Hybrid data assimilation for model error estimation and correction at ECMWF

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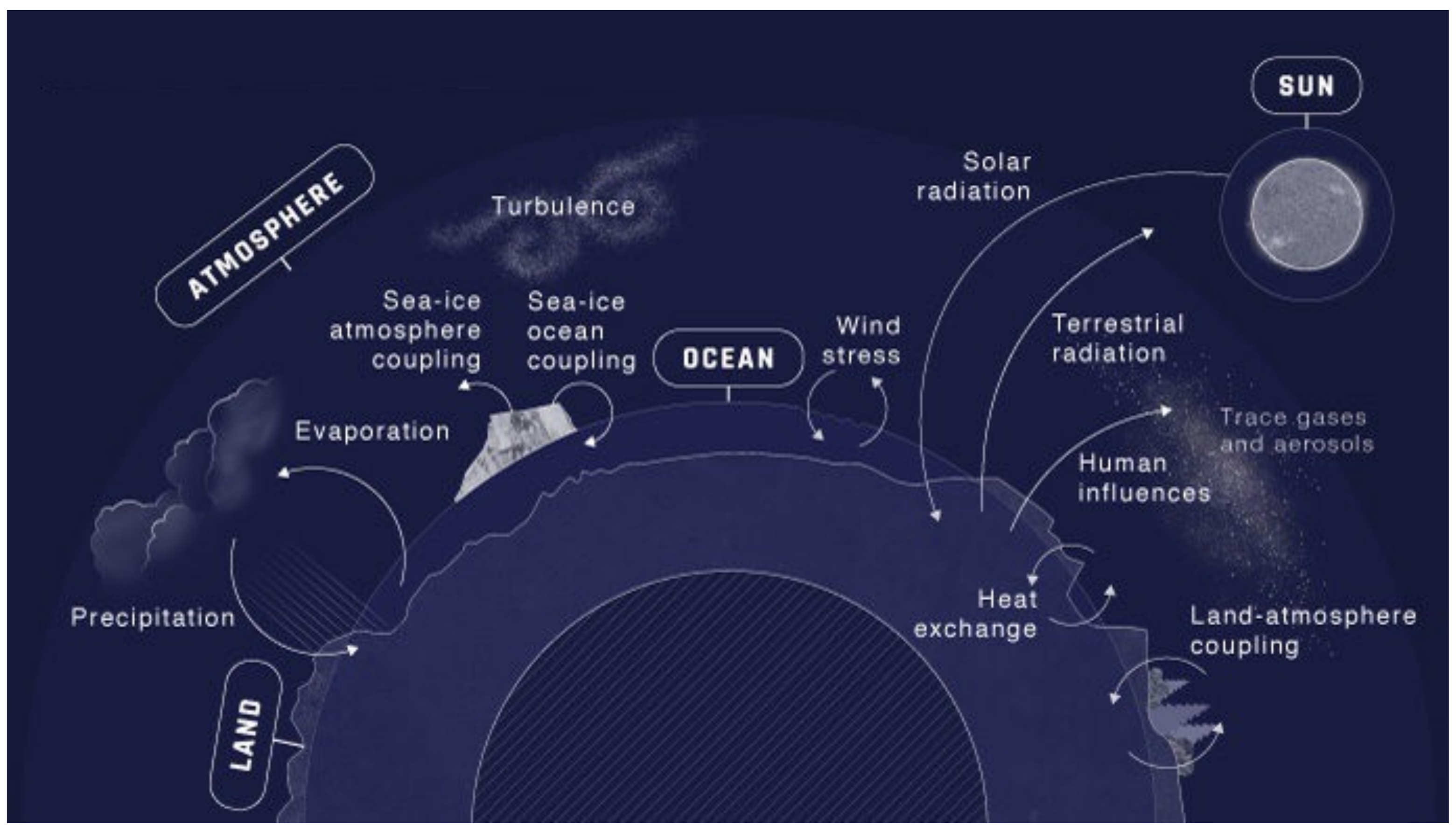
Model error

- Systematic errors are considered to be a limiting factor for the predictive skill of the ECMWF Earth System Model.
- These errors have many origins. For example, the use of imperfect parameterisations of complex and not fully resolved physical processes, as shown in the figure below.



