Linearized physics at ECMWF: 25 years old and counting

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Linearized models in NWP

- different applications:
  - variational data assimilation ⟵ *e.g., incremental 4D-Var*
  - singular vector computations ⟵ *initial perturbations for ensemble prediction*
  - sensitivity analysis ⟵ *forecast errors*
  - other ⟵ *model parameter estimation,*
    sensitivity of the parametrization scheme to input parameters

- **initially, applications were using adiabatic linearized model**

- **nowadays, the physical processes included in the linearized models help to:**
  - produce physically consistent initial atmospheric state
  - represent some atmospheric features *(processes in PBL, tropical & baroclinic instabilities, …)*
  - reduce forecast error

4D-Var – Four-dimensional Variational Data Assimilation
PBL – Planetary Boundary Layer

Tangent-linear (TL) and adjoint (AD) models – definition

- Tangent-linear model

If M is a model such as:

$$\mathbf{x}(t_{i+1}) = M[\mathbf{x}(t_i)]$$

then the tangent linear model of M, called M', is:

$$\delta\mathbf{x}(t_i + 1) = M'[\mathbf{x}(t_i)]\,\partial\mathbf{x}(t_i) = \frac{\partial M[\mathbf{x}(t_i)]}{\partial\mathbf{x}}\,\delta\mathbf{x}(t_i)$$

- Adjoint model

The adjoint of a linear operator M' is the linear operator M^* such that, for the inner product <,> :

$$\forall\mathbf{x}, \forall\mathbf{y} \qquad \langle M'\,\mathbf{x}, \mathbf{y}\rangle = \langle \mathbf{x}, \mathbf{M}^*\,\mathbf{y}\rangle$$

For the inner product in the Euclidean space:

$$\mathbf{M}^* = M'^T$$

Physical parametrization in the linearized models

- **Development of the linearized physical parametrizations:**
  - work started in the mid-1990s, initially focusing on its potential usage in sensitivity studies as well as on non-differentiability issues
  - gathered pace at the turn of the century with the operational implementation of four-dimensional variational (4D-Var) data assimilation

- **History: from simple to comprehensive physical parametrization schemes for adjoint models:**
  - started from very simple schemes, such as *Buizza (1994)*, just aimed at removing very strong increments produced by the adiabatic adjoint models
  - more complex, but still incomplete schemes were developed by *Zou et al. (1993)*, *Zupanski and Mesinger (1995)*, *Janisková et al. (1999)*, *Mahfouf (1999, 2005)* and *Laroche et al. (2002)*
  - at ECMWF, comprehensive schemes implemented describing the whole set of physical processes and their interactions almost as in the non-linear model, but with simplifications and/or regularization (*Janisková et al. 2002*; *Tompkins & Janisková 2004*; *Lopez & Moreau 2005*; *Janisková & Lopez 2013*).

Linearized physics at ECMWF

- **ECMWF 4D-Var uses the most detailed linearized physics package (LP) in the world:**
  - the following processes are included:
    - radiation
    - vertical diffusion
    - surface
    - orographic gravity wave drag
    - moist convection
    - large-scale condensation/precipitation
    - non-orographic gravity wave

4D-Var became operational at ECMWF in 1997:

25 years of operational 4D-Var

↓

25 years of operational use of linearized physics

Physical parametrizations in data assimilation (DA) needed:

- to convert the model state variables (as temperature, wind, humidity, surface pressure) into observed equivalents (e.g. radiances, reflectivities, precipitation, cloud, …)
- to describe the time evolution of the model state over the assimilation window as accurately as possible

- ***Using physics (full & simplified) in incremental 4D-Var system:***

4D-Var →

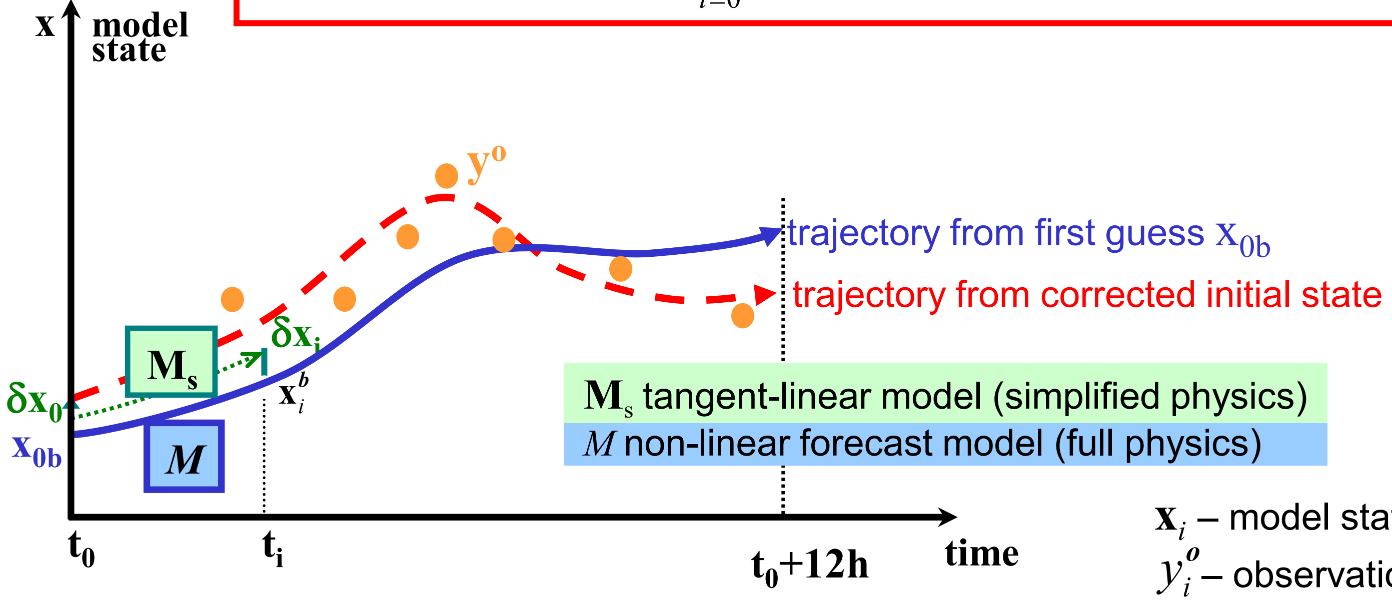
$$\min J(\delta\mathbf{x}_0) = \frac{1}{2}\delta\mathbf{x}_0^T\mathbf{B}^{-1}\delta\mathbf{x}_0 + \frac{1}{2}\sum_{i=0}^{n}(\mathbf{H}_i(\delta\mathbf{x}_i)-\mathbf{d}_i)^T\mathbf{R}_i^{-1}(\mathbf{H}_i(\delta\mathbf{x}_i)-\mathbf{d}_i)$$

$$\Leftrightarrow \nabla_{\delta\mathbf{x}_0}J = \mathbf{B}^{-1}\delta\mathbf{x}_0 + \frac{1}{2}\sum_{i=0}^{n}\mathbf{M}^T(t_i,t_0)\mathbf{H}_i^T\mathbf{R}_i^{-1}(\mathbf{H}_i(\delta\mathbf{x}_i)-\mathbf{d}_i) = 0$$

$\mathbf{d}_i = y_i^o - H_i(\mathbf{x}_i^b)$ - innovation vector

H_i non-linear observation operator

$\mathbf{H}_i$ tangent-linear observation operator



$\mathbf{d}_i$ ← using non-linear model M at high resolution & full physics

$\delta\mathbf{x}_i$ ← using tangent-linear model $\mathbf{M}$ at low resolution & simplified physics

$\nabla_{\delta\mathbf{x}_0}J$ ← using a low resolution adjoint model $\mathbf{M}^T$ & simplified physics

$\mathbf{x}_i$ – model state at t_i

y_i^o – observations at t_i

Constraints when developing the linearized parametrization schemes

Solving the 4D-Var minimization requires the linearization of the forecast model's physical parameterizations so that their tangent-linear (TL) and adjoint (AD) versions can be used to describe the (forward, respectively backward) time evolution of the model state during the minimization:

- The minimization of the 4D-Var cost function being solved with an iterative algorithm is computationally rather demanding → simplifications required to **reduce computational cost**:
  - by retaining only the most significant physical processes represented in the full forecast model
- Physical processes might be highly non-linear & often discontinuous → linearity considerations require:
  - either discarding processes which could lead to instabilities
  - or regularizing by smoothing out discontinuities

AT THE SAME TIME

- To keep the description of atmospheric processes sound → parametrization schemes used in the linearized model must remain realistic enough
- Thorough validation is also required and must be done for non-linear, TL and AD versions of parametrizations schemes

Simplifications of the linearized models for practical applications

- In the most common applications:
  - *incremental 4D-Var (ECMWF, Météo-France, …),*
  - *simplified gradients in 4D-Var (Zupanski 1993),*
  - *the initial perturbations computed for ensemble prediction (ECMWF),*

linearized versions of forecast models are run at lower resolution

⇓

the linear model may not be “the exact tangent” to the full model

(different resolution and geometry, different physics)

⇓

simplified approach to include physical processes step-by-step in TL and AD models

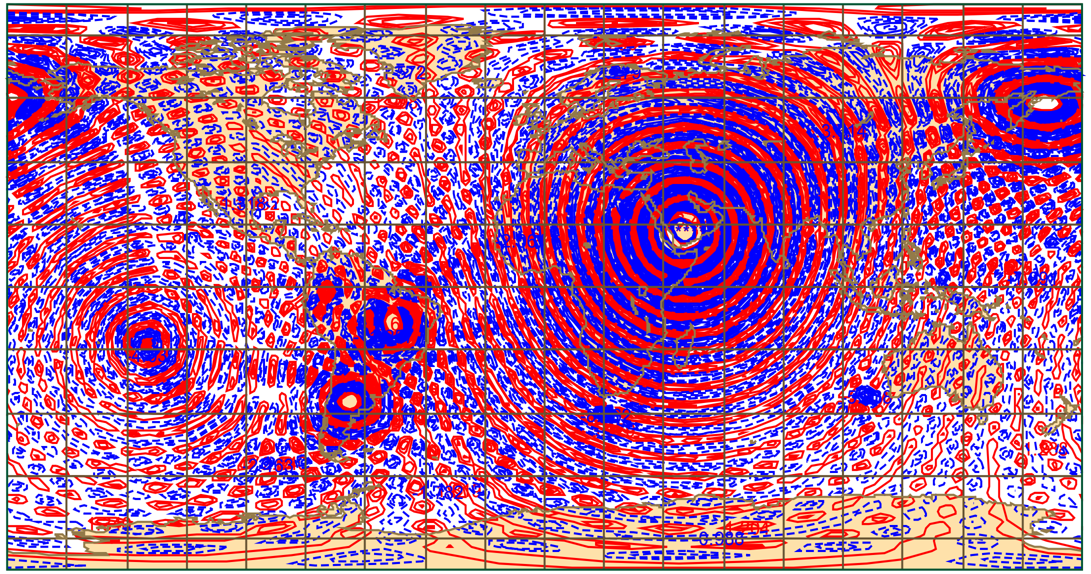
- **to build a physical package which is:**
  - **easy to linearize**
  - **regular** – to avoid strong non-linearities and thresholds
  - **realistic enough**
  - **computationally affordable**

Regularization of the linearized physical parametrization

WHY REGULARIZATION IS SO IMPORTANT

Cont.int: 0.5e+07

lv31 T* 1999-03-15 12h fc t+6 - TL with vdif (no regularization applied) [cont.int: 0.5e+07]



Without any treatment of most serious threshold processes, the TL approximation can become useless.

