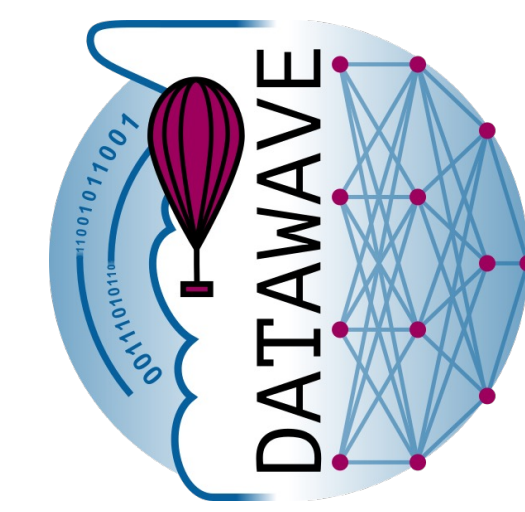


A 1-D QBO model testbed for data-driven gravity wave parameterization: Generalization and calibration

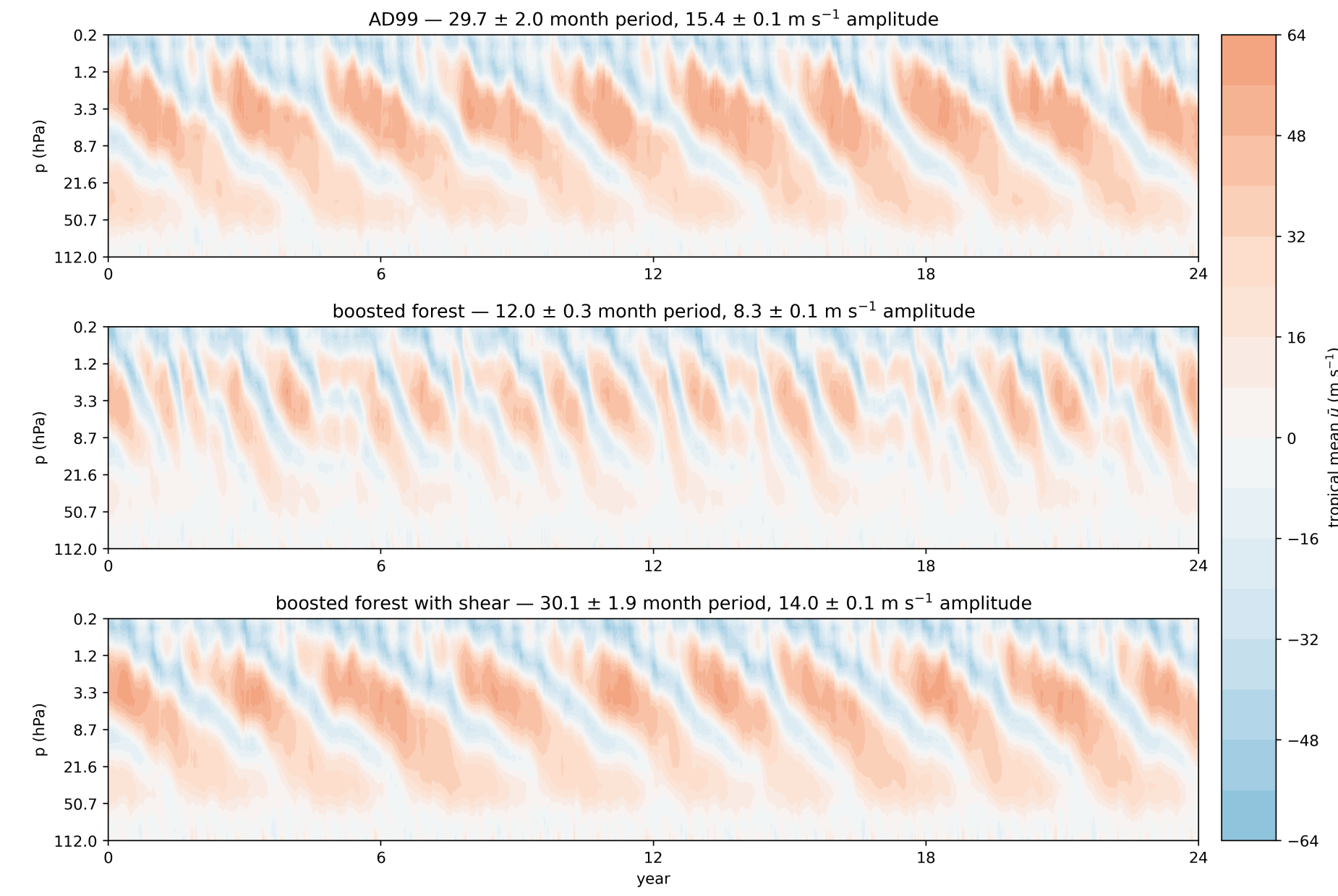


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(1) Motivation

- A key metric of gravity wave (GW) parameterization tuning is the fidelity of the simulated Quasi-Biennial Oscillation (QBO).
- Simulated QBOs in an intermediate complexity atmospheric model (MiMA), forced with emulators of physics-based GW parameterization (AD99¹), are highly variable.