Diagnosing Model Error

- Model error can be diagnosed using observations that are known to be unbiased, like radiosonde temperature or wind measurements, or of radio occultation from GPS satellites.

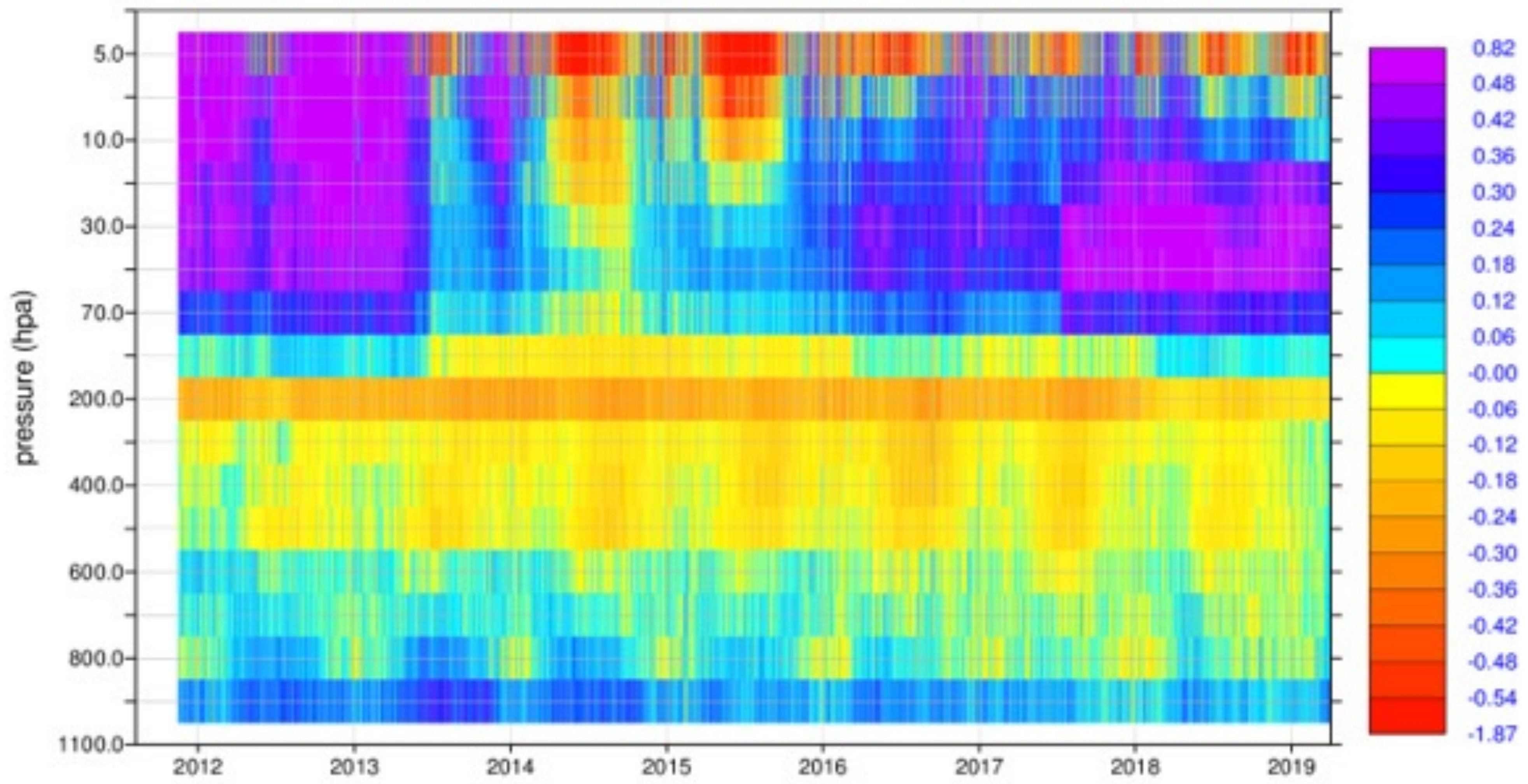


Fig. Time series of the IFS temperature model bias (courtesy of Patrick Laloyaux).

