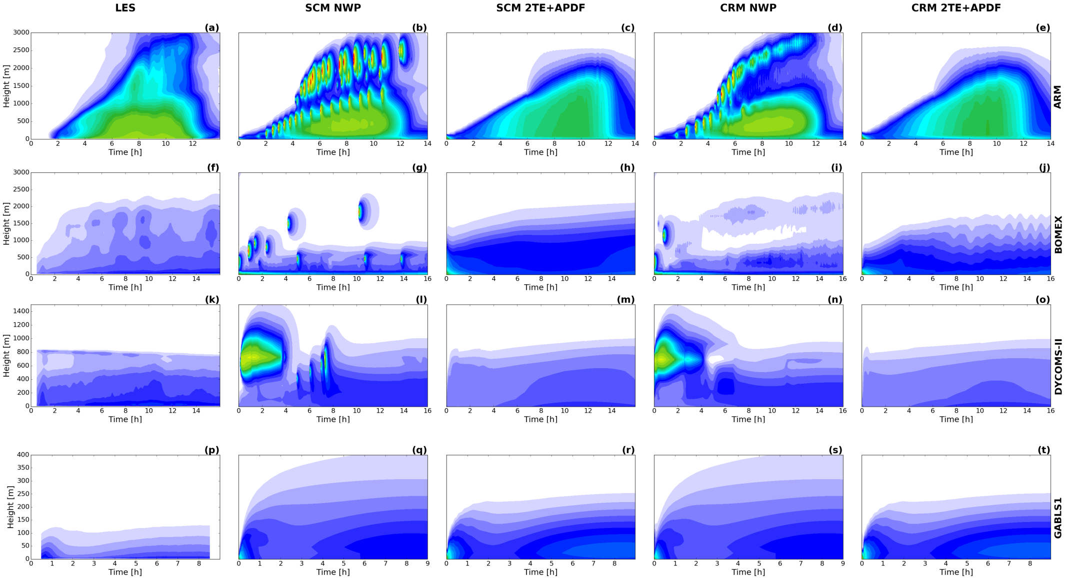
GABLS(1)



