Orographic drag and gravity wave drag

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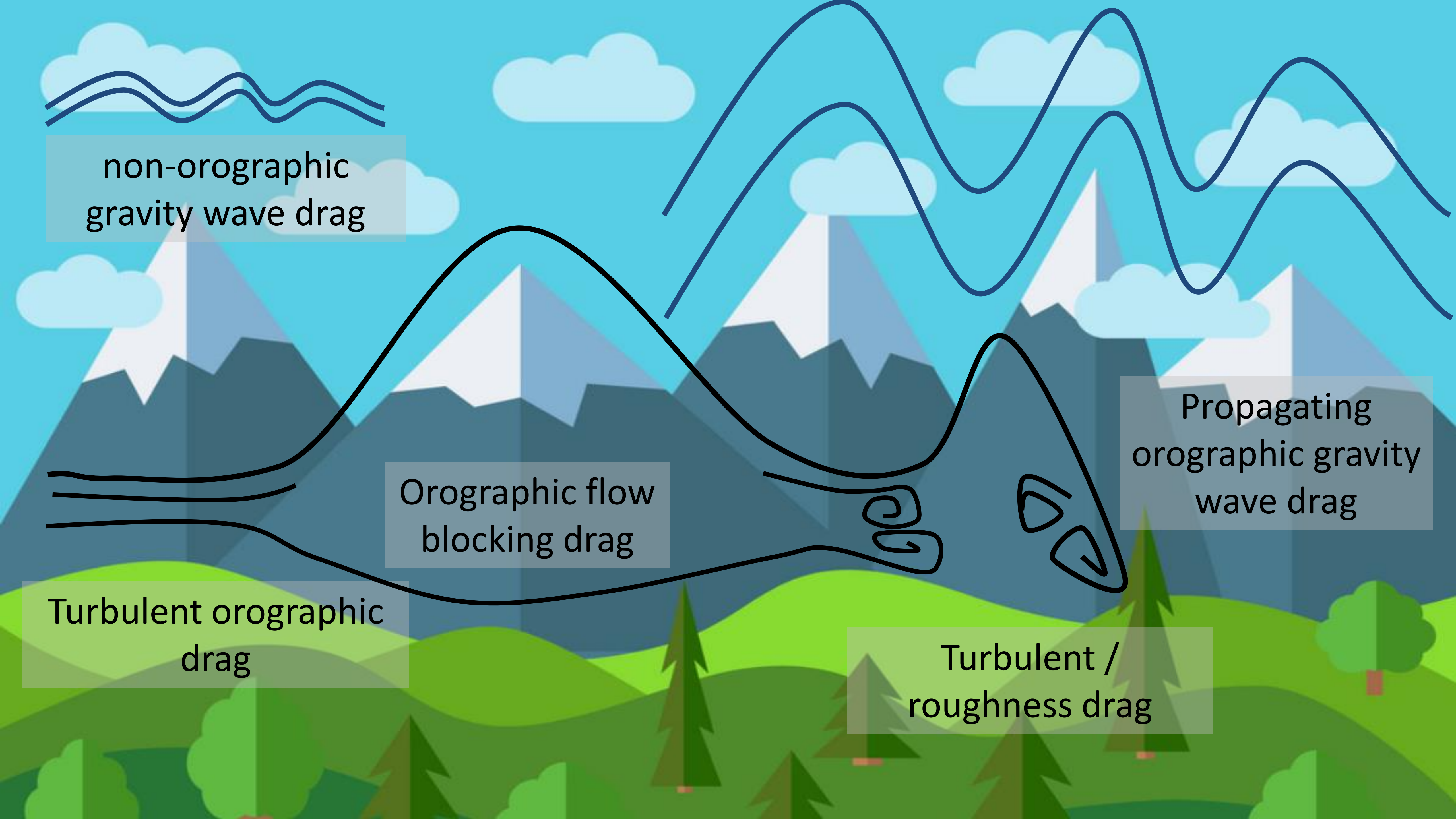
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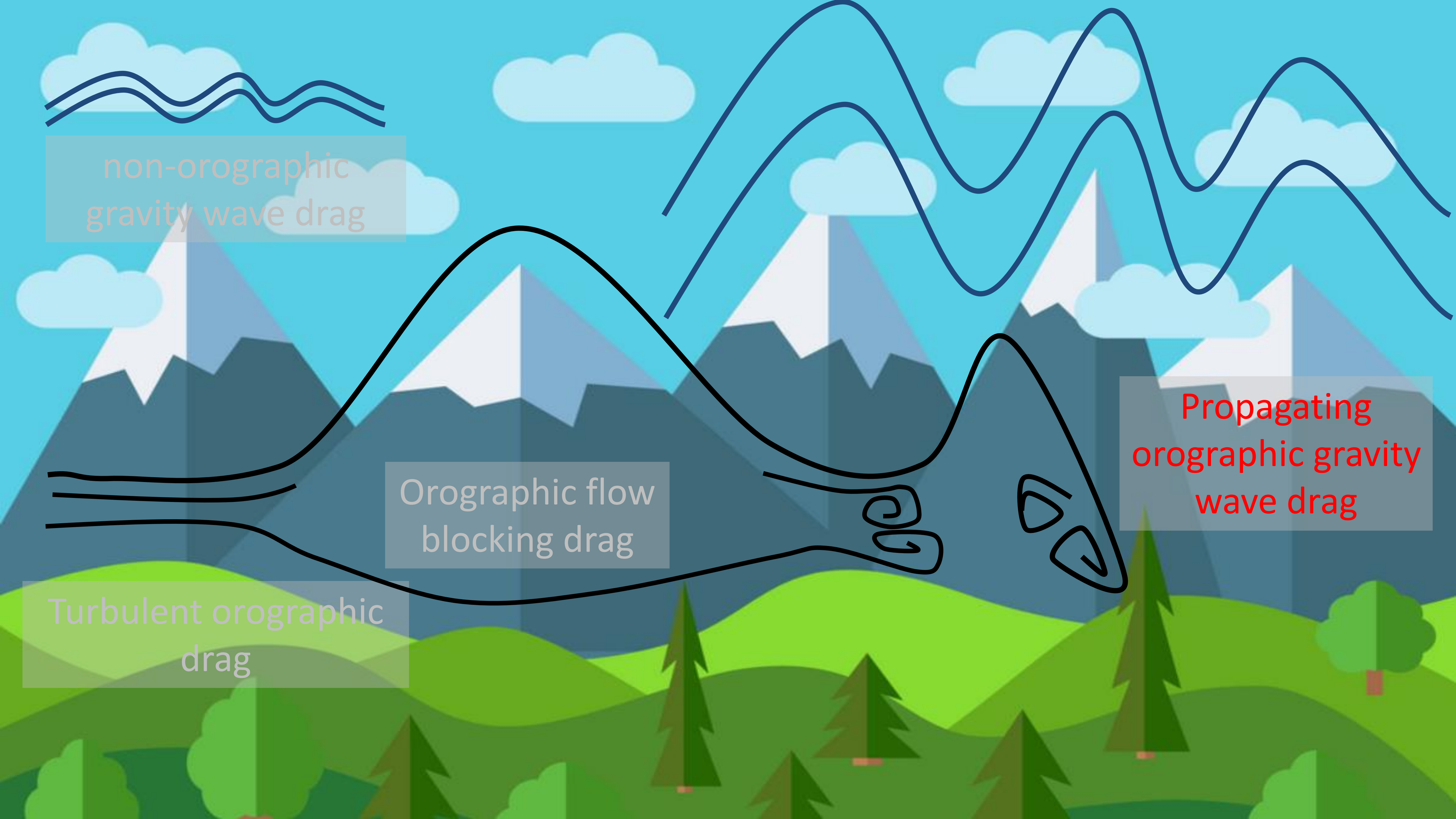
Turbulent / roughness drag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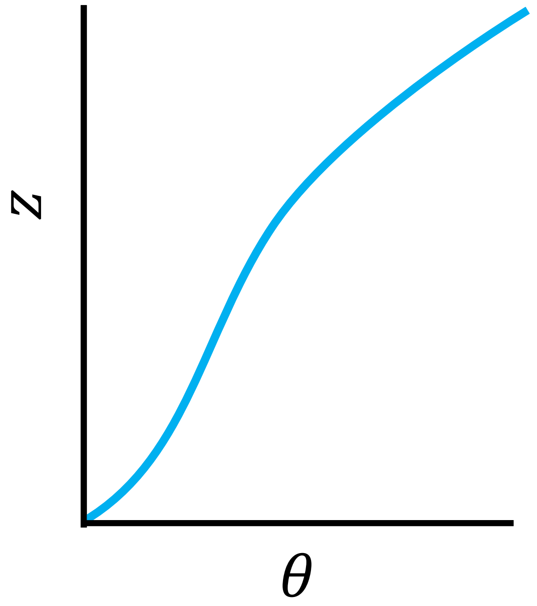
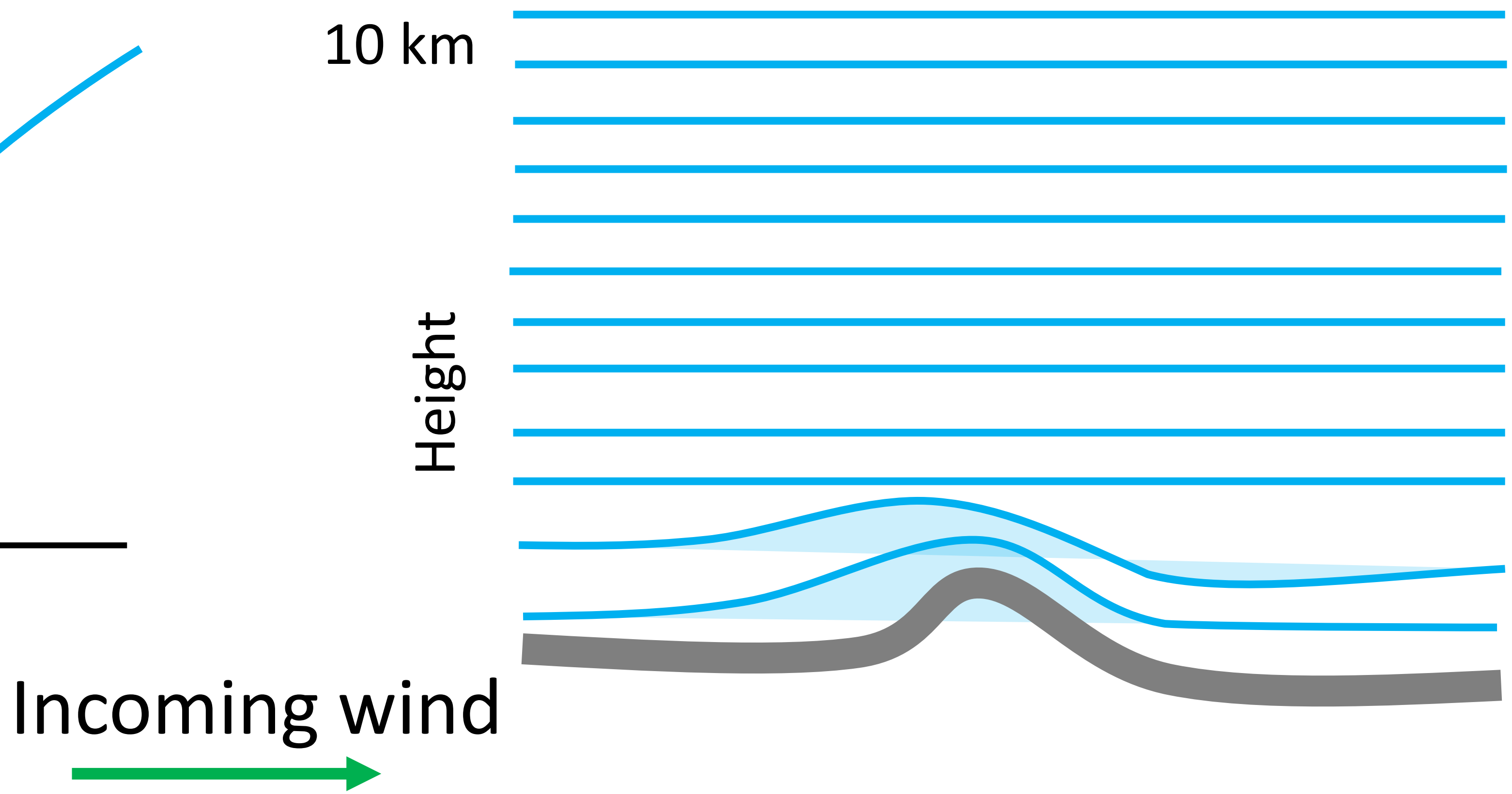
What are orographic gravity waves?

Potential temperature



Incoming wind forces air over mountain



What are orographic gravity waves?

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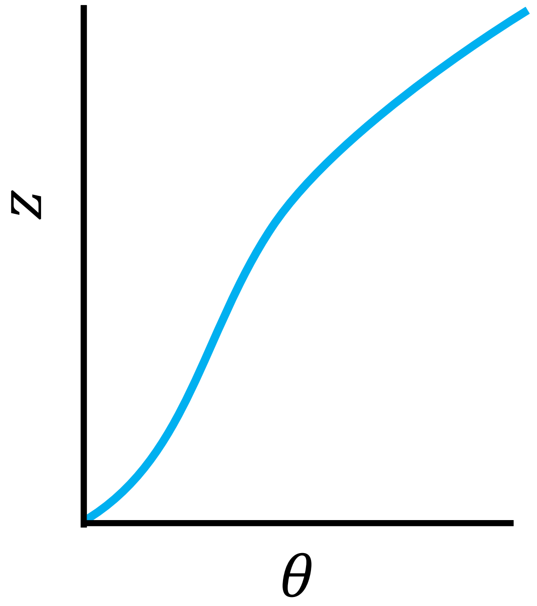
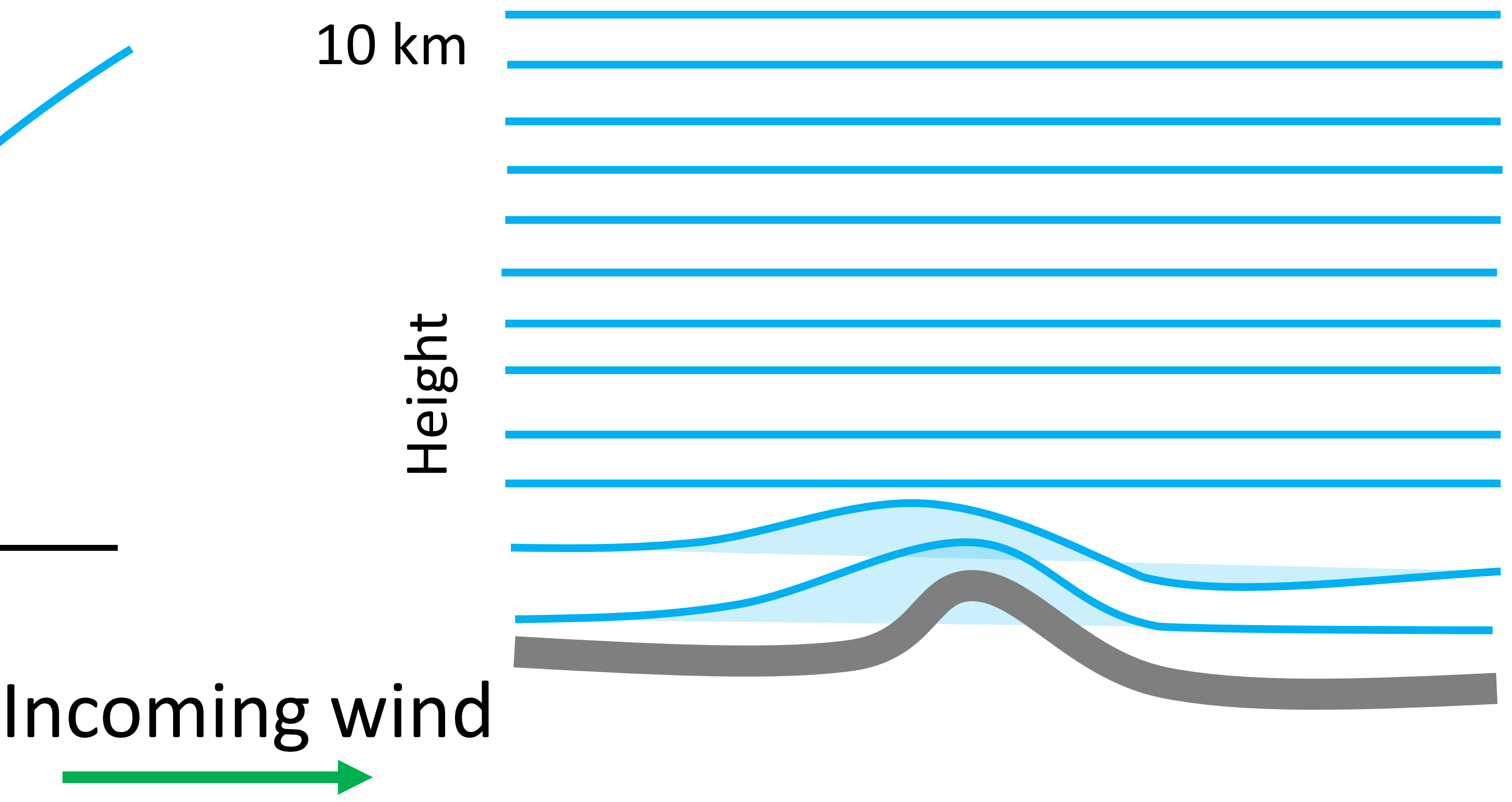


In stably stratified atmosphere, this leads to denser air being pushed up



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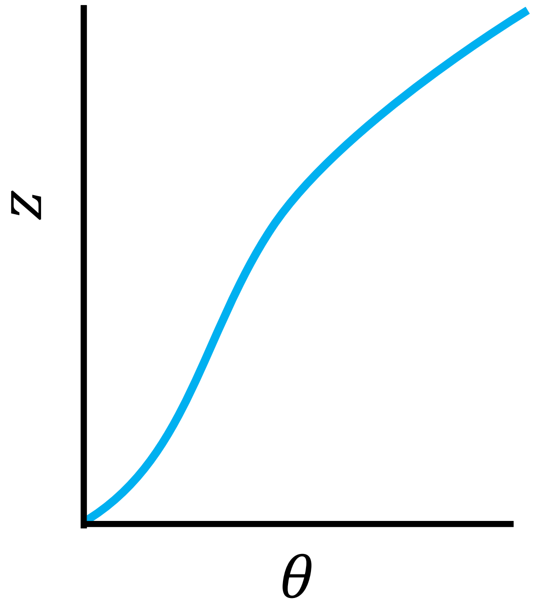
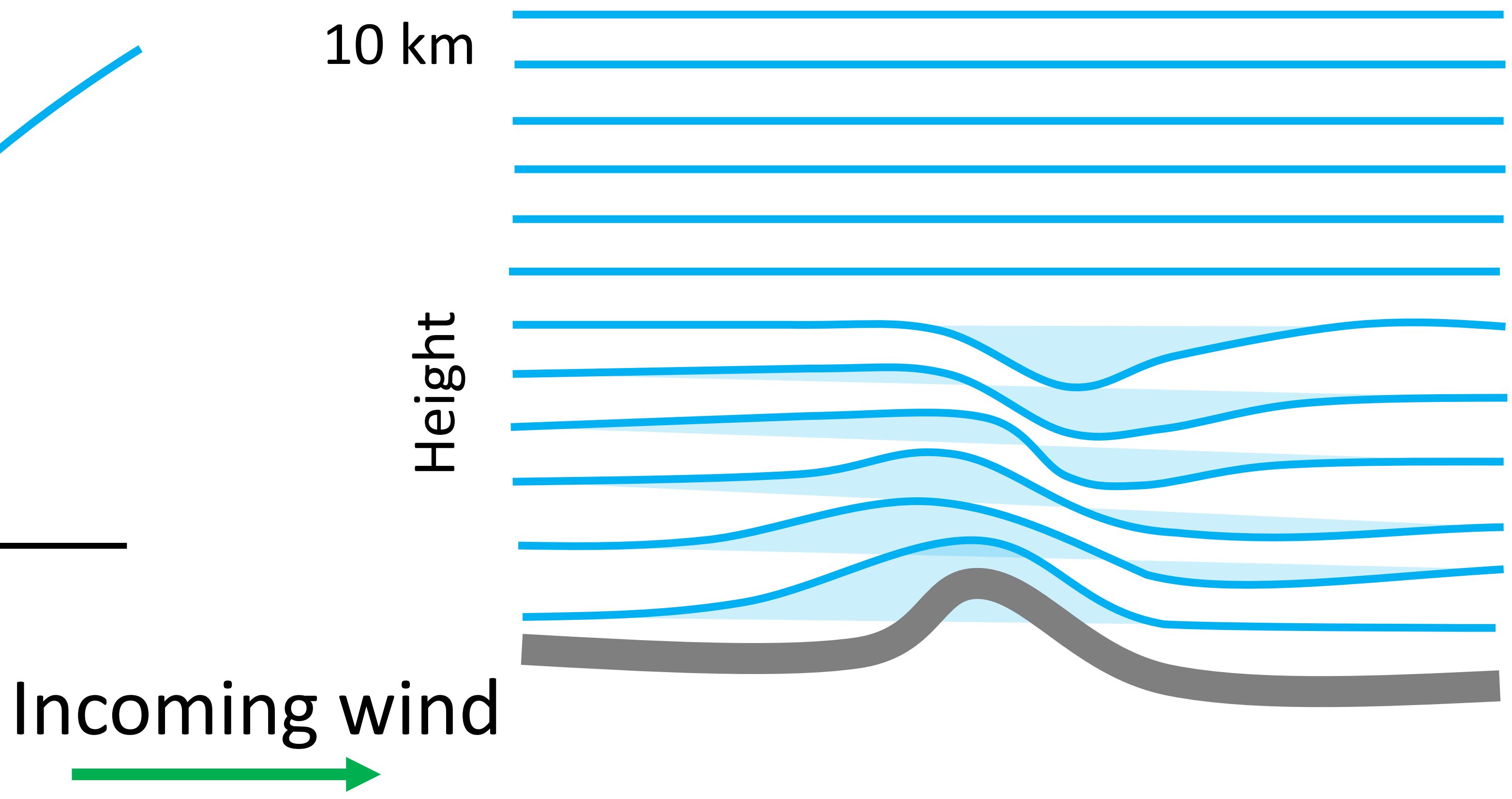
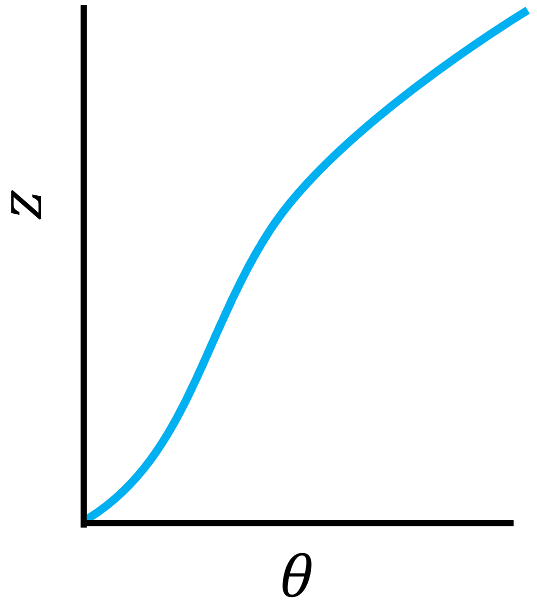


