Variational Data Assimilation with Deep Prior

Arthur Filoche [1] Dominique Béréziat [1] Anastase Charantonis [2] Théo Archambault [1]

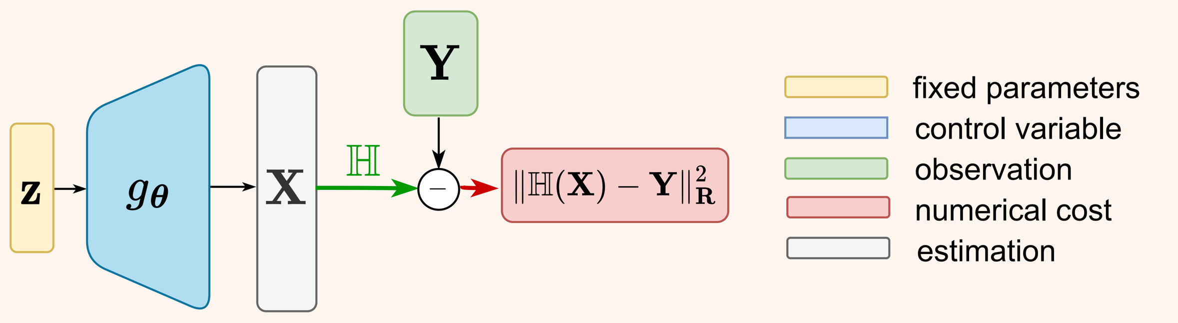
[1]Sorbonne Université, CNRS, LIP6, France [2]ENSIIE, CNRS, LAMME, France France

Motivations

- The variational data assimilation cost function is derived from Gaussian error modeling, then relying on covariances as hyper-parameters.
- Instead of a Gaussian background prior, we propose to use a deep prior as regularizer, circumventing the need for the background covariance matrix.

3DVAR with Deep Prior

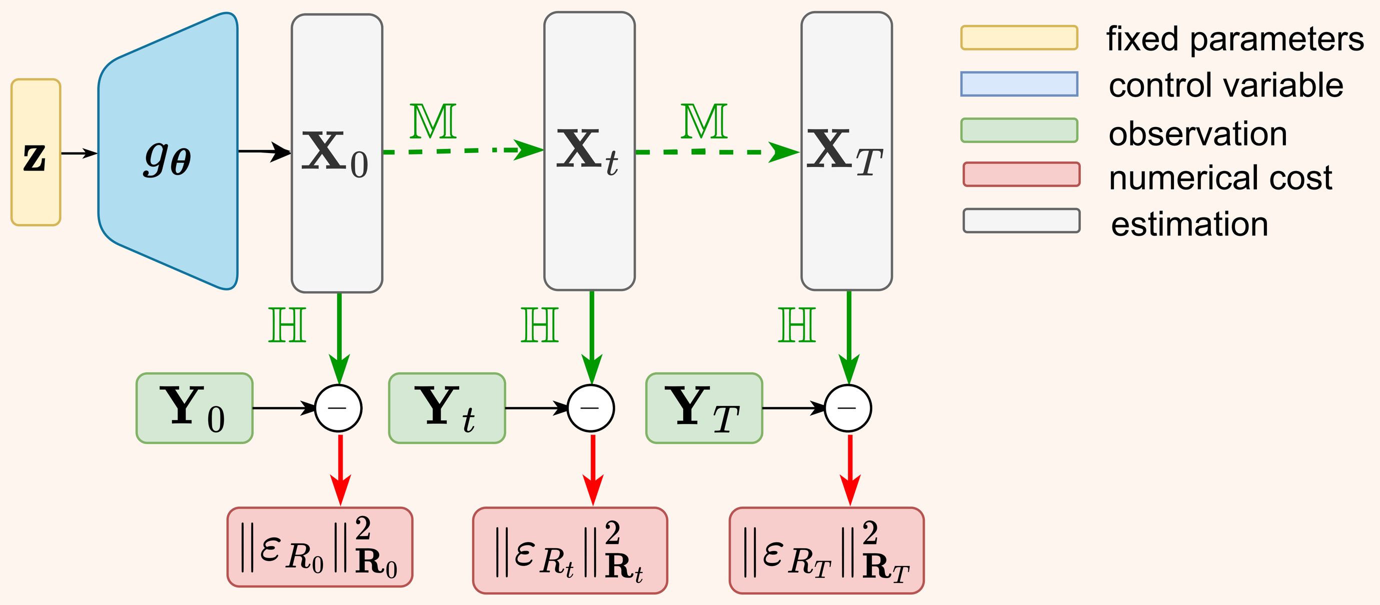
- Vanilla 3DVAR: $\min_{\mathbf{X}} \quad \mathcal{J}(\mathbf{X}) = \|\mathbf{Y}-\mathbb{H}(\mathbf{X})\|^2_{\mathbf{R}} + \|\mathbf{X}\|^2_{\mathbf{B}}$
- 3DVAR-DP: $\min_{\theta} \quad \mathcal{J}(\theta) = \|\mathbf{Y}-\mathbb{H}\circ g_{\theta}(z)\|^2_{\mathbf{R}}$ s.t. $\mathbf{X} = g_{\theta}(z)$