GABLS1

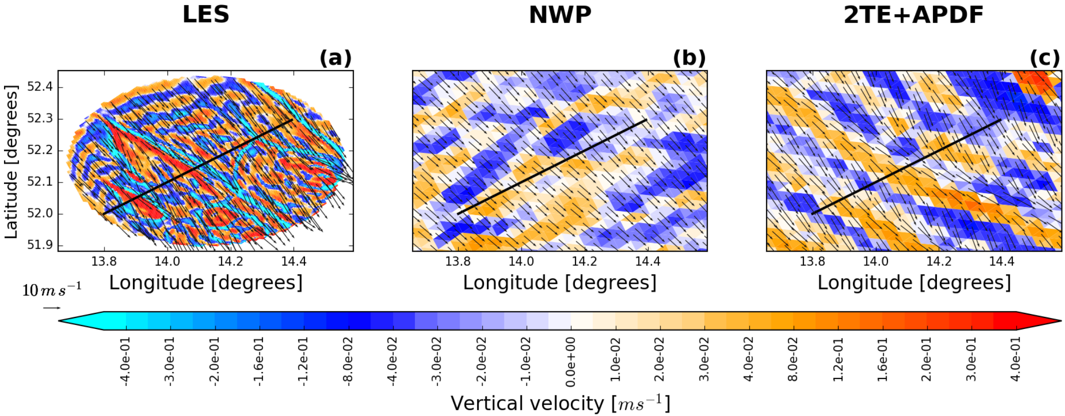
2TE+APDF scheme

Evolution of TKE



2TE+APDF scheme

Real case - 13.06.2021 : cloud streets Horizontal cross section at 1900 m



2TE+APDF scheme

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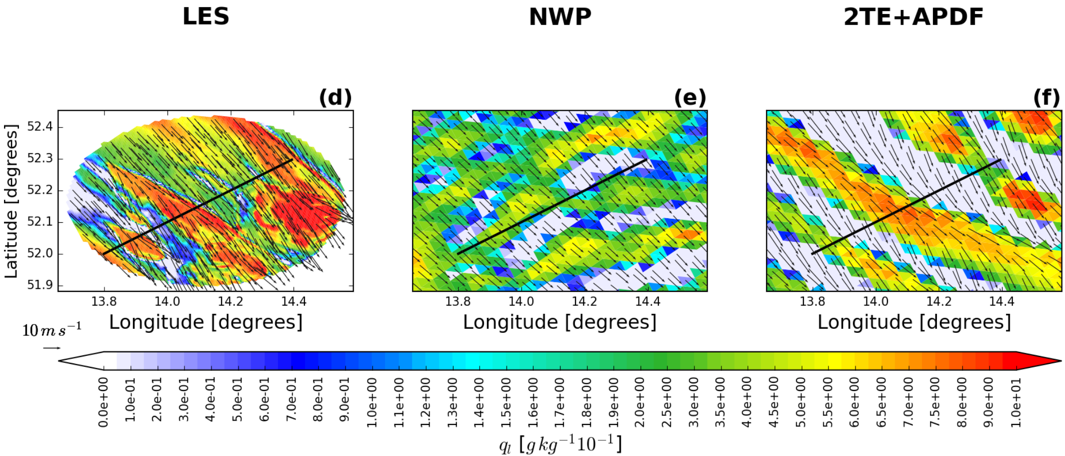

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2TE+APDF scheme

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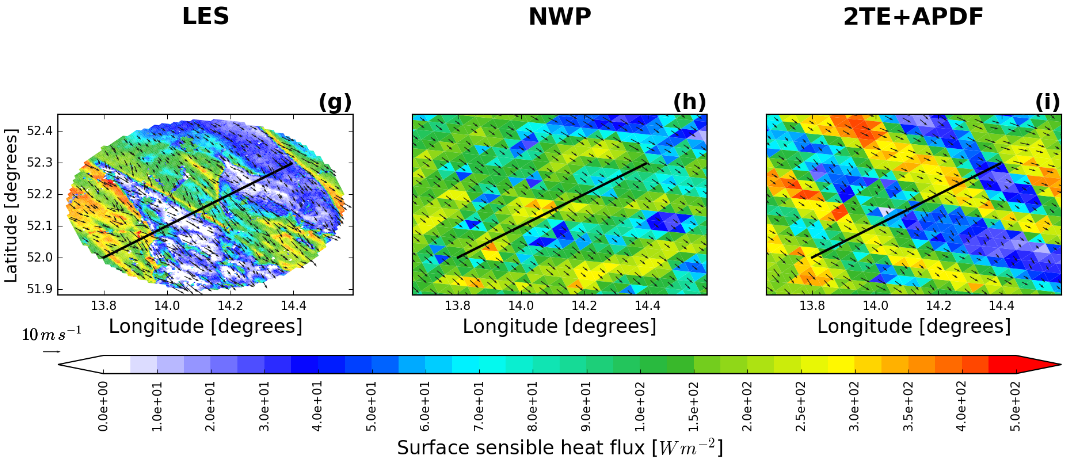