Hybrid Data Assimilation (DA)

- We combined a physical model with a neural network based model correction (see Farchi et al., 2022) with the aim of accounting for systematic errors in DA.
- Input to the neural network (see Bonavita&Laloyaux, 2020) consists of climatological parameters (time of day/year, coordinates) and vertical columns of prognostic variables (T, LNSP, U, V).

$$\mathbf{x}_{k+1} = \boldsymbol{\mathcal{M}}^{\mathsf{nn}}_{k+1:k}(\mathbf{p}, \mathbf{x}_k) = \boldsymbol{\mathcal{M}}_{k+1:k}(\mathbf{x}_k) + \boldsymbol{\mathcal{F}}(\mathbf{p}, \mathbf{x}_k)$$

where $\mathcal{F}$ is a neural network correction added to $\mathcal{M}_{k+1:k}$, the resolvent of the physics based model from time t_k to t_{k+1}, and p are the parameters of the neural network model.

Training the Neural Network in the incremental 4D-Var framework

- Parameters of the neural network are optimized in the incremental 4D-Var cost function along with the atmospheric state:

$$\begin{aligned}
\mathcal{J}^{\mathsf{nn}}(\mathbf{p}, \mathbf{x}_0) &= \mathcal{J}^{\mathsf{nn}}\left(\mathbf{p}^{\mathsf{i}} + \delta\mathbf{p}, \mathbf{x}_0^{\mathsf{i}} + \delta\mathbf{x}_0\right), \\
&= \frac{1}{2}\left\|\mathbf{x}_0^{\mathsf{i}} - \mathbf{x}_0^{\mathsf{b}} + \delta\mathbf{x}_0\right\|^2_{\mathbf{B}^{-1}} + \frac{1}{2}\left\|\mathbf{p}^{\mathsf{i}} - \mathbf{p}^{\mathsf{b}} + \delta\mathbf{p}\right\|^2_{\mathbf{P}^{-1}} \\
&\quad + \frac{1}{2}\sum_{k=0}^{L}\left\|\mathbf{y}_k - \boldsymbol{\mathcal{H}}_k \circ \boldsymbol{\mathcal{M}}^{\mathsf{nn}}_{k:0}\left(\mathbf{p}^{\mathsf{i}} + \delta\mathbf{p}, \mathbf{x}_0^{\mathsf{i}} + \delta\mathbf{x}_0\right)\right\|^2_{\mathbf{R}_k^{-1}}, \\
&\approx \frac{1}{2}\left\|\mathbf{x}_0^{\mathsf{i}} - \mathbf{x}_0^{\mathsf{b}} + \delta\mathbf{x}_0\right\|^2_{\mathbf{B}^{-1}} + \frac{1}{2}\left\|\mathbf{p}^{\mathsf{i}} - \mathbf{p}^{\mathsf{b}} + \delta\mathbf{p}\right\|^2_{\mathbf{P}^{-1}} \\
&\quad + \frac{1}{2}\sum_{k=0}^{L}\left\|\mathbf{d}_k - \mathbf{H}_k\mathbf{M}^{\mathsf{nn}}_{k:0}\left(\delta\mathbf{p}, \delta\mathbf{x}_0\right)^{\top}\right\|^2_{\mathbf{R}_k^{-1}}, \\
&\triangleq \widehat{\mathcal{J}}^{\mathsf{nn}}(\delta\mathbf{p}, \delta\mathbf{x}_0).
\end{aligned}$$

Algorithm 1 Gradient of the incremental cost function $\widehat{\mathcal{J}}^{\mathsf{nn}}$ eq. (16d).