- Sensitivity analysis of the QBO response to external forces (e.g., CO₂) and GW parameters is computationally taxing.
- We explore the generalization and calibration of data-driven GW parameterization in a 1D QBO model testbed.

(2) Model and stochastic wave forcing

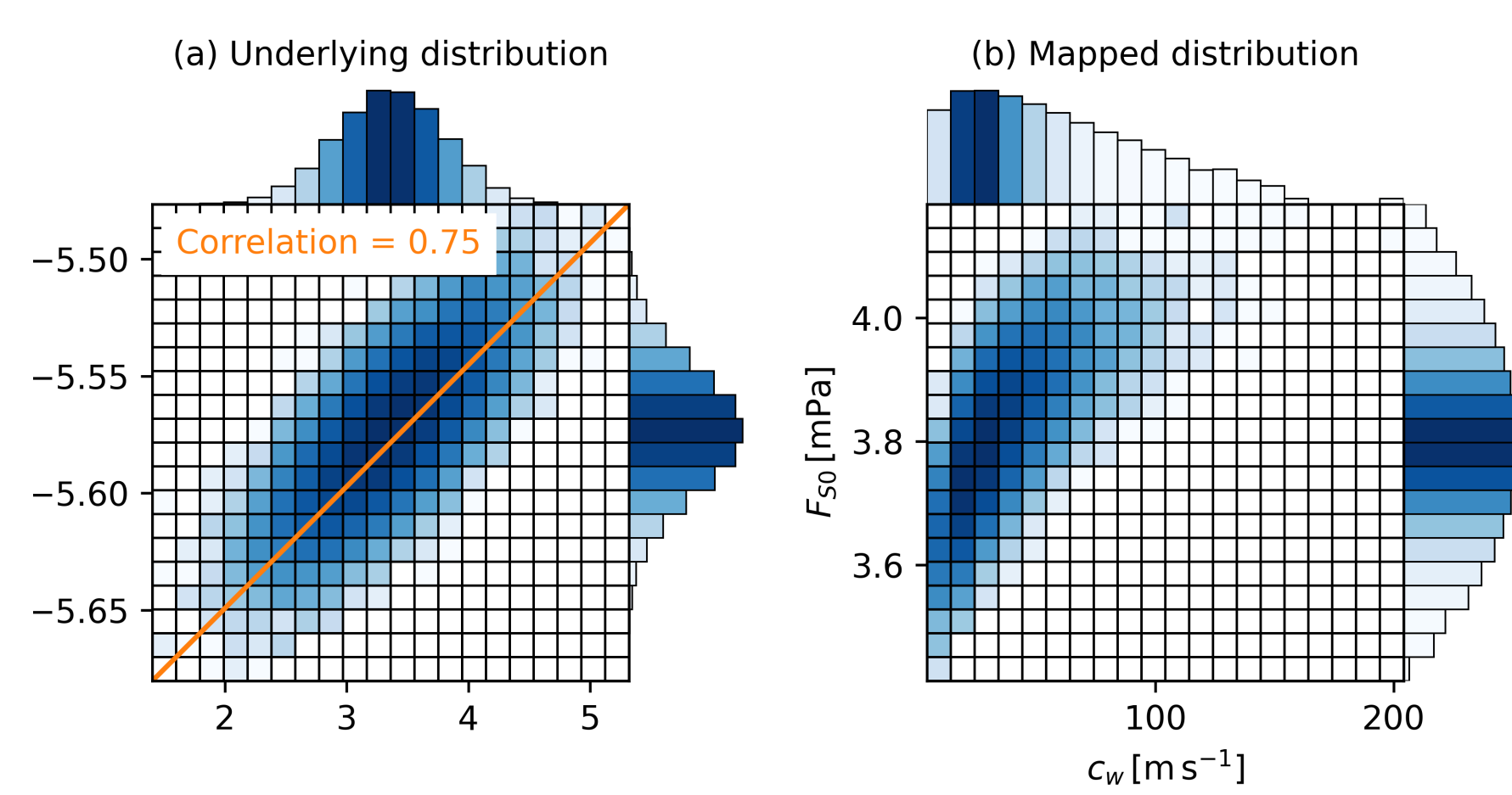
- A hybrid of the 1D QBO models studied in HL72² and P77³, forced by a collection of monochromatic waves packets:

$$\frac{\partial u}{\partial t} + w \frac{\partial u}{\partial z} - \kappa \frac{\partial^2 u}{\partial z^2} = -\frac{1}{\rho} \frac{\partial}{\partial z} \sum_i A_i \exp \left\{ -\int_{z=z_1}^z \frac{\alpha(z') N}{k_i (u - c_i)^2} dz' \right\}$$