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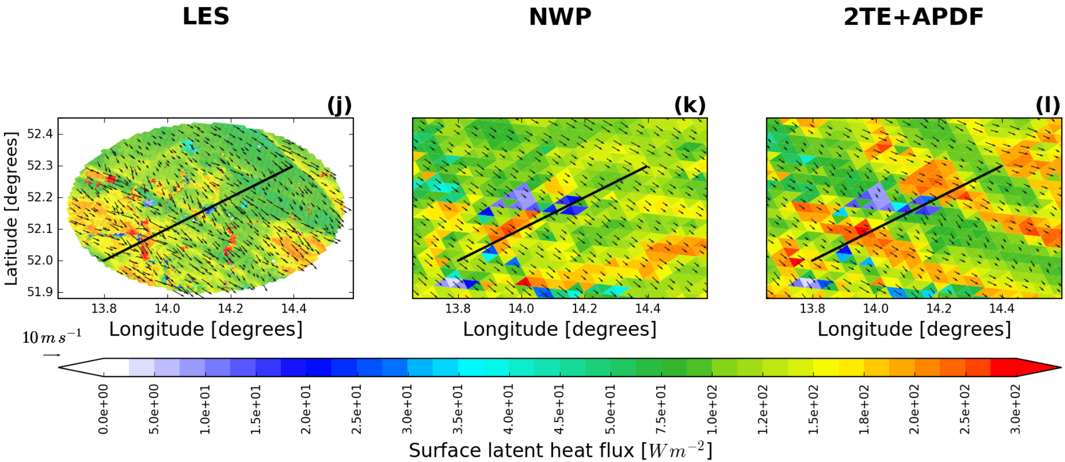

ECMWF

2TE+APDF scheme

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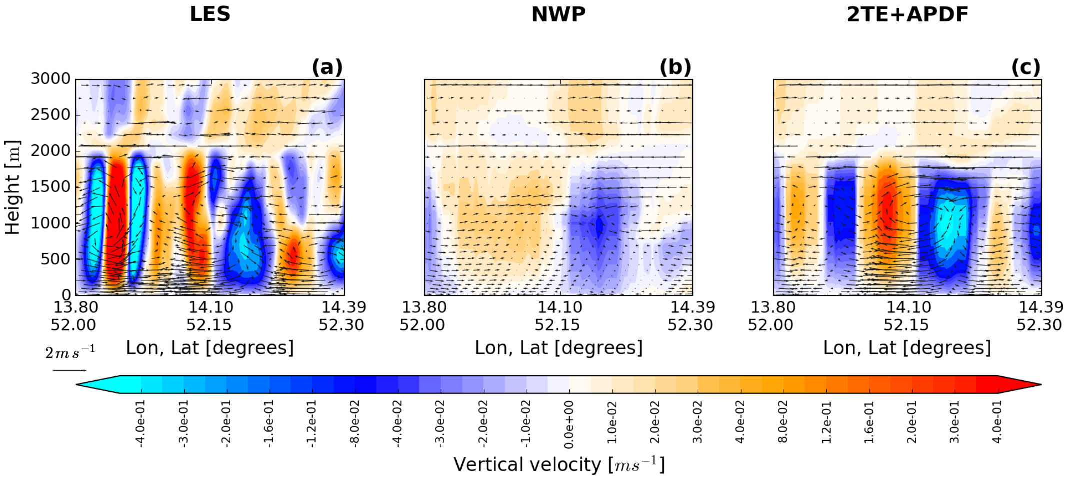
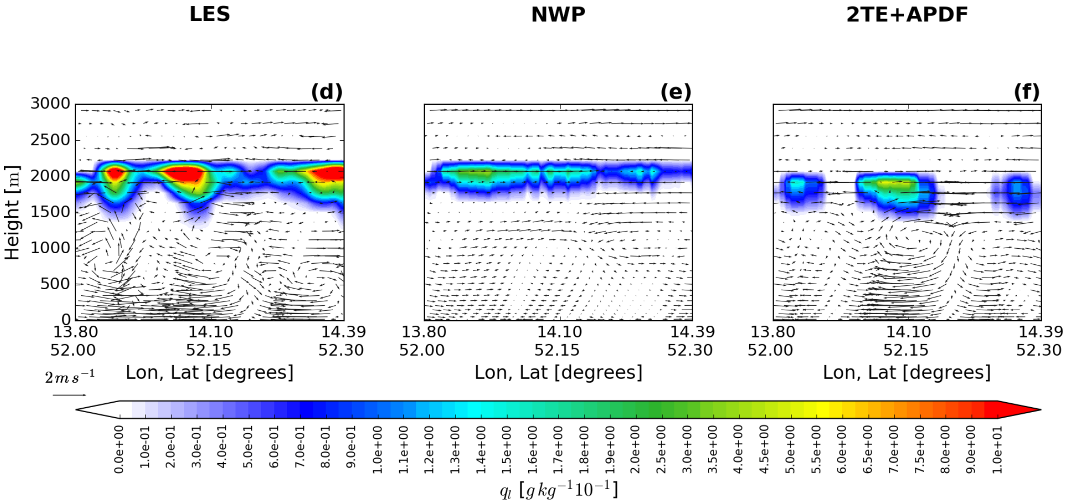