Input: $\delta\mathbf{p}$ and $\delta\mathbf{x}_0$

$\delta\mathbf{w} \leftarrow \mathbf{F}^{\mathsf{p}}\delta\mathbf{p} + \mathbf{F}^{\mathsf{x}}\delta\mathbf{x}_0$ ▷ TL of the NN $\boldsymbol{\mathcal{F}}$
$\mathbf{z}_0 \leftarrow \mathbf{R}_0^{-1}(\mathbf{H}_0\delta\mathbf{x}_0 - \mathbf{d}_0)$
for $k = 1$ **to** $L-1$ **do**
  $\delta\mathbf{x}_k \leftarrow \mathbf{M}_{k:k-1}\delta\mathbf{x}_{k-1} + \delta\mathbf{w}$ ▷ TL of the dynamical model $\boldsymbol{\mathcal{M}}_{k:k-1}$
  $\mathbf{z}_k \leftarrow \mathbf{R}_k^{-1}(\mathbf{H}_k\delta\mathbf{x}_k - \mathbf{d}_k)$
end for
$\delta\tilde{\mathbf{x}}_{L-1} \leftarrow \mathbf{0}$ ▷ AD variable for system state
$\delta\tilde{\mathbf{w}}_{L-1} \leftarrow \mathbf{0}$ ▷ AD variable for model error
for $k = L-1$ **to** 1 **do**
  $\delta\tilde{\mathbf{x}}_k \leftarrow \mathbf{H}_k^{\top}\mathbf{z}_k + \delta\tilde{\mathbf{x}}_k$
  $\delta\tilde{\mathbf{w}}_{k-1} \leftarrow \delta\tilde{\mathbf{x}}_k + \delta\tilde{\mathbf{w}}_k$
  $\delta\tilde{\mathbf{x}}_{k-1} \leftarrow \mathbf{M}_{k:k-1}^{\top}\delta\tilde{\mathbf{x}}_k$ ▷ AD of the dynamical model $\boldsymbol{\mathcal{M}}_{k:k-1}$
end for
$\delta\tilde{\mathbf{x}}_0 \leftarrow \mathbf{H}_0^{\top}\mathbf{z}_0 + \delta\tilde{\mathbf{x}}_0$
$\delta\tilde{\mathbf{x}}_0 \leftarrow \left[\mathbf{F}^{\mathsf{x}}\right]^{\top}\delta\tilde{\mathbf{x}}_0$ ▷ AD of the NN $\boldsymbol{\mathcal{F}}$
$\delta\tilde{\mathbf{p}} \leftarrow \left[\mathbf{F}^{\mathsf{p}}\right]^{\top}\delta\tilde{\mathbf{w}}_0$ ▷ AD of the NN $\boldsymbol{\mathcal{F}}$
$\delta\tilde{\mathbf{x}}_0 \leftarrow \mathbf{B}^{-1}\left(\mathbf{x}_0^{\mathsf{i}} - \mathbf{x}_0^{\mathsf{b}} + \delta\mathbf{x}_0\right) + \delta\tilde{\mathbf{x}}_0$
$\delta\tilde{\mathbf{p}} \leftarrow \mathbf{P}^{-1}\left(\mathbf{p}^{\mathsf{i}} - \mathbf{p}^{\mathsf{b}} + \delta\mathbf{p}\right) + \delta\tilde{\mathbf{p}}$