On the lee of the mountain, less dense air is pulled down

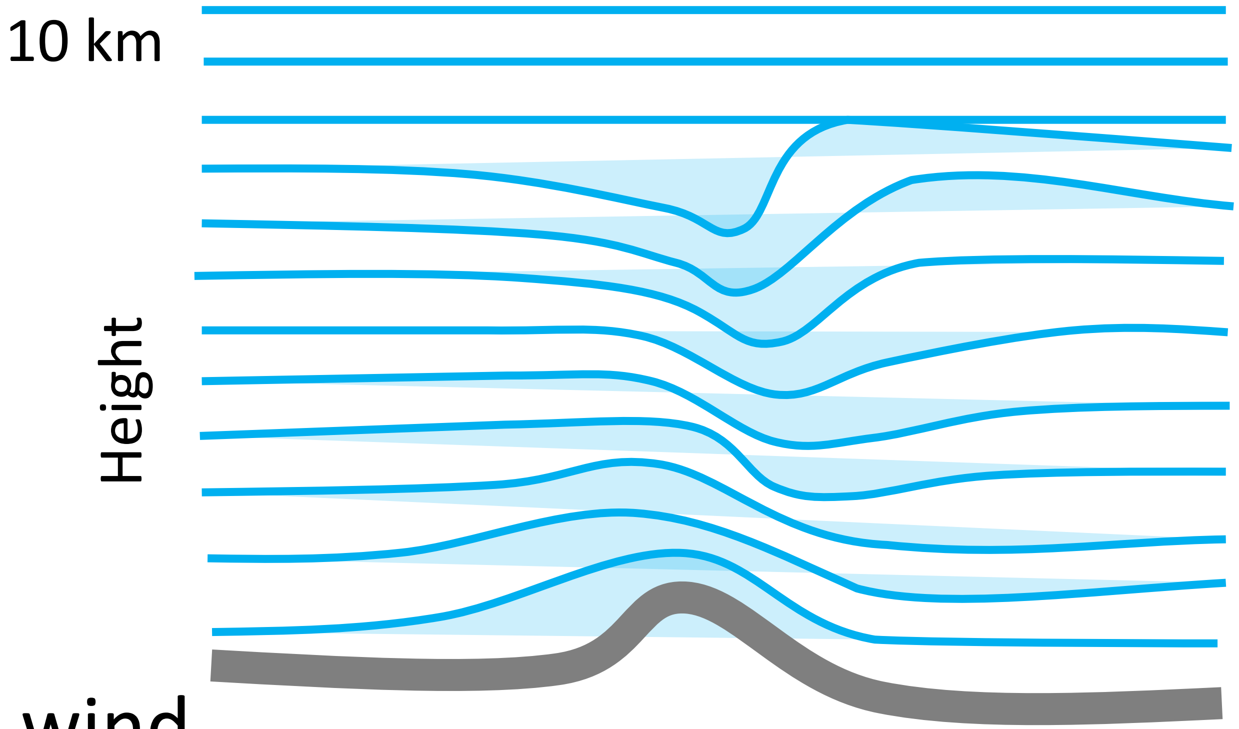


What are orographic gravity waves?

Potential temperature



This creates a vertically propagating wave throughout the atmosphere



Incoming wind

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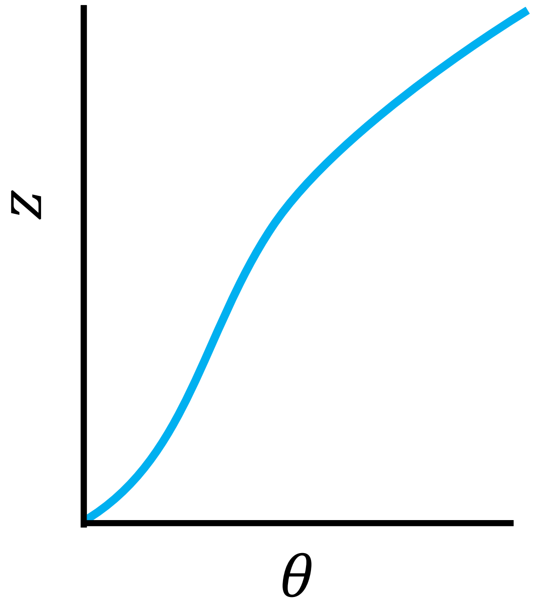
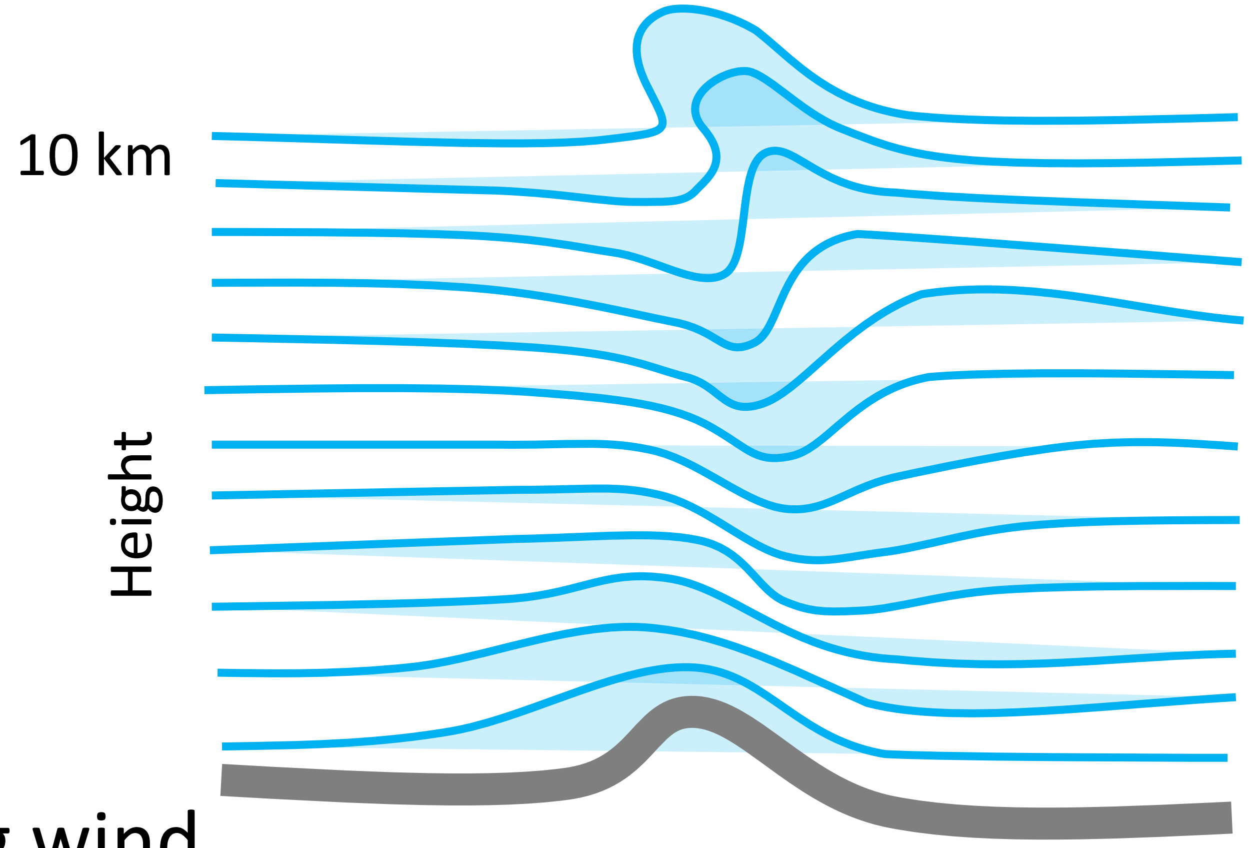
What are orographic gravity waves?

As density decreases with height, the amplitude grows, until the wave breaks

Potential temperature

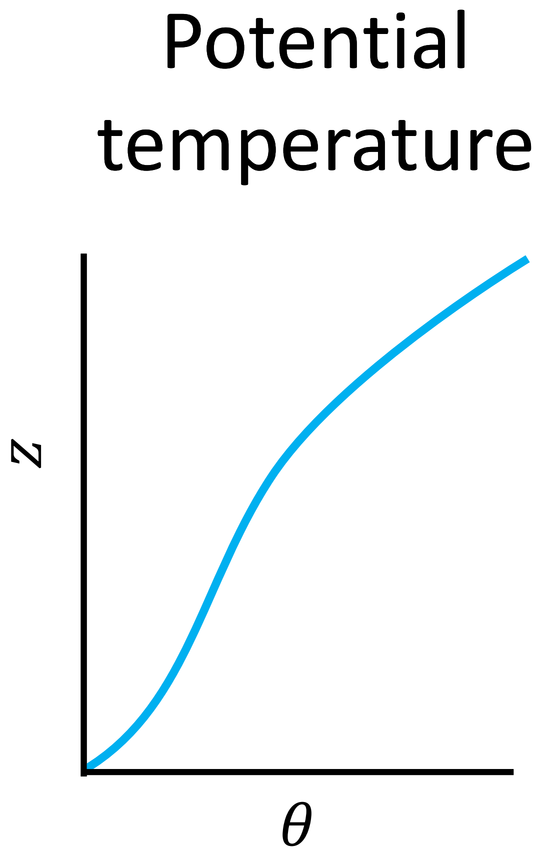


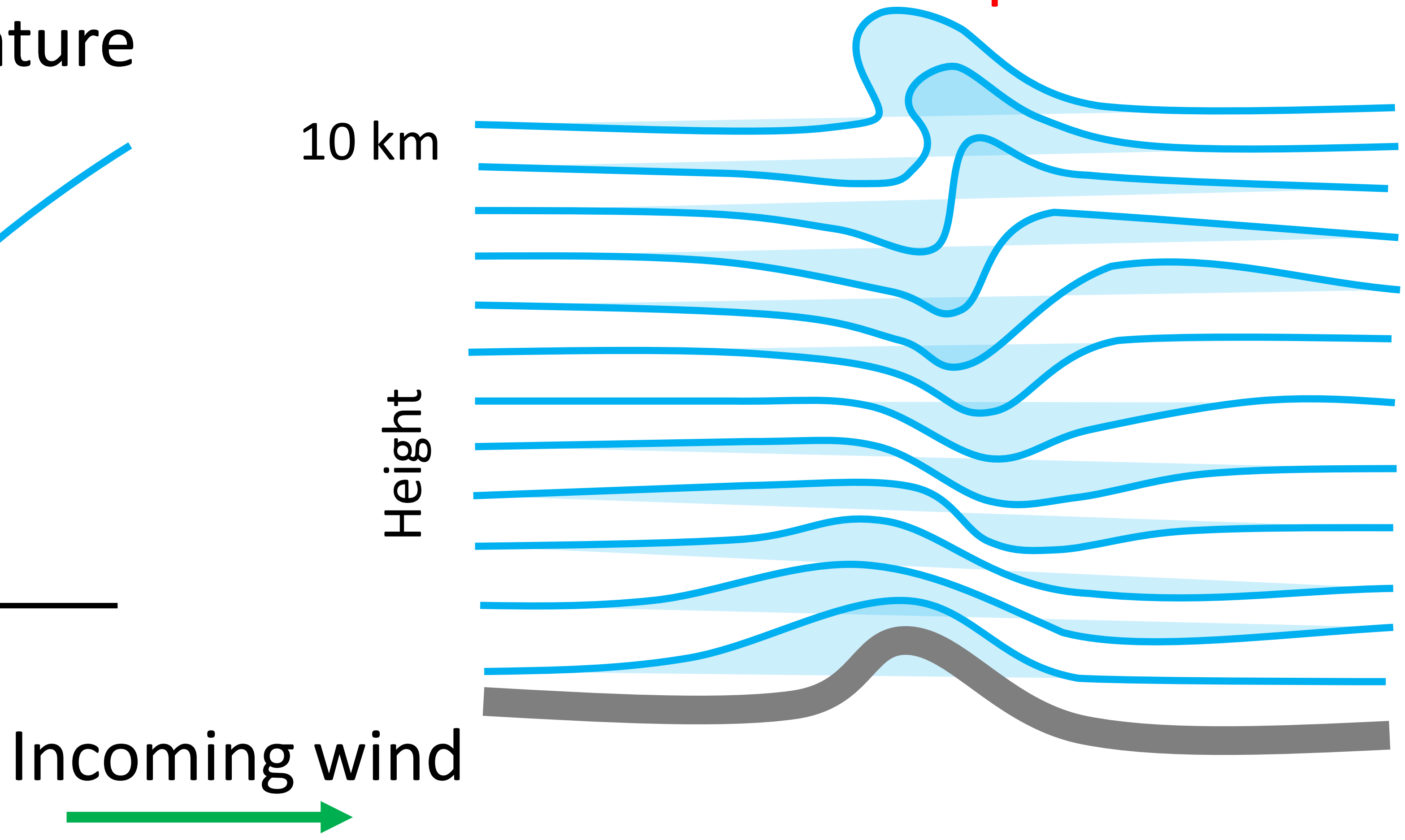


Incoming wind

What are orographic gravity waves?

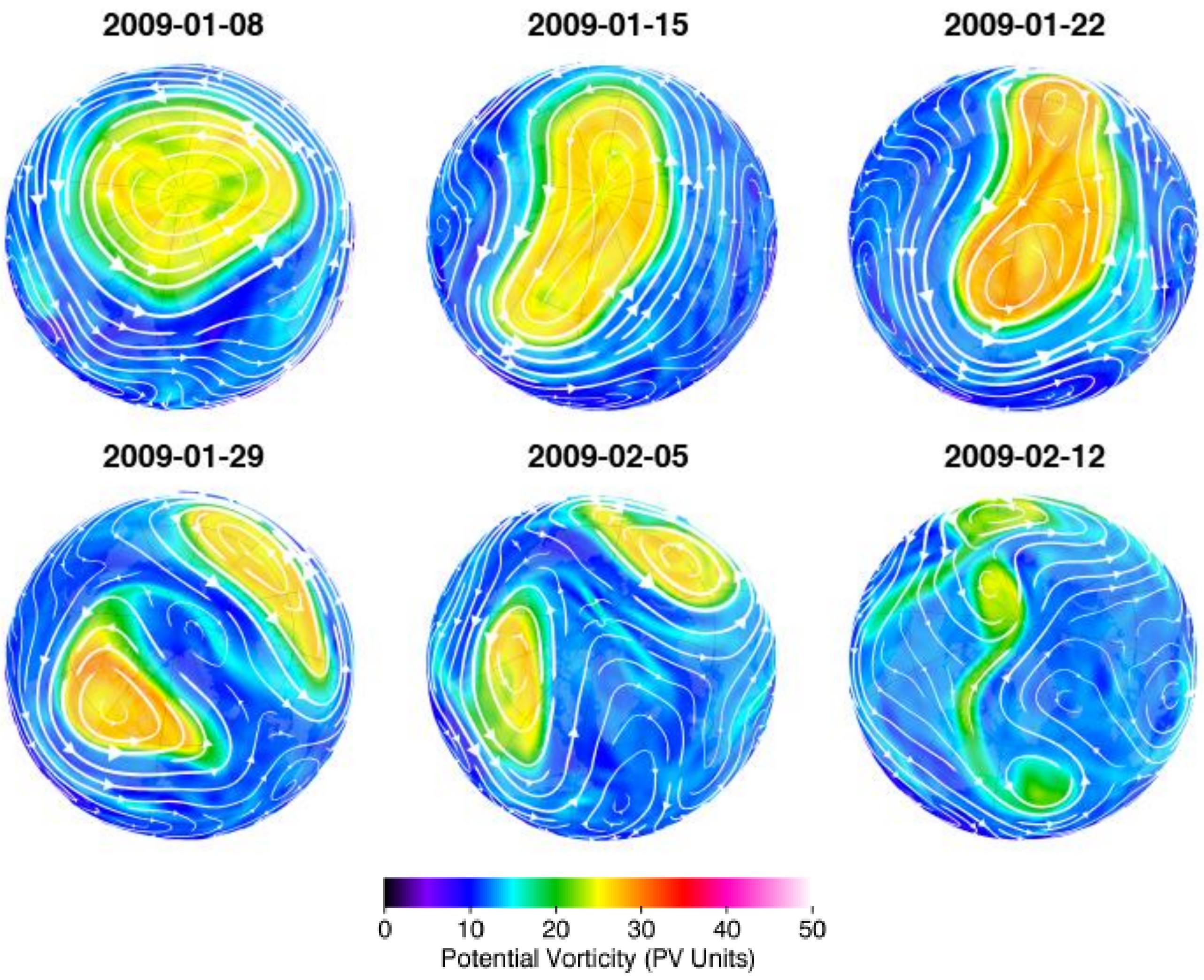
This causes a turbulent drag force on the atmosphere



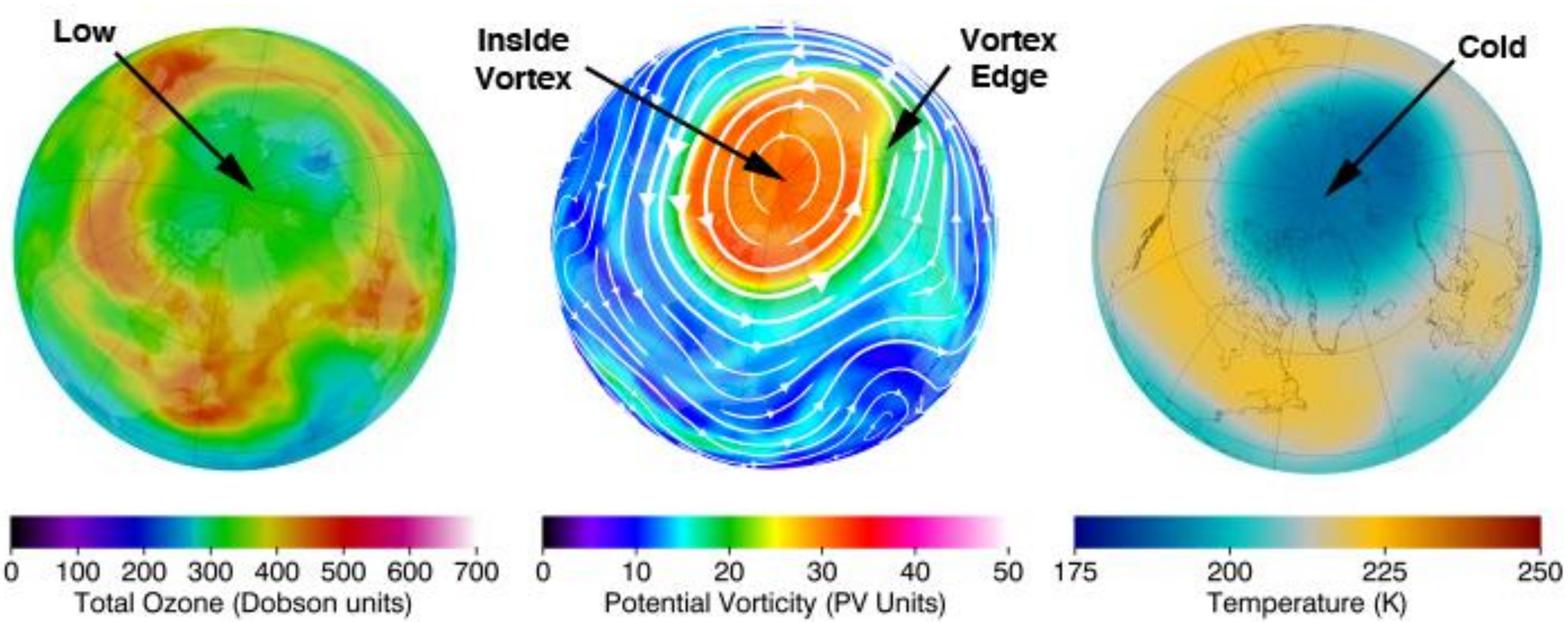


They affect Polar Vortex Variability

During Vortex breakdown



Gravity waves change the winds and temperatures in the Polar Vortex



NASA Ozone watch

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Stratosphere is important for surface predictability

Polar vortex death toll rises to 21 as US cold snap continues

1 February 2019

US polar vortex

Chicago's frozen shoreline

At least 21 people have died in one of the worst cold snaps to hit the US Midwest in decades.