- The wave spectrum follows AD99¹:

$$A(c) \propto \text{sgn}(c) \exp \left[-\ln 2 \left(\frac{c}{c_w} \right)^2 \right]$$

- We add stochasticity to the wave forcing: at each time step the total source flux $F_{S0} = \sum_i |A_i|$ and spectral width c_w are drawn from a bi-variate log-normal distribution.

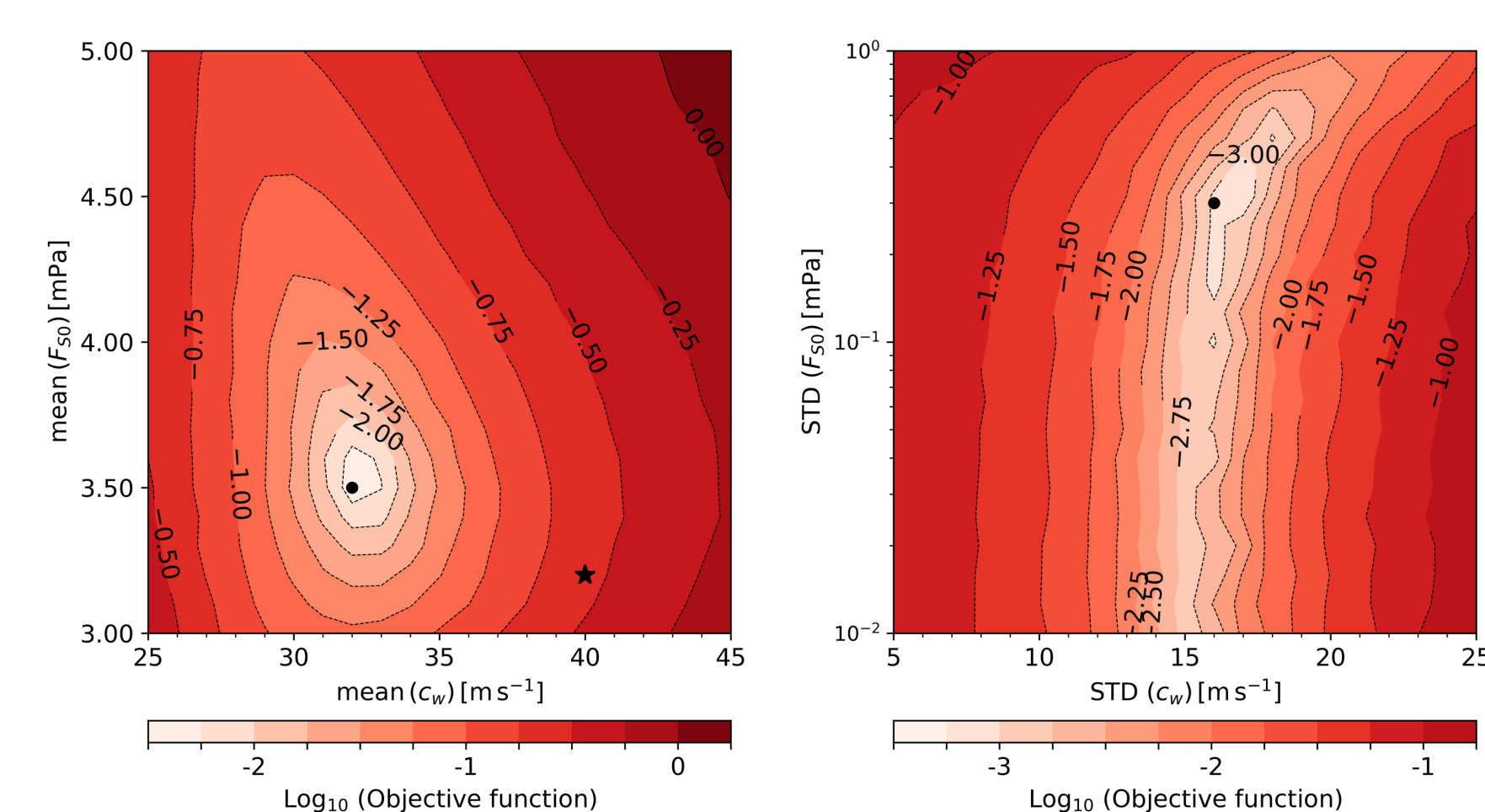


- Physically, F_{S0} (related to precipitation²) and c_w (related to convection depth) are positively correlated.