- Only the observational cost is optimized

4DVAR with Deep Prior

- Vanilla 4DVAR: $\min_{\mathbf{X}_0} \quad \mathcal{J}(\mathbf{X}_0) = \|\mathbf{X}_B-\mathbf{X}_0\|^2_{\mathbf{B}} + \sum_{t=0}^{T}\|\mathbb{H}_t\circ\mathbb{M}_{0\to t}(\mathbf{X}_0)-\mathbf{Y}_t\|^2_{\mathbf{R}_t}$
- 4DVAR-DP: $\min_{\theta} \quad \mathcal{J}(\theta) = \sum_{t=0}^{T}\|\mathbb{H}_t\circ\mathbb{M}_{0\to t}(\mathbf{X}_0)-\mathbf{Y}_t\|^2_{\mathbf{R}_t}$ s.t. $\mathbf{X}_0 = g_{\theta}(z)$

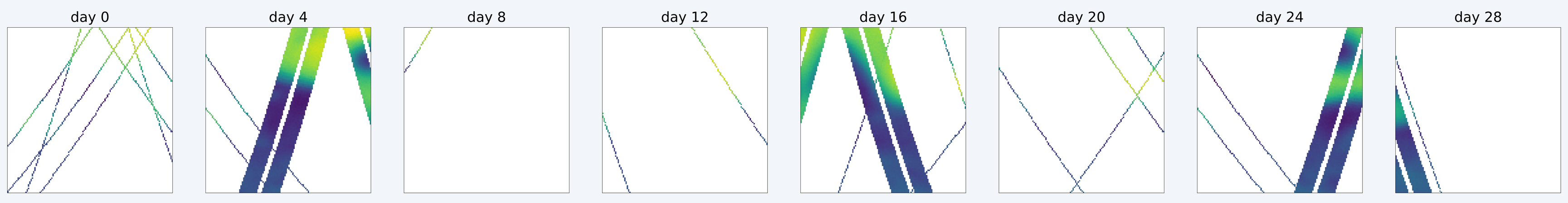


- Only the observational cost is optimized

Case study: Spatio-temporal interpolation of Sea Surface Height

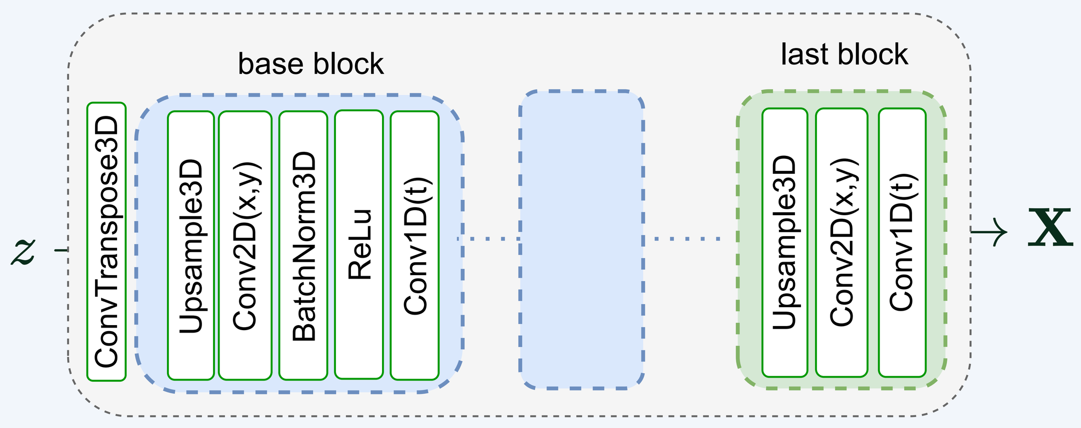
Observation

- NATL60 high-resolution ocean simulation rescaled at $(1/20)^{\circ}$
- Simulated altimeter satellites tracks: 4 nadirs + SWOT