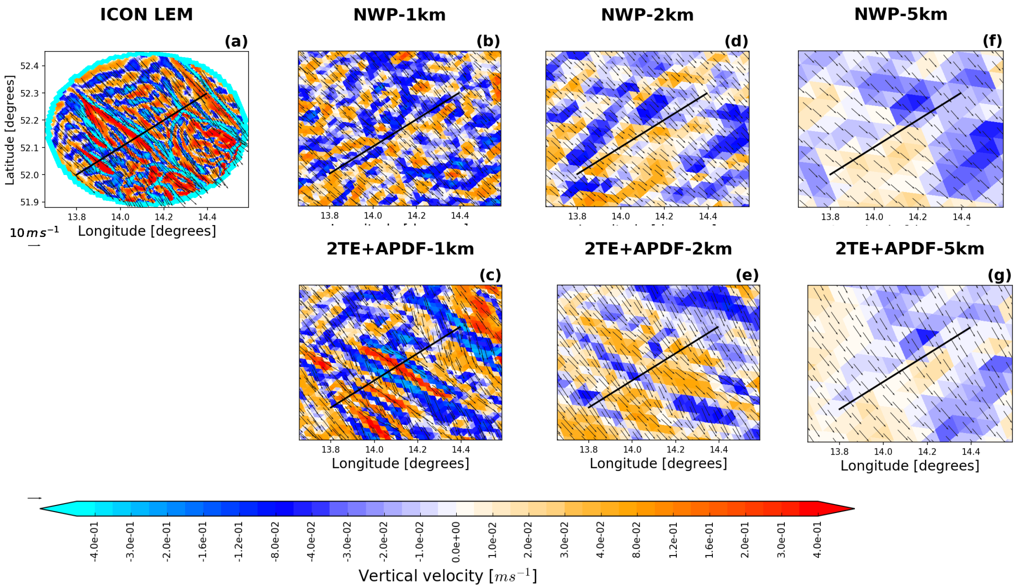
Real case - 13.06.2021 : cloud streets

Vertical cross section



2TE+APDF scheme

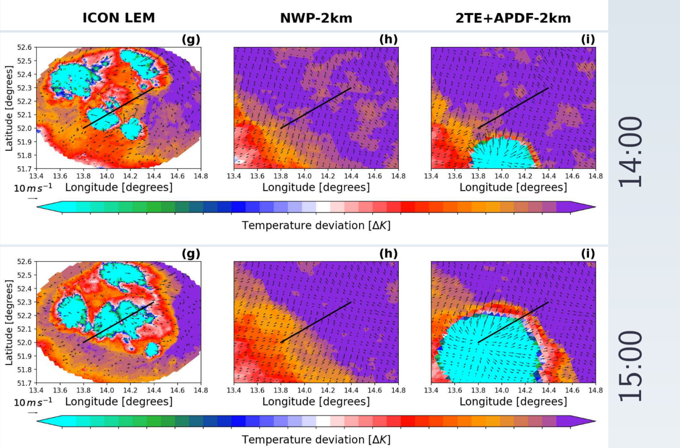
Real case - 13.06.2021 Resolution dependence