(3) "Optimal" / "observed" wave forcing

- The control GW spectrum corresponds to the unique combination of wave flux and spectral width that yields the "observed" QBO amplitude (σ) and period (τ) according to (*).

$$\frac{[\sigma(25 \text{ km}) - 33 \text{ m/s}]^2}{[33 \text{ m/s}]^2} + \frac{[\sigma(20 \text{ km}) - 19 \text{ m/s}]^2}{[19 \text{ m/s}]^2} + \frac{[\tau(25 \text{ km}) - 28 \text{ months}]^2}{[28 \text{ months}]^2} \quad (*)$$

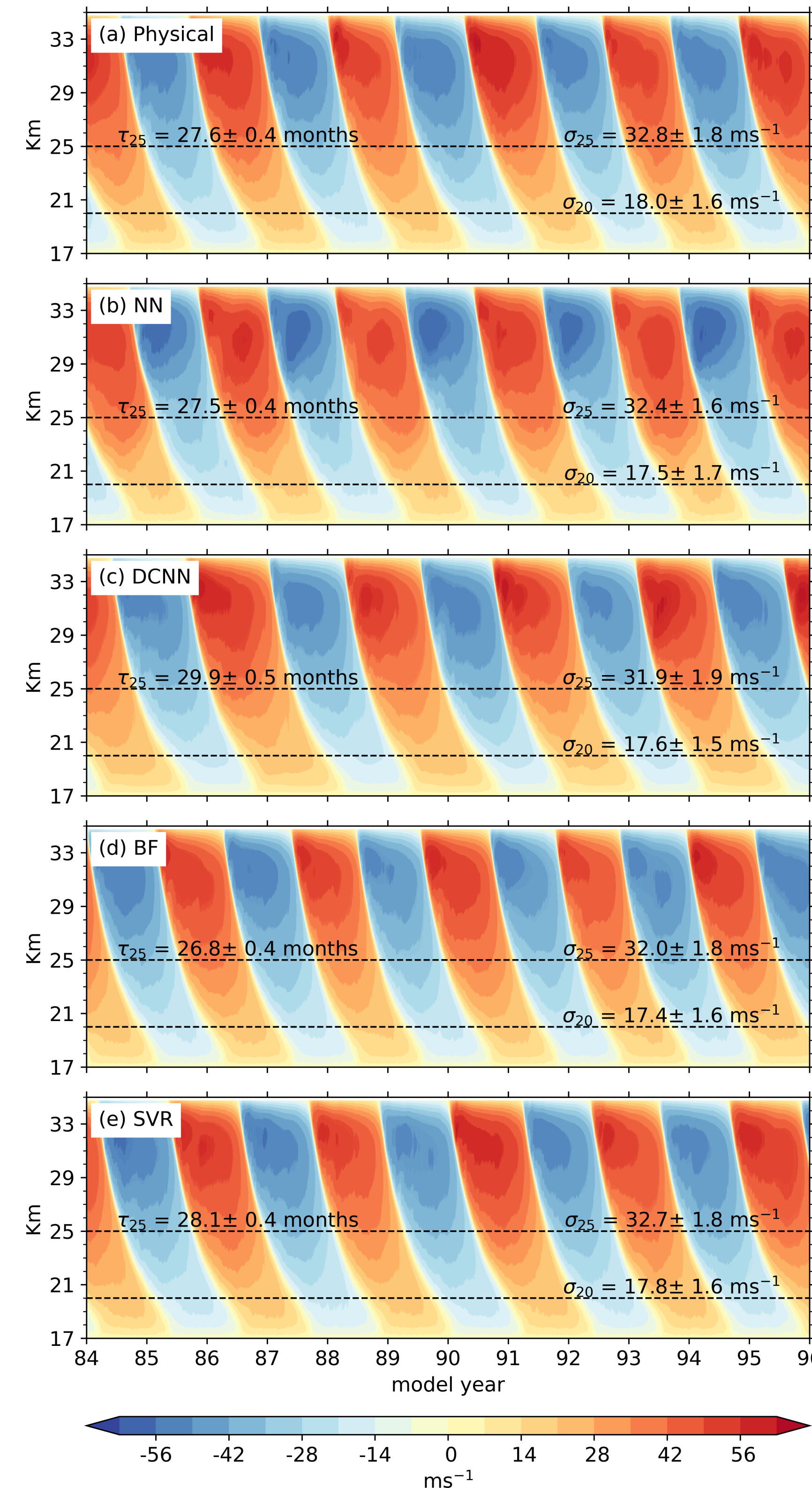


- The mean wave flux and spectral width required to capture the "observed" QBO in the 1D model are remarkably similar to those found in higher complexity models.

(4) Emulation

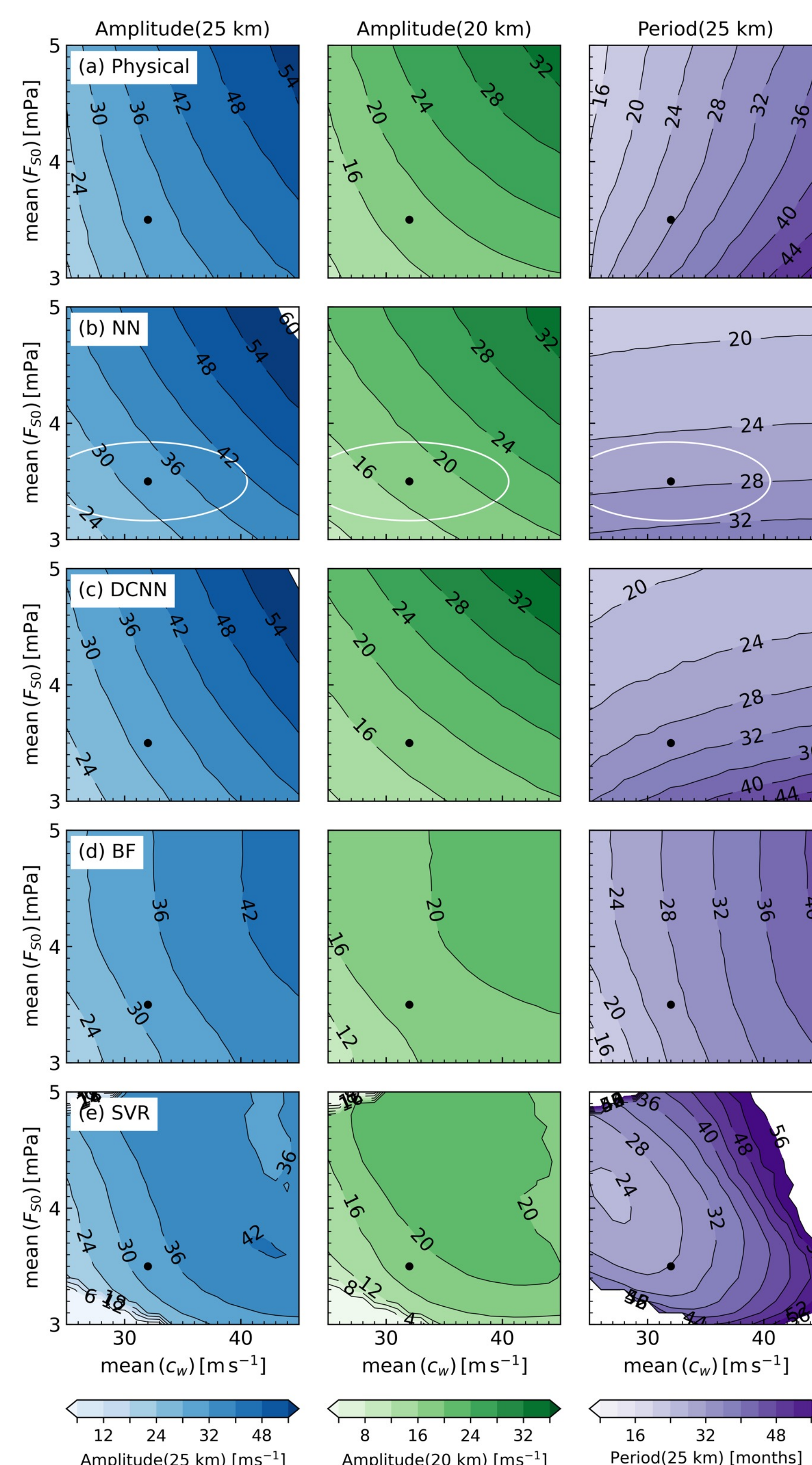
- We train different emulators using the "optimal" GW flux distribution for the training data.

$$-\frac{1}{\rho} \frac{\partial}{\partial z} \sum_i A_i \exp \left\{ -\int_{z=z_1}^z \frac{\alpha(z') N}{k_i (u - c_i)^2} dz' \right\} \rightarrow \text{Emulator}(u, F_{S0}, c_w)$$



(5) Generalization

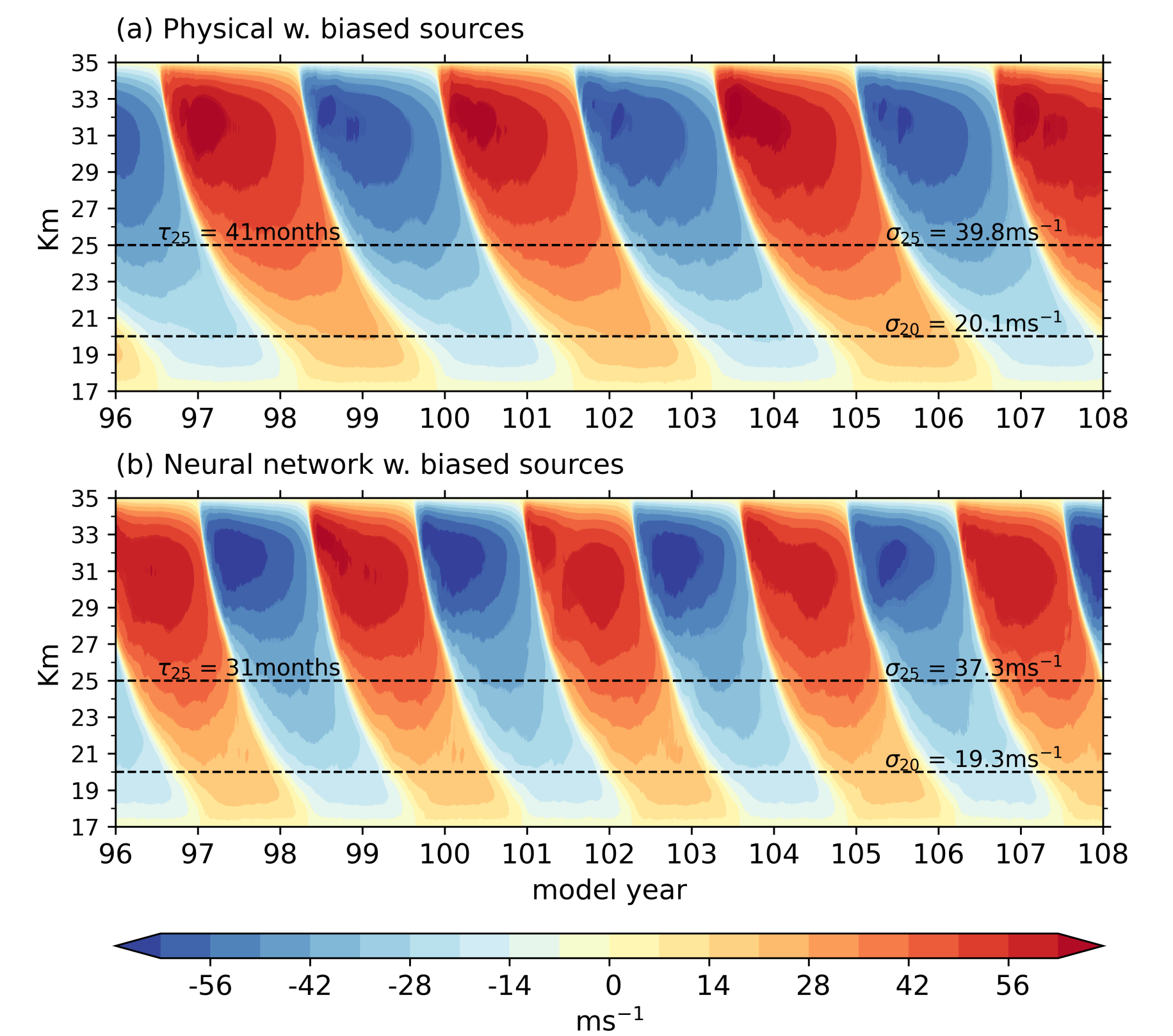
- How well do emulators trained on a single source distribution generalize to nearby source distributions?



- The emulators capture the qualitative sensitivity of the QBO's amplitude to changes in F_{S0} and c_w , but struggle to capture the qualitative sensitivity of the QBO's period.