Spatio-temporal Deep Prior

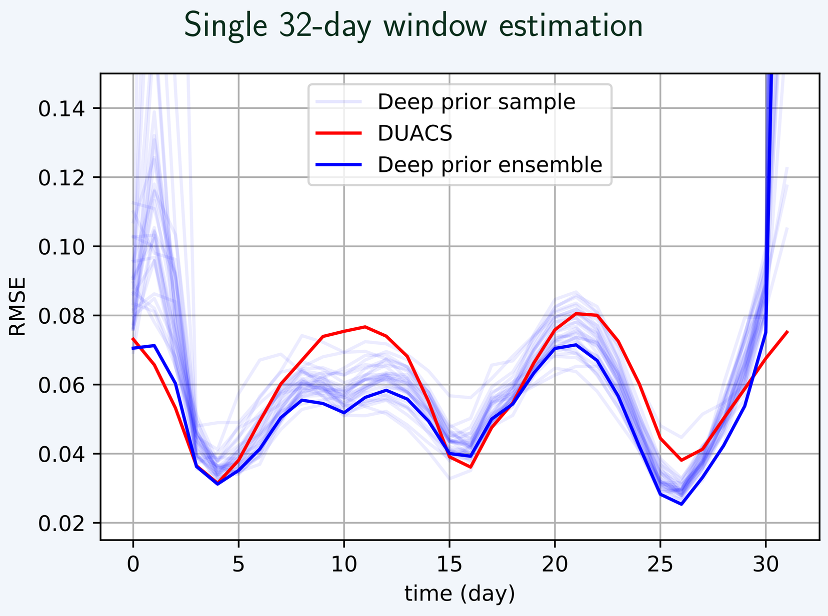
$$g_{\theta} : \mathbb{R}^{n_z} \to \mathbb{R}^{n_x \times n_y \times T}$$



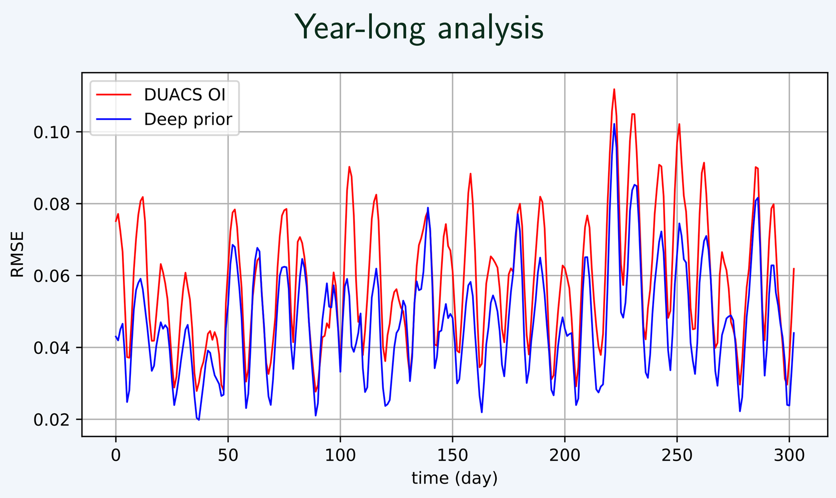
Results

Optimal Interpolation baseline: Data Unification and Altimeter Combination System

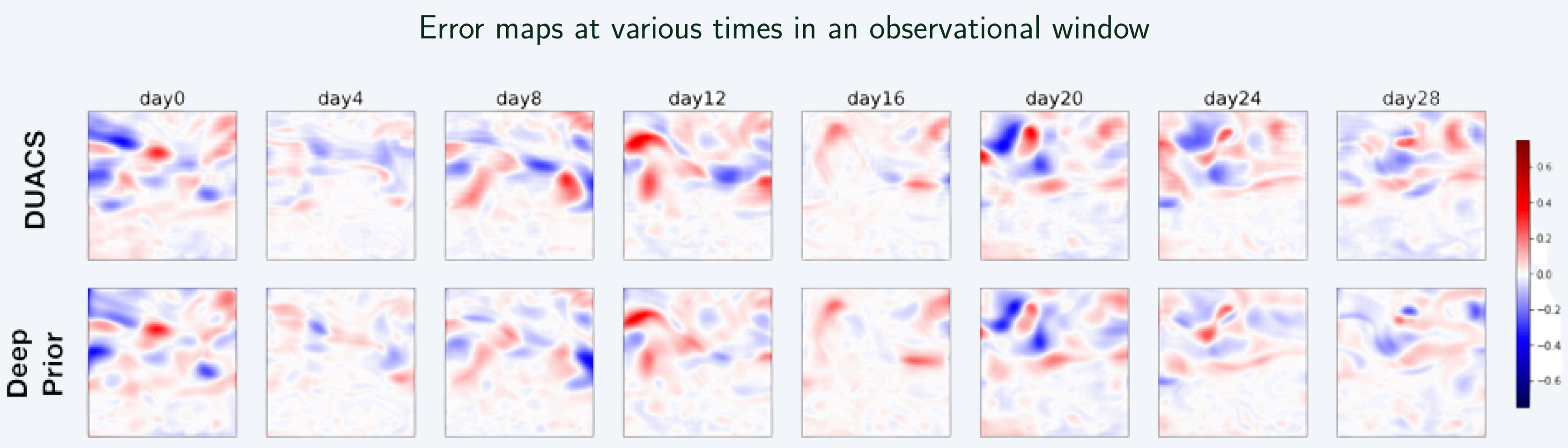
- DUACS, leverage 25 years of reprocessed sea level altimetry to estimate $\mathbf{B}$