2TE+APDF scheme

29.06.2021 14:00 and 15:00 - cold pool

Horizontal cross sections - 10 m



Summary

- 2TE+APDF scheme is implemented in full ICON code
- based on selected cases, the 2TE+APDF scheme can be considered as an alternative to the operational turbulence and shallow convection scheme in ICON
- 2TE+APDF scheme improves the coupling with dynamics, which is beneficial for the modeling of coherent flow structures in the ABL

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Budget-Based Turbulence Length Scale Diagnostics

- More objective reference for turbulence length scale, L, is required, especially in the **gray zone** of turbulence, where the **cross-scale transfer** of TKE is not negligible.
- L can be estimated from the TKE or scalar variance **budget** using high resolution **LES data** (Bastak Duran et al. 2020).

L **diagnostics**:

Based on TKE:

$$L_{C,e_k} = \frac{(e_k)^{\frac{3}{2}}}{\widetilde{\epsilon}} C_\epsilon$$

where $\widetilde{\epsilon}$ is the **effective molecular dissipation rate**, C_ϵ and C_p are closure constants, and e_k is the TKE.

Based on scalar variances:

$$L_{C,\phi} = (e_k)^{\frac{1}{2}} \frac{\overline{\phi'^2}}{\widetilde{\epsilon}_\phi} \frac{C_\epsilon}{C_p}$$

where $\phi \in \{q_t, \Theta_l\}$, and $\widetilde{\epsilon}_\phi$ is the effective molecular dissipation rates of the variance of q_t and Θ_l.

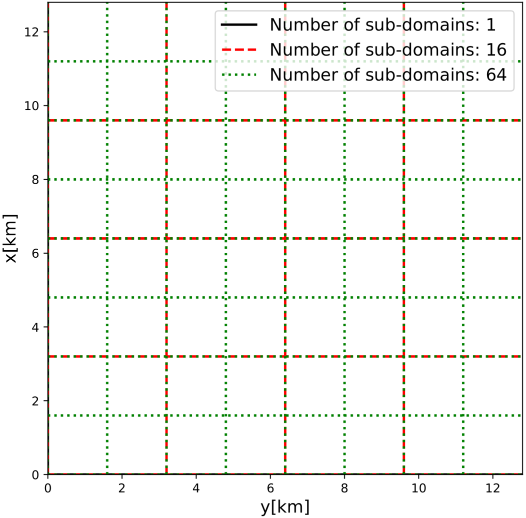
Five idealized LES cases (Micro-HH LES model):

Case	**Domain Size (km^3)**	**Grid Size (m^2)**
ARM	12.8x12.8x4.4	12.5x12.5x34.4
BOMEX	12.8x12.8x3.0	12.5x12.5x23.4
RICO	12.8x12.8x1.5	12.5x12.5x2.93
DYCOMS-II	12.8x12.8x4.4	12.5x12.5x31.25
GABLS1	0.4x0.4x0.4	12.5x12.5x0.78

The scale dependence is analysed using a coarse graining method of the LES domain.

Coarse graining method

The scale dependence is analysed using a coarse graining method of the LES domain.



Length scale formulations

Blackadar (1962) **(L_B)**: $l_B = \frac{k \cdot z}{1+\frac{k \cdot z}{\lambda}}$,

Bastak Duran et al. (2018) **(L_{BD1})**:

$$L_{BD1} = \frac{(C_K \cdot C_\epsilon)^{\frac{1}{4}}}{C_K} \frac{k \cdot z}{1 + \frac{k \cdot z}{\lambda_m} \left[\frac{1+exp\left(-a_m \cdot \sqrt{\frac{z}{H_{abl}}} + b_m\right)}{\beta_m + exp\left(-a_m \cdot \sqrt{\frac{z}{H_{abl}}} + b_m\right)} \right]},$$

Nakanishi and Niino (2009) **(L_{NN})**:

$$\frac{1}{L_{NN}} = \left(\frac{1}{L_{Surf}} + \frac{1}{L_T} + \frac{1}{L_{Bouy}} \right).$$

Length scale formulations (2)

Bougeault and Lacarrere (1989) $\mathbf{(}L_{BL}\mathbf{)}$:

$$
\begin{aligned}
L_{BL} &= \frac{(C_K\, C_\epsilon)^{\frac{1}{4}}}{C_K} \left(L_{up} \cdot L_{down}\right)^{\frac{1}{2}}, \\
& \int_{z}^{z+L_{up}} \frac{g}{\theta_{vr}} (\theta_v(z) - \theta_v(z'))\, dz' = \mathbf{e_k}(z), \\
& \int_{z-L_{down}}^{z} \frac{g}{\theta_{vr}} (\theta_v(z') - \theta_v(z))\, dz' = \mathbf{e_k}(z).
\end{aligned}
$$

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Length scale formulations (3)

Honnert et al. (2021) (L_{H21}):

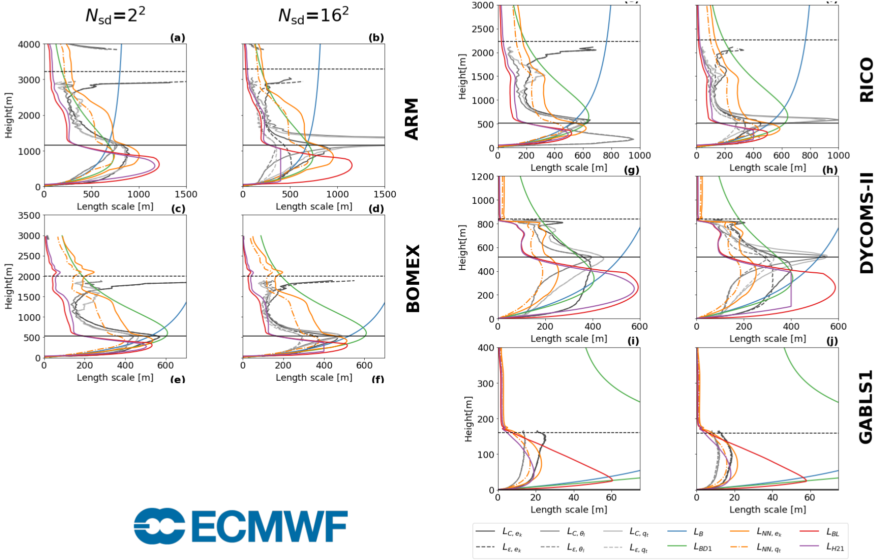
$$
\begin{aligned}
L_{H21} &= min(L_R, \alpha \cdot L_{LES}), \\
L_{LES} &= (\Delta x \cdot \Delta y)^{\frac{1}{2}} \\
L_R &= \frac{(C_K\, C_\epsilon)^{\frac{1}{4}}}{C_K} \left(\frac{L_{Rup}^{-\frac{2}{3}} + L_{Rdown}^{-\frac{2}{3}}}{2} \right)^{-\frac{3}{2}},
\end{aligned}
$$

$$
\int_z^{z+L_{Rup}} \frac{g}{\theta_{vr}} \left(\theta_v(z) - \theta_v(z') + \mathbf{C_0} \cdot \sqrt{\mathbf{e_k}} \cdot \sigma(\mathbf{z'})\right) dz' = \mathbf{e_k}(z),
$$

$$
\int_{z-L_{Rdown}}^{z} \frac{g}{\theta_{vr}} \left(\theta_v(z) - \theta_v(z') + \mathbf{C_0} \cdot \sqrt{\mathbf{e_k}} \cdot \sigma(\mathbf{z'})\right) dz' = \mathbf{e_k}(z),
$$