Article | Open Access | Published: 23 July 2021

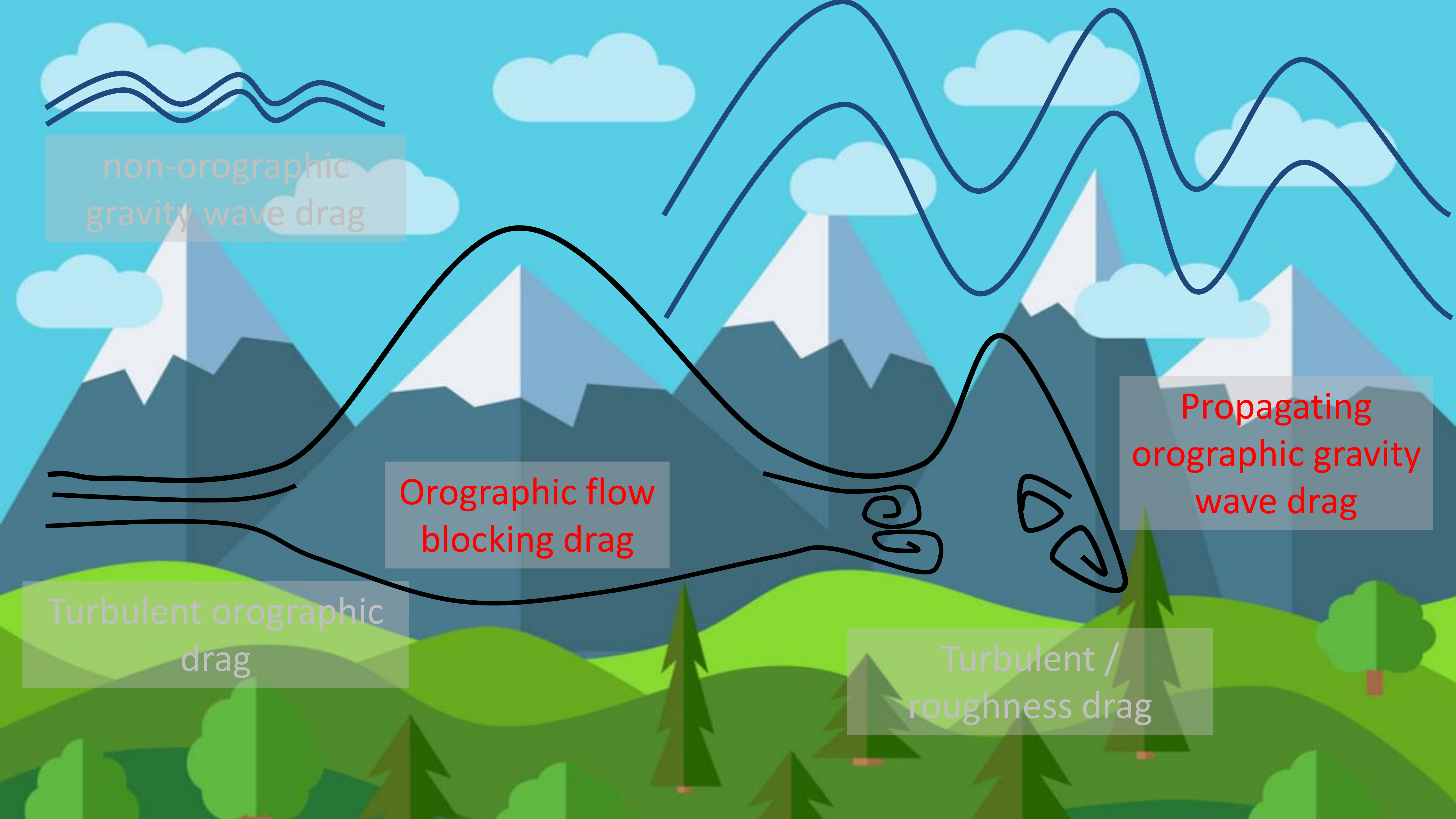
Northern hemisphere cold air outbreaks are more likely to be severe during weak polar vortex conditions

Jinlong Huang, Peter Hitchcock, Amanda C. Maycock, Christine M. McKenna & Wenshou Tian

Communications Earth & Environment **2**, Article number: 147 (2021) | Cite this article

Abstract

Severe cold air outbreaks have significant impacts on human health, energy use, agriculture, and transportation. Anomalous behavior of the Arctic stratospheric polar vortex provides an important source of subseasonal-to-seasonal predictability of Northern Hemisphere cold air outbreaks. Here, through reanalysis data for the period 1958–2019 and climate model simulations for preindustrial conditions, we show that weak stratospheric polar vortex conditions increase the risk of severe cold air outbreaks in mid-latitude East Asia by 100%, in contrast to only 40% for moderate cold air outbreaks. Such a disproportionate increase is also found in Europe, with an elevated risk persisting more than three weeks. By analysing the stream of polar cold air mass, we show that the polar vortex affects severe cold air outbreaks by modifying the inter-hemispheric transport of cold air mass. Using a novel method to assess Granger causality, we show that the polar vortex provides predictive information regarding severe cold air outbreaks over multiple regions in the Northern Hemisphere, which may help with mitigating their impact.



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Orographic flow blocking drag
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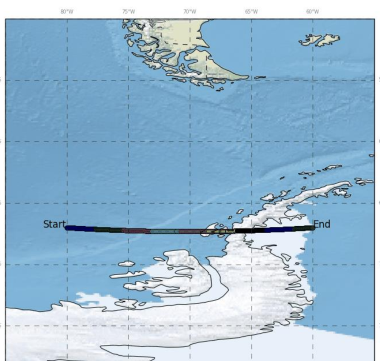
Turbulent / roughness drag

Orographic flow blocking and gravity wave drag

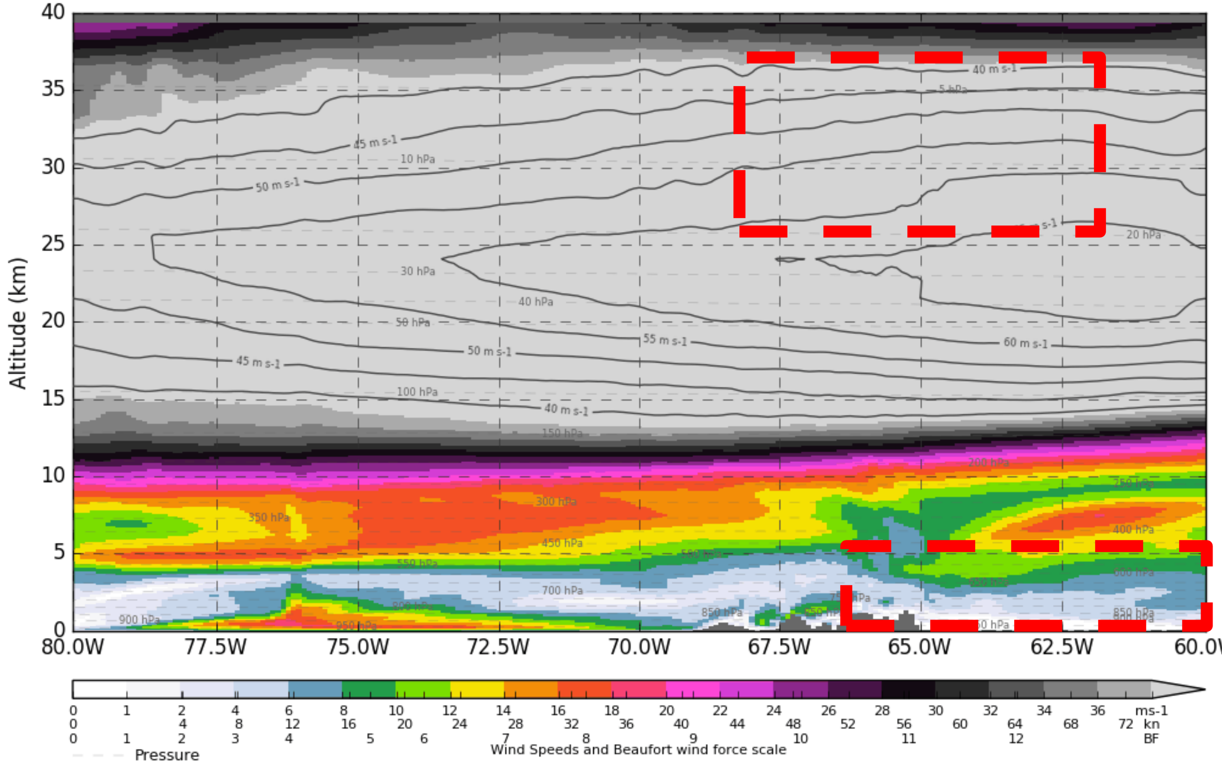


2.5 km model simulation over the Antarctic Peninsula with Met Office Unified Model

Strong surface wind → large amplitude waves

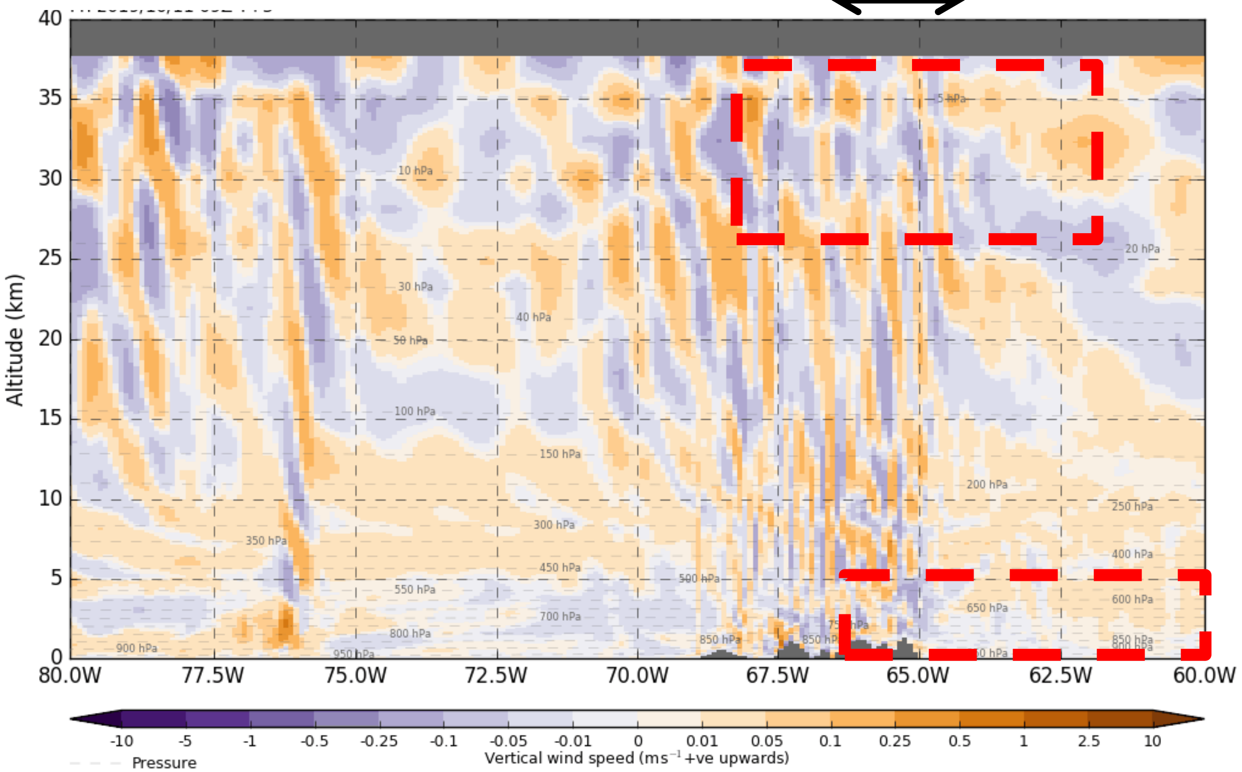
Weak surface wind → flow is blocked

Wind speed



~100 km

Vertical Velocity



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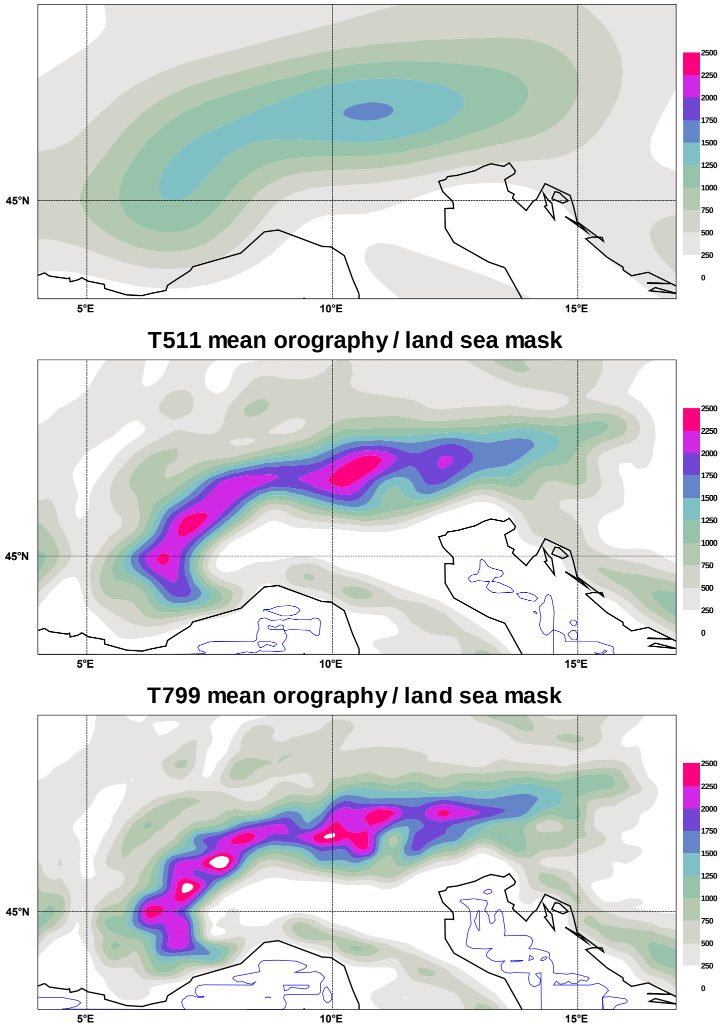
Orography and model resolution

Grid-mean orography

125 km

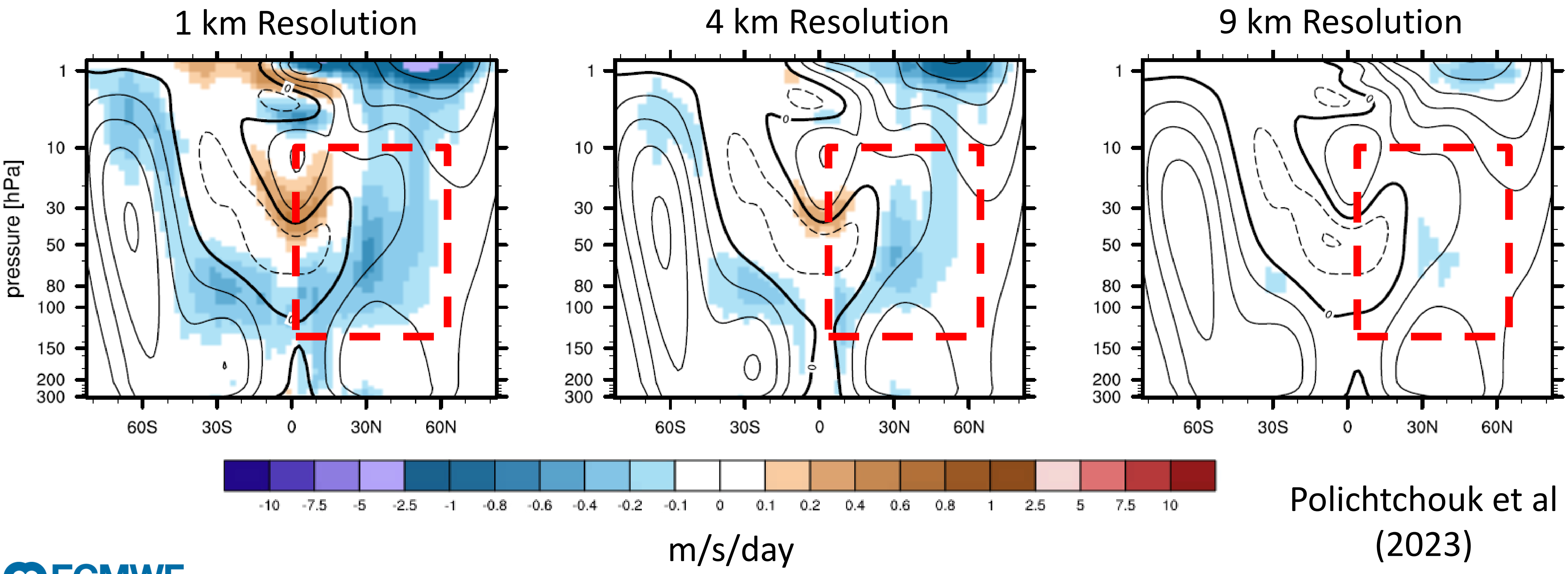
40 km

25 km



Resolved gravity wave drag increases when more mountains are resolved

Plots show zonal mean gravity wave drag from resolved waves in ECMWF IFS global simulations



Polichtchouk et al (2023)

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Orography and model resolution

Grid-mean orography

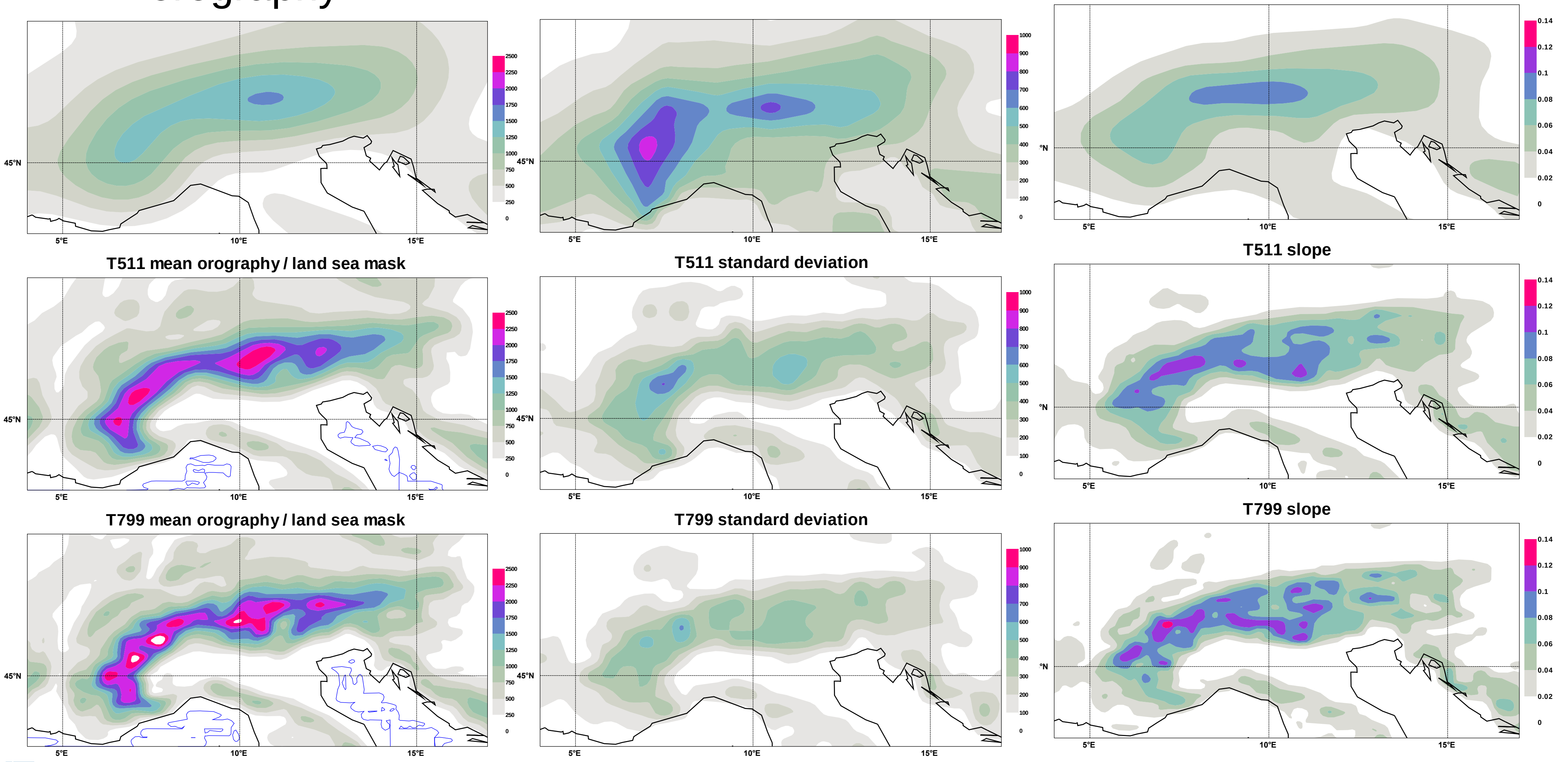
Sub-grid standard deviation

Sub-grid slope

125 km

40 km

25 km



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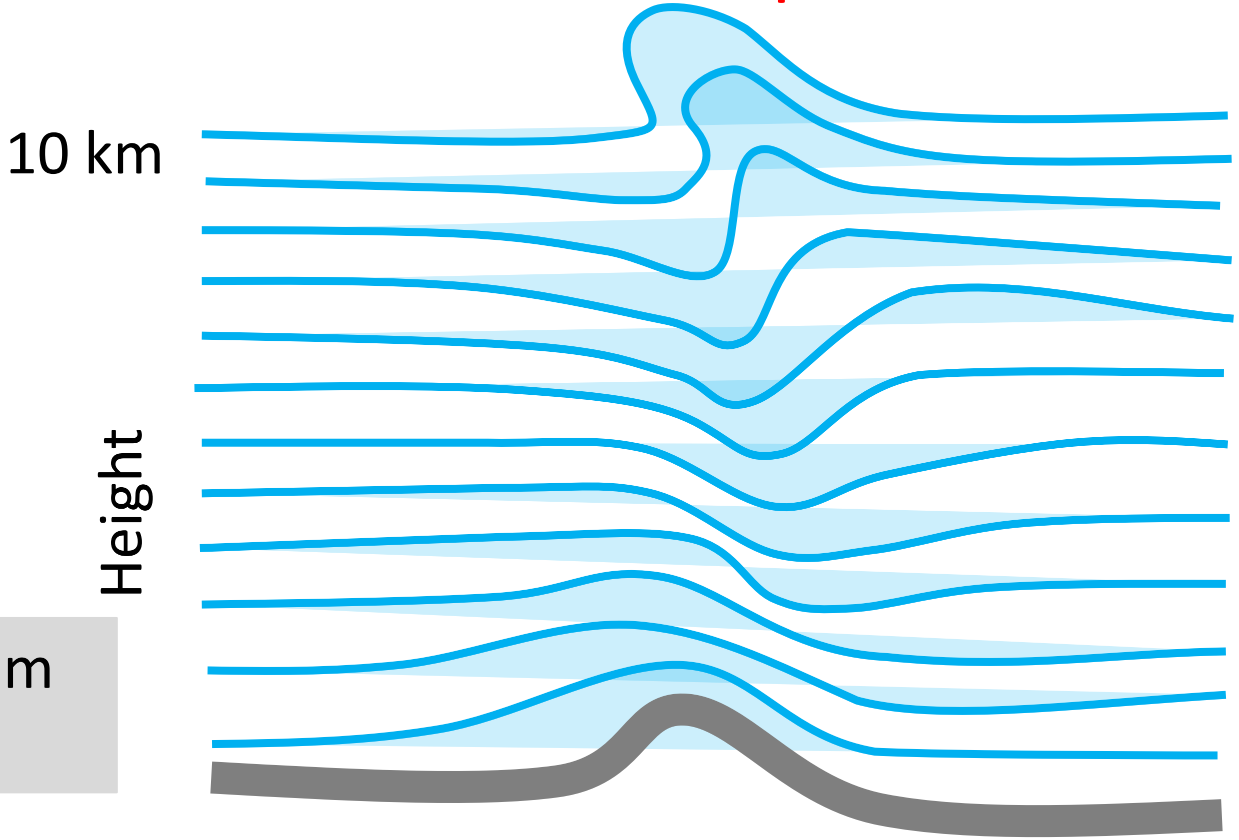
What are orographic gravity waves?

This causes a turbulent drag force on the atmosphere

$$\frac{d\,(U,V)}{dt} = -\frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho\overline{u'w'},\rho\overline{v'w'}\right)$$

Assume that vertical momentum flux dominates



Derivation of gravity wave momentum fluxes

Momentum

$$\frac{Du}{Dt} = -\frac{uw}{r} - 2\Omega w cos\phi + \frac{uv tan\phi}{r} + 2\Omega sin\phi v - \frac{1}{\rho r cos\phi}\frac{\partial p}{\partial \lambda}$$

$$\frac{Dv}{Dt} = -\frac{vw}{r} - \frac{u^2 tan\phi}{r} - 2\Omega sin\phi u - \frac{1}{\rho r}\frac{\partial p}{\partial \phi}$$

$$\frac{Dw}{Dt} = \frac{(u^2+v^2)}{r} + 2\Omega cos\phi u - g - \frac{1}{\rho}\frac{\partial p}{\partial r}$$

Mass Continuity

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \boldsymbol{u} = 0$$

Thermodynamics

$$\frac{D\theta}{Dt} = \frac{\theta}{T}\frac{\dot{Q}}{c_p}$$

Derivation of gravity wave momentum fluxes

Momentum

$$\boldsymbol{u} \cdot \nabla u = -\frac{1}{\rho}\frac{\partial p}{\partial x}$$

$$\boldsymbol{u} \cdot \nabla v = -\frac{1}{\rho}\frac{\partial p}{\partial y}$$

$$\frac{\partial p}{\partial r} = -\rho g$$

Mass Continuity

$$\nabla \cdot \boldsymbol{u} = 0$$

Thermodynamics

$$\frac{D\theta}{Dt} = 0$$

Following approximations are made:

Cartesian coordinates
Shallow atmosphere
No rotation
Adiabatic + incompressible
Hydrostatic
Steady state

Derivation of gravity wave momentum fluxes

Momentum

$$U\frac{\partial u'}{\partial x} + V\frac{\partial u'}{\partial y} + w'\frac{\partial U}{\partial z} = -\frac{1}{\rho}\frac{\partial p'}{\partial x}$$

$$U\frac{\partial v'}{\partial x} + V\frac{\partial v'}{\partial y} + w'\frac{\partial V}{\partial z} = -\frac{1}{\rho}\frac{\partial p'}{\partial y}$$

$$\frac{\partial p'}{\partial z} = -\rho g$$

Mass Continuity

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0$$

Thermodynamics

$$U\frac{\partial \theta'}{\partial x} + V\frac{\partial \theta'}{\partial y} + w'\frac{\partial \Theta}{\partial z} = 0$$

Following approximations are made:

Cartesian coordinates
Shallow atmosphere
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Steady state

Linearised :

$$u = U(z) + u'(x,y,z),\ u'u' \sim 0$$

Derivation of gravity wave momentum fluxes

Momentum

$$U\,\hat{u}ik + V\,\hat{u}il + \hat{w}\frac{\partial U}{\partial z} = -\frac{1}{\rho}\,\hat{p}ik$$

$$U\,\hat{v}ik + V\,\hat{v}il + \hat{w}\frac{\partial V}{\partial z} = -\frac{1}{\rho}\,\hat{p}il$$

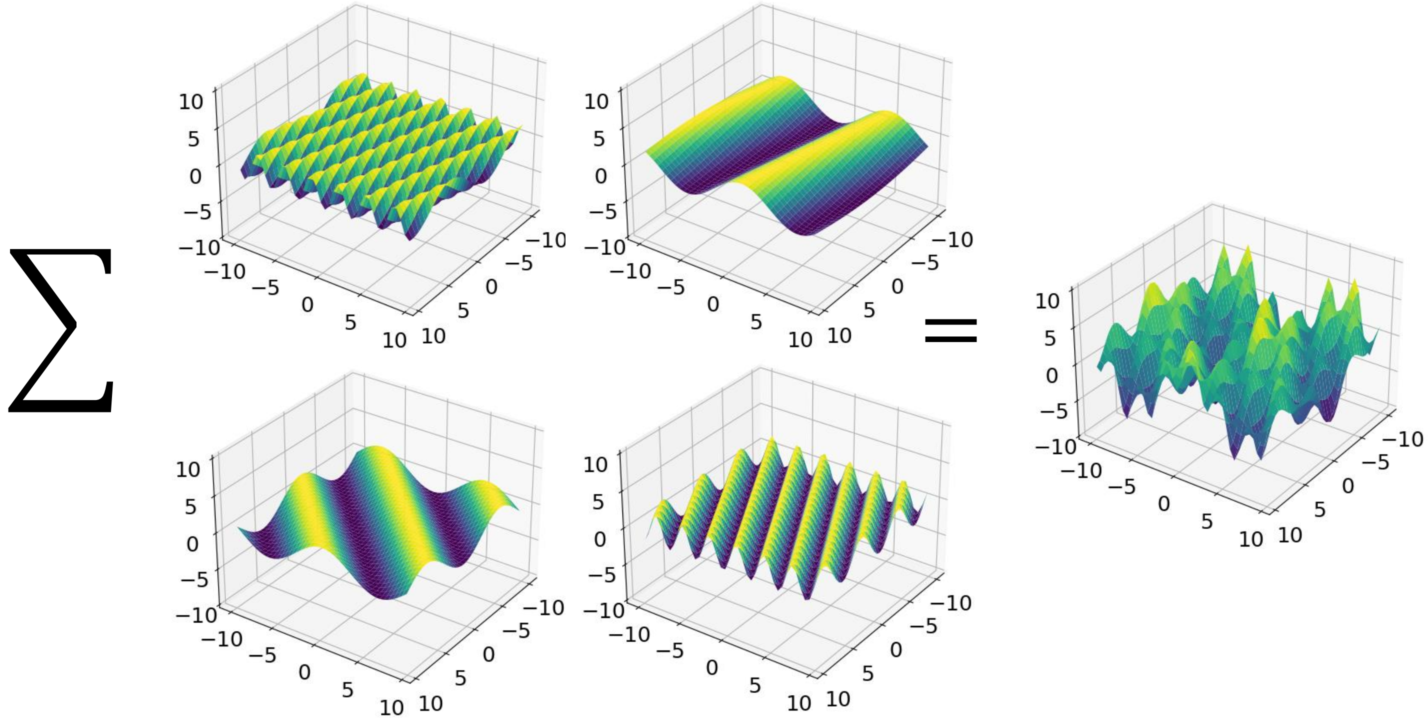
$$\frac{\partial\,\hat{p}}{\partial z} = -\rho g$$

Mass Continuity

$$\hat{u}ik + \hat{v}il + \frac{\partial \hat{w}}{\partial z} = 0$$

Thermodynamics

$$U\,\hat{\theta}ik + V\,\hat{\theta}il + \hat{w}\frac{\partial \Theta}{\partial z} = 0$$



Transform to spectral space:

$$w' \sim \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \hat{w}\,\exp\big(i(kx + ly)\big)\,dk\,dl$$

...

Derivation of gravity wave momentum fluxes

Momentum

$$U\,\hat{u}ik + V\,\hat{u}il + \hat{w}\frac{\partial U}{\partial z} = -\frac{1}{\rho}\,\hat{p}ik$$

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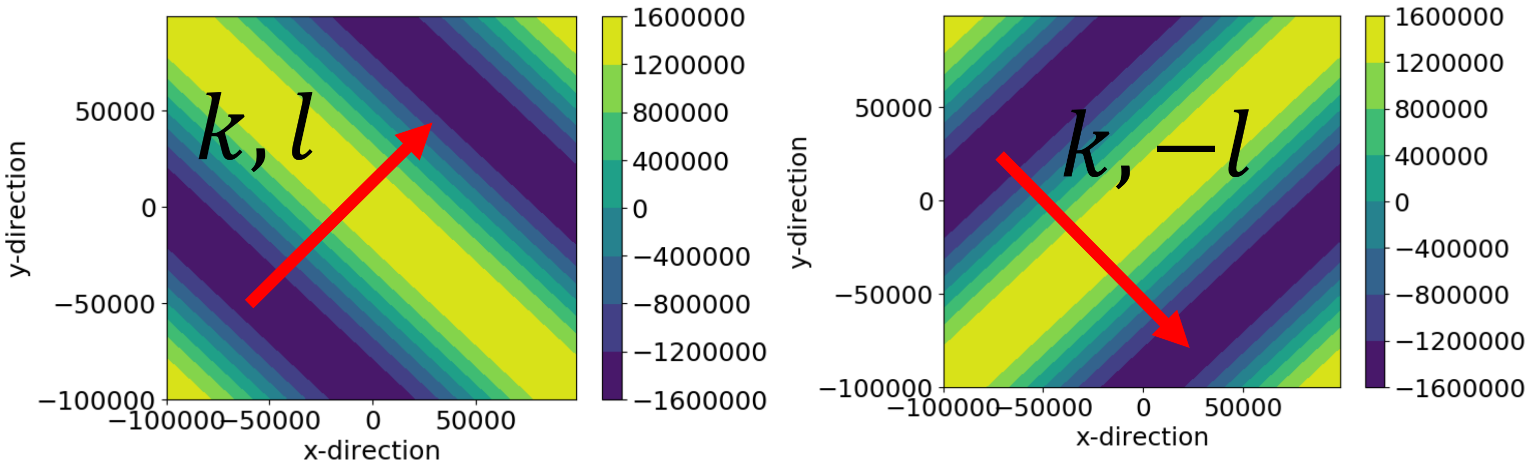

Mass Continuity

$$\hat{u}ik + \hat{v}il + \frac{\partial \hat{w}}{\partial z} = 0$$

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Thermodynamics

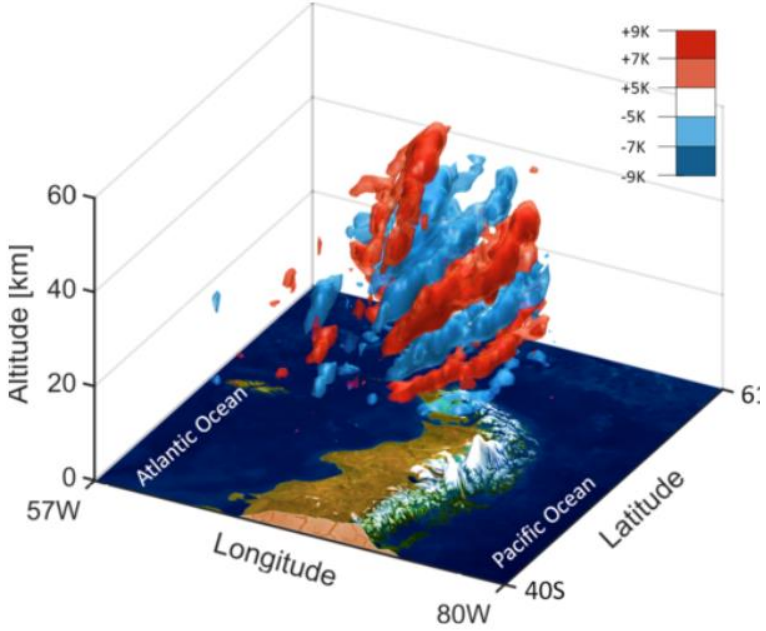
$$U\,\hat{\theta}ik + V\,\hat{\theta}il + \hat{w}\frac{\partial \Theta}{\partial z} = 0$$

Combine equations:

$$\frac{\partial^2\,\hat{w}}{\partial z^2} + \left[\frac{N^2(k^2+l^2)}{(Uk+Vl)^2}\right]\hat{w} = 0$$

Solution:

$$\hat{w} = \hat{w}_0 \exp(imz)\,, m^2 = \left[\frac{N^2(k^2+l^2)}{(Uk+Vl)^2}\right]$$



Satellite derived image of temperature perturbations from a gravity wave

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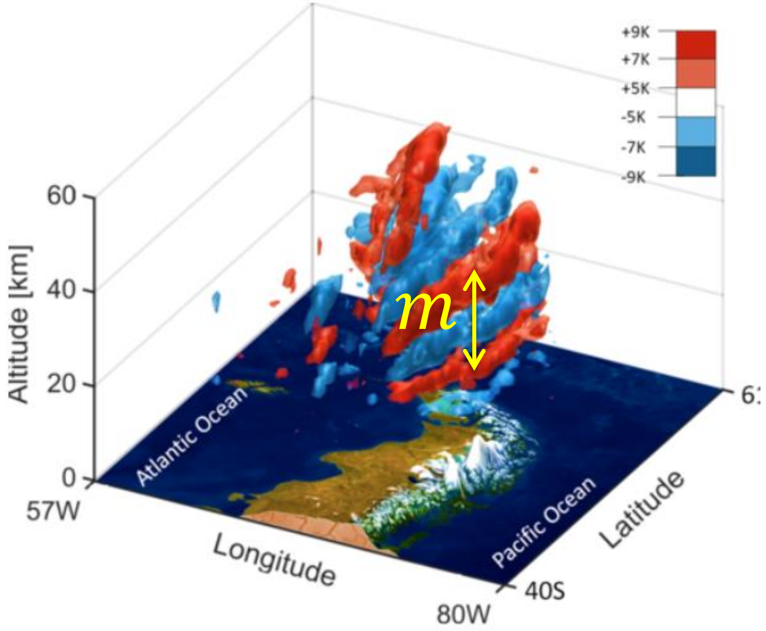

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Solution:

$$\hat{w} = \hat{w}_0 \exp(imz)\,,\, m^2 = \left[\frac{N^2(k^2+l^2)}{(Uk+Vl)^2}\right]$$

At surface the flow follows the mountain:

$$w'(x,y,0) = \boldsymbol{U}\cdot\nabla h$$

Surface vertical velocity:

$$\hat{w}_0 \sim i(Uk+Vl)\hat{h}$$

Derivation of gravity wave momentum fluxes

Momentum

$$U\,\hat{u}ik + V\,\hat{u}il + \hat{w}\frac{\partial U}{\partial z} = -\frac{1}{\rho}\,\hat{p}ik$$

$$U\,\hat{v}ik + V\,\hat{v}il + \hat{w}\frac{\partial V}{\partial z} = -\frac{1}{\rho}\,\hat{p}il$$

$$\frac{\partial\,\hat{p}}{\partial z} = -\rho g$$

Mass Continuity

$$\hat{u}ik + \hat{v}il + \frac{\partial \hat{w}}{\partial z} = 0$$

Thermodynamics

$$U\,\hat{\theta}ik + V\,\hat{\theta}il + \hat{w}\frac{\partial \Theta}{\partial z} = 0$$

$$\frac{d\,(U,V)}{dt} = -\frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho\overline{u'w'}, \rho\overline{v'w'}\right)$$

Assume that vertical momentum flux dominates

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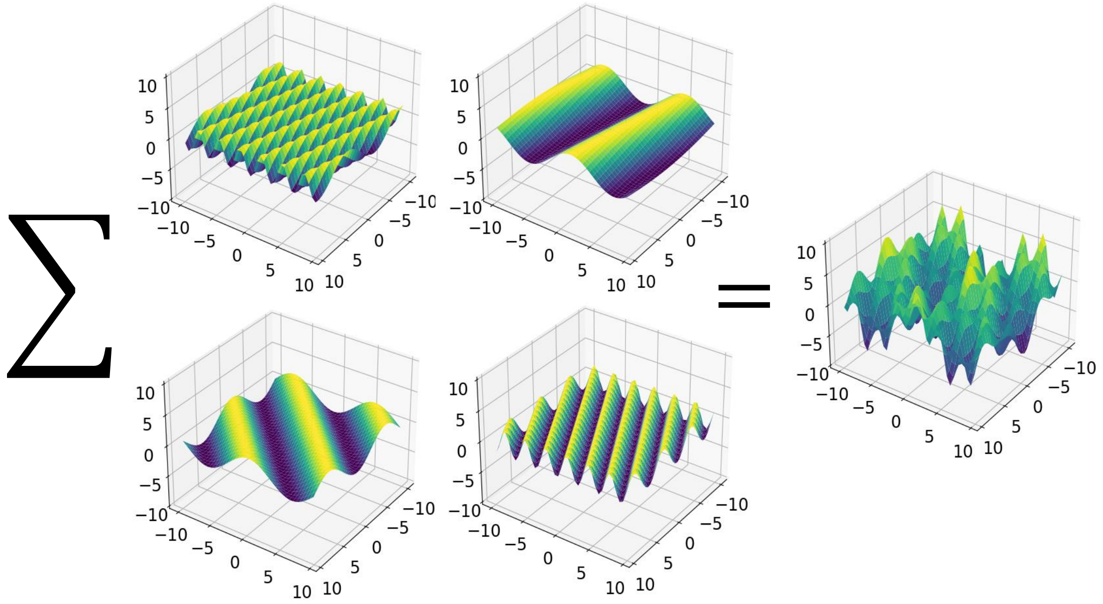
Expression for the surface momentum flux is given by mountain height

Linear hydrostatic gravity wave surface stress in spectral space:



$$\tau_x, \tau_y = \left(\rho_0 \overline{u'w'}, \rho_0 \overline{v'w'}\right) = \left(\rho_0 \overline{\hat{u}\hat{w}^*}, \rho_0 \overline{\hat{v}\hat{w}^*}\right)$$

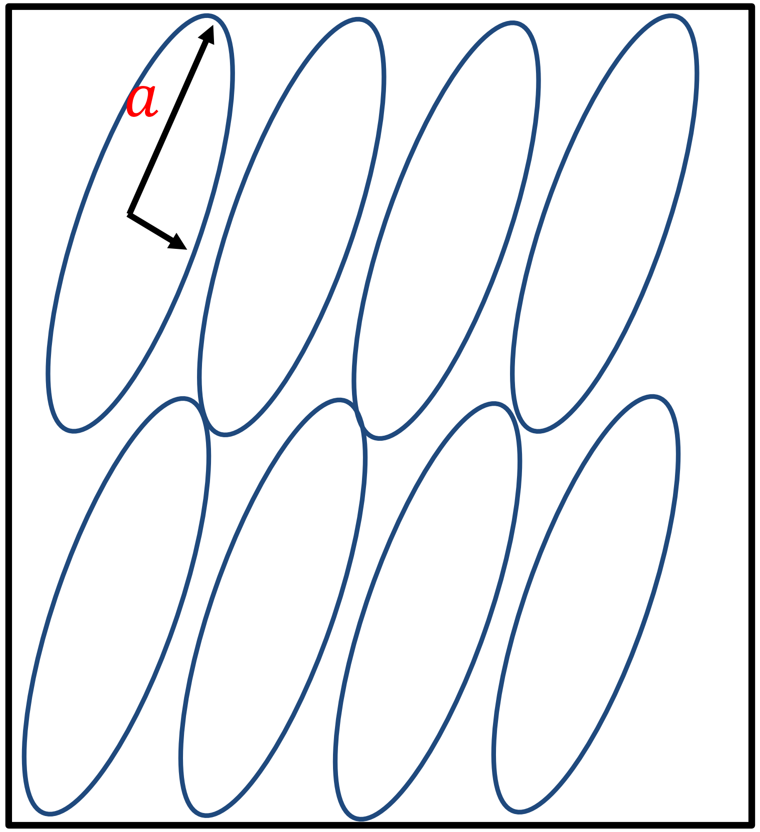
$$= A^{-1} \rho_0 N_o 4\pi^2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{(k,l)}{K} (U_0 k + V_0 l) \left|\hat{h}\right|^2 dk\, dl$$

ρ_0 = Density
N_0 = Stability
k, l = zonal and meridional wavenumber
$K = (k+l)^{\frac{1}{2}}$
A = Area
U_0, V_0 = Surface wind

$\left|\hat{h}\right|$ = Spectral transform of mountain height

Mountains are assumed to be ellipses

Grid-box



Linear hydrostatic gravity wave surface stress:

$$\tau_x, \tau_y = A^{-1}\rho_0 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (u', v')w' dxdy$$

$$= A^{-1}\rho_0 N_o 4\pi^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{k,l}{K}(U_0 k + V_0 l)\left|\hat{h}\right|^2 dk\, dl$$

$|\hat{h}|$ = Fourier transform of surface height

Assume elliptical mountains (Lott and Miller 1997, Phillips 1984):

$$\boldsymbol{\tau} = G\rho N \frac{1}{4a} h_{eff}^2 (\boldsymbol{UD})$$

$$\mathrm{h}_{eff} = min\left(h, \frac{U}{NF_c}\right)$$

Mountain half-width

Effective mountain height

Mountain anisotropy

Parametrizing flow blocking drag

Gravity wave drag:

$$\boldsymbol{\tau} = G\rho N \frac{1}{4a} h_{eff}^{2}(\boldsymbol{UD})$$

Mountain half-width
Effective mountain height
Mountain anisotropy

$$\mathrm{h}_{eff} = min\left(h, \frac{U}{NF_c}\right)$$

$$h = n\sigma$$

σ = standard deviation of subgrid orography

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Parametrizing flow blocking drag

Gravity wave drag:

$$\boldsymbol{\tau} = G\rho N \frac{1}{4a} h_{eff}^2 (\boldsymbol{U}\boldsymbol{D})$$

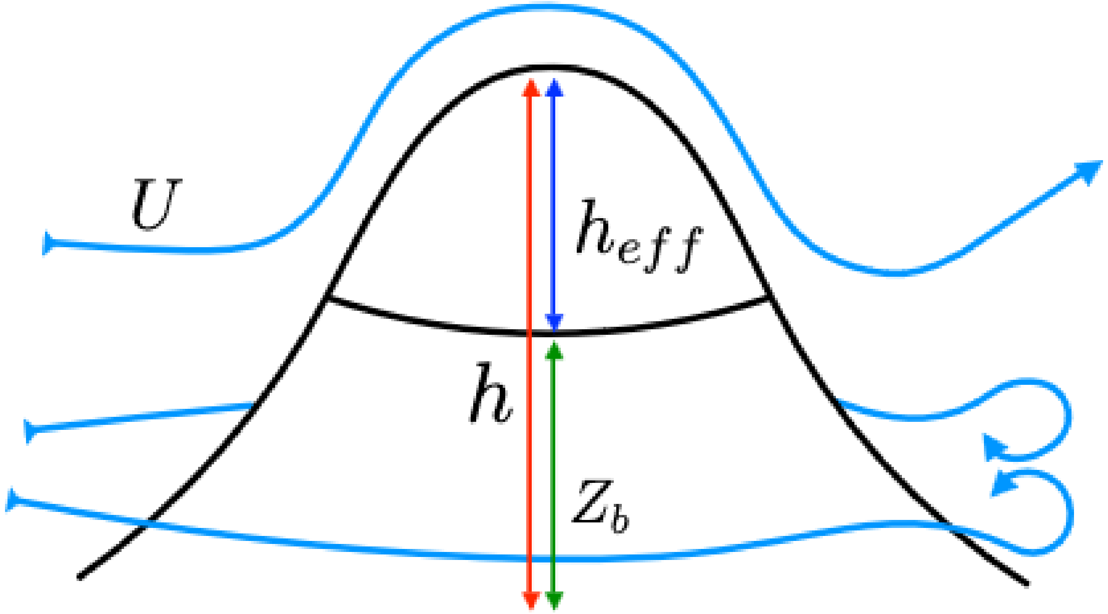
Flow blocking drag:

$$\frac{d\boldsymbol{U}}{dt} \sim -C_d \rho |U| \boldsymbol{U} max\left(1-\frac{1}{r},0\right)\frac{1}{a}\left(\frac{Z_{blk}-z}{z+\sigma}\right)^{\frac{1}{2}} \boldsymbol{D}$$