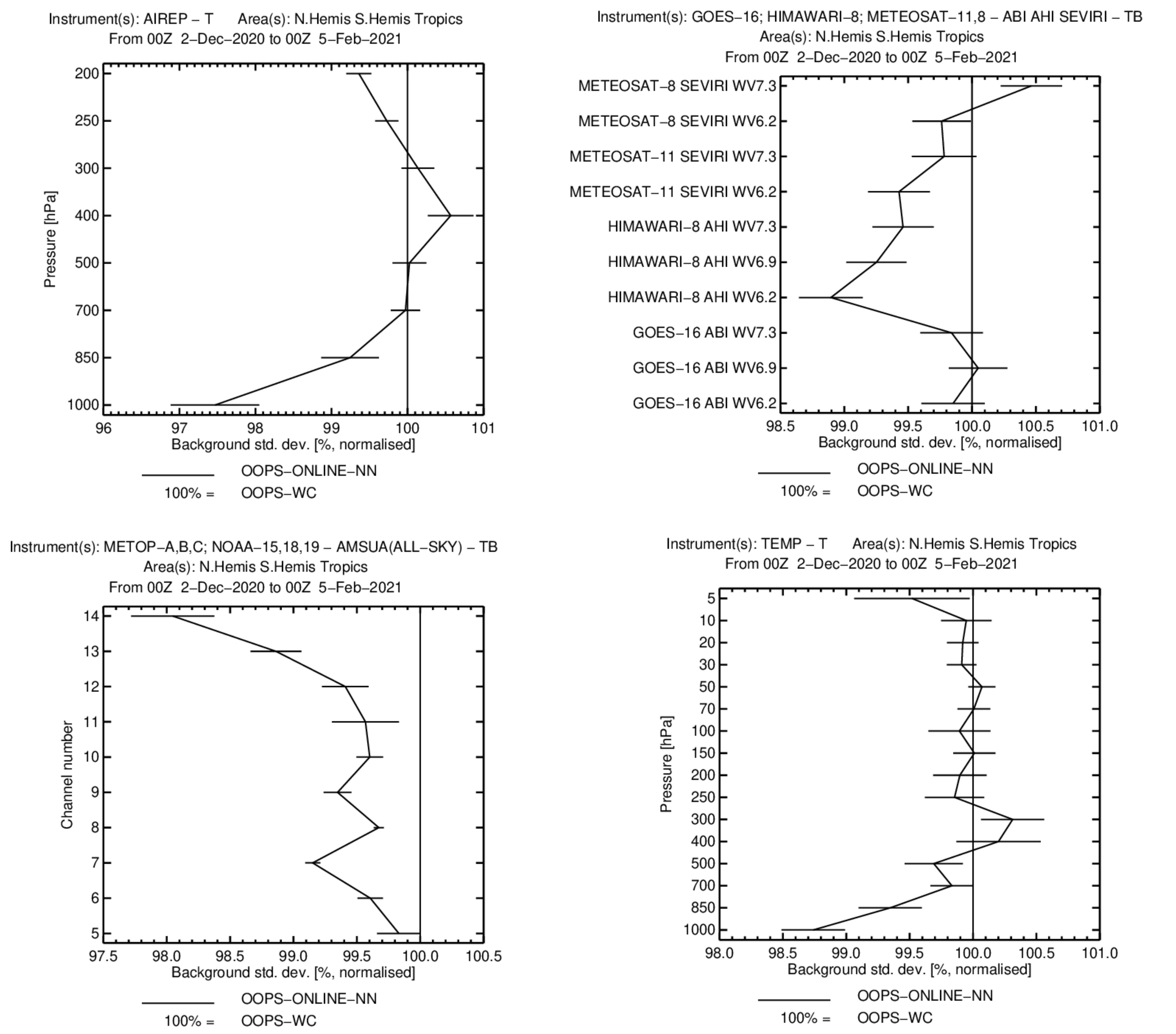
Output: $\nabla_{\delta\mathbf{p}}\widehat{\mathcal{J}}^{\mathsf{nn}} = \delta\tilde{\mathbf{p}}$ and $\nabla_{\delta\mathbf{x}_0}\widehat{\mathcal{J}}^{\mathsf{nn}} = \delta\tilde{\mathbf{x}}_0$

FNN (Fortran Neural Network) library

- Fortran implementation of sequential neural networks equipped with tangent linear and adjoint operators required by incremental 4D-Var: https://github.com/cerea-daml/fnn.

Hybrid Data Assimilation performance

- Joint estimation of the parameters of the neural network and atmospheric state allows for statistically significant reduction of first guess error fits to observations.
- Residual biases in observations can cause issues, similar as in weak constrain 4D-Var.



References

- Farchi, A., Chrust, M., Bocquet, M., Laloyaux P., Bonavita, M. (2022), Online model error correction with neural networks in incremental 4D-Var framework. *arXiv preprint*, arXiv:2210.13817.
- Bonavita, M., & Laloyaux, P. (2020). Machine learning for model error inference and correction. *Journal of Advances in Modeling Earth Systems*, 12, e2020MS002232. https://doi.org/10.1029/2020MS002232.