- Estimation depends on weights initialization



- Sliding averaging/ensembling window



- Performances on par with DUACS

Case study: Motion Estimation with a Shallow water model

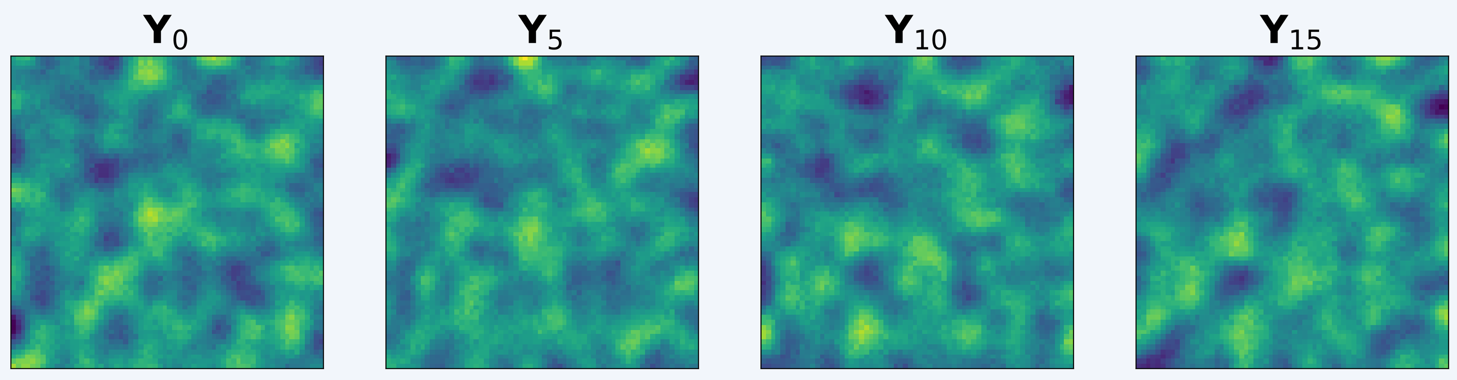
Shallow water toy model

State variables of the considered system are η, the height deviation of the horizontal pressure surface from its mean height, and $\mathbf{w}$, the associated velocity field.

$$\frac{\partial \eta}{\partial t} + \frac{\partial(\eta+H)u}{\partial x} + \frac{\partial(\eta+H)v}{\partial y} = 0$$
$$\frac{\partial u}{\partial t} + g\frac{\partial \eta}{\partial x} = 0$$
$$\frac{\partial v}{\partial t} + g\frac{\partial \eta}{\partial y} = 0$$