Mountain half-width
Effective mountain height
Mountain anisotropy
Mountain aspect ratio
Blocking depth

$$\mathrm{h}_{eff} = min\left(h, \frac{U}{NF_c}\right)$$

$$Z_{blk} = h - h_{eff}$$



$$h = n\sigma$$

σ = standard deviation of subgrid orography

Parametrizing flow blocking drag

Gravity wave drag:

$$\boldsymbol{\tau} = G\rho N \frac{1}{4a} h_{eff}^2 (\boldsymbol{U}\boldsymbol{D})$$

Flow blocking drag:

$$\frac{d\boldsymbol{U}}{dt} \sim -C_d \rho |U| \boldsymbol{U} max\left(1 - \frac{1}{r}, 0\right) \frac{1}{a} \left(\frac{Z_{blk} - z}{z + \sigma}\right)^{\frac{1}{2}} \boldsymbol{D}$$

Mountain half-width

Effective mountain height

Mountain anisotropy

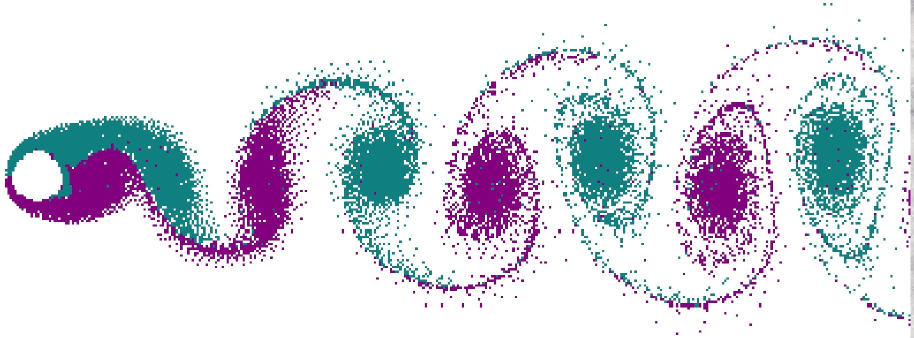
Mountain aspect ratio

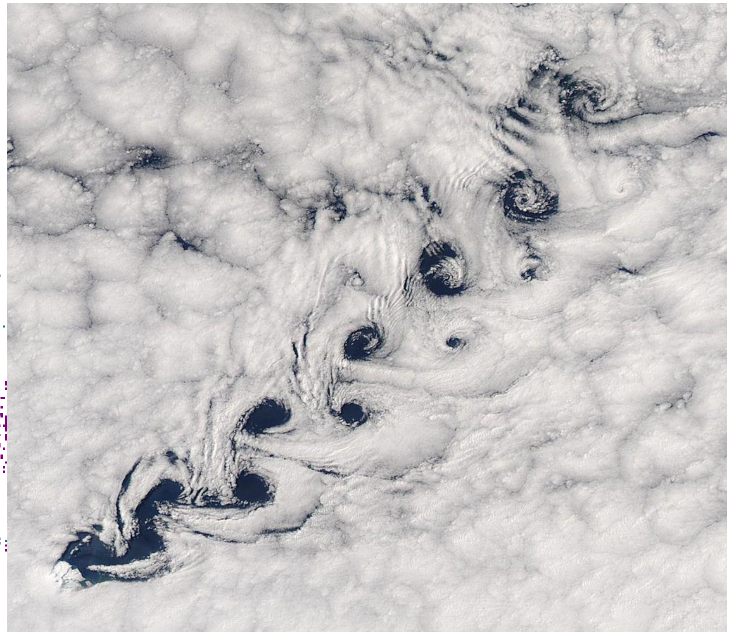
Blocking depth

$$\mathrm{h}_{eff} = min\left(h, \frac{U}{NF_c}\right)$$

$$Z_{blk} = h - h_{eff}$$

Flow past a bluff body:



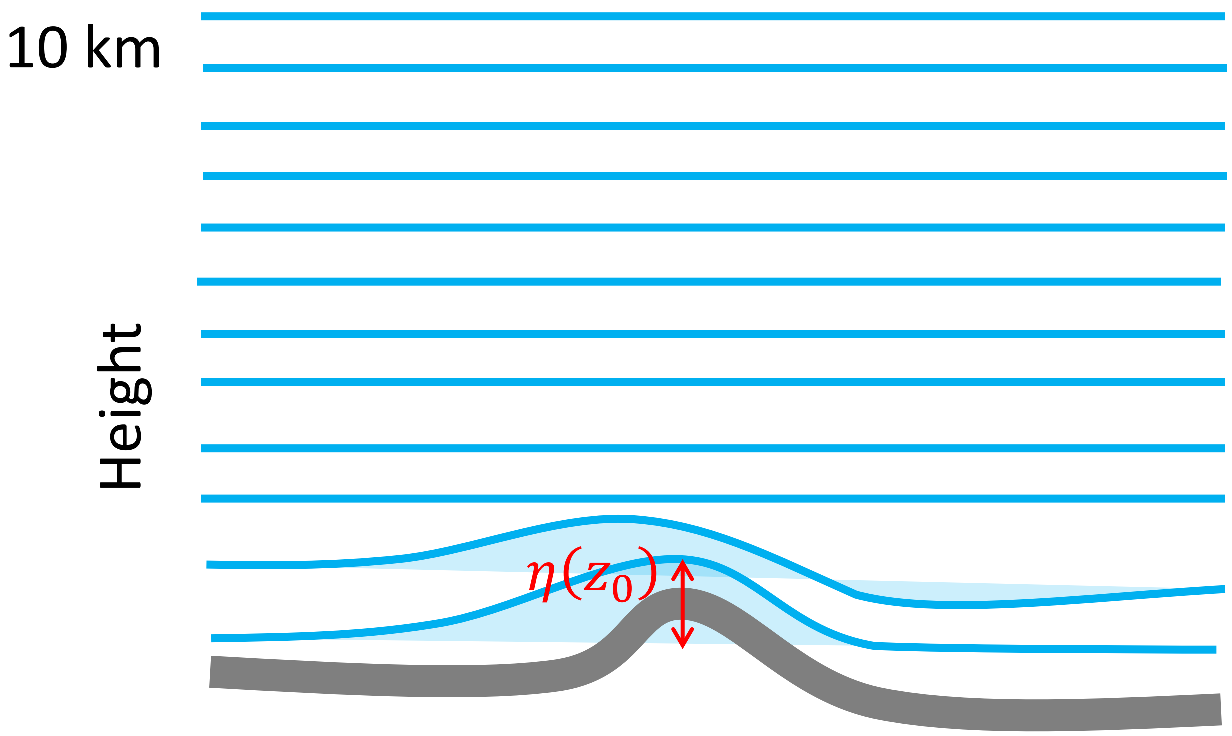


$$h = n\sigma$$

σ = standard deviation of subgrid orography

Parametrizing gravity wave propagation and breaking

Incoming wind forces air over mountain



$\eta(z_0) = h_{eff}$, wave amplitude at surface

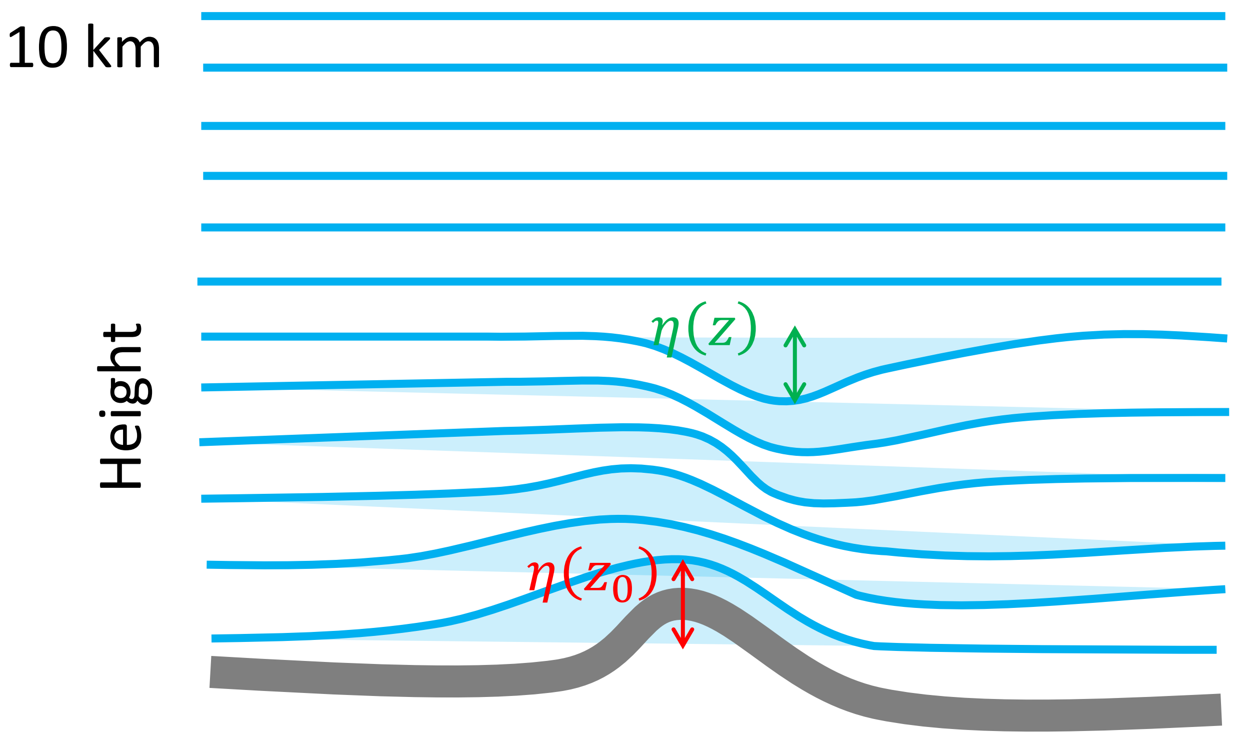
Parametrizing gravity wave propagation and breaking

$$\eta(z) = \eta(z-1)\sqrt{\frac{\rho(z-1)N(z-1)U(z-1)}{\rho(z)N(z)U(z)}}$$

$\eta(z)$ = Amplitude at particular height

U = wind in direction of wave vector
N = Brunt-Vaisala frequency (stability)
ρ = density

A vertically propagating wave is generated



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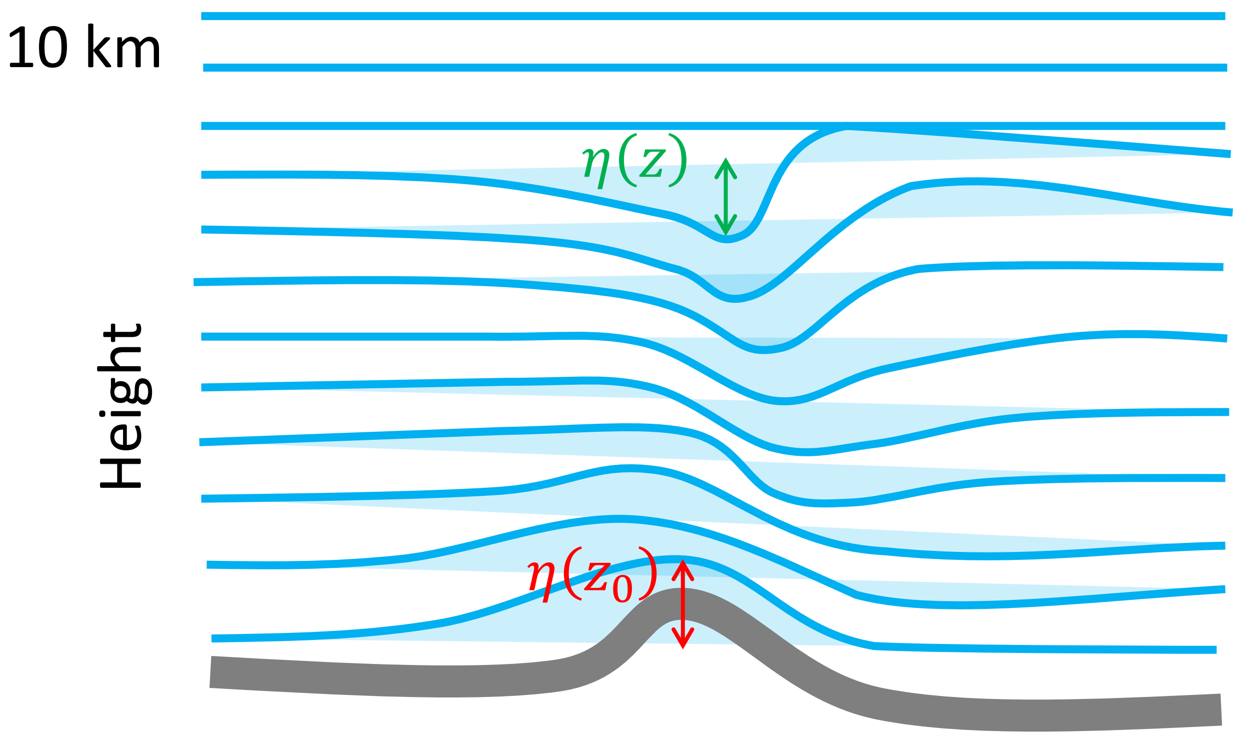
As density decreases with height, the amplitude grows



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Parametrizing gravity wave propagation and breaking

$$\eta(z) = \eta(z-1)\sqrt{\frac{\rho(z-1)N(z-1)U(z-1)}{\rho(z)N(z)U(z)}}$$

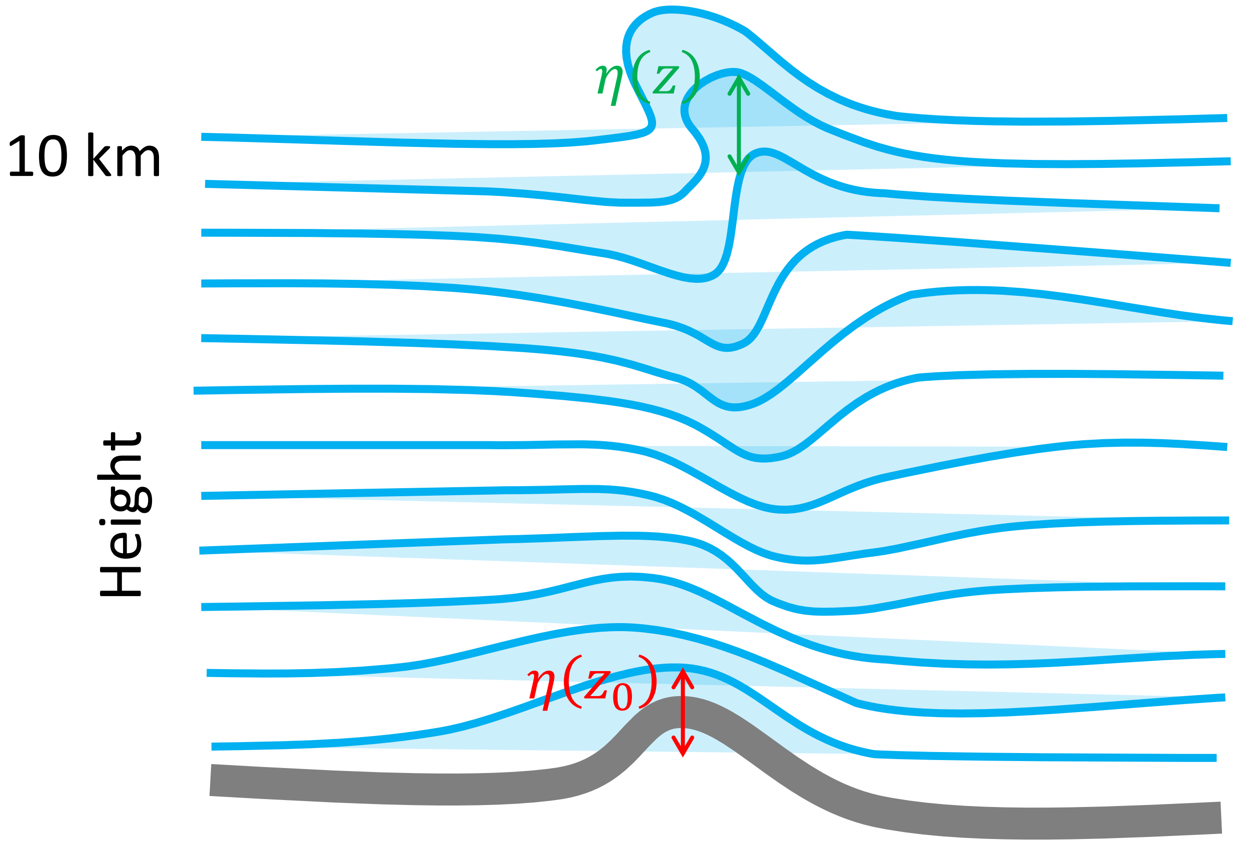
As density decreases with height, the amplitude grows, until the wave breaks

$\eta(z)$ = Amplitude at particular height

U = wind in direction of wave vector
N = Brunt-Vaisala frequency (stability)
ρ = density

When $\mathrm{Ri}\left\{\frac{1-\left(\frac{N\eta}{U}\right)}{\left(1+\mathrm{Ri}^{\frac{1}{2}}\left(\frac{N\eta}{U}\right)^{2}\right)^{2}}\right\} > \mathrm{Ri}_{crit}$,

η is reduced



$\eta(z_0) = h_{eff}$, wave amplitude at surface

ECMWF

Parametrizing gravity wave propagation and breaking

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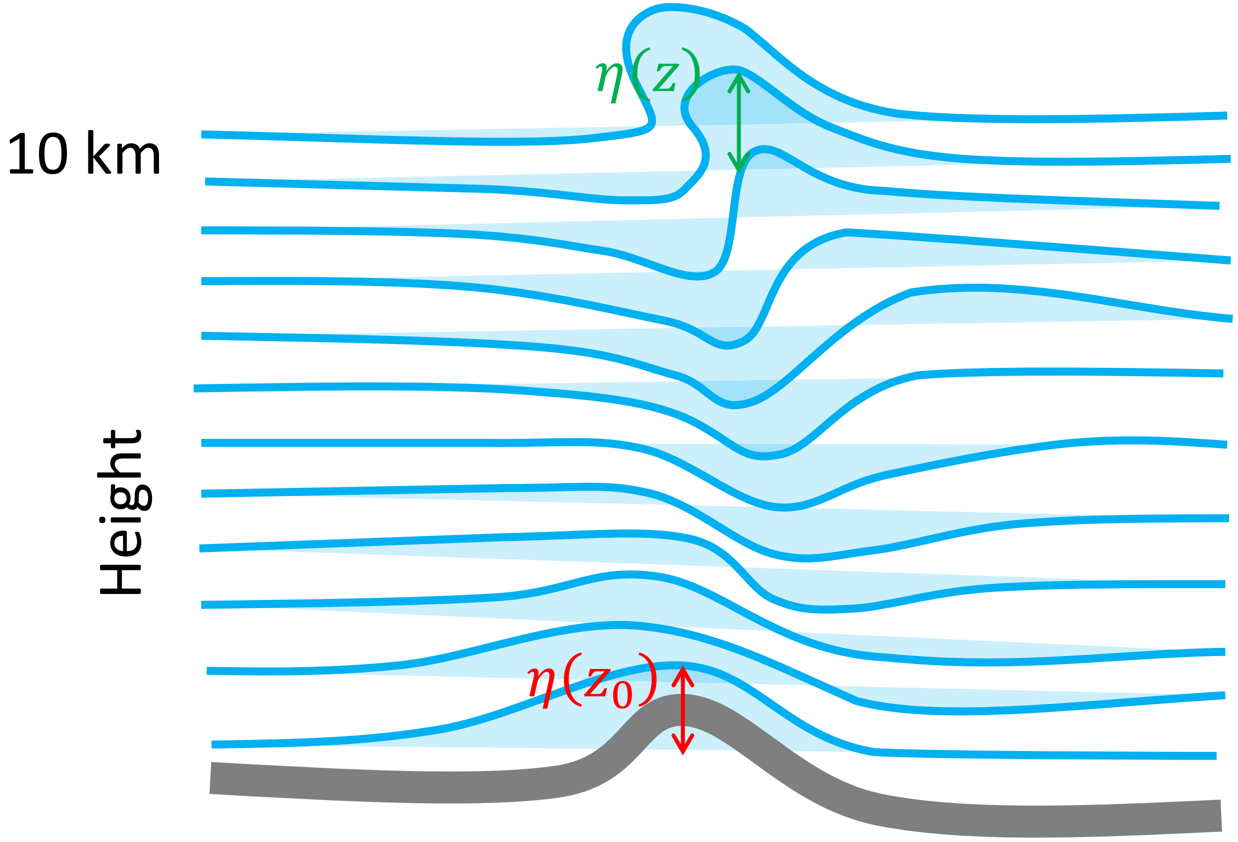

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η is reduced

$$\frac{d\,(U,V)}{dt} = -\frac{1}{\rho}\frac{\partial}{\partial z}(\tau_x, \tau_y)$$

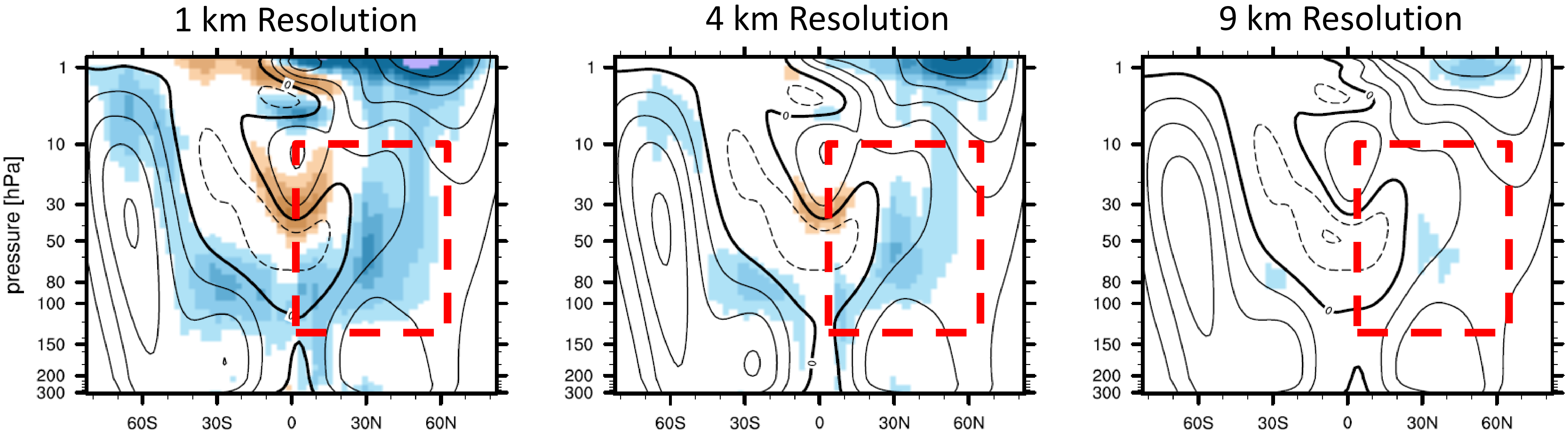
$$\tau_x, \tau_y(z) \propto \eta^2(z)$$

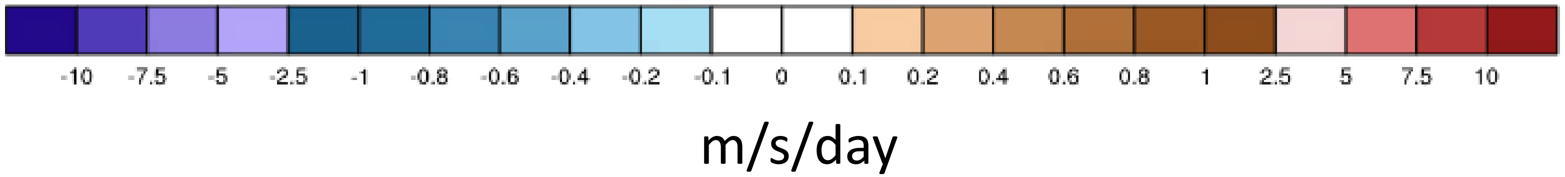
$\eta(z_0) = h_{eff}$, wave amplitude at surface

ECMWF

Resolved gravity wave drag increases when more mountains are resolved

Plots show zonal mean gravity wave drag from resolved waves in ECMWF IFS global simulations

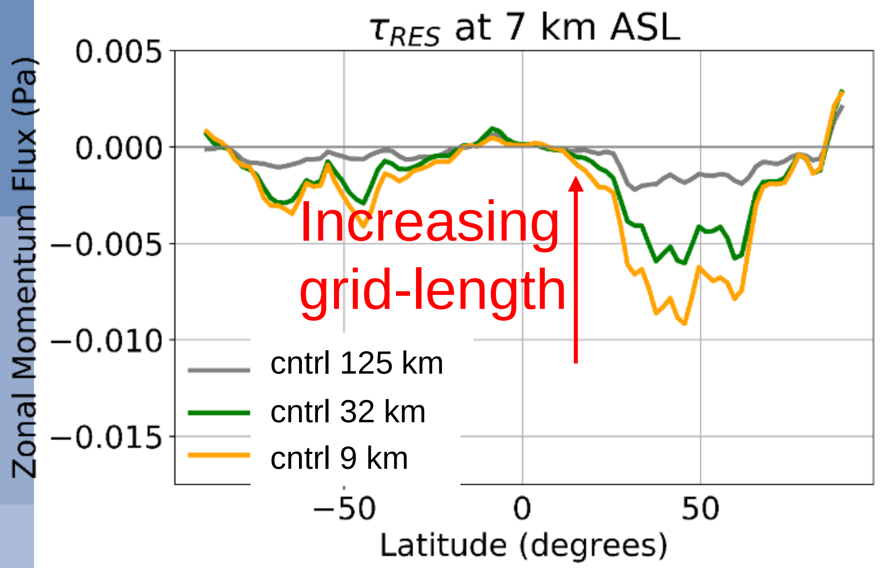




Polichtchouk et al (2023)

Resolution sensitivity of gravity wave drag parametrization

Resolved GW momentum flux



Resolved GW momentum flux decreases at larger grid-lengths

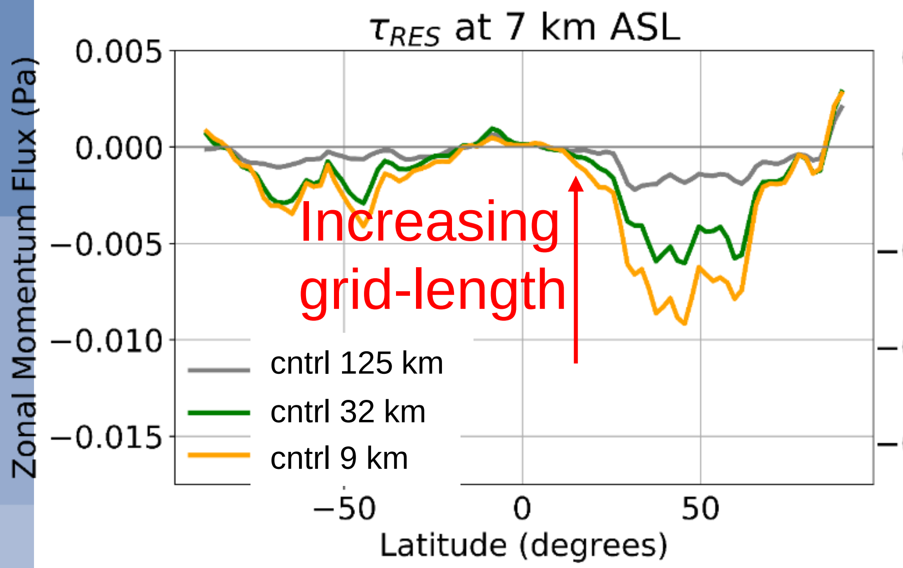
Plots show: zonal mean zonal gravity wave momentum fluxes at 7 km above sea level

van Niekerk et al (2021)

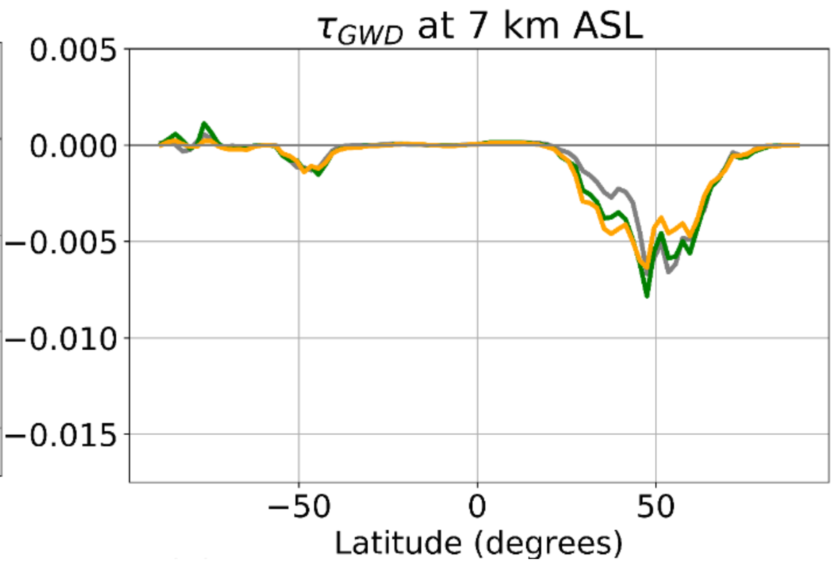
ECMWF

Resolution sensitivity of gravity wave drag parametrization

Resolved GW momentum flux



Parametrized GW momentum flux



Resolved GW momentum flux decreases at larger grid-lengths

Parametrized GW momentum flux is almost insensitive to grid-length

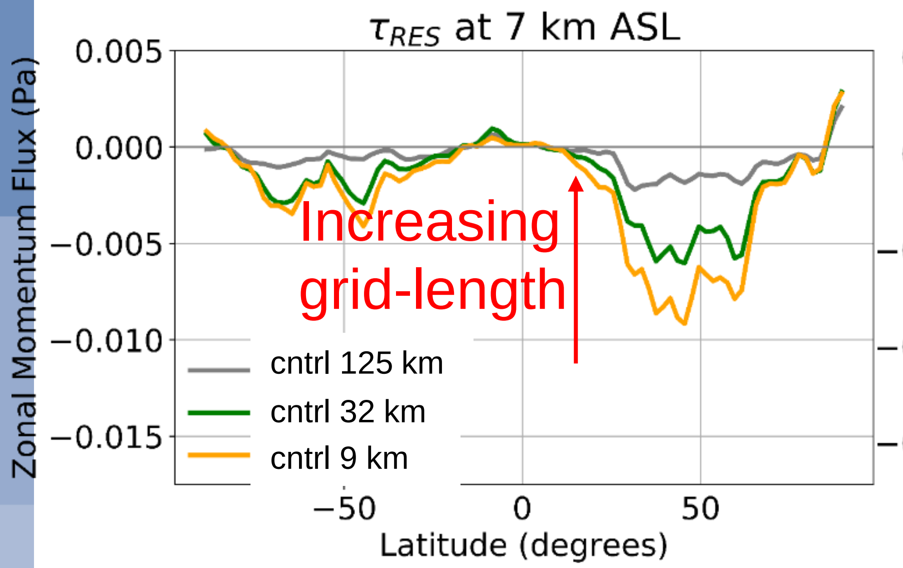
Plots show: zonal mean zonal gravity wave momentum fluxes at 7 km above sea level

van Niekerk et al (2021)

ECMWF

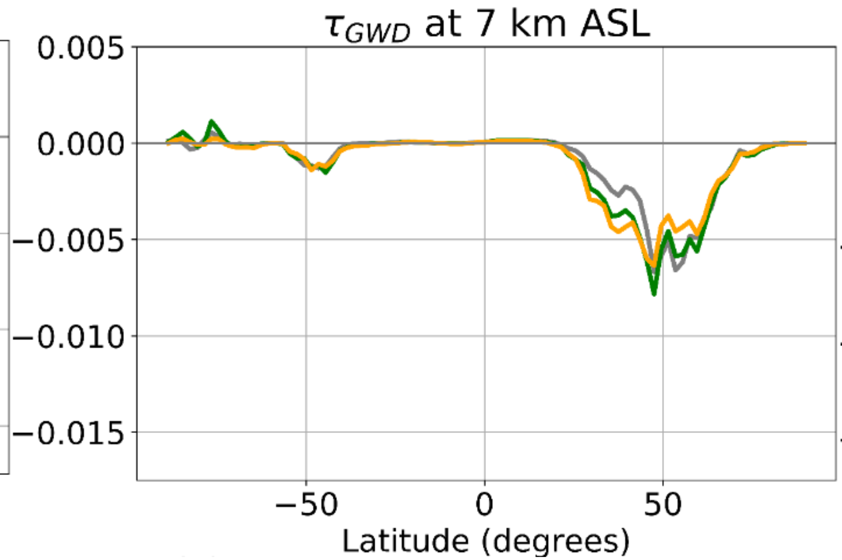
Resolution sensitivity of gravity wave drag parametrization

Resolved GW momentum flux



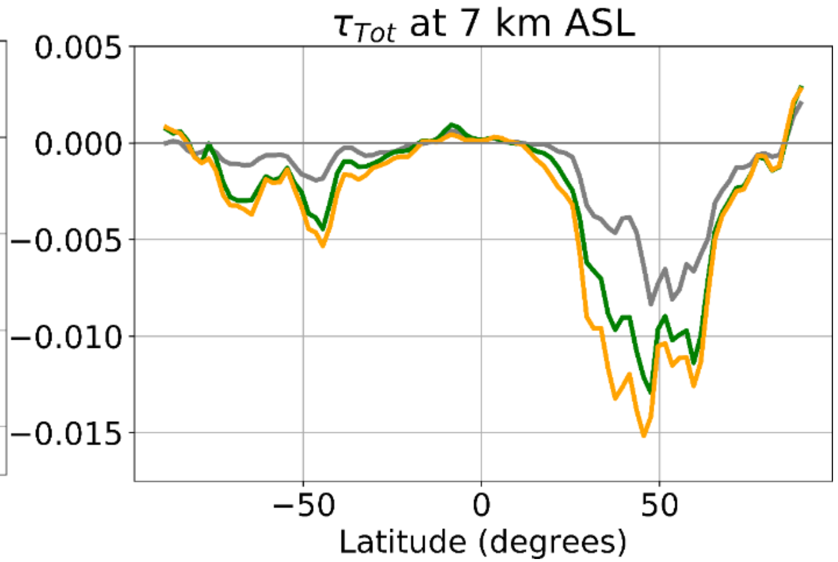
Resolved GW momentum flux decreases at larger grid-lengths

Parametrized GW momentum flux



Parametrized GW momentum flux is almost insensitive to grid-length

Total GW momentum flux



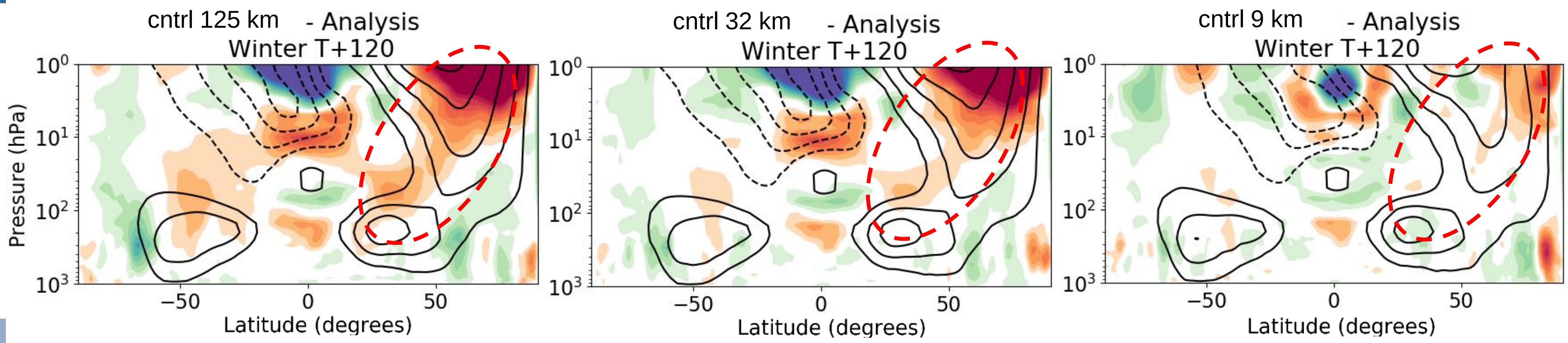
Total GW momentum flux is significantly underestimated at large grid-lengths

Plots show: zonal mean zonal gravity wave momentum fluxes at 7 km above sea level

van Niekerk et al (2021)

ECMWF

Resolution sensitivity of gravity wave drag parametrization



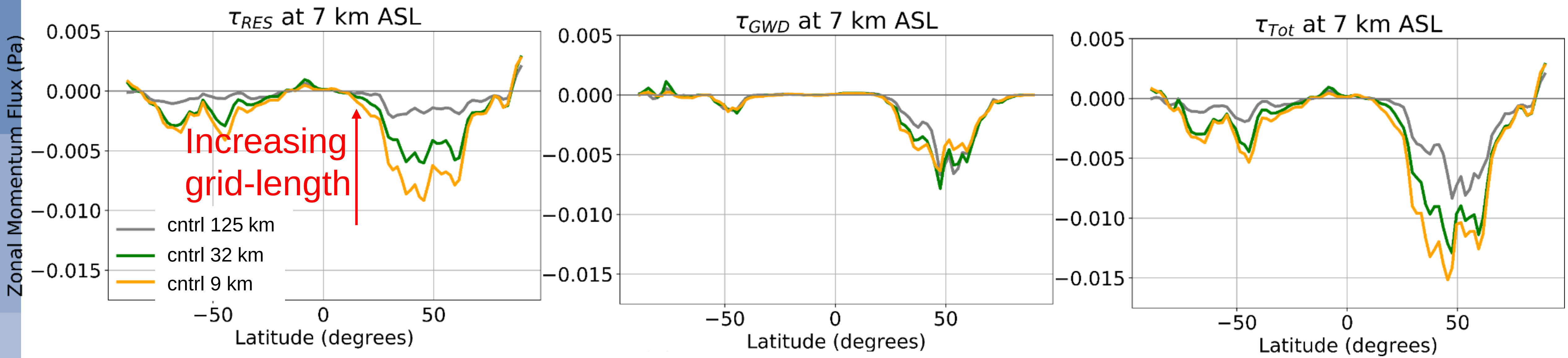
Plots show: zonal mean zonal wind error relative to analysis at lead time of 5 days

van Niekerk et al (2021)

ECMWF

Resolution sensitivity of gravity wave drag parametrization

Resolved GW momentum flux | Parametrized GW momentum flux | Total GW momentum flux



Resolved GW momentum flux decreases at larger grid-lengths

Parametrized GW momentum flux is almost insensitive to grid-length

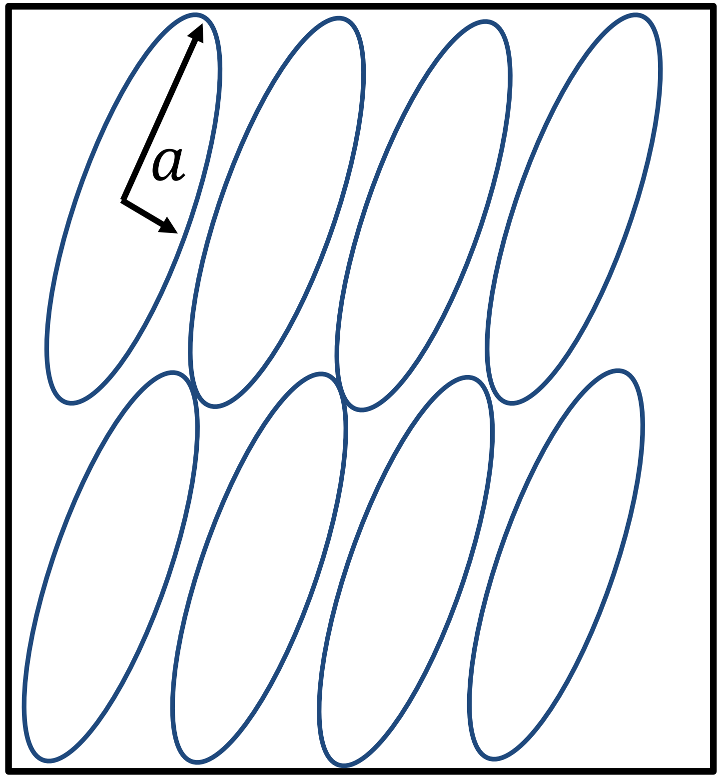
Total GW momentum flux is significantly underestimated at large grid-lengths

Plots show: zonal mean zonal gravity wave momentum fluxes at 7 km above sea level

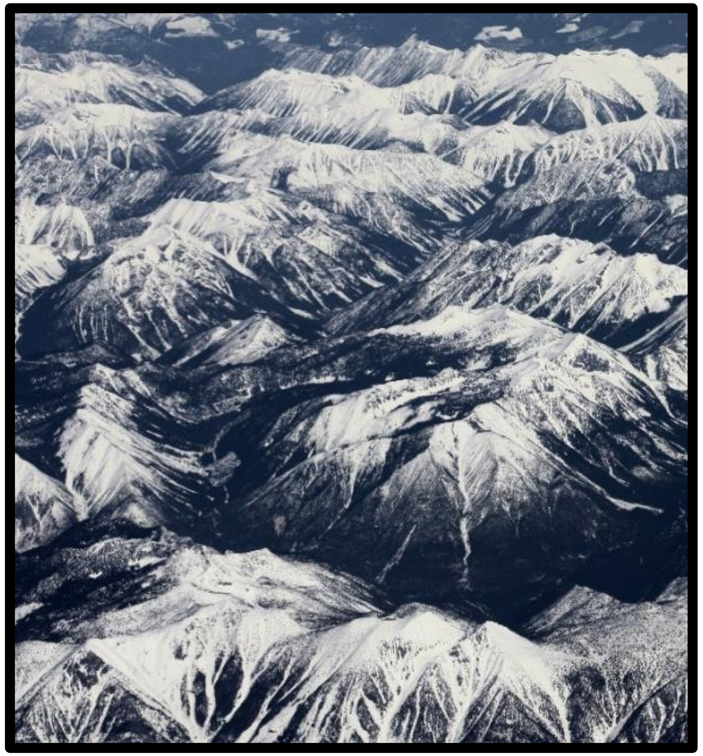
van Niekerk et al (2021)

ECMWF

Parametrization

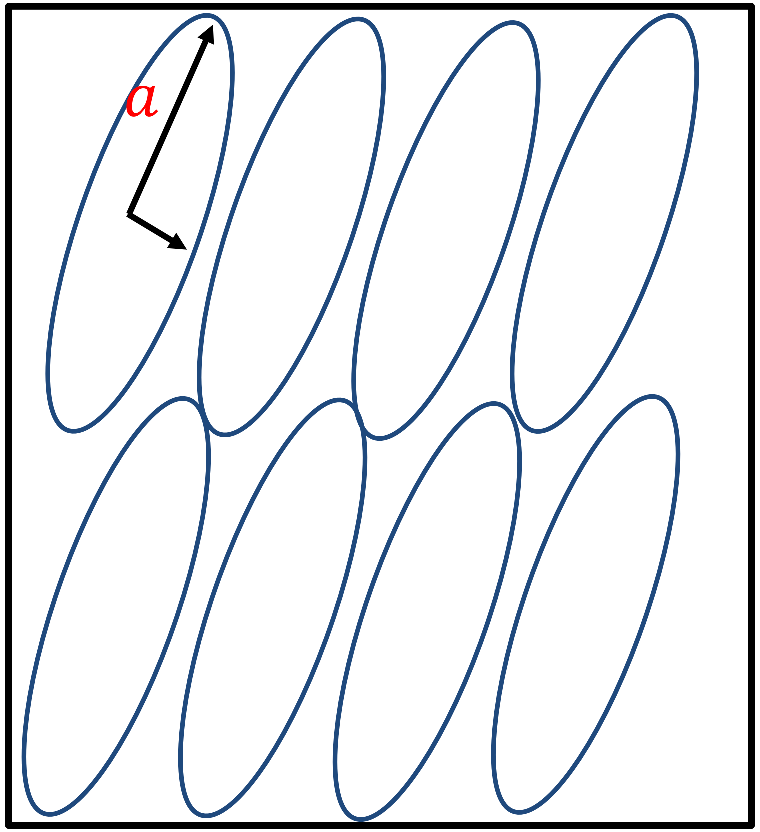


Reality



Mountains are assumed to be ellipses

Grid-box



Linear hydrostatic gravity wave surface stress:

$$\tau_x, \tau_y = A^{-1}\rho_0 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (u', v')w'\,dx\,dy$$

$$= A^{-1}\rho_0 N_o 4\pi^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{k,l}{K}(U_0 k + V_0 l)\left|\hat{h}\right|^2 dk\,dl$$

$\left|\hat{h}\right|$ = Fourier transform of surface height

Assume elliptical mountains (Lott and Miller 1997, Phillips 1984):

$$\boldsymbol{\tau} = G\rho N \frac{1}{4a} h_{eff}^2 (\boldsymbol{U}\boldsymbol{D})$$

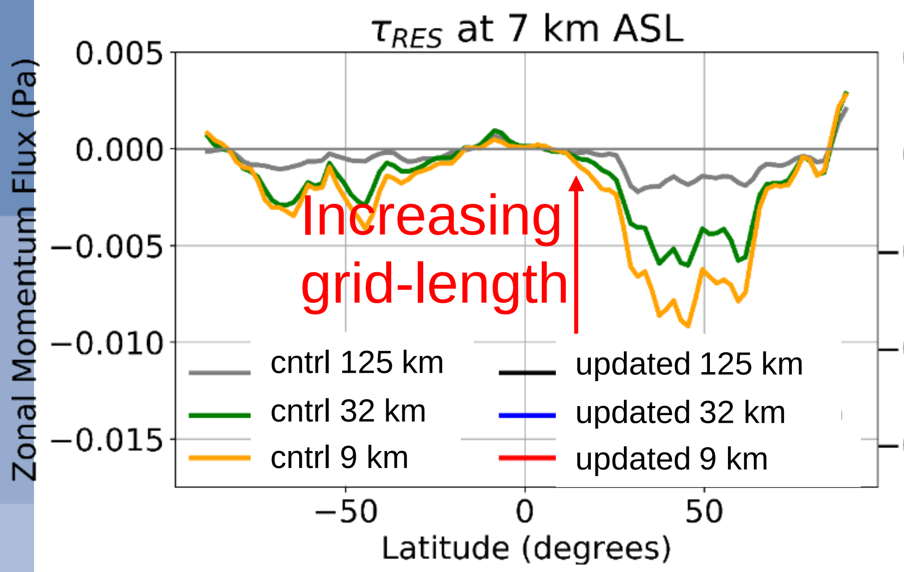
Mountain half-width
Effective mountain height
Mountain anisotropy

$$\mathrm{h}_{eff} = min\left(h, \frac{U}{NF_c}\right)$$

ECMWF

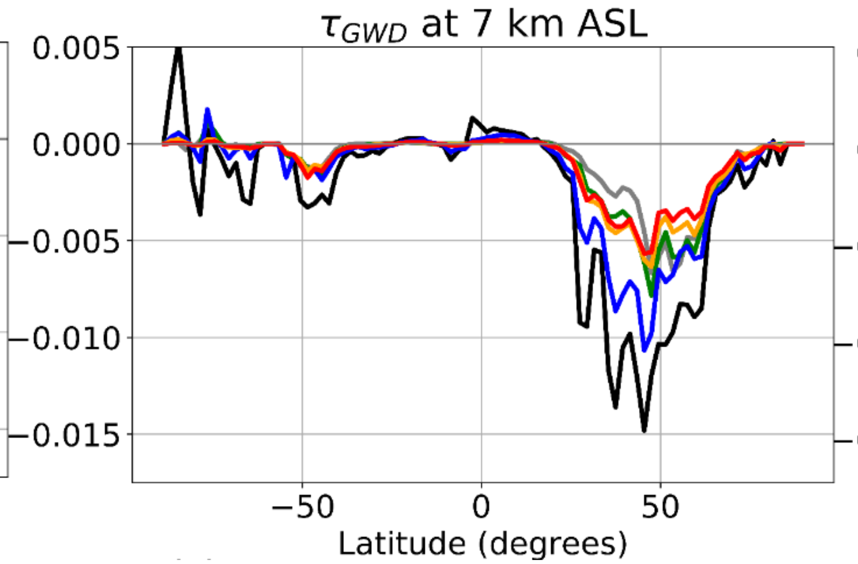
Resolution sensitivity of gravity wave drag parametrization

Resolved GW momentum flux



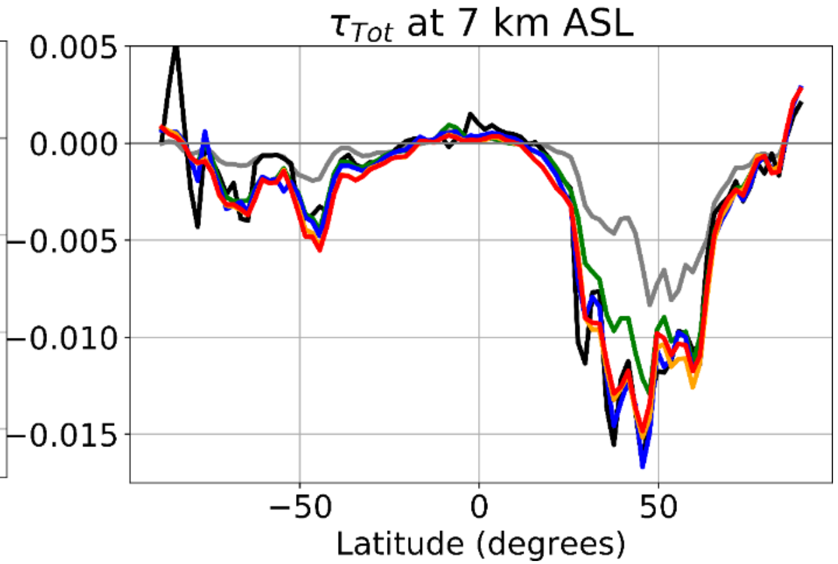
Resolved GW momentum flux decreases at larger grid-lengths

Parametrized GW momentum flux



Parametrized GW momentum flux increases at larger grid-length

Total GW momentum flux



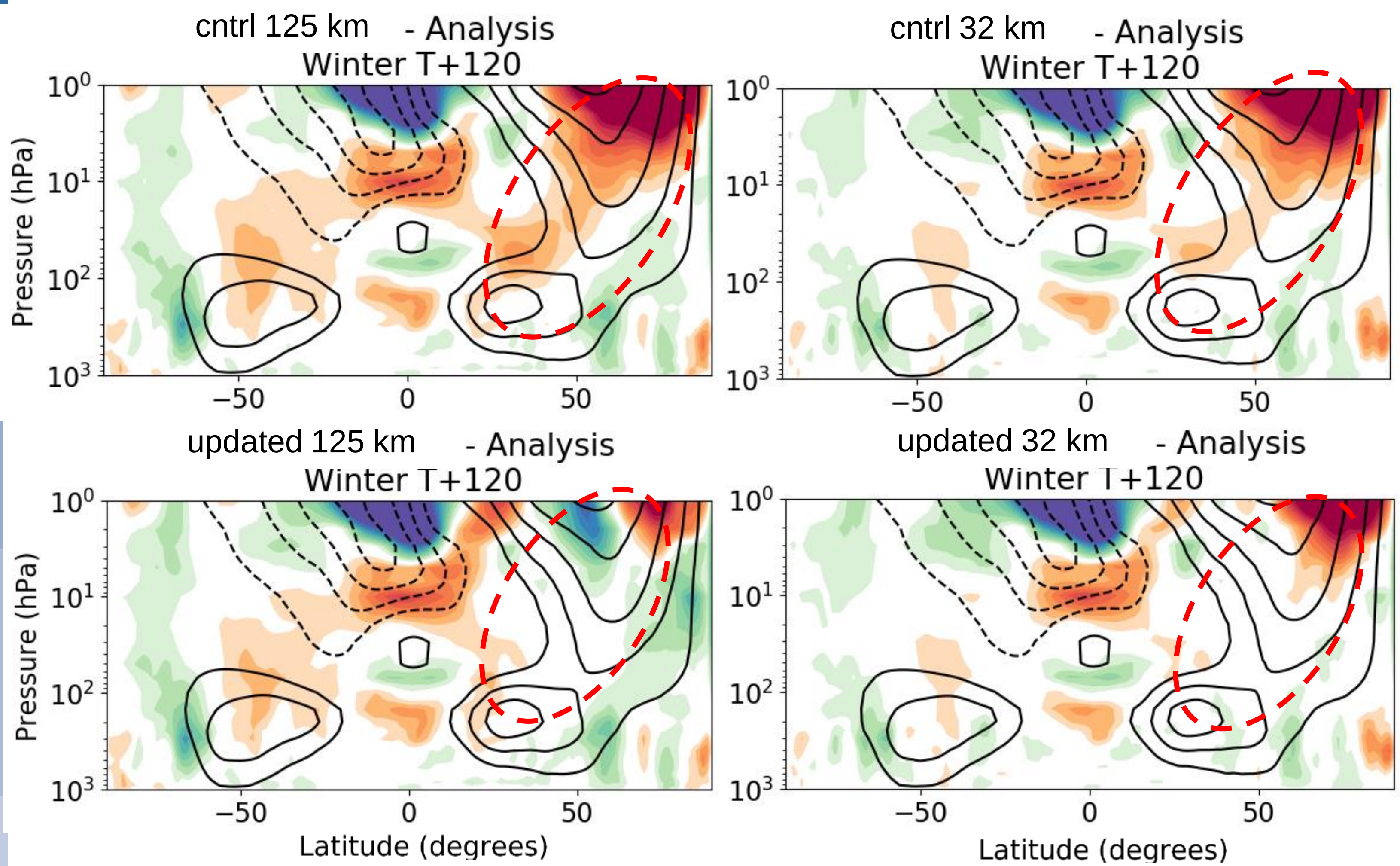
Total GW momentum flux is almost constant at different grid-lengths

Plots show: zonal mean zonal gravity wave momentum fluxes at 7 km above sea level

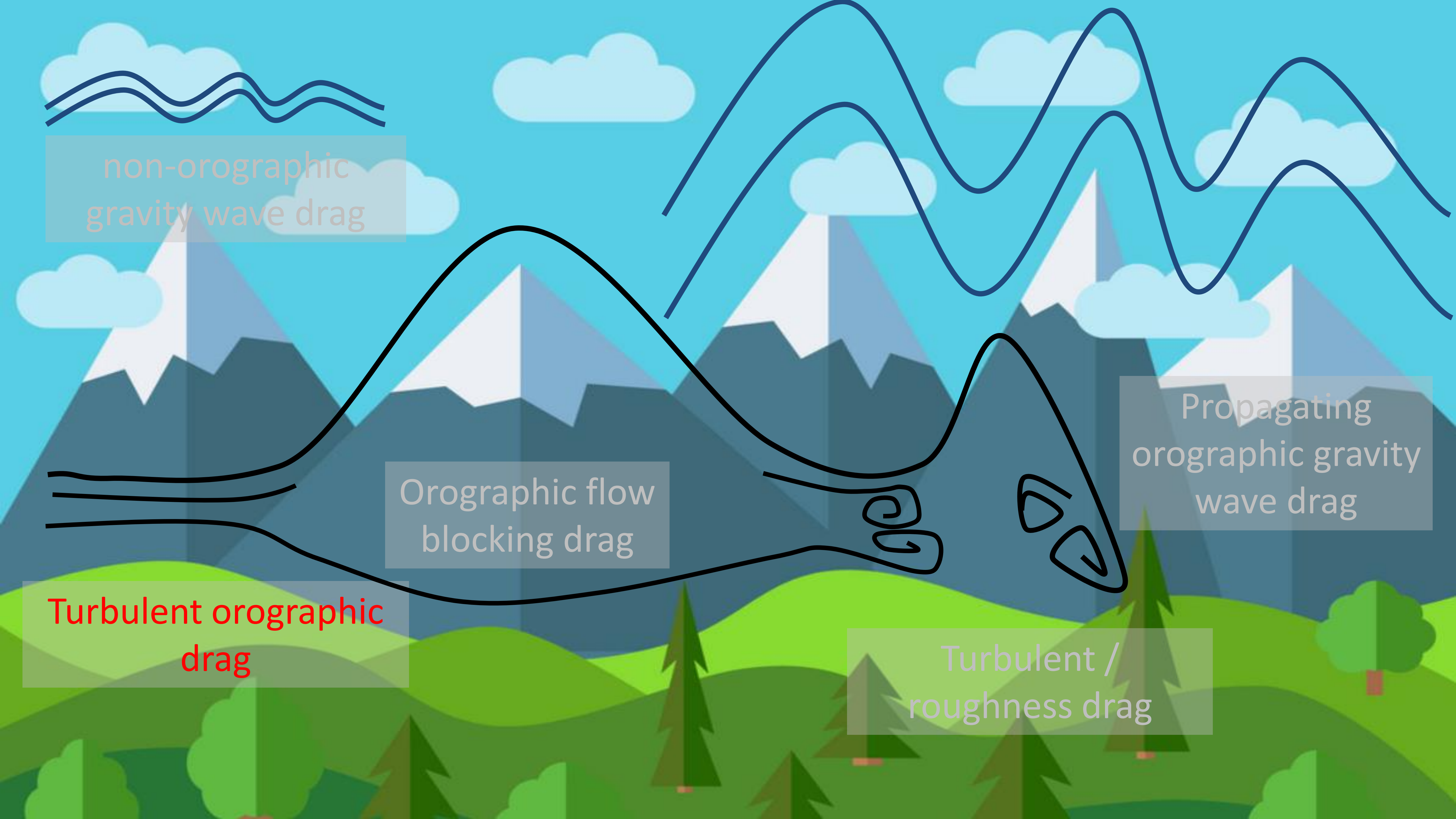
van Niekerk et al (2021)

ECMWF

Resolution sensitivity of gravity wave drag parametrization



Day 5 error at 125km and 32 km now much closer to 9km


non-orographic
gravity wave drag
Orographic flow
blocking drag
Propagating
orographic gravity
wave drag
Turbulent orographic
drag
Turbulent /
roughness drag

Derivation of gravity wave momentum fluxes

Momentum

$$U\,\hat{u}ik + V\,\hat{u}il + \hat{w}\frac{\partial U}{\partial z} = -\frac{1}{\rho}\,\hat{p}ik$$

$$U\,\hat{v}ik + V\,\hat{v}il + \hat{w}\frac{\partial V}{\partial z} = -\frac{1}{\rho}\,\hat{p}il$$

$$U\,\hat{w}ik + V\,\hat{w}il = -\frac{1}{\rho}\frac{\partial\,\hat{p}}{\partial z} - g\frac{\hat{\theta}}{\theta_0}$$

Mass Continuity

$$\hat{u}ik + \hat{v}il + \frac{\partial \hat{w}}{\partial z} = 0$$

Thermodynamics

$$U\,\hat{\theta}ik + V\,\hat{\theta}il + \hat{w}\frac{\partial \Theta}{\partial z} = 0$$

Combine equations:

$$\frac{\partial^2\,\hat{w}}{\partial z^2} + \left[\frac{N^2(k^2+l^2)}{(Uk+Vl)^2} - k^2\right]\hat{w} = 0$$

Non-hydrostatic solution:

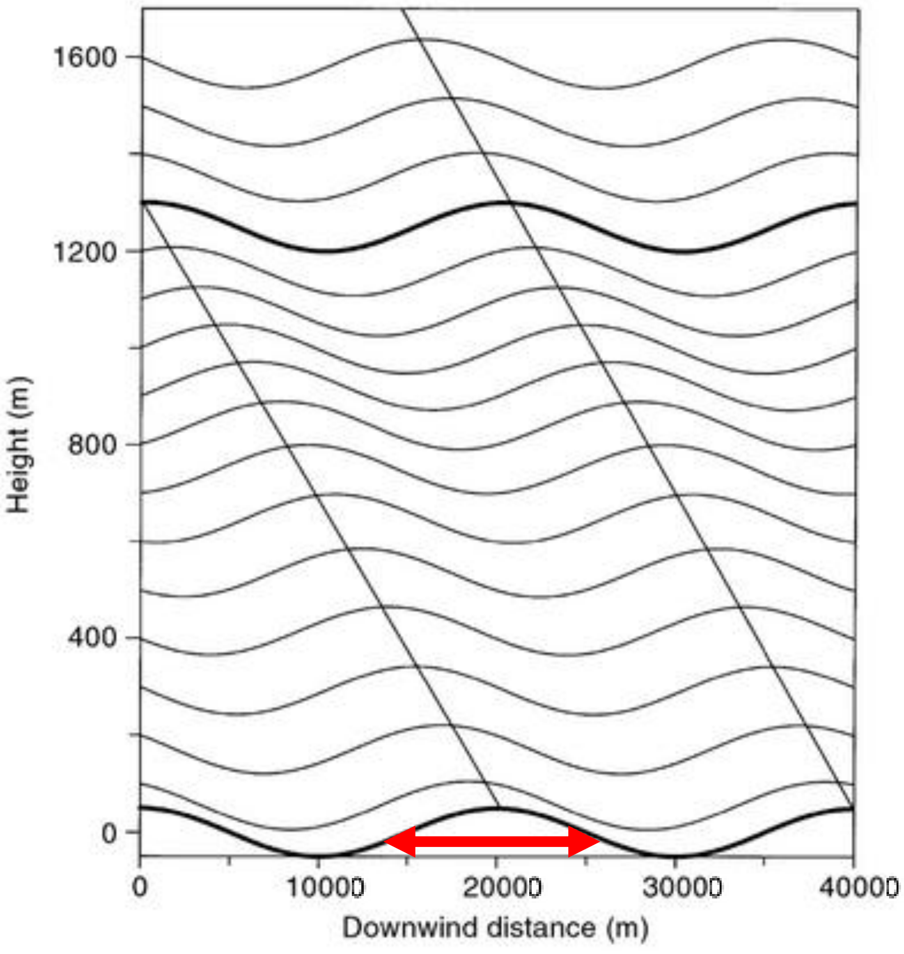
$$\hat{w} = \hat{w}_0 \exp(imz)\,,\, m^2 = \left[\frac{N^2(k^2+l^2)}{(Uk+Vl)^2} - k^2\right]$$

If $m^2 < 0$, the wave is not propagating

Non-propagating (evanescent) waves

Plots show the streamline displacement induced by the wave

Propagating wave



Nappo, 2002

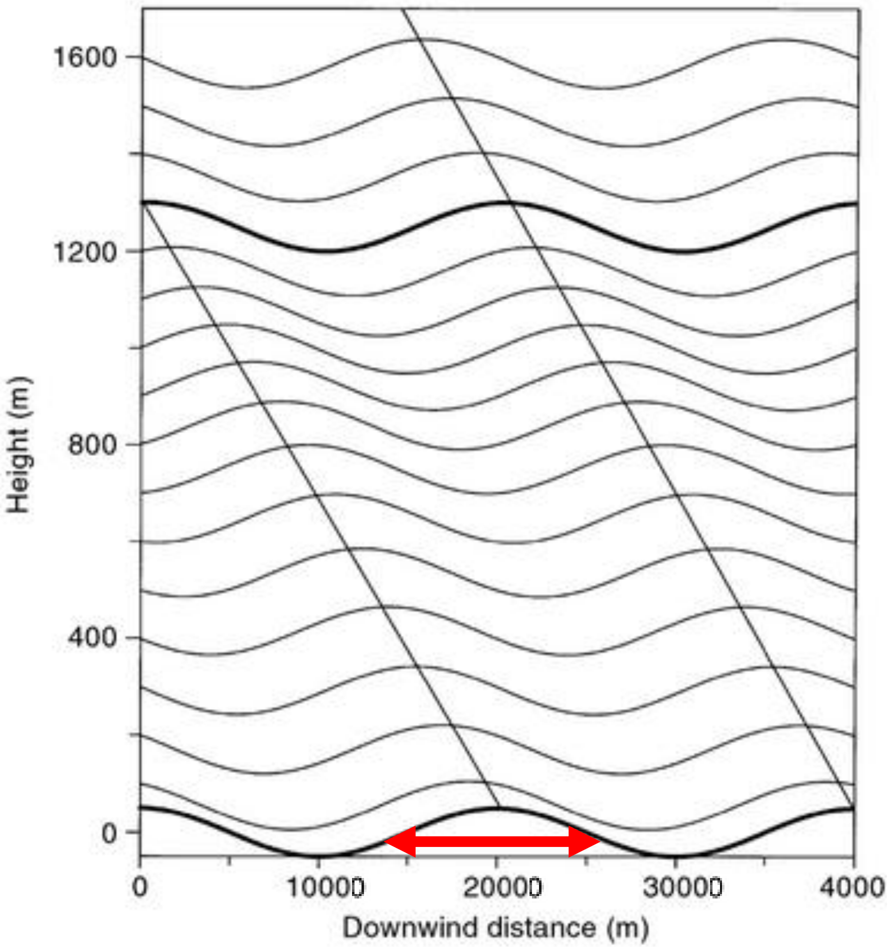
Non-propagating (evanescent) waves

Plots show the streamline displacement induced by the wave

Waves that decay with height (non-propagating waves) have $\lambda_x \ll \frac{U}{N}$

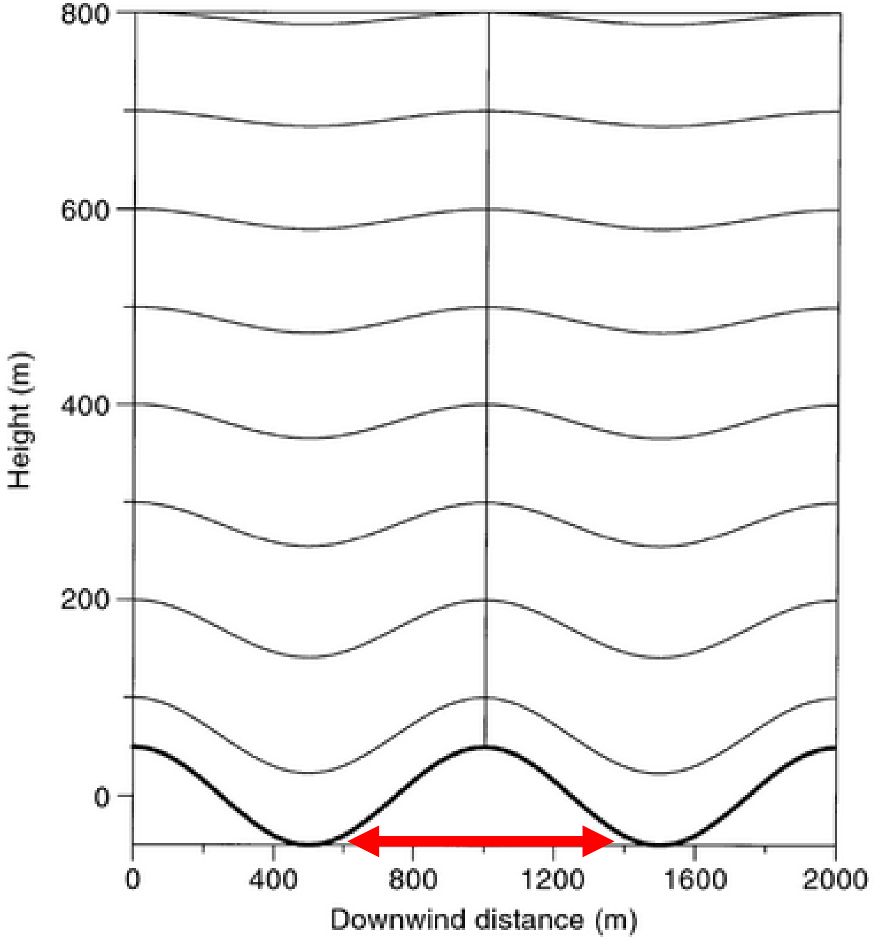
In a typical atmosphere this is for $\lambda_x < 6\ \mathrm{km}$

Propagating wave



20 km

Non-propagating wave

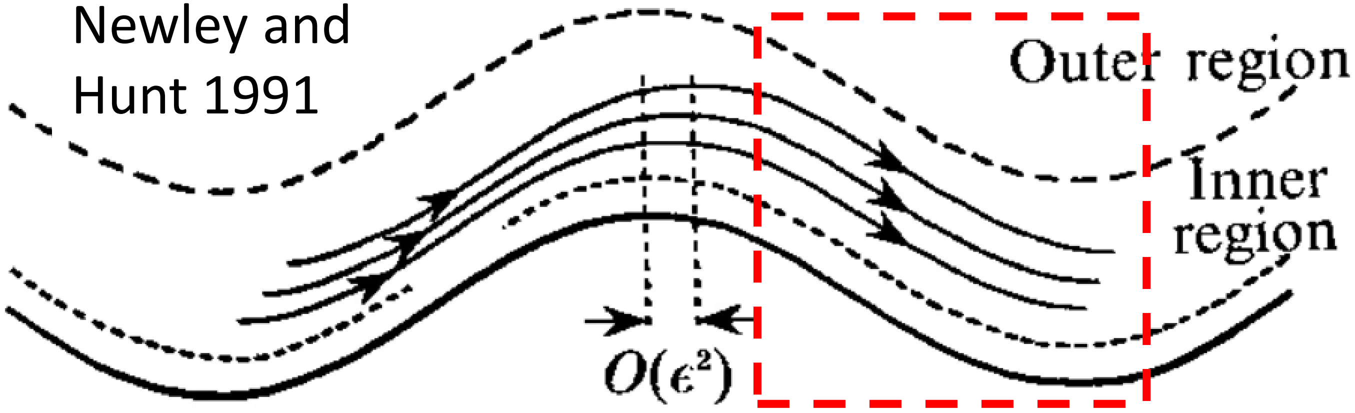


1 km

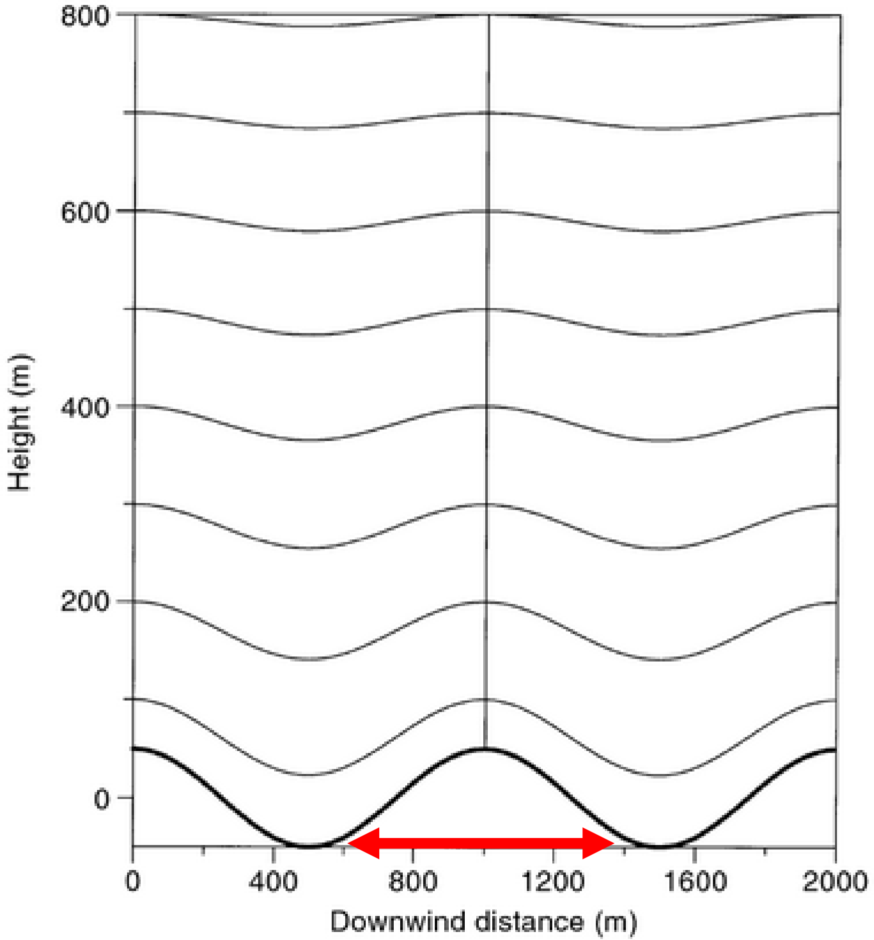
Nappo, 2002

Turbulent orographic form drag

Belcher, Newley and Hunt 1991



Non-propagating wave



1 km

In evanescent waves, the near-surface turbulent stress causes a deepening of the boundary layer on the leeside of the hill

This deepening leads to an asymmetry in the flow over the mountain, which results in a drag on the atmosphere – termed turbulent orographic form drag

Parametrizing turbulent orographic form drag

Turbulent surface stress for one mountain:

$$\tau_{TOFD} = \rho 2\alpha\beta C_{TOFD} |\nabla \cdot h|^2 |U|^2$$

Parametrizing turbulent orographic form drag

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$$\tau_{TOFD} = \rho 2\alpha\beta C_{TOFD} |\nabla \cdot h|^2 |U|^2$$

Vertically distributed drag for one mountain:

$$\frac{\partial \boldsymbol{U}}{\partial t}_{TOFD} = -\rho 2\alpha\beta C_{TOFD} |\nabla \cdot h|^2 |U| \boldsymbol{U} exp\left(-\frac{z}{l}\right)$$

Parametrizing turbulent orographic form drag

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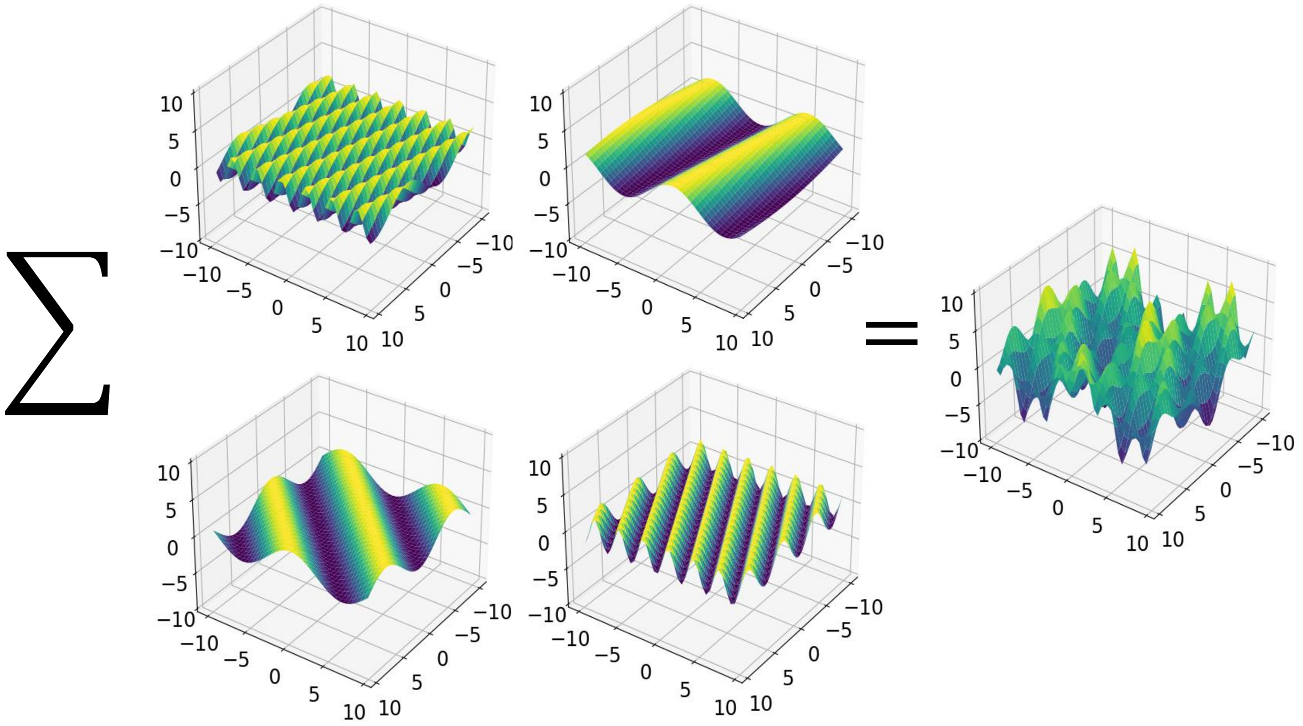

Drag from several mountain waves:

$$\frac{\partial \boldsymbol{U}}{\partial t}_{TOFD} = -\rho 2\alpha\beta C_{TOFD} |U| \boldsymbol{U} \int_{k_0}^{\infty} k^2 \left|\hat{h}\right|^2 exp\left(-\frac{zk}{2}\right) dk$$

$\left|\hat{h}\right|$ = Spectral transform of mountain height

Parametrizing turbulent orographic form drag

Turbulent surface stress for one mountain:

$$\tau_{TOFD} = \rho 2\alpha\beta C_{TOFD} |\nabla \cdot h|^2 |U|^2$$

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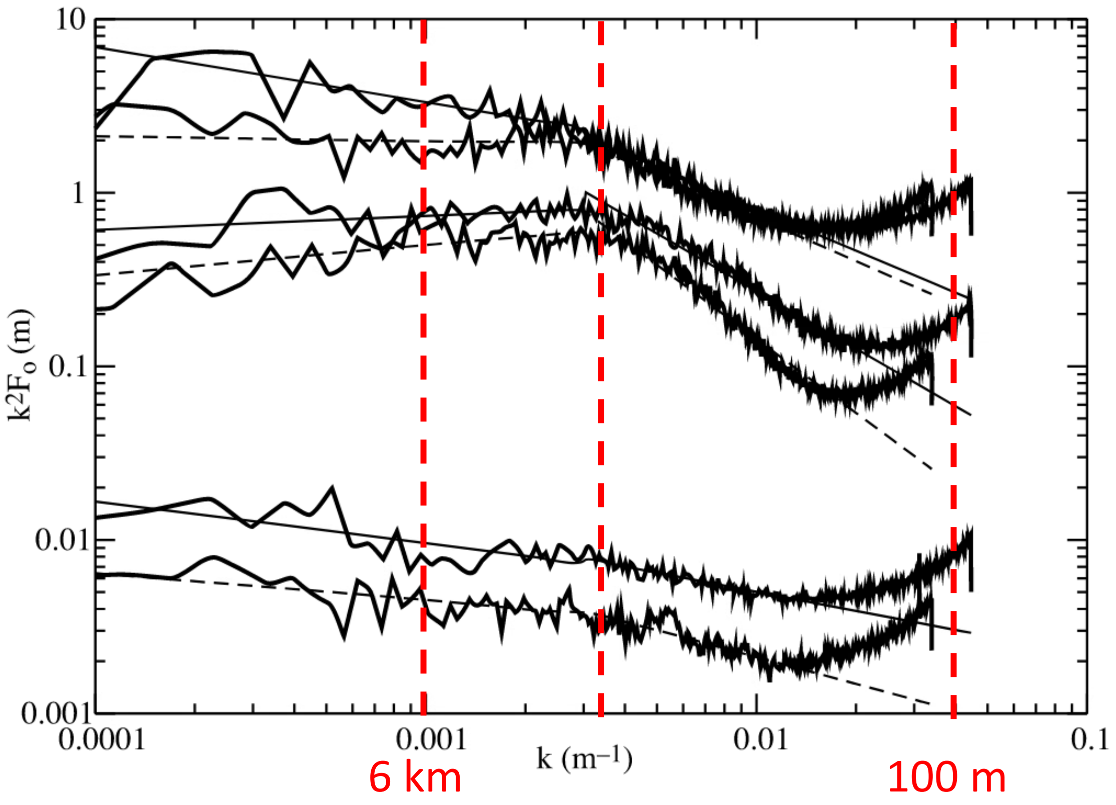
Drag from several mountain waves:

$$\frac{\partial \boldsymbol{U}}{\partial t}_{TOFD} = -\rho 2\alpha\beta C_{TOFD} |U| \boldsymbol{U} \int_{k_0}^{\infty} k^2 |\hat{h}|^2 exp\left(-\frac{zk}{2}\right) dk$$

Power spectrum of orography from 100m data



$|\hat{h}|$ = Spectral transform of mountain height

ECMWF

Parametrizing turbulent orographic form drag

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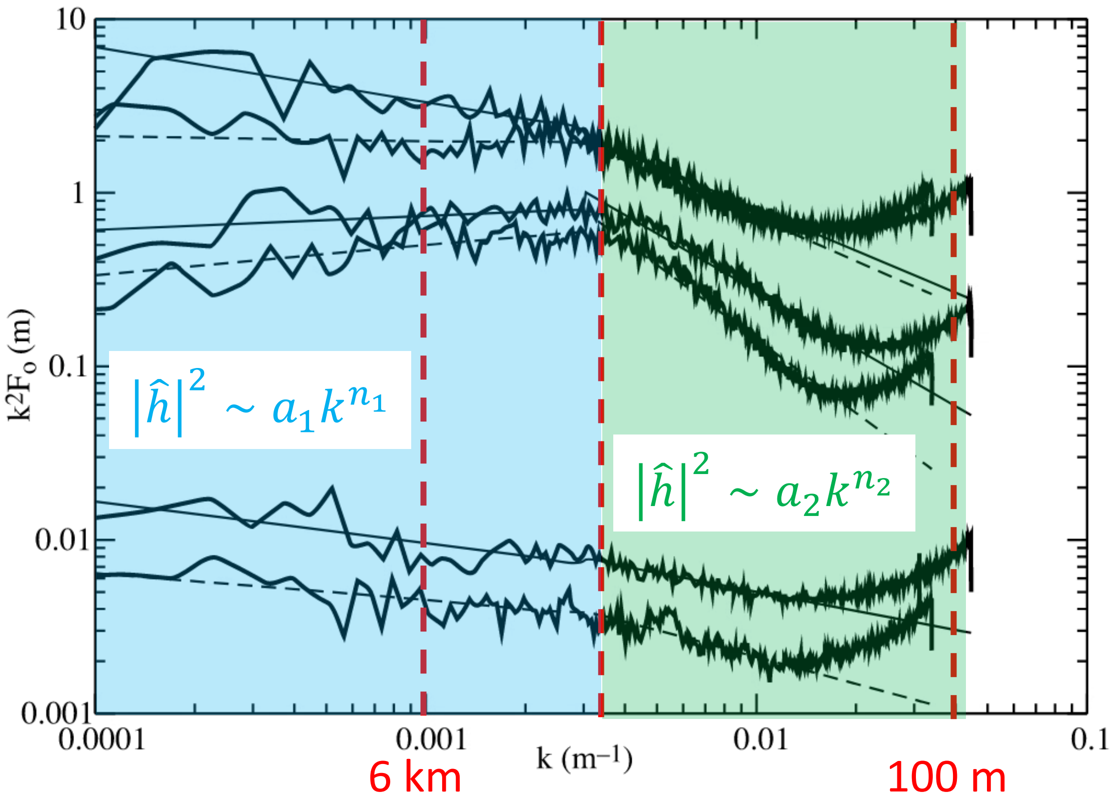


Approximate the shape of the power spectrum

$\left|\hat{h}\right|$ = Spectral transform of mountain height

Parametrizing turbulent orographic form drag

Turbulent surface stress for one mountain:

$$\tau_{TOFD} = \rho 2\alpha\beta C_{TOFD}|\nabla \cdot h|^2|U|^2$$

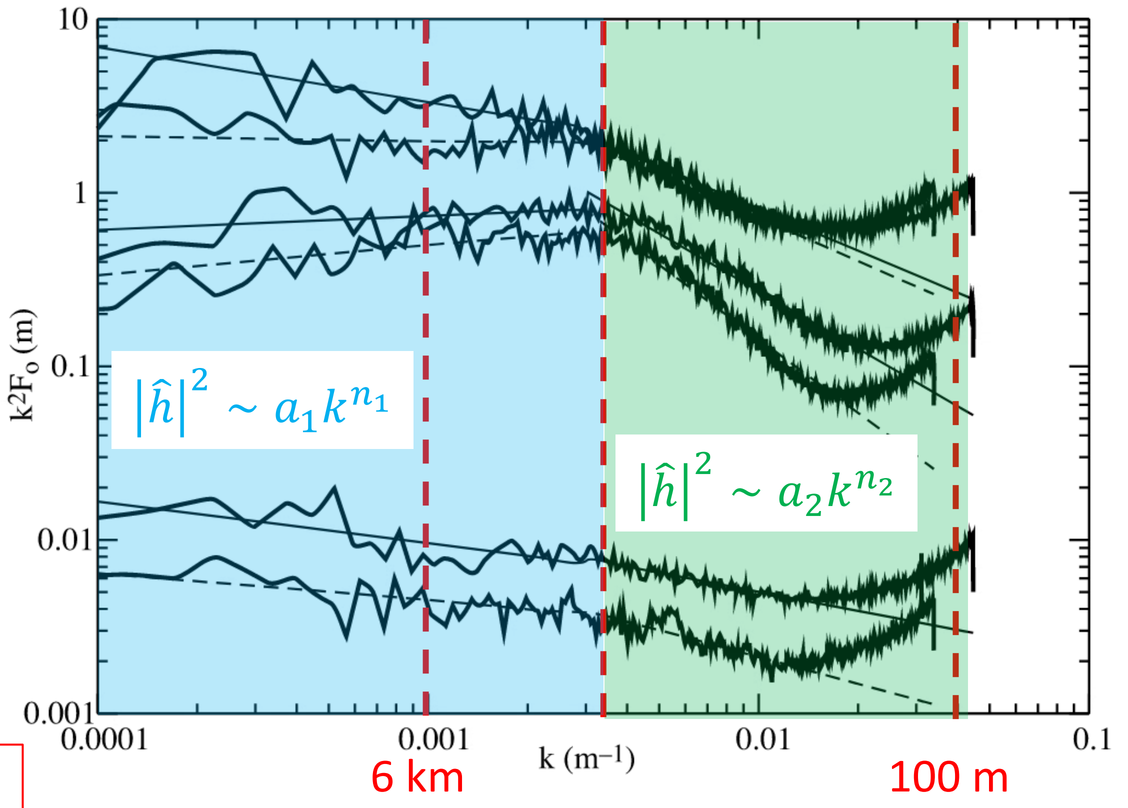
Vertically distributed drag for one mountain:

$$\frac{\partial \boldsymbol{U}}{\partial t}_{TOFD} = -\rho 2\alpha\beta C_{TOFD}|\nabla \cdot h|^2|U|\boldsymbol{U} exp\left(-\frac{z}{l}\right)$$

Drag from several mountain waves:

$$\frac{\partial \boldsymbol{U}}{\partial t}_{TOFD} = -\rho 2\alpha\beta C_{TOFD}|U|\boldsymbol{U} 2.109\, exp\left(-\left(\frac{z}{1500}\right)^{1.5}\right) a_2 z^{-1.2}$$

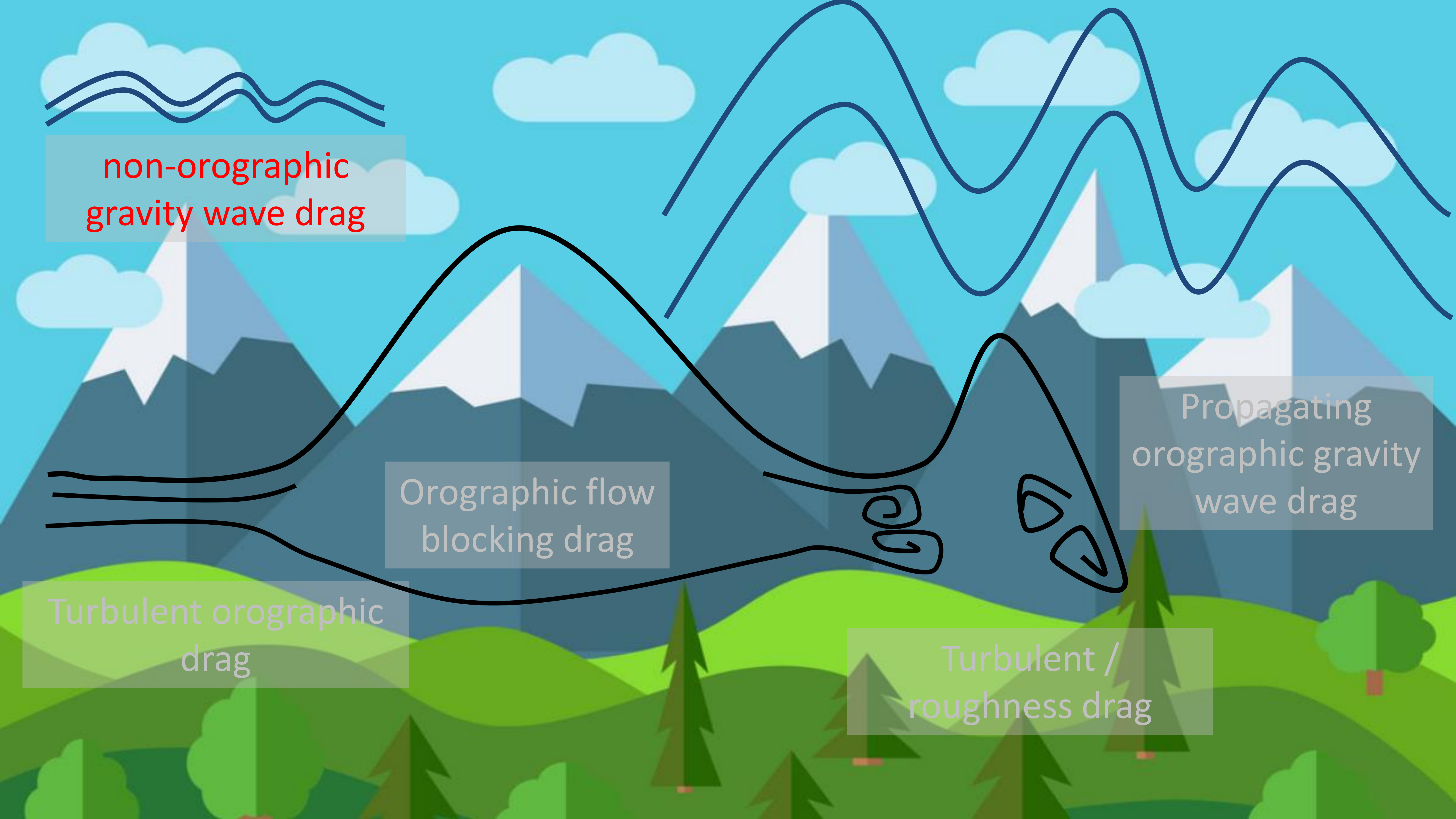
Power spectrum of orography from 100m data



Approximate the shape of the power spectrum – and integrate

$|\hat{h}|$ = Spectral transform of mountain height

ECMWF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