ECMWF

Vertical profiles



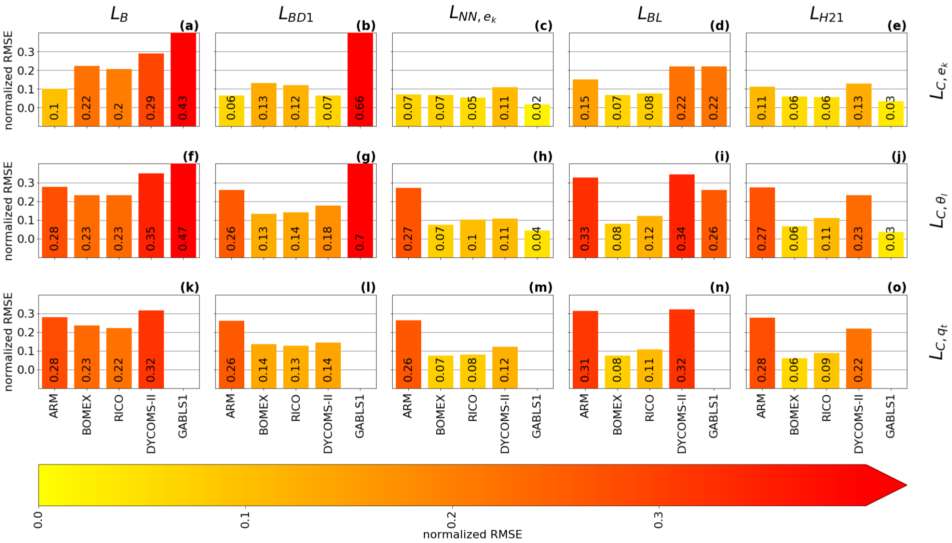
Normalized (by H_{abl}) RMSE

$$\mathrm{RMSE}_n = \sqrt{\frac{1}{n_z\, n_t} \sum_{j=1}^{j=n_t} \sum_{i=1}^{i=n_z} \left(\frac{L_F\left(z_i, t_j\right) - L_D\left(z_i, t_j\right)}{H_{\mathrm{abl}}} \right)^2},$$

z_i is the height of the i-th model level, n_z is number of model levels in the ABL, t_j is the time after j time steps, n_t is number of time steps, L_F is an algebraic length scale, L_D is diagnosed length scale.

Budget-Based Turbulence Length Scale Diagnostics

Normalized RMSE



Three-component score

Amplitude:

$$\mathbf{A} = \frac{1}{n_t}\sum_{j=1}^{j=n_t}\left(\mathrm{L_F P}(t_j) - \mathrm{L_D P}(t_j)\right),$$

$$\mathrm{L_F P} = \frac{1}{H_{abl}^2}\int_{z=0}^{z=H_{abl}} L_F(z)\,dz, \qquad \mathrm{L_D P} = \frac{1}{H_{abl}^2}\int_{z=0}^{z=H_{abl}} L_D(z)\,dz,$$

Peak location and magnitude:

$$\mathbf{S_l} = \frac{1}{n_t}\sum_{j=1}^{j=n_t}\frac{\mathrm{z_{F,max}}(t_j) - \mathrm{z_{D,max}}(t_j)}{H_{abl}},$$

$$\mathbf{S_m} = \frac{1}{n_t}\sum_{j=1}^{j=n_t}\frac{\mathrm{L_{F,max}}(t_j) - \mathrm{L_{D,max}}(t_j)}{H_{abl}},$$

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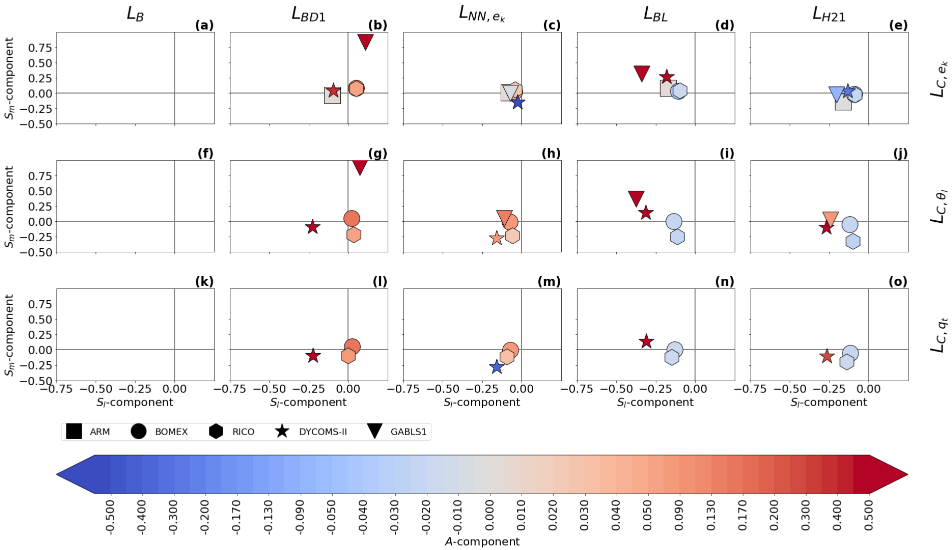
Budget-Based Turbulence Length Scale Diagnostics

Three-component score



Summary

- L can be estimated from LES data using budget based diagnostic (Bastak Duran et al. 2020).
- Typical shape of L: proportional to the distance from surface near surface, peak in the ABL, asymptotic value (positive or zero) at the top of the ABL.
- TKE-based L formulations show better performance.

TKE scheme in IFS

- TKE scheme from ARPEGE physics (CBR scheme) already implemented
- Calibration in IFS is required:
  - **Under-estimation of turbulent mixing in free atmosphere - particularly tropical jet region**
  - Under-estimation of Stratocumulus cloudiness
  - ...

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TKE in free atmosphere

- TKE source terms are computed using TKE from previous time step:

  $$\text{Shear term} = K_M\left(\mathbf{e_k^-}\right)\cdot S^2,\ \text{Buoyancy term} = K_H\left(\mathbf{e_k^-}\right)\cdot N^2$$

- Problem with **temporal discretization**: $e_k^- = 0 \Rightarrow e_k^+ = 0$

- Solution: assume **TKE equilibrium** to estimate the TKE source terms:

  $$\text{Shear term} = K_M\left(\widetilde{\mathbf{e_k}}\right)\cdot S^2,\ \text{Buoyancy term} = K_H\left(\widetilde{\mathbf{e_k}}\right)\cdot N^2$$

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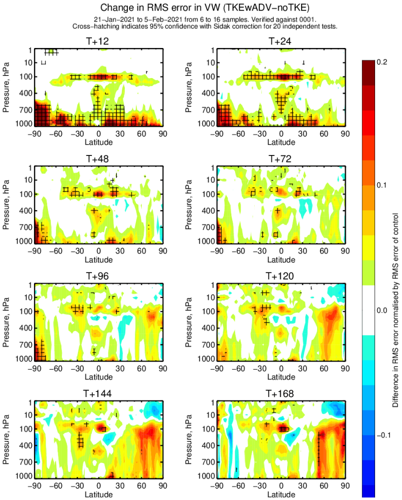
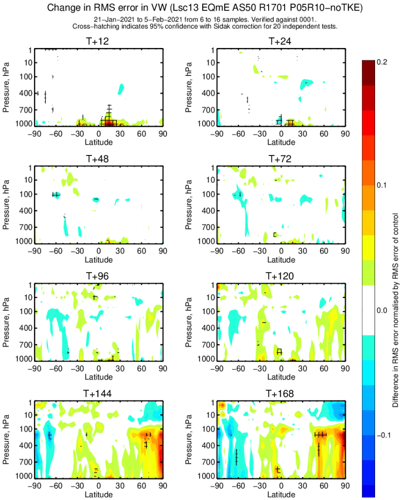
TKE scheme in IFS

TKE in free atmosphere (change in RMSE of wind vector)

Original implementation



Equilibirum TKE source terms



Thank you for your attention!

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