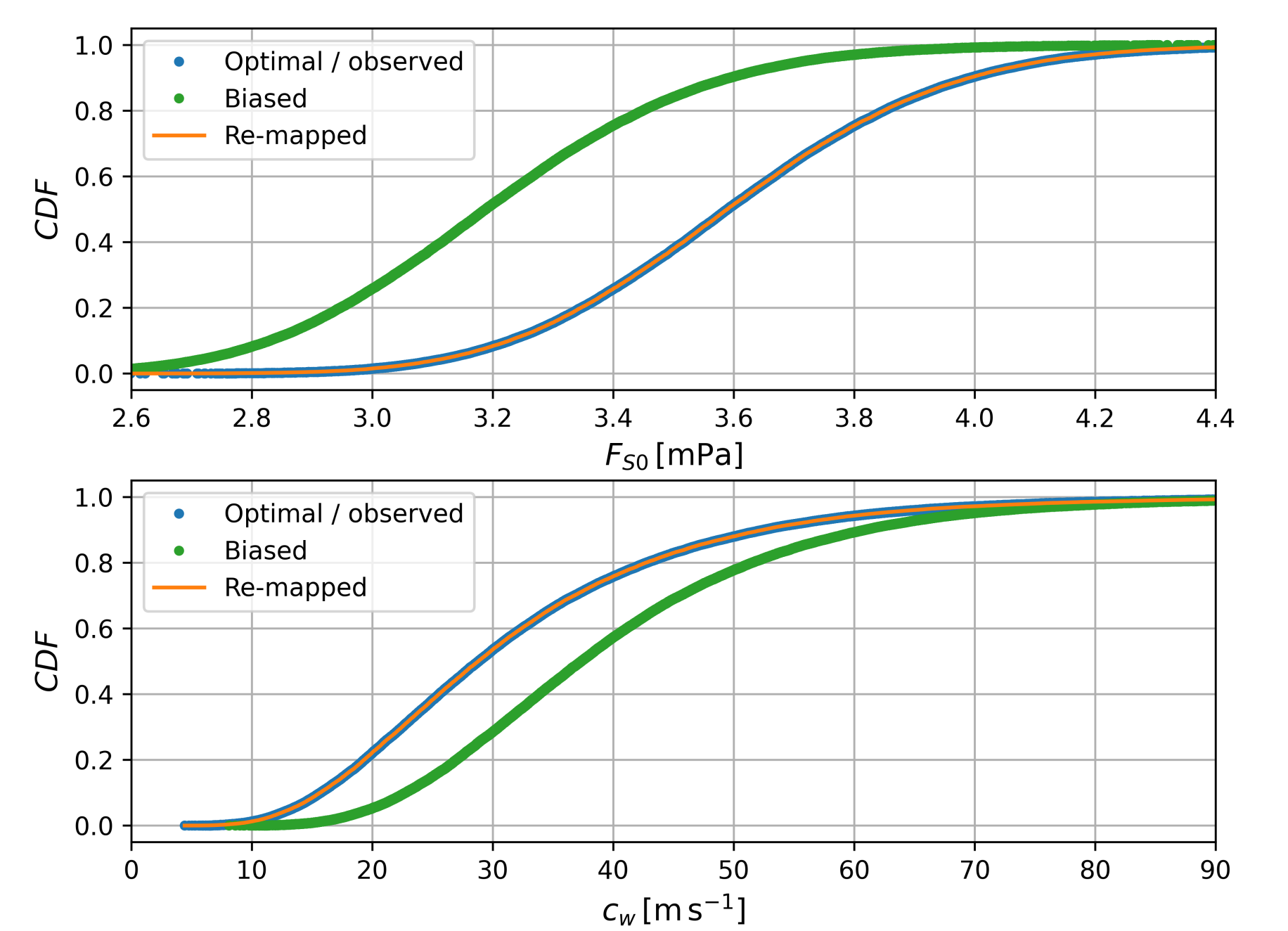
(6) Calibration

- Our emulators were trained on the optimal/observed source distribution, but a model's source distribution can be biased.
- Consider the emulated solution forced by the biased source distribution indicated by black * in box (3):