BAD NEWS !!!

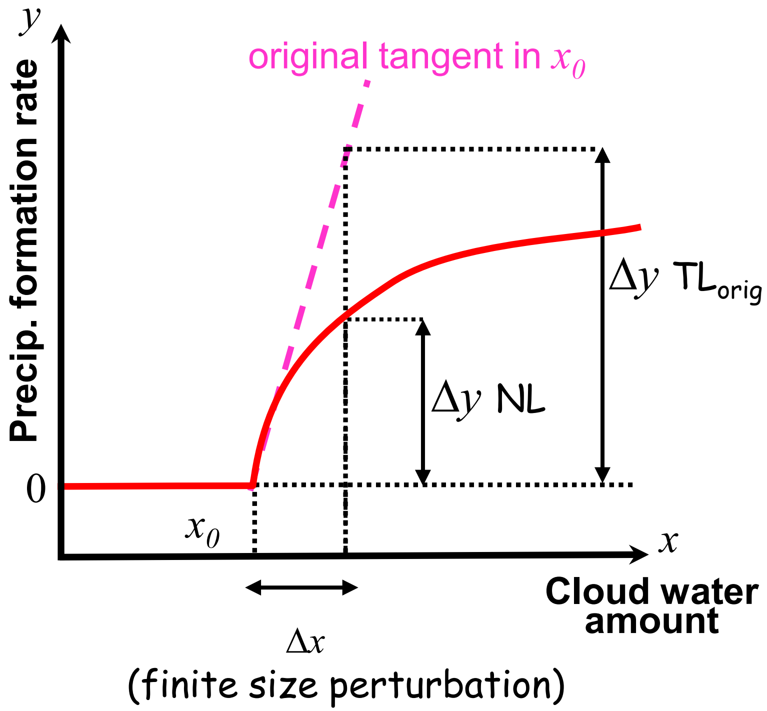
Unless one wants to use model for generating modern art.

Wednesday 6 October 2004 15UTC ECMWF Forecast t+1 VT: Wednesday 6 October 2004 16UTC Model Level 45 temperature

e+02

Importance of the regularization of TL model (1)

- physical processes are characterized by:
  - * threshold processes:
    - discontinuities of some functions describing the physical processes *(on/off processes)*
    - discontinuities of the derivative of a continuous function
  - * strong nonlinearities



u-wind increments
fc t+12, ~700 hPa

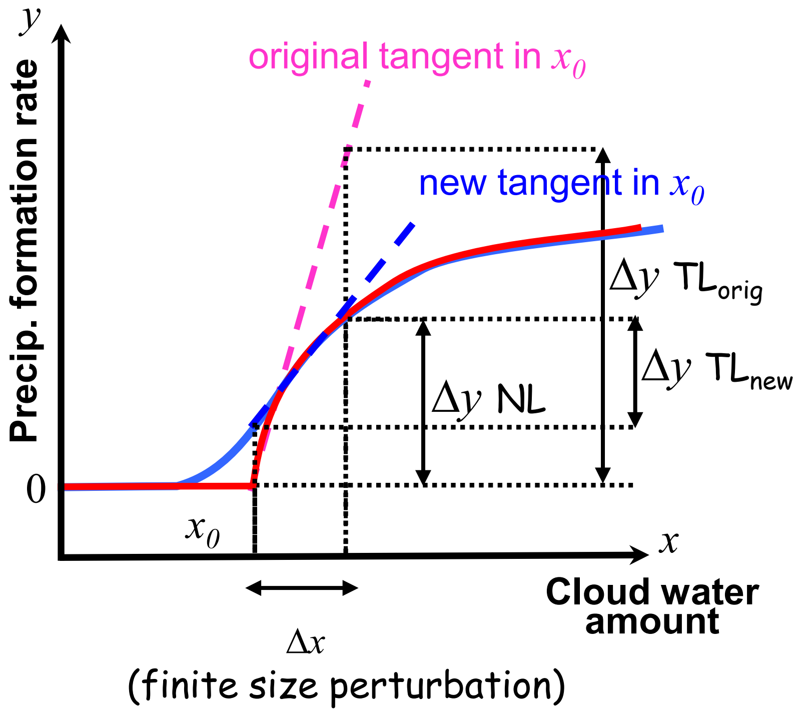
x 10^5

finite difference (FD)

TL integration without regularization

Importance of the regularization of TL model (2)

- regularizations help to remove the most important threshold processes in physical parametrizations affecting the range of validity of TL approximation



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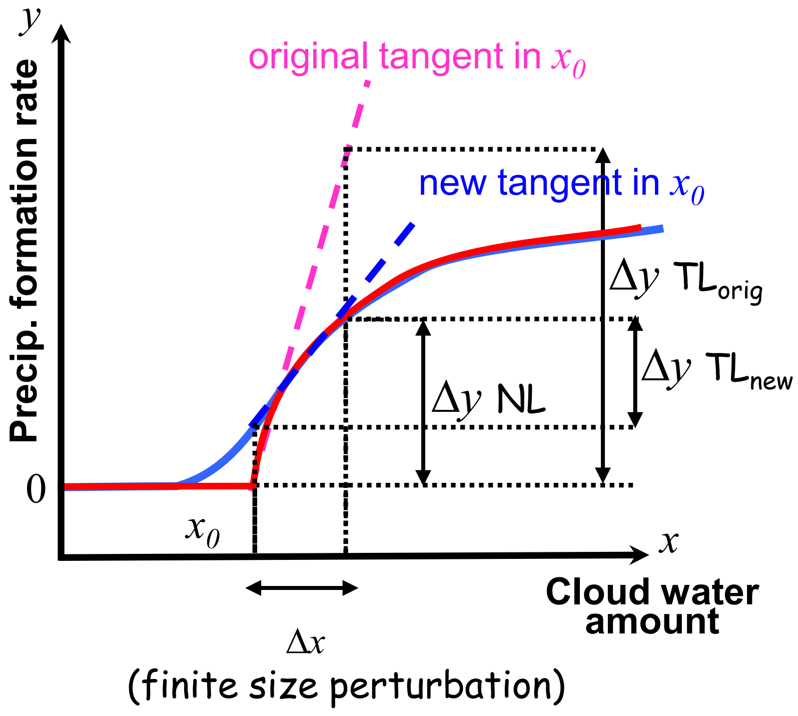


u-wind increments
fc t+12, ~700 hPa

finite difference (FD)

TL integration without regularization

Tangent-linear (TL) approximation

Comparison:

finite differences (FD) $\leftrightarrow$ tangent-linear (TL) integration

$$M(\mathbf{x}_{an}) - M(\mathbf{x}_{fg}) \quad \leftrightarrow \quad M'(\mathbf{x}_{an} - \mathbf{x}_{fg})$$

$$(an = analysis, \quad fg = first\ guess)$$

TL_{ADIAB} – adiabatic TL model

$TL_{ADIABSVD}$ – TL model with very simple vertical diffusion *(Buizza 1994)*

TL_{WSPHYS} – TL model with the whole set of simplified physics *(Mahfouf 1999, Janisková et al 1999)*

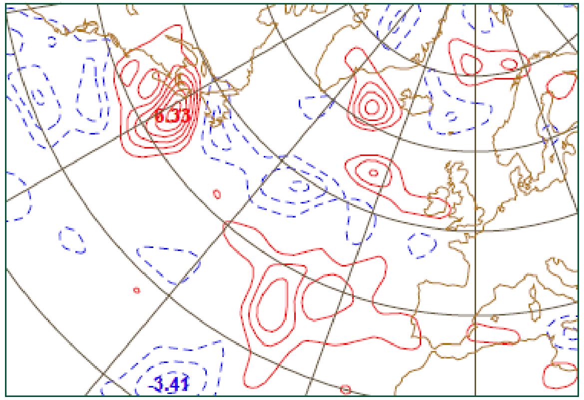
GREAT !!!