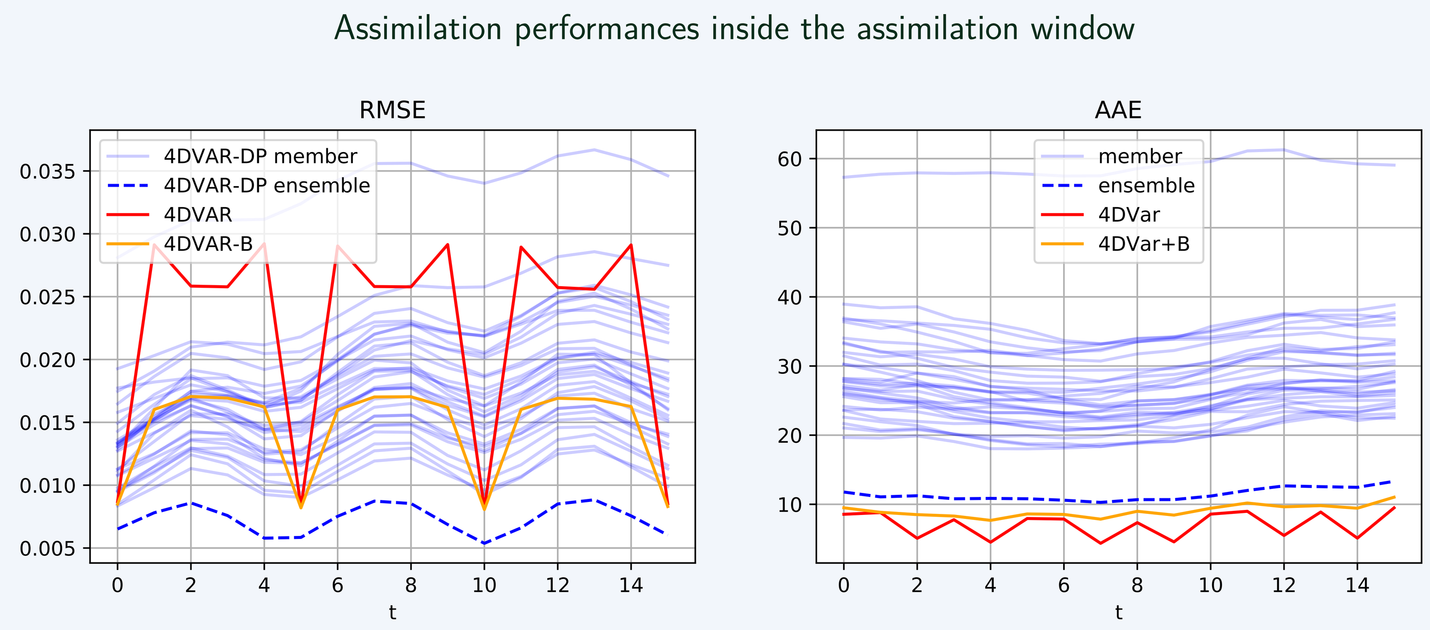
Observation

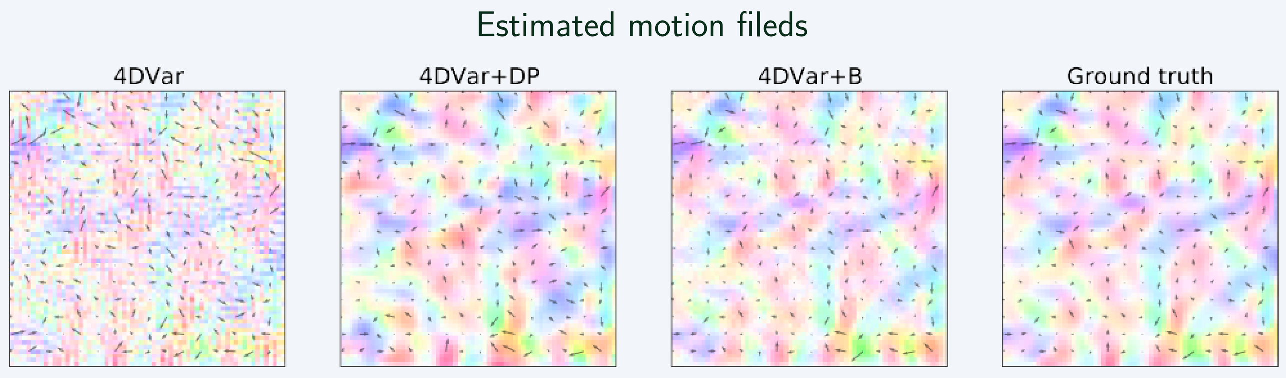
- Observations are regularly sparse in time
- $\mathbf{w}$ is never observed
- At observational date, η is fully observed up to a Gaussian white noise



Results

- Compare 4DVAR with various priors: No prior, Gaussian background, deep convolutional





- Deep prior has a strong regularizing effect, on par with background regularization

Conclusion

- A well-suited deep architecture can act like a handcrafted background matrix
- Convolutional prior account for a correlation in space and time

Going deeper

Filoche, A., T. Archambault, et al. (Sept. 2022). "Statistics-free interpolation of ocean observations with deep spatio-temporal prior". In: *ECML/PKDD Workshop on Machine Learning for Earth Observation and Prediction (MACLEAN)*.

Filoche, A. and D. Béréziat (July 2022). "Simultaneous Assimilation and Downscaling of Simulated Sea Surface Heights with Deep Image Prior". In:.

Filoche, A., D. Béréziat, and A. A. Charantonis (May 2022). "Deep prior in variational assimilation to estimate ocean circulation without explicit regularization". In: *Climate Informatics*. Asheville, NC, United States.

Github: