Representing uncertainty in initial conditions: experimentation from lagged average forecasting to singular vectors

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Outline (a story about four papers)

- **1984-85:** Early experiments with 30-day time-lagged ensembles (Molteni, Cubasch, Tibaldi 1986)
- **1986-89:** Further experimentation on extended-range forecasts, including time-lagged ensembles (Brankovic, Palmer, Molteni, Tibaldi, Cubasch 1990)
- **1990-91:** A new approach to initial perturbations: fastest-growing singular vectors in barotropic and quasi-geostrophic models (Molteni and Palmer 1993)
- **1991:** Testing quasi-geostrophic singular vectors in ensembles run with the ECMWF model (Mureau, Molteni, Palmer 1993)
- The legacy, and some comments about the future of ensembles.

The road to ensembles: Stochastic-dynamic and Monte-Carlo forecasting

- Epstein, *Tellus* 1969: Stochastic dynamic predictions
- Gleeson, *J. Appl. Met.* 1970: Statistical-dynamical predictions
- Leith, *Mon. Wea. Rev.* 1974: Theoretical skill of Monte Carlo forecasts
- Hollingsworth 1979: An experiment in Monte Carlo forecasting (ECMWF Wks. on Stochastic Dynamic Forecasting)

Tellus (1983), *35A*, 100–118

Lagged average forecasting, an alternative to Monte Carlo forecasting

By ROSS N. HOFFMAN and EUGENIA KALNAY, *Goddard Laboratory for Atmospheric Sciences, NASA/Goddard Space Flight Center, Greenbelt, MD 20771, U.S.A.*

(Manuscript received March 26; in final form August 2, 1982)

ABSTRACT

In order to use the information present in past observations and simultaneously to take advantage of the benefits of stochastic dynamic prediction we formulate the lagged average forecast (LAF) method. In a LAF, just as in a Monte Carlo forecast (MCF), sample statistics are calculated from an ensemble of forecasts. Each LAF ensemble member is an ordinary dynamical forecast (ODF) started from the initial conditions observed at a time lagging the start of the forecast period by a different amount. These forecasts are averaged at their proper verification times to obtain an LAF. The LAF method is operationally feasible since the LAF ensemble members are produced during the normal operational cycle.

Dynamical Predictability of Monthly Means

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(Manuscript received 14 January 1981, in final form 10 July 1981)

These results suggest that the evolution of long waves remains sufficiently predictable at least up to one month, and possibly up to 45 days, so that the combined effects of their own nonpredictability and their depredictabilization by synoptic-scale instabilities is not large enough to degrade the dynamical prediction of monthly means. The Northern Hemisphere appears to be more predictable than the Southern Hemisphere.

The 1984-85 experimentation with time-lagged ensembles

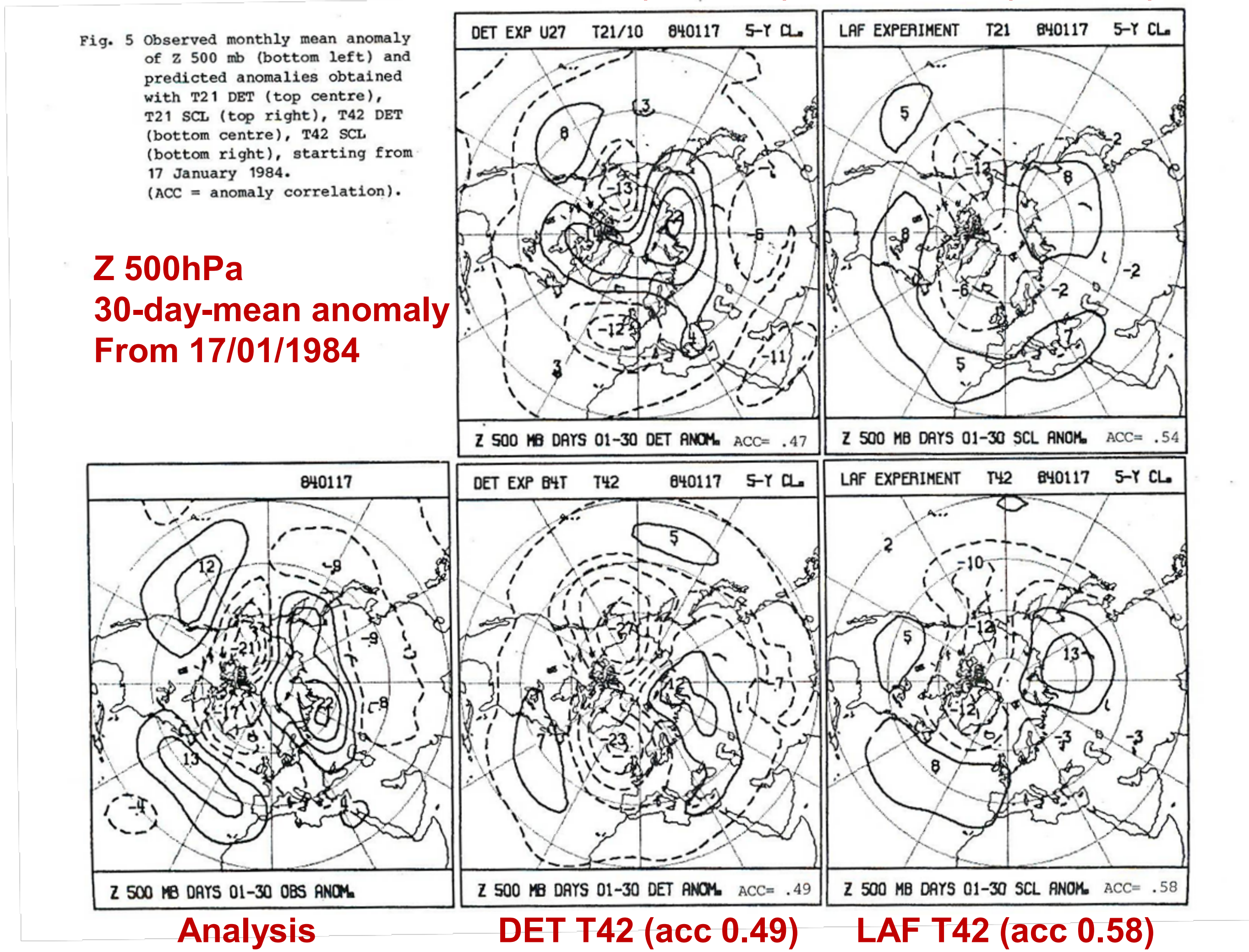
- 4 case studies from initial conditions in winter 1983/84
- 9-member lagged-average forecasts, I.C. from operational analysis at 6-hour intervals
- T21 and T42 spectral model
- Fixed SST, persisted from I.C. (as in operational forecasts)
- Correction for systematic error, based on 10 30-day integrations in winters 1981/82 and 1982/83, started at 10-day intervals
- Comparison with deterministic forecast from last I.C. and persistence

Molteni, Cubasch,Tibaldi:
30- and 60-day forecast experiments
with the ECMWF spectral models

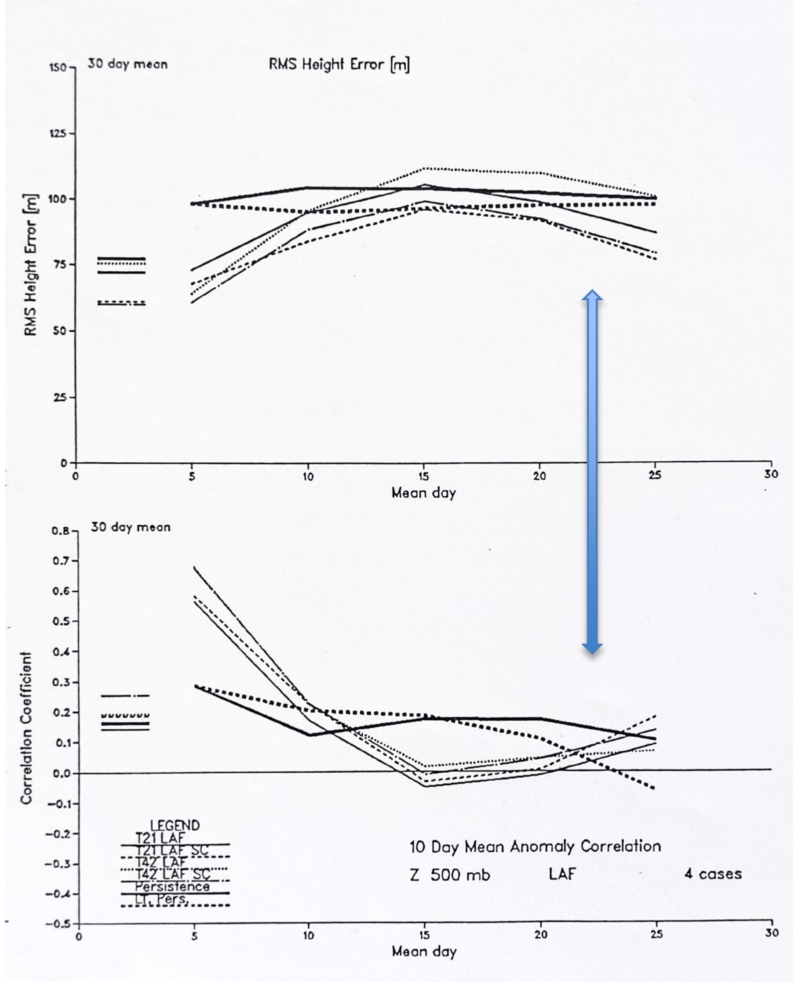
The first lagged-average experiment (17 Jan. 1984)