TL_{WSPHYS} better than TL_{ADIAB}

Zonal wind increments at model level 31 (~ 1000 hPa)
24-hour integration

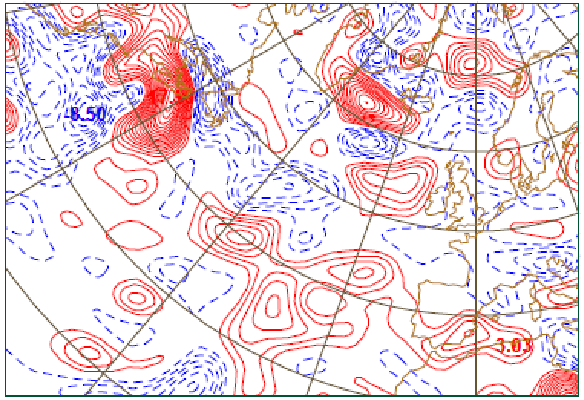
FD

rms=1.01222 stdev=0.96653



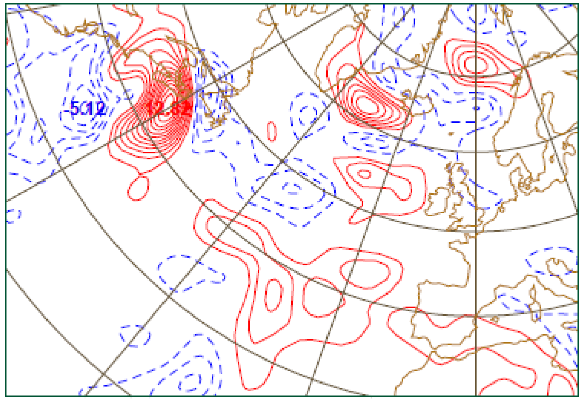
TL_{ADIAB}

rms=2.90740 stdev=2.88242



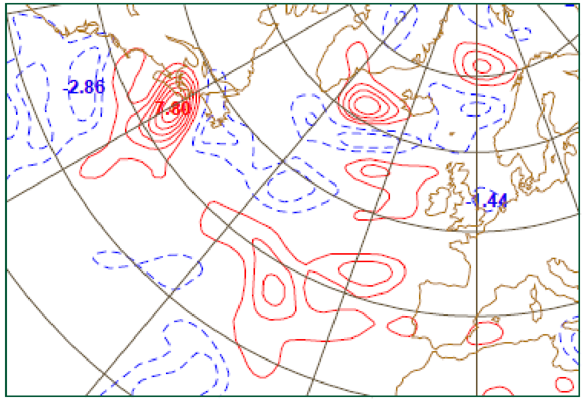
$TL_{ADIABSVD}$

rms=1.27768 stdev=1.27739



TL_{WSPHYS}

rms=1.08662 stdev=1.07506



T63L31 run: 15/03/1999 T+24
T63 ~ 320 km

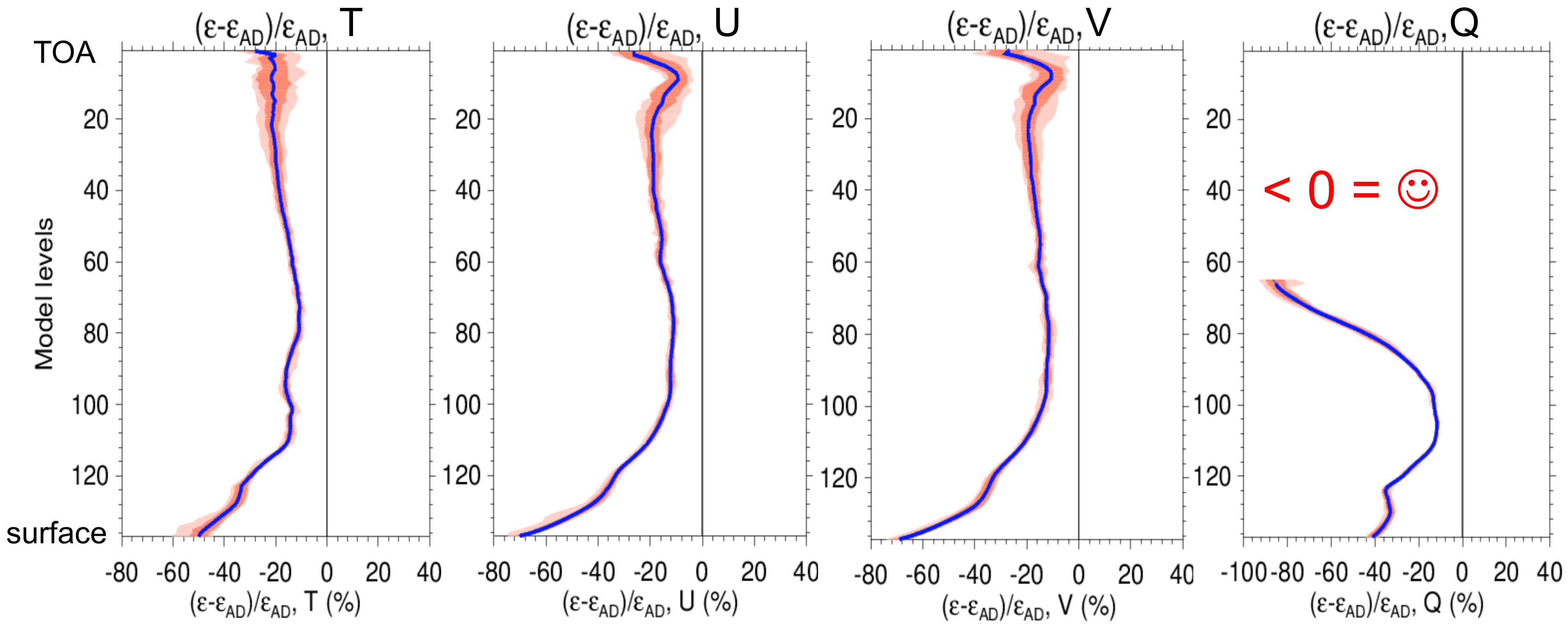
Impact of linearized physics on TL approximation

$$\frac{\varepsilon_{EXP} - \varepsilon_{REF}}{\varepsilon_{REF}}.100\%$$

where

$$\varepsilon = \left| \left[M(\mathbf{x}+\delta\mathbf{x}) - M(\mathbf{x}) \right] - M'(\delta\mathbf{x}) \right|$$

non-linear (NL) difference ↔ tangent-linear (TL) integration



Mean vertical profile of change in TL error when full linearized physics included in TL.

Relative to adiabatic TL run (50-km resolution; twenty runs, 12h integ.)

Inclusion of linearized physics leads to better TL approximation.

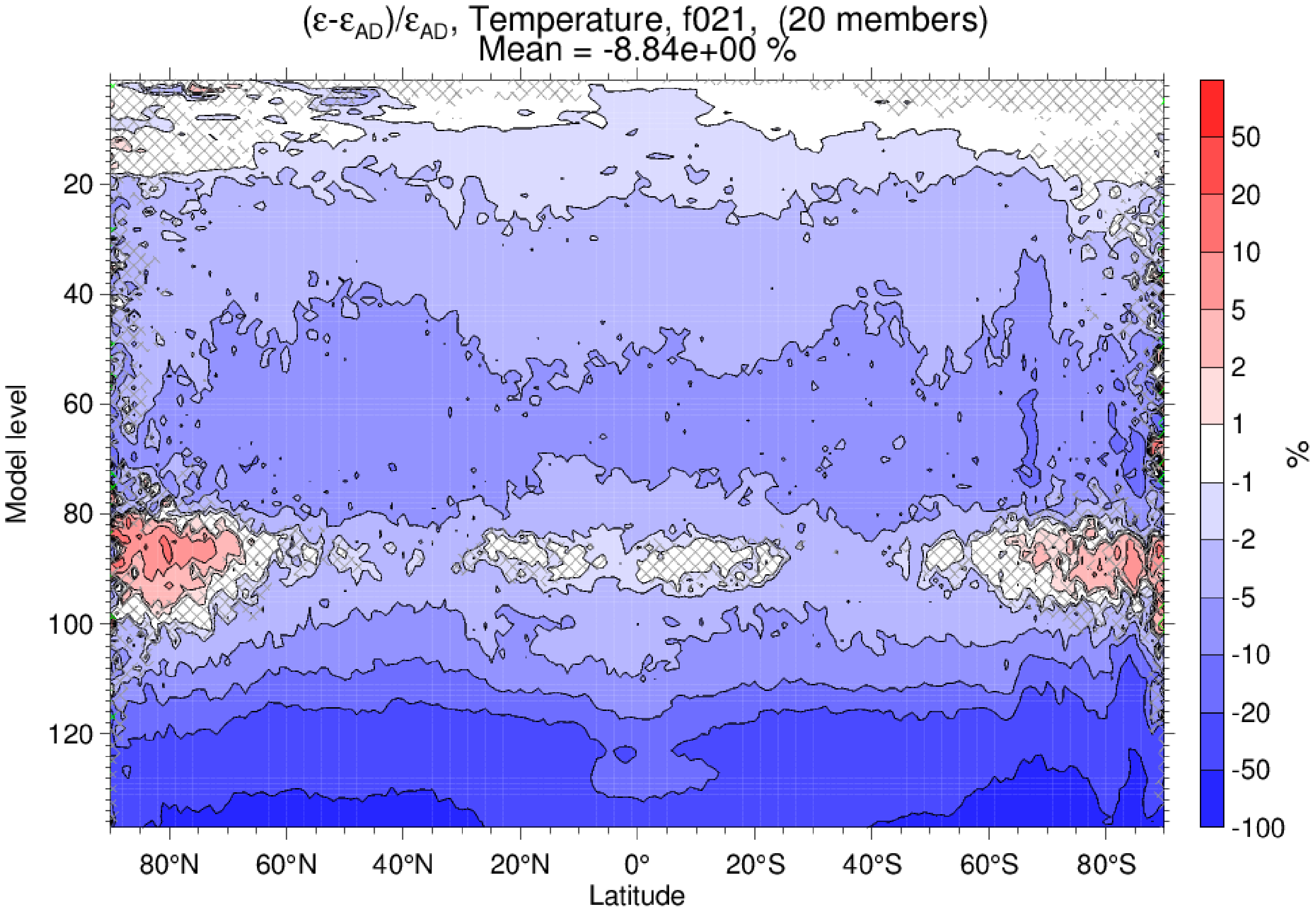
Impact of linearized physics on TL approximation – contribution from different processes (1)

Zonal mean cross-section of change in TL error when TL includes:

VDIF + orog. GWD + SURF

Blue = TL error reduction = ☺

Temperature



Relative to adiabatic TL run:

- 50-km resolution
- 20 runs
- after 12h integr.