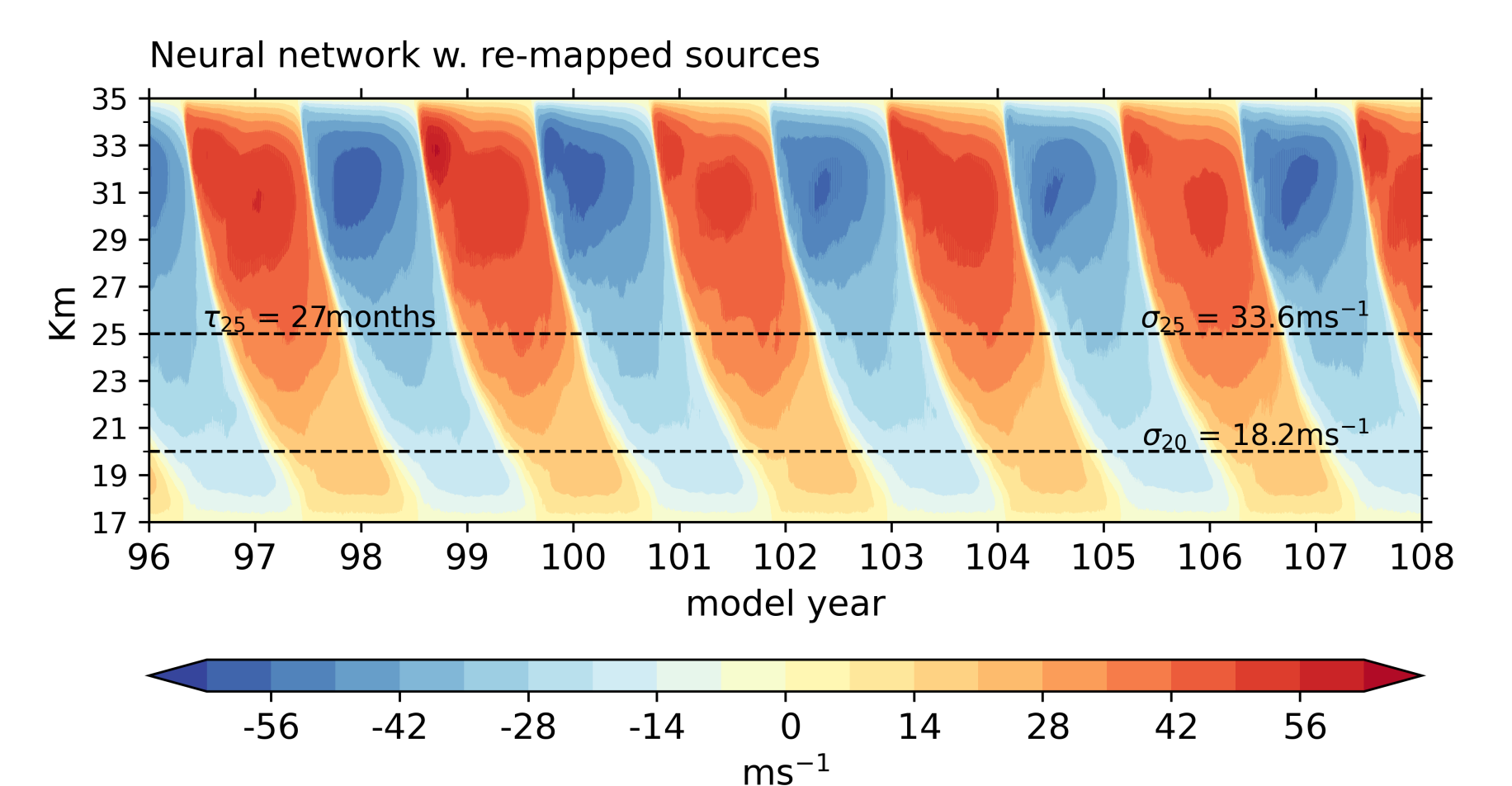


- How can we adjust the data-driven GW parameterization to yield the desired QBO statistics as in (*)?
- One approach is to re-map the biased source distribution to the optimal source distribution:

$$\{F_{S0}, c_w\} \rightarrow CDF_{Optimal}^{-1}(CDF_{Biased}(\{F_{S0}, c_w\}))$$



- Yielding the desired QBO period and amplitudes:



(7) Conclusions

- Machine learning methods show promise in replicating a physics based GWP, yielding stable, accurate simulations when coupled under the current climatological conditions.
- Our results demonstrate the challenge of generalizing to out of sample data, a major challenge of data-driven methods.
- Training on more complex data, having an annual cycle in w , does not improve our emulators' ability to generalize.
- If our emulators struggle to capture the sensitivity of the QBO's period to changes in the GW spectrum of a known physics-based parameterization, how well can we trust data-driven methods trained on observation to capture this sensitivity?

References

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⁴Garfinkel, C. I., Gerber, E. P., Shamir, O., Rao, J., Jucker, M., White, I., & Paldor, N. (2022). A QBO Cookbook: Sensitivity of the Quasi-Biennial Oscillation to Resolution, Resolved Waves, and Parameterized Gravity Waves. *Journal of Advances in Modeling Earth Systems*, 14(3), e2021MS002568.