DET T21 (acc 0.47) LAF T21 (acc 0.54)

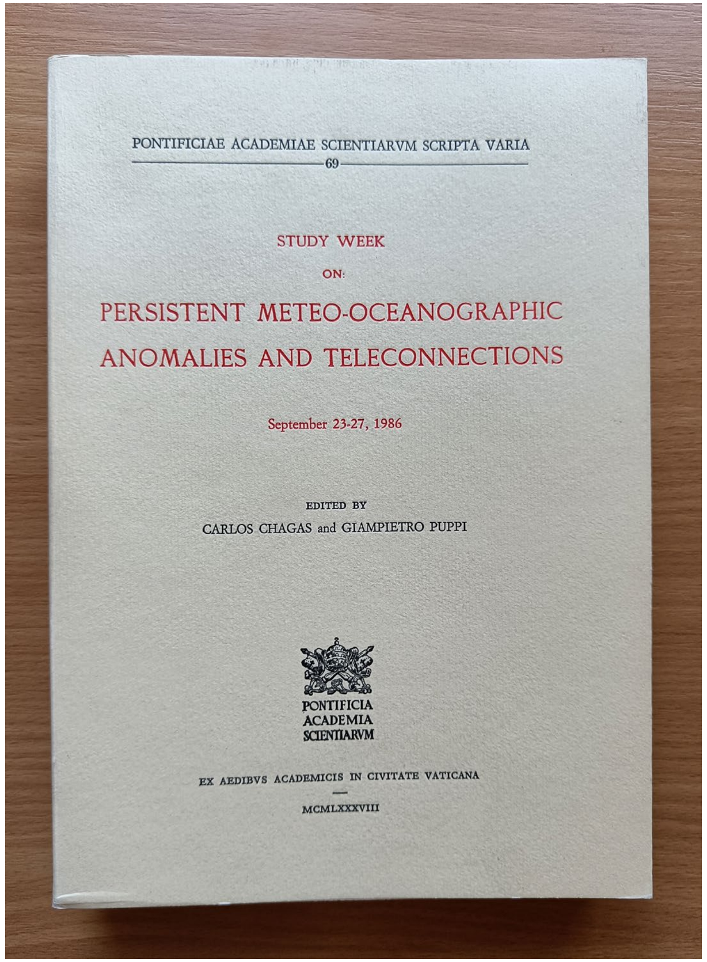
Z 500hPa
30-day-mean anomaly
From 17/01/1984



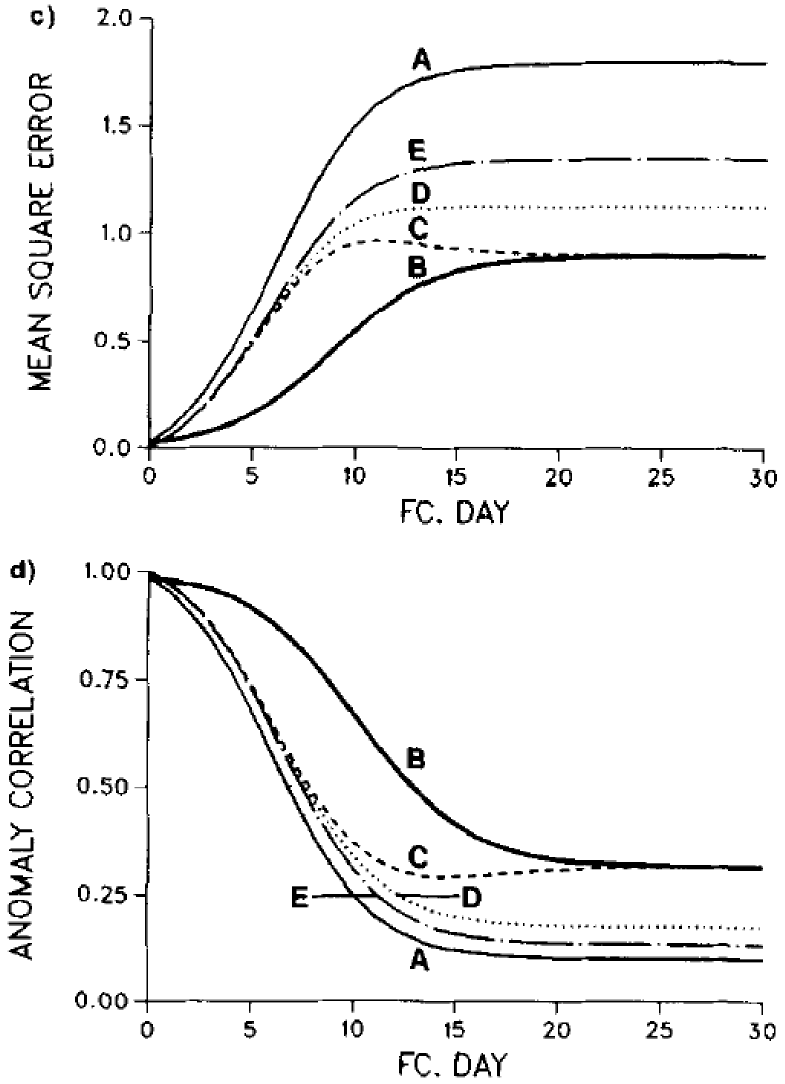
Analysis DET T42 (acc 0.49) LAF T42 (acc 0.58)



Stefano and Tim look for high-level endorsement …



Understanding the “return of skill” in time-lagged ensembles



Brankovic et al., QJRMS 1990: Extended-range predictions with the ECMWF model: Time-lagged ensemble forecasting

16 30-day, 9-member ensembles with the T63 model

Singular vectors: fastest-growing perturbation of a linearised model

(Farrell, *J.Atmos.Sci.* 1988, 1989, 1990; Lacarra and Talagrand, *Tellus-A* 1988)

Given a non-linear dynamical system: $d\,\mathbf{X} / dt = \mathbf{F}\,(\mathbf{X})$

and the evolution of its state vector **X** from time t_0 to t_1 : $\mathbf{X_0}\,(t_0\,,\,t_1)$

we can write the linear evolution of a perturbation δ**X** around the basic-state trajectory as: $d\,\delta\mathbf{X} / dt = \mathbf{L}\,(\mathbf{X_0}\,(t))\,\delta\mathbf{X}$

Integrating the tangent-linear model from time t_0 to t_1 we have: $\delta\mathbf{X}\,(t_1) = \mathbf{P}\,(t_0\,,\,t_1)\,\delta\mathbf{X}\,(t_0)$

where **P** is called the *linear propagator* for δ**X**

Defining a norm for the perturbation δ**X** and the associated inner product, the squared norm of δ**X** (t_1) is given by:

$$< \delta\mathbf{X}\,(t_1), \delta\mathbf{X}\,(t_1)> = < \mathbf{P}\,\delta\mathbf{X}\,(t_0), \mathbf{P}\,\delta\mathbf{X}\,(t_0)>$$

$$= < \mathbf{P^*}\,\mathbf{P}\,\delta\mathbf{X}\,(t_0), \delta\mathbf{X}\,(t_0)>$$

where **P*** is the adjoint of the propagator **P**

The eigenvectors **U** of **P* P** with the largest eigenvalues λ^2 are the initial perturbations that maximise the ratio between the final and initial norms of δ**X**

The vectors **U** are also the 'right' singular vectors of **P,** while the "evolved" vectors **V = P U** are the 'left' singular vectors

$$\mathbf{P} = \mathbf{V}\,\boldsymbol{\Lambda}\,\mathbf{U^*} \;\longrightarrow\; \mathbf{P^*}\,\mathbf{P} = \mathbf{U}\,\boldsymbol{\Lambda}^2\,\mathbf{U^*}$$

see also: Ehrendorfer and Tribbia, *J.Atmos.Sci.* 1997
Optimal prediction of fc. error covariances through S.V.s

Exploring singular vectors in barotropic and quasi-geostrophic models

Molteni and Palmer, QJRMS 1993: Predictability and finite-time instability of the northern winter circulation

Fastest-growing SV in a barotropic model

↓

Fastest-growing SV in a 3-level QG model →

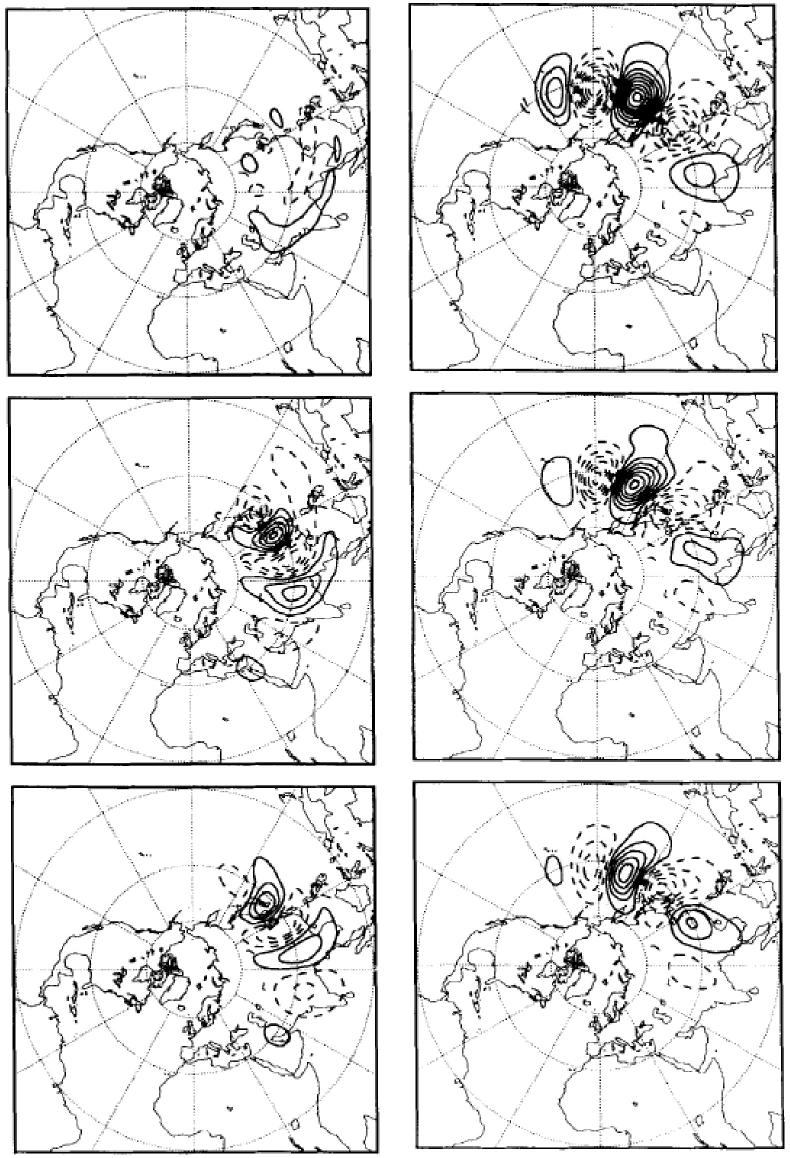
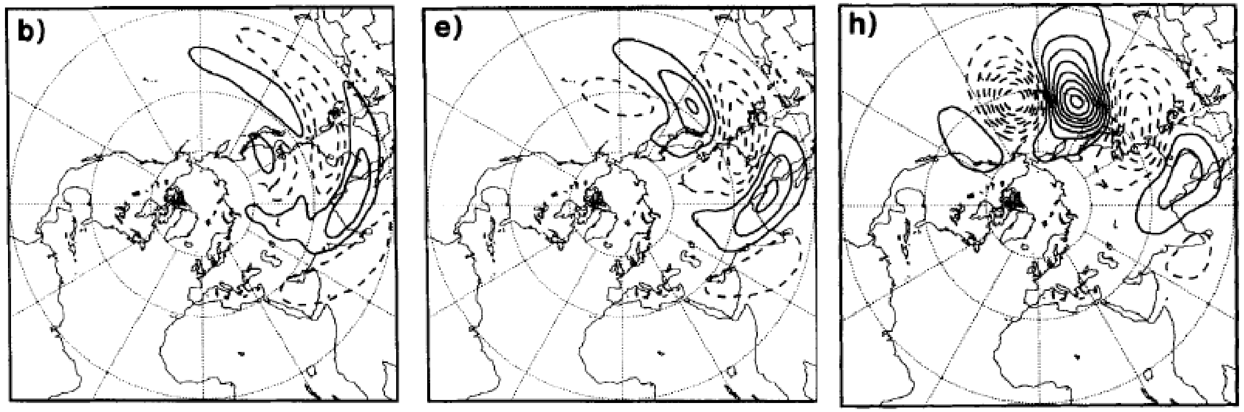


Figure 6. Stream function of the fastest-growing baroclinic day-2 singular vector evaluated at day 0 (column (a), contour interval $1.5 \times 10^6\ m^2\ s^{-1}$) and day 2 (column (b), contour interval $7 \times 10^6\ m^2\ s^{-1}$). Top row 200 hPa, middle row 500 hPa, and bottom row 800 hPa.



Exploring singular vectors in barotropic and QG models

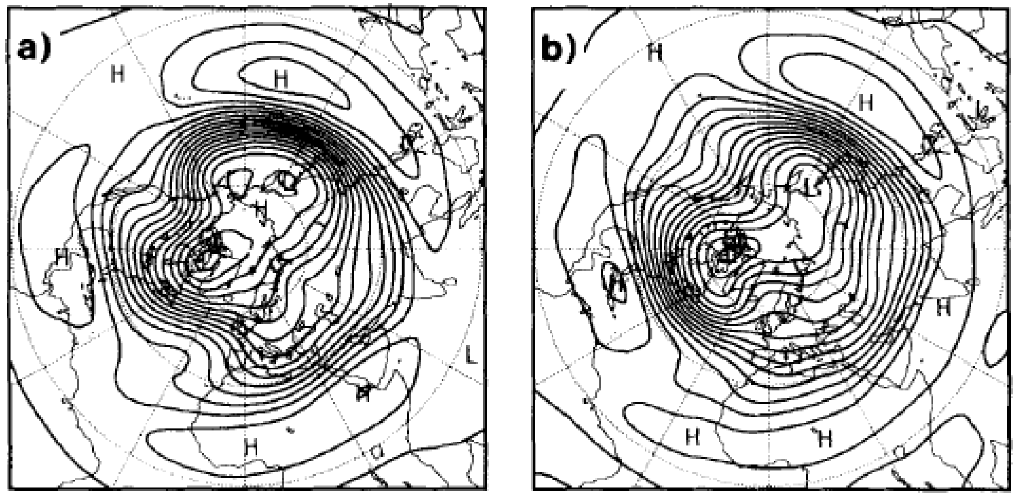


Figure 8. 500 hPa stream function associated with cluster centroid numbers (a) 2 and (b) 5 of Molteni *et al.* (1990). Contour interval 6×10^6 m^2 s^{-1}.

Instability of basic states with positive and negative PNA:

- PNA- more unstable than PNA+ according to growth rate of normal-modes and barotropic SVs
- Baroclinic SVs grow faster in PNA+ in opt. time range 12h – 4 days because of the stronger Pacific jet
- The opposite is true with 8-day opt. time, but the difference between PNA+ and PNA- is not as large as with barotropic SVs

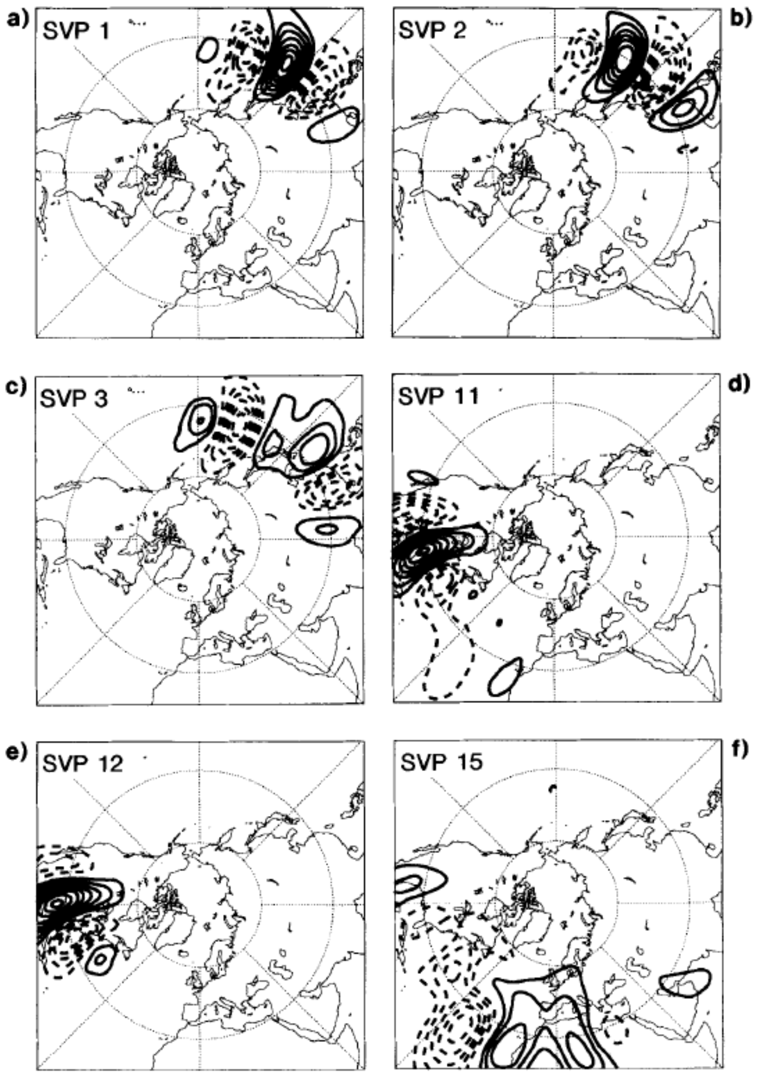
TABLE 4. AMPLIFICATION FACTOR (RELATIVE TO THE INITIAL AMPLITUDE) OF THE MOST UNSTABLE NORMAL MODE AND OPTIMAL SINGULAR VECTOR CALCULATED WITH THE BAROTROPIC MODEL FOR CLUSTER-2 FLOW AND FOR CLUSTER-5 FLOW

	Cluster 2		Cluster 5	
Integration time	Normal mode	Singular vector	Normal mode	Singular vector
12 hours	1.0	1.7	1.0	1.6
1 day	1.0	2.5	1.1	2.3
2 days	1.1	3.8	1.2	3.7
4 days	1.2	5.3	1.3	6.3
8 days	1.5	5.9	1.7	11.2

TABLE 5. AS TABLE 4 BUT FOR THE QUASI-GEOSTROPHIC MODEL

	Cluster 2		Cluster 5	
Integration time	Normal mode	Singular vector	Normal mode	Singular vector
12 hours	1.1	2.3	1.1	2.2
1 day	1.1	4.1	1.2	3.8
2 days	1.2	8.2	1.3	7.2
4 days	1.6	17.7	1.8	14.0
8 days	2.5	25.7	3.2	28.8

Testing QG singular vectors in a quasi-operational environment



Mureau, Molteni, Palmer QJRMS 1993:
Ensemble prediction using dynamically conditioned perturbations

QG singular vectors were interpolated to the 3D grid of the T63-L19 ECMWF primitive-equation model to create initial perturbations

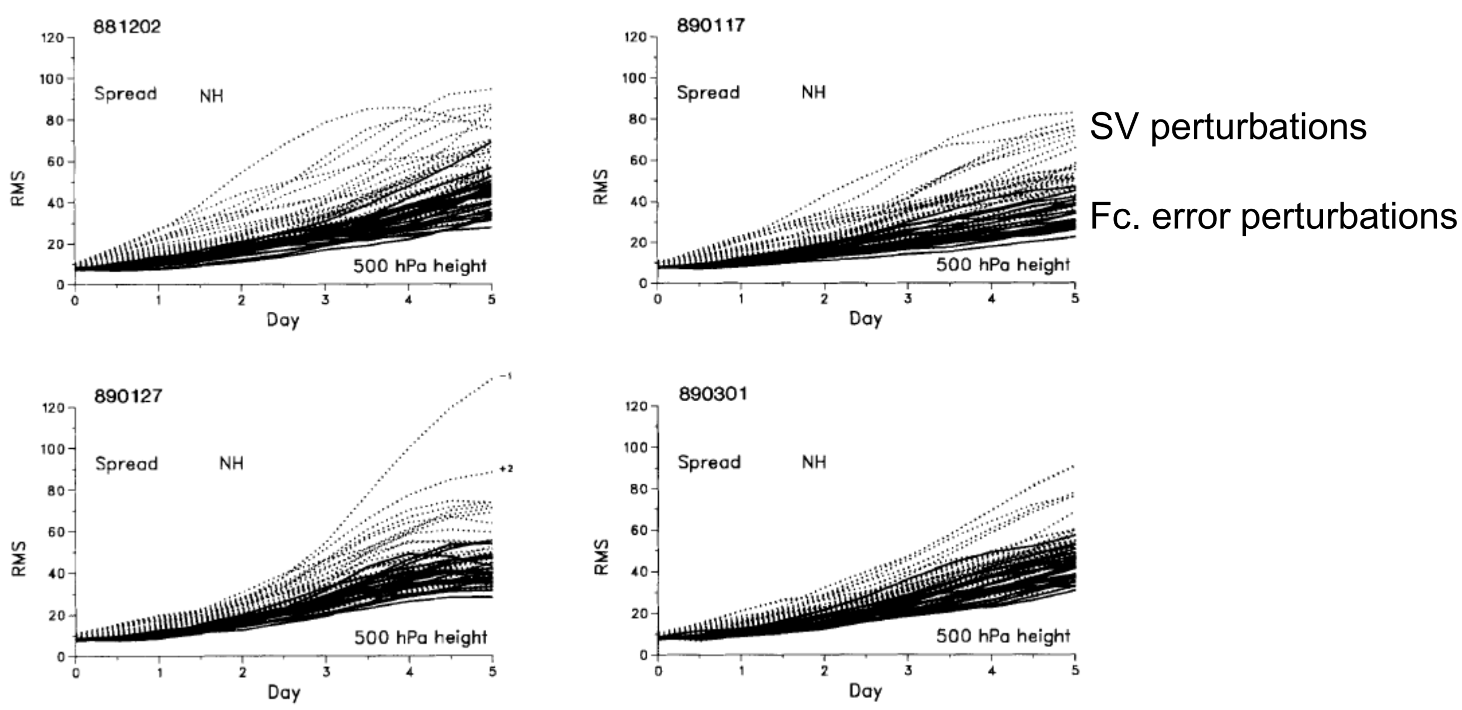


Figure 5. Root-mean-square height difference at 500 hPa between perturbed and control integrations averaged between 30°N and 80°N for the four initial dates. Dotted lines are for SVPs and full lines for FEPs.

The legacy, and some comments about the future of ensembles

- Computation of SVs in the PE model and operational implementation of medium-range ensemble forecasts in December 1992 (see Roberto's talk)

  Among many other milestones:

  - Stochastic perturbation of physical parametrization tendencies (1998)
  - Ensemble of data assimilations (2010)
  - First medium-range ensemble forecasts with a coupled model (2013)

- Experimentation with a hierarchy of models provided important guidance to the implementation of SV computation in the IFS
- Ensemble forecasts are now used from the short-range to the decadal time scale
- However, when the 3-way balance among resolution / complexity / ensemble size is assessed in climate modelling, single-model ensembles are often seen as "optional" in climate change projects
- For regional-scale variability and teleconnections, a single multi-decadal run is not sufficient to assess model fidelity with a high level of significance *(see papers from the PRIMAVERA project)*