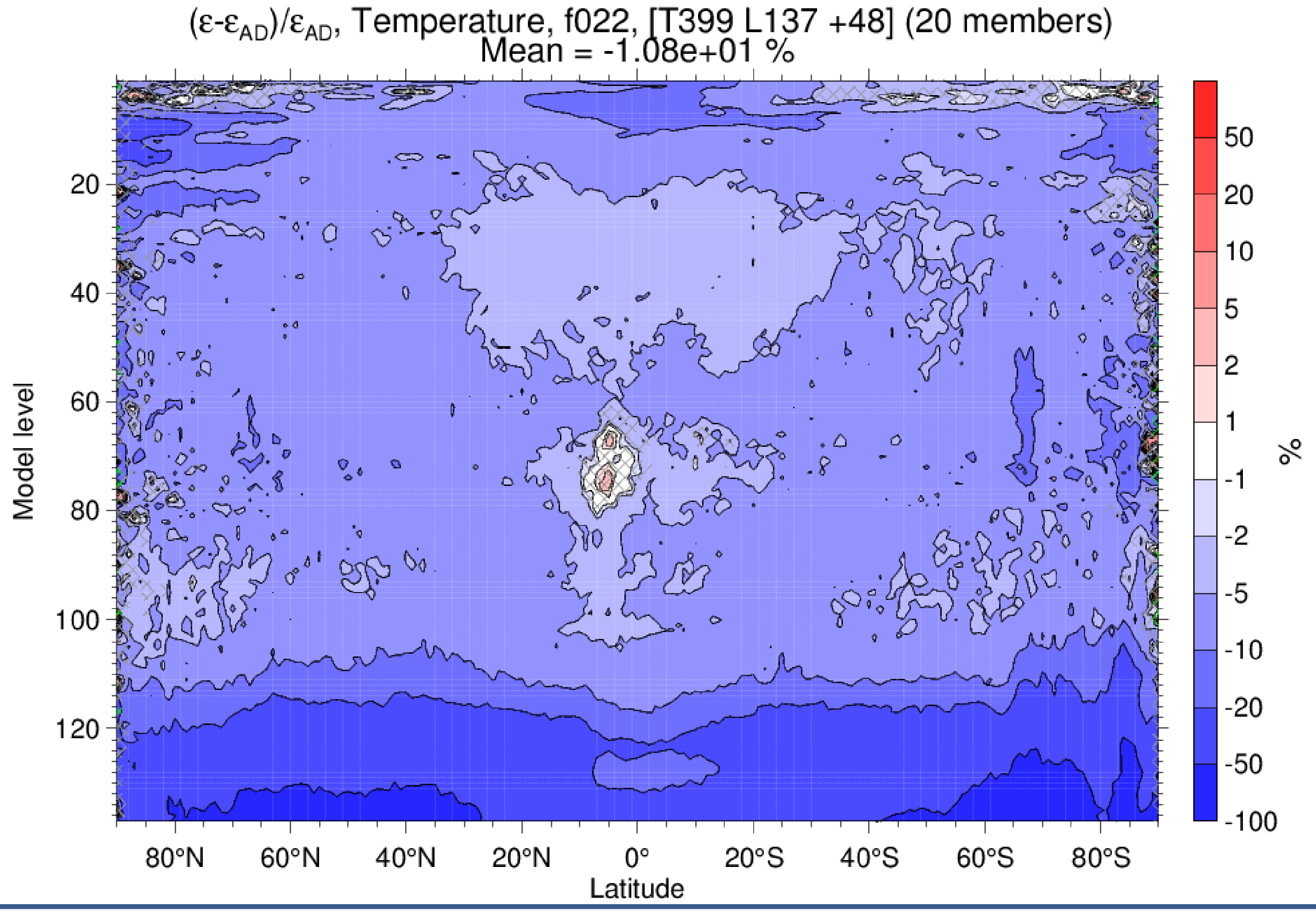
Impact of linearized physics on TL approximation – contribution from different processes (2)

Zonal mean cross-section of change in TL error when TL includes:

VDIF + orog. GWD + SURF + RAD

Blue = TL error reduction = ☺

Temperature



Relative to adiabatic TL run:

- 50-km resolution
- 20 runs
- after 12h integr.

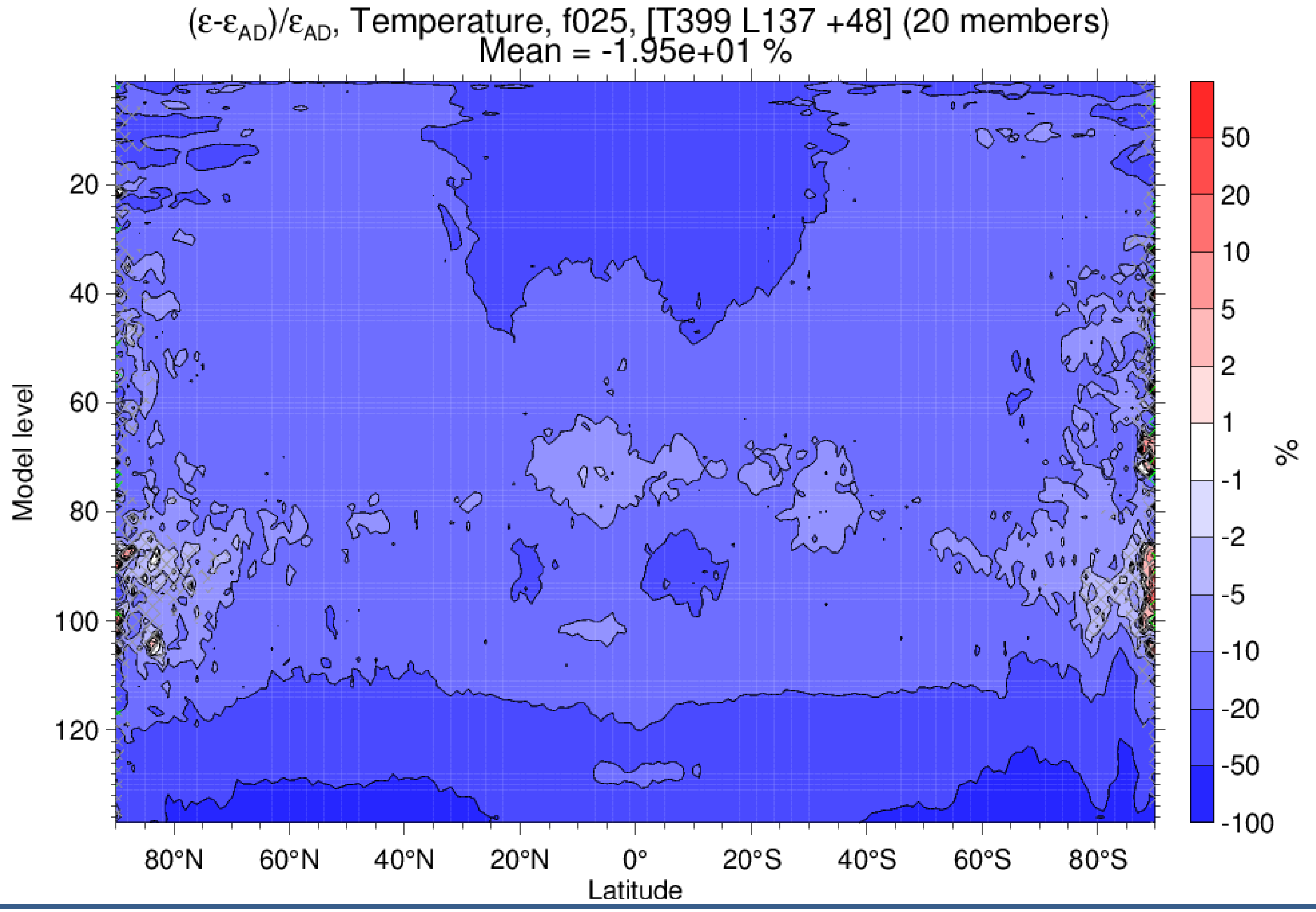
Impact of linearized physics on TL approximation – contribution from different processes (3)

Zonal mean cross-section of change in TL error when TL includes:

VDIF + orog. GWD + SURF + RAD + non-orog GWD + moist physics

Blue = TL error reduction = ☺

Temperature

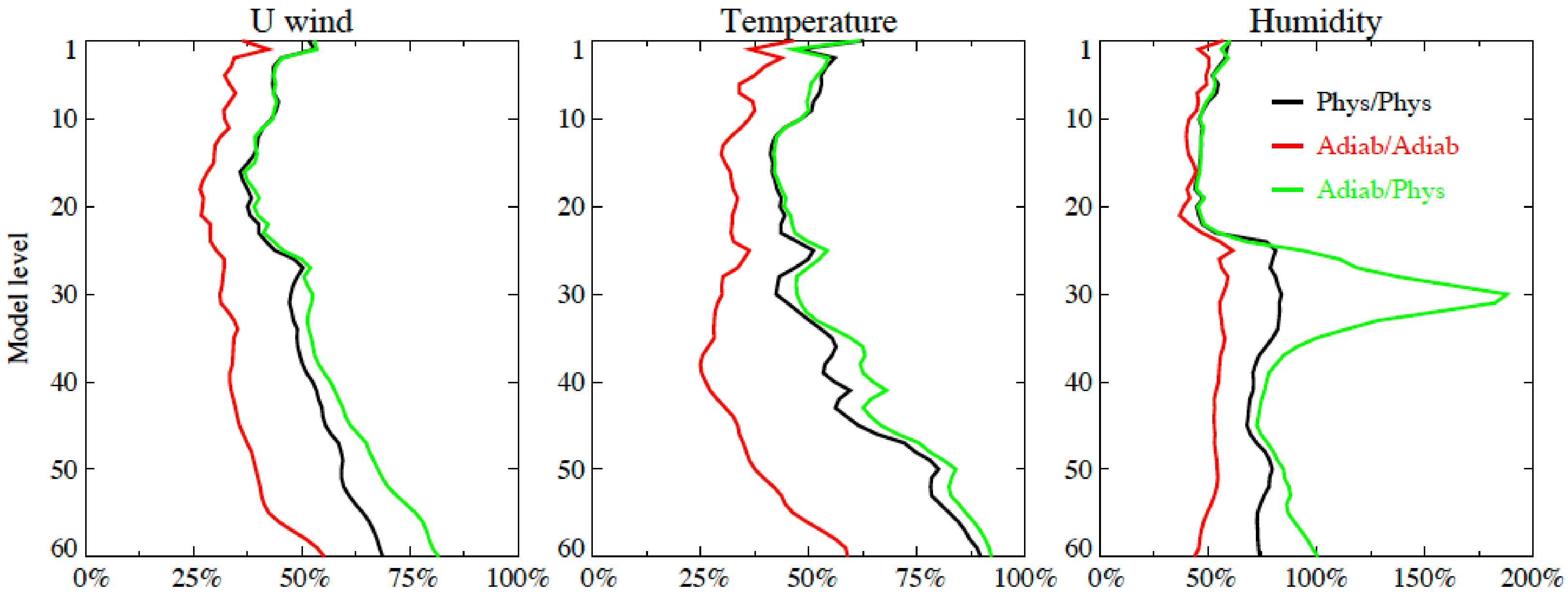


Relative to adiabatic TL run:

- 50-km resolution
- 20 runs
- after 12h integr.

Impact of the linearized model used in incremental 4D-Var system

- **In incremental 4D-Var:**
  - outer loops run at higher resolution & using all non-linear (NL) physical parametrization
  - inner loops (several iterations) run at lower resolution & using simplified (linearized) parametrization
  - the model used (adiabatic vs physics) and change of the resolution impacting linear approximation



Relative error of T159 linear model (TL) with respect to T511 nonlinear (NL) model:

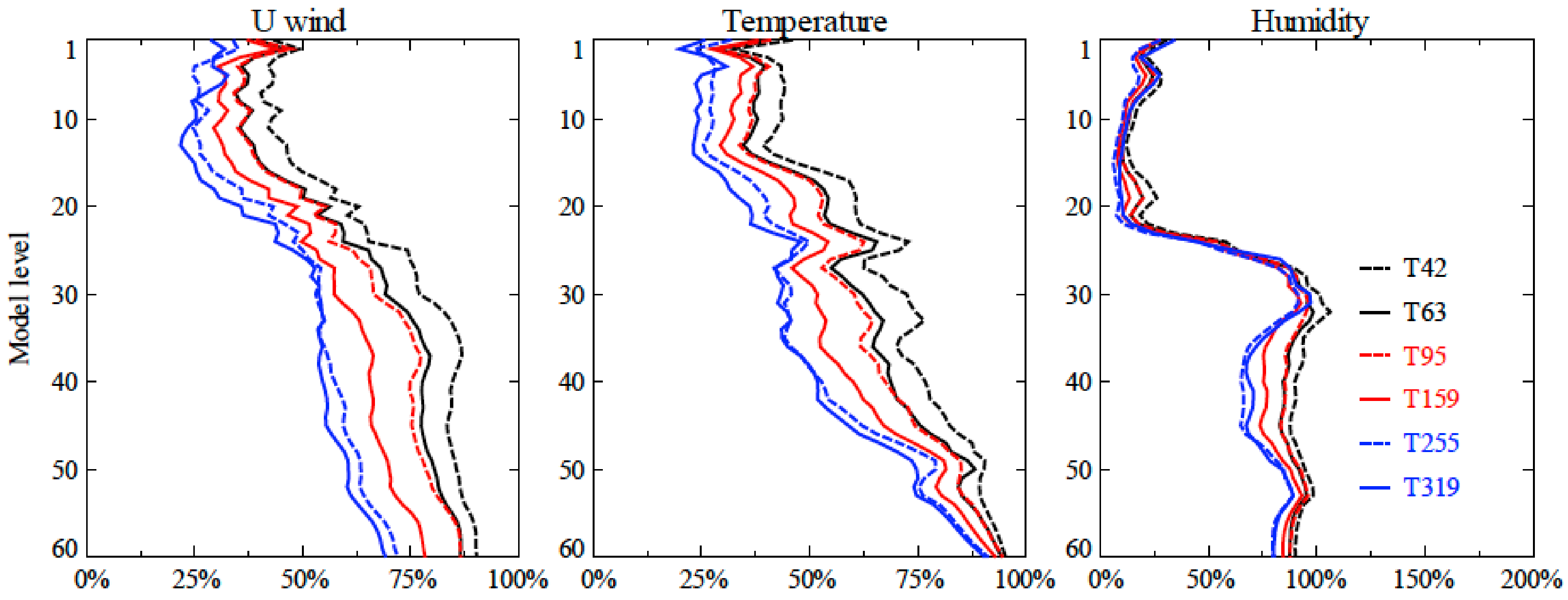
- Adiab NL / Adiab TL
- Phys_full NL / Adiab TL
- Phys_full NL / Phys_simp TL

- presence of the linearized physics in inner loop improves its accuracy
- running NL & TL models adiabatically gives the smallest TL approximation error, but leads to highly suboptimal analysis & forecast

T159 ~ 125 km
T511 ~ 40 km

Impact of inner loop resolution in incremental 4D-Var system

- **In incremental 4D-Var:**
  - inner loops run at lower resolution than outer loops to reduce computational cost



Relative error of the TL model run with the simplified physics using various resolutions with respect to the full T511 nonlinear (NL) model after 12-hour integration:

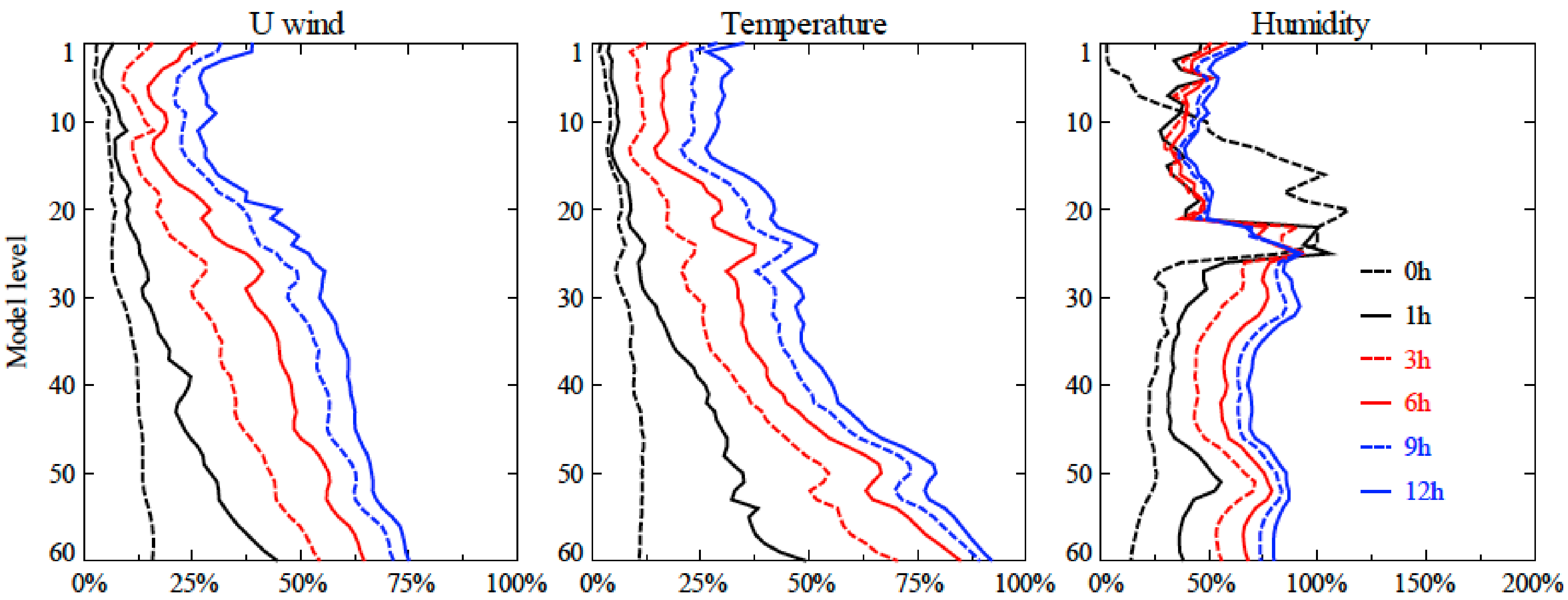
T42	~ 490 km
T63	~ 320 km
T95	~ 210 km
T159	~ 125 km
T255	~ 80 km
T319	~ 65 km

Decreasing the gap between outer and inner loop resolutions decreases relative error

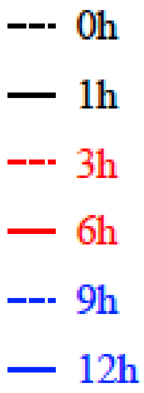
⟶ improves TL/AD approximation in 4D-Var

Radnóti et al. 2005

Impact of integration length in incremental 4D-Var system



Relative error of the TL model run with the simplified physics at the resolution T159 with respect to the full T511 model using all NL physical parametrizations for various integration lengths:



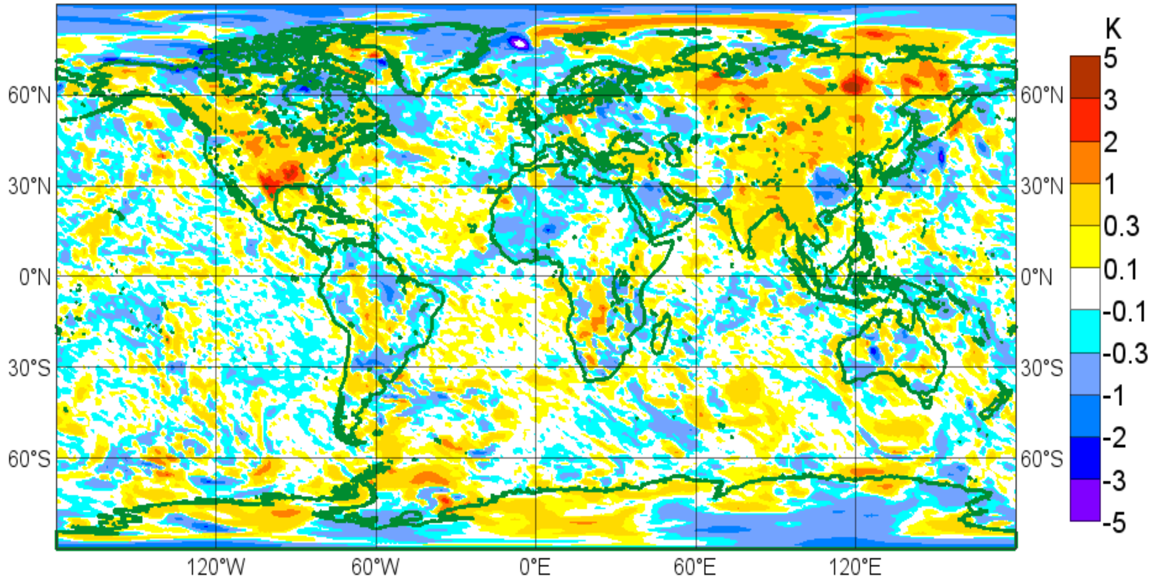
Relative error increases with integration time

⟶ the linearity assumptions becomes less valid

Radnóti et al. 2005

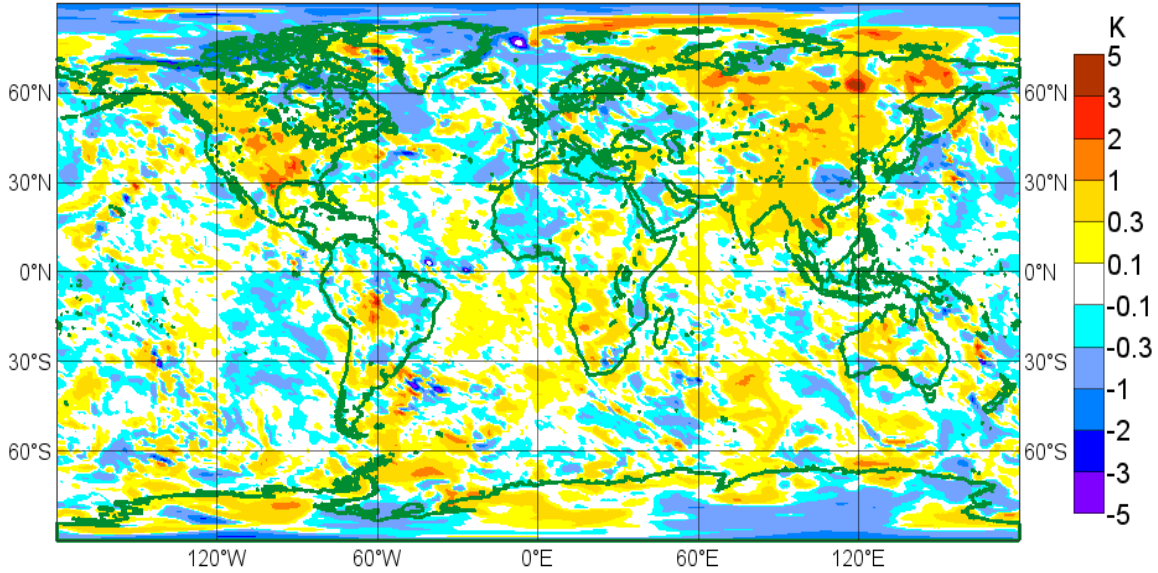
TL approximation at high resolution: ~ 18 km

$M(x+\delta x)-M(x)$



TCo639
~ 18 km

M’δx



Temperature at level 125 (~950 hPa) on 20140105 at 12Z.

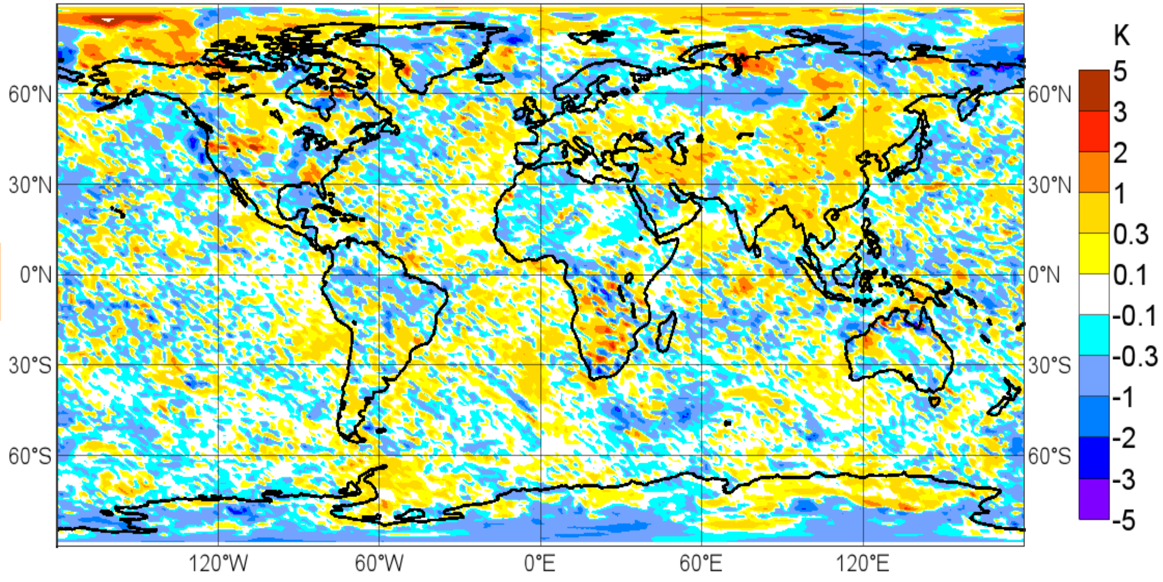
Comparison of NL difference $M(x+\delta x)-M(x)$ with perturbation evolved using the TL model M’δx after 12h of integration.

Thanks to stabilization of both the dynamics and the physics in the TL model, resolutions as fine as 18 km might be considered in 4D-Var minimizations, provided some (minor) sources of noise can be eliminated.

TL approximation at high resolution: ~ 9 km

Temperature at level 125 (~950 hPa) on 20140105 at 12Z.

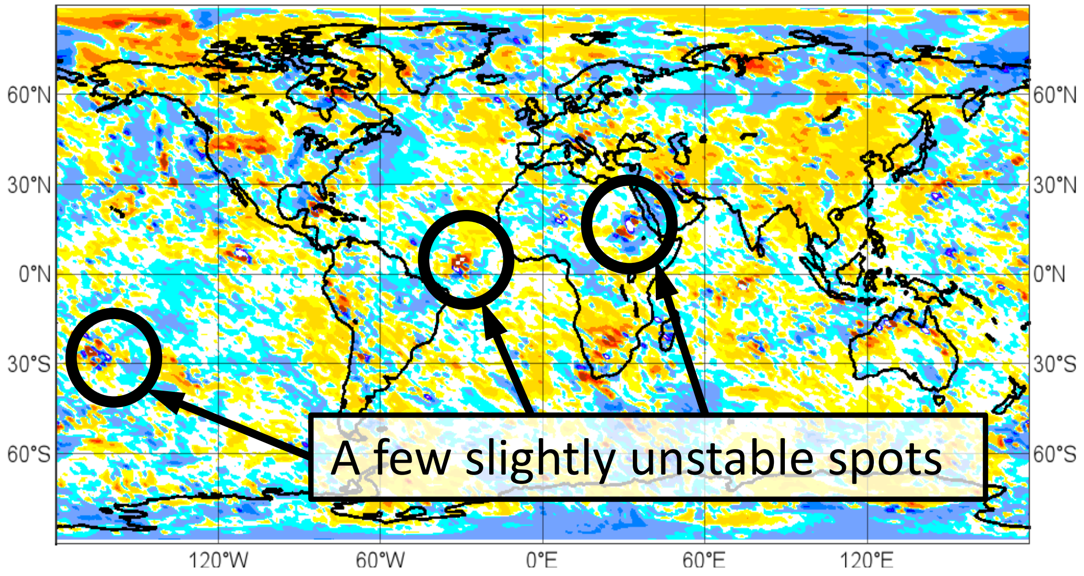
$M(x+\delta x) - M(x)$



Comparison of NL difference $M(x+\delta x) - M(x)$ with perturbation evolved using the TL model $M'\delta x$ after <u>12h of integration</u>.

TCo1279
~ 9 km

$M'\delta x$



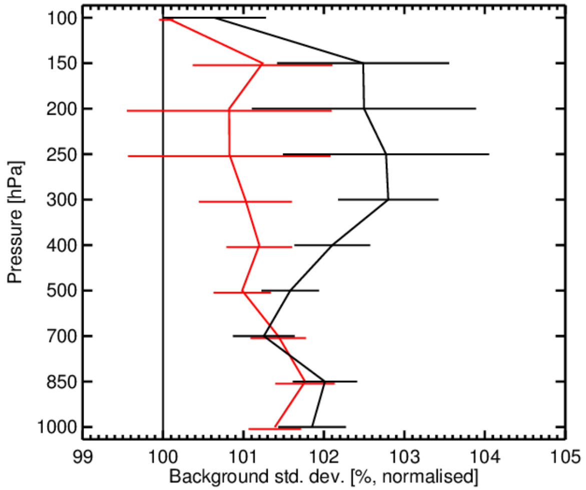
First time our TL model tested at such high resolution and the results surprisingly encouraging.

Benefits of using linearized physics in 4D-var

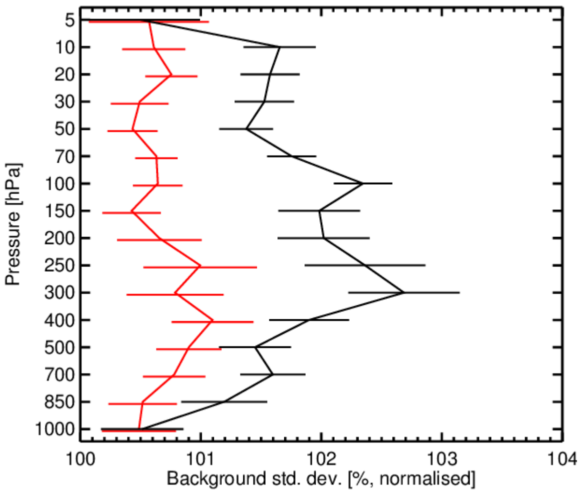
- inclusion of physical processes in the evolution of the model state during 4D-Var minimization
- inclusion in TL/AD versions of observation operators
- assimilation of observations directly linked to the physical processes (e.g. rain, clouds)

Improved fit to assimilated observations in analysis when using ECMWF linearized physics

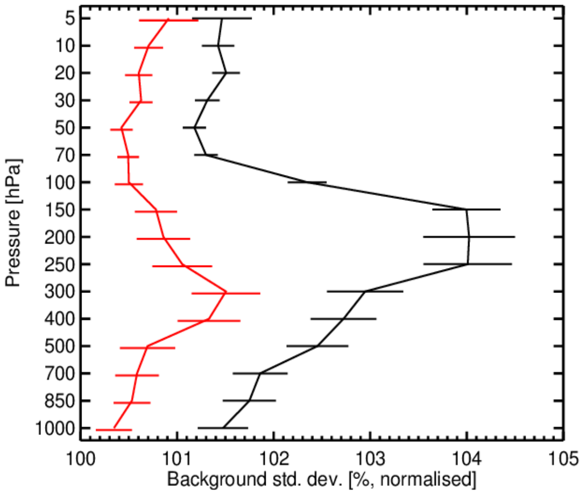
TEMP – q profiles



TEMP – T profiles



Conventional U/V wind profiles



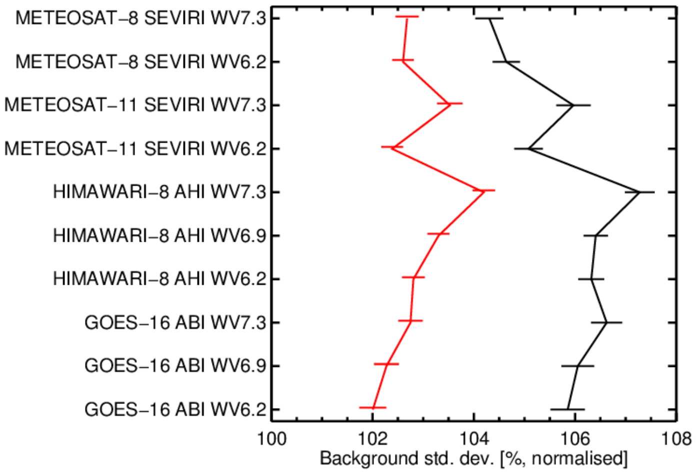
adiab
nophysobs
100% = reference

adiab – physics not included in the linearized model

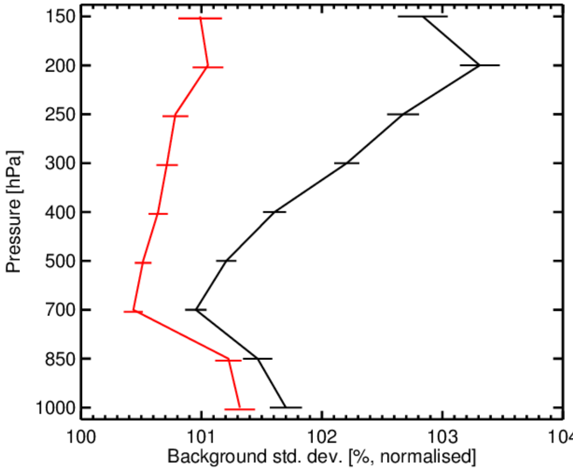
nophysobs – obs related to physical processes not assimilated

Run at:
TCo639 ~ 18 km, L137

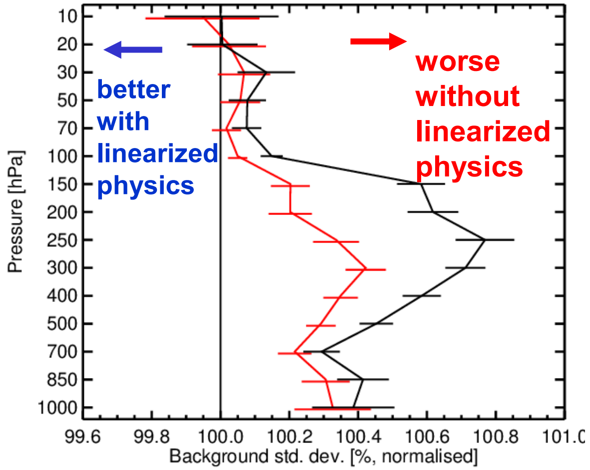
GOES



SATOB – U/V wind



AEOLUS – Mie cloudy, Rayleigh clear



Using linearized physics in data assimilation provides better model fit to observations

3-month period: July – September 2021

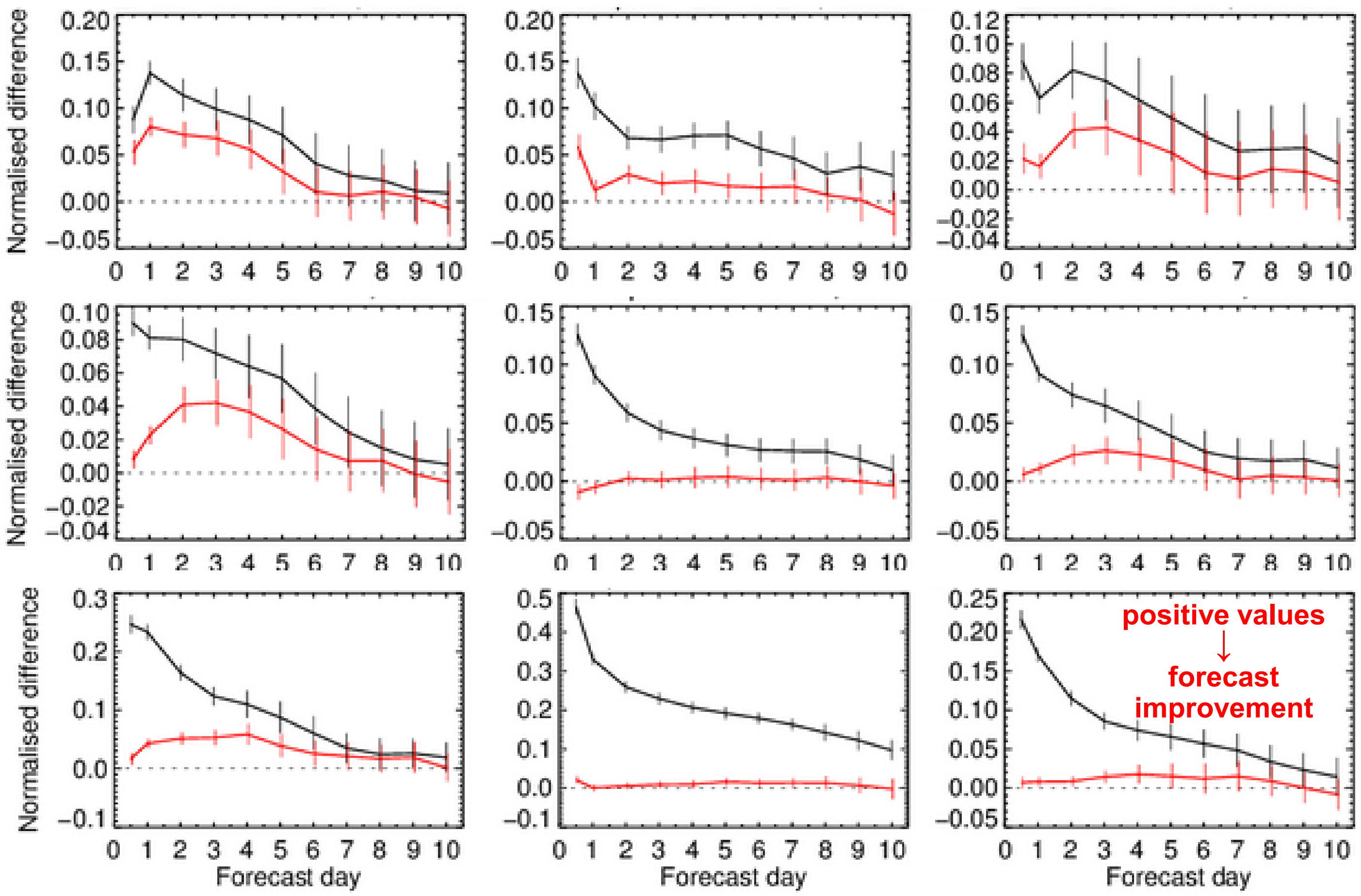
Relative improvement of forecast scores from ECMWF linearized physics

Southern Hemisphere | Tropics | Northern Hemisphere

Geopotential 500 hPa

Vector Wind 850 hPa

Temperature 1000 hPa



adiab – reference
nophysobs – reference

Forecast scores against <u>own analysis</u>:

- RMS errors normalised by the control
- 3-months period: July – Sept. 2021
- bars indicate significance at 95% confidence level

adiab – physics not included in the linearized model
nophysobs – obs related to physical processes not assimilated

Run at:

TCo639 ~ 18 km, L137

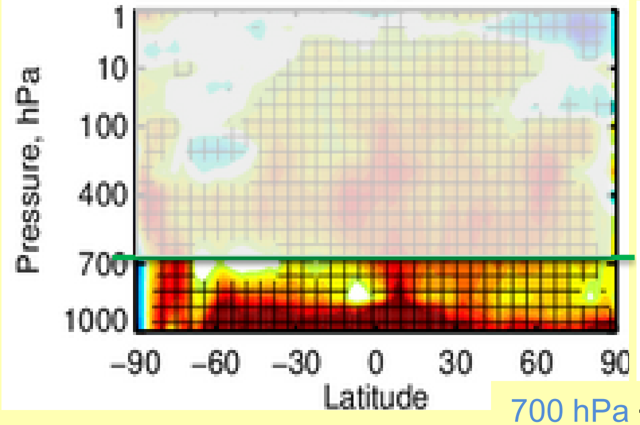
Including linearized physics in data assimilation improves forecast scores.

Relative improvement of forecast scores from ECMWF linearized physics – zonal means

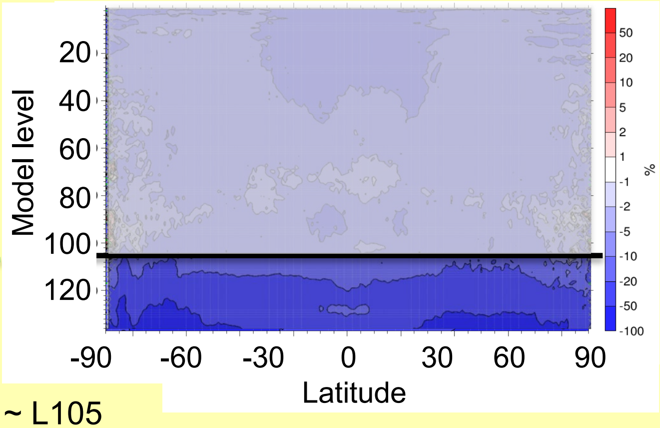
Temperature

Maximum forecast improvement where TL error is the most reduced by linearized physics

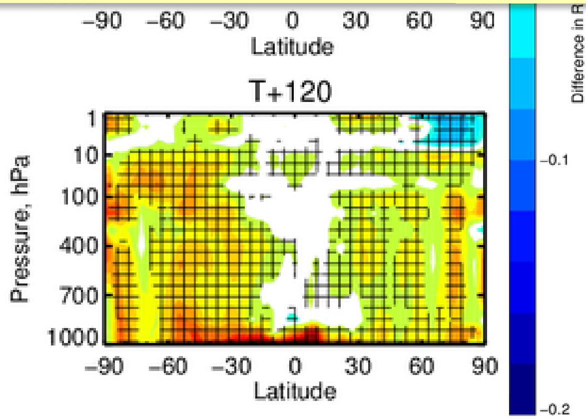
Forecast score



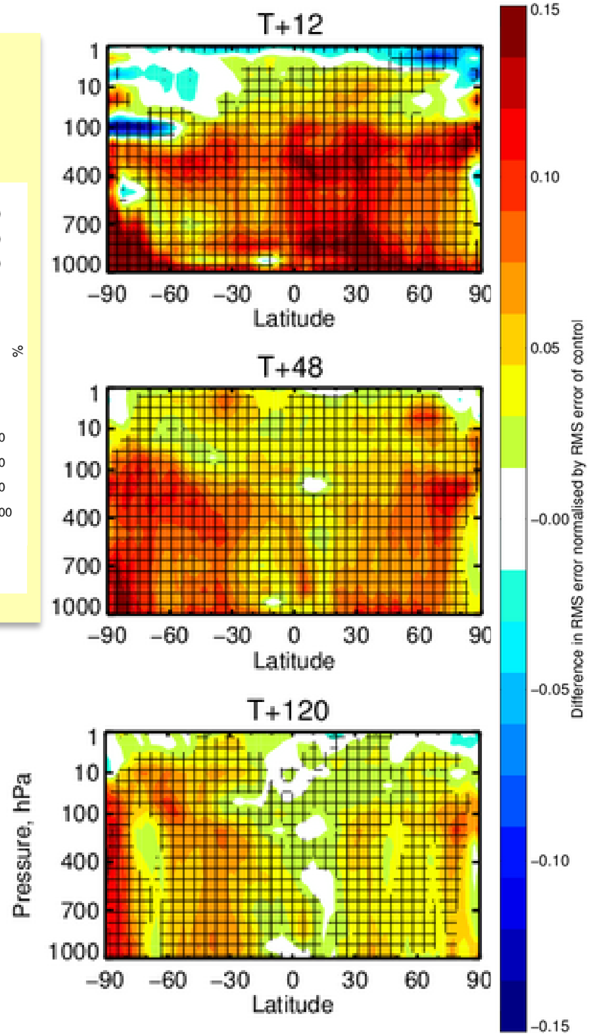
TL approximation



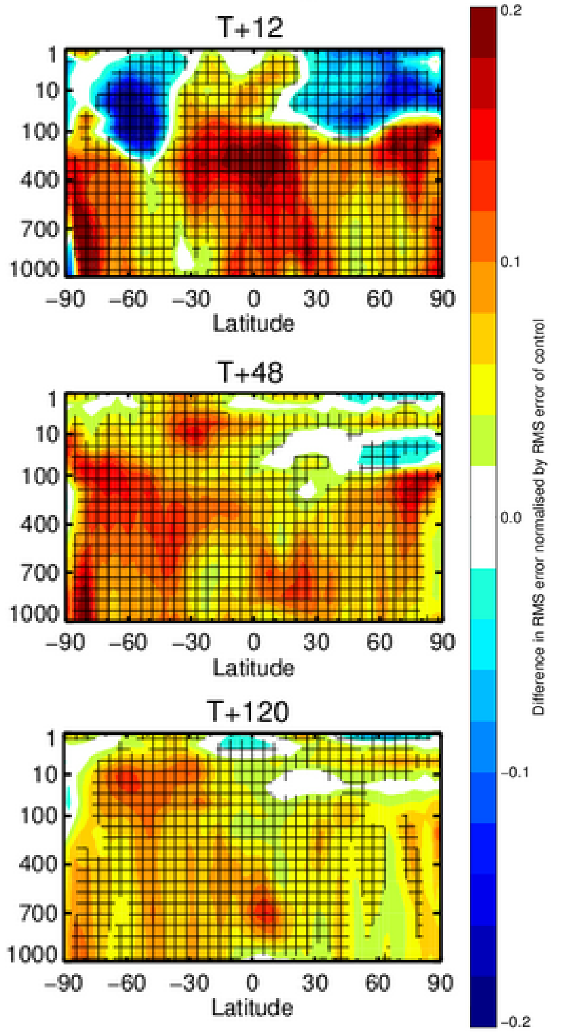
700 hPa ~ L105



Vector Wind



Geopotential



Forecast scores:

Positive values (red)

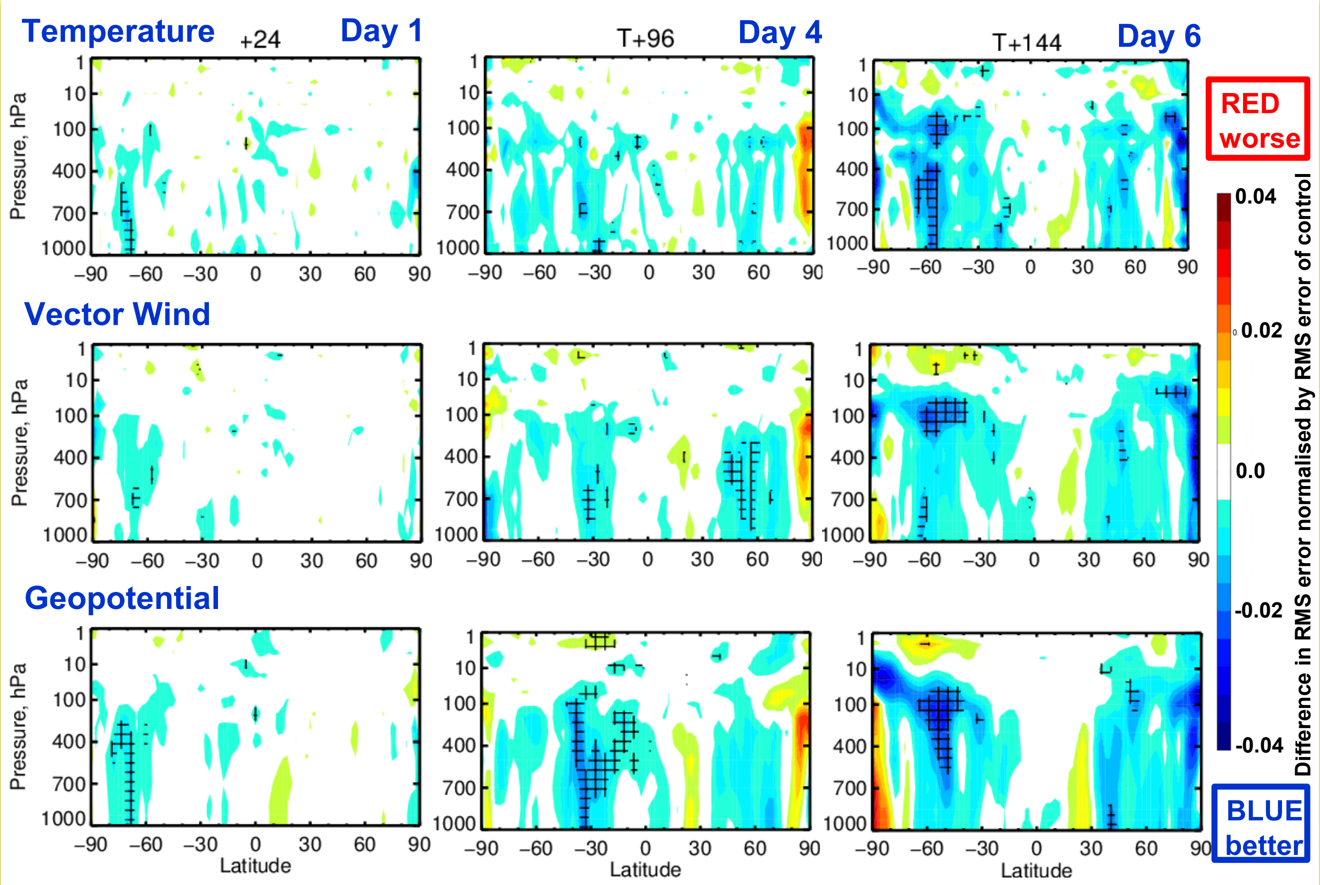
↓

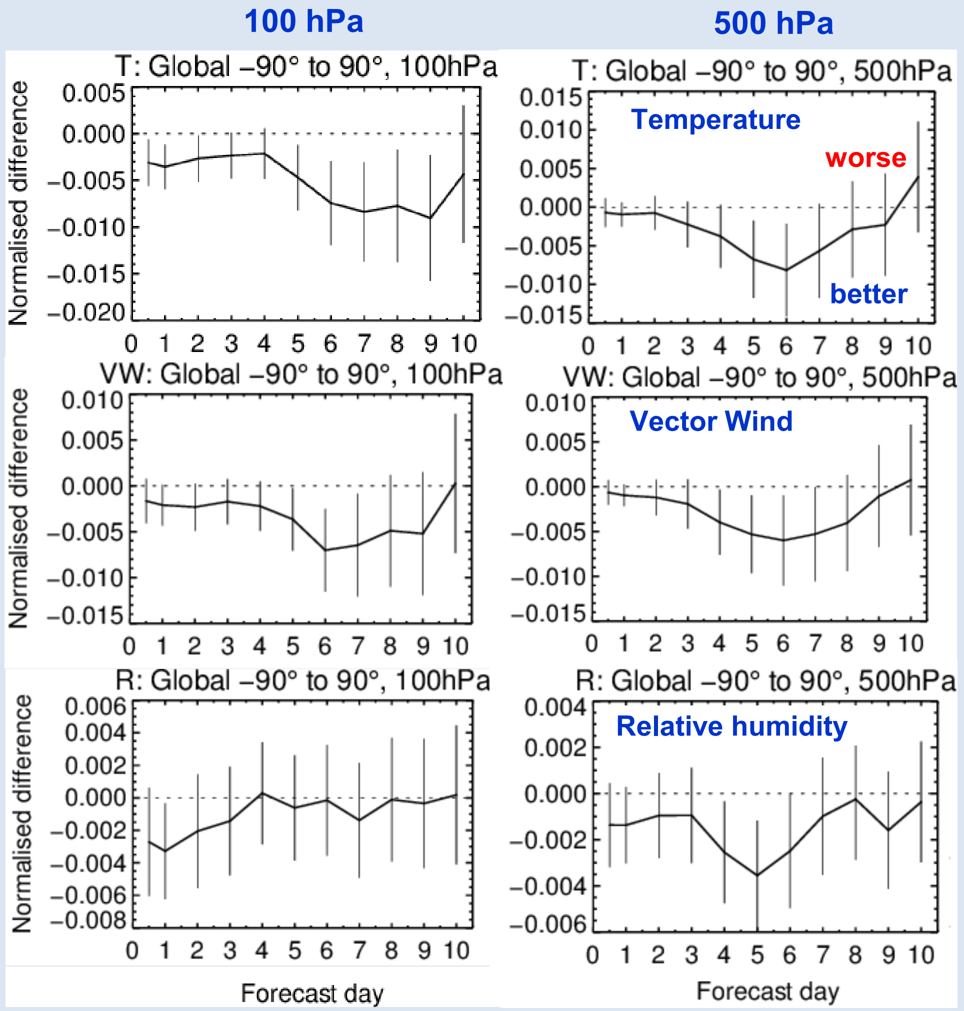
Improvement when using linearized physics

Difference in RMS error normalized by RMS error of control: July - September 2021

Expected improvements from assimilating new types of observations related to physics

Assimilation of CloudSat cloud radar reflectivity at 94 GHz and CALIPSO lidar backscatter at 532 nm as preparation for EarthCARE observations





Forecast error reduction grows with forecast lead time

significant 0.5 - 1% improvements in global upper tropospheric temperature & winds at day 4-7!

11-month combined period:
1 August 2007 – 31 August 2008
Run at: TCo639 ~ 18 km, L137

ECMWF

Fielding & Janisková 2020, 2022

Operational constraints for using TL/AD models

Imply:

- **permanent testing of the validity of TL approximation and necessary revisions:**
  - when the NL physics or dynamics changes significantly
  - higher horizontal and vertical resolutions, longer time-integrations
  - to ensure good match with reference non-linear forecast model
- **ensure robust stability of the linearized model:**
  - regularizations / simplifications to eliminate any source of instability
  - non-noisy behaviour in all situations and for different model resolutions
- **code optimizations to reduce computational cost**
- **finding best compromise between complexity, linearity and cost**

Summary

- **Nowadays, linearized physics is used in data assimilation, singular vector computation** (ensemble forecast) **and sensitivity studies.**

- **Positive impact from including physical parametrization schemes into the linearized model has been demonstrated.**

- **Physical parametrizations have become important components in current variational data assimilation systems:**
  - better representation of the evolution of the atmospheric state during the minimization of the cost function (via the adjoint model integration)
  - enabling to assimilate observations related to physical processes (rain, clouds, ...)
  - positive impact on analysis and subsequent forecast

- **Optimizing the performance of linearized physics in 4D-Var requires finding the best compromise between:**

  linearity, computational cost $\Leftrightarrow$ realism (accuracy of representing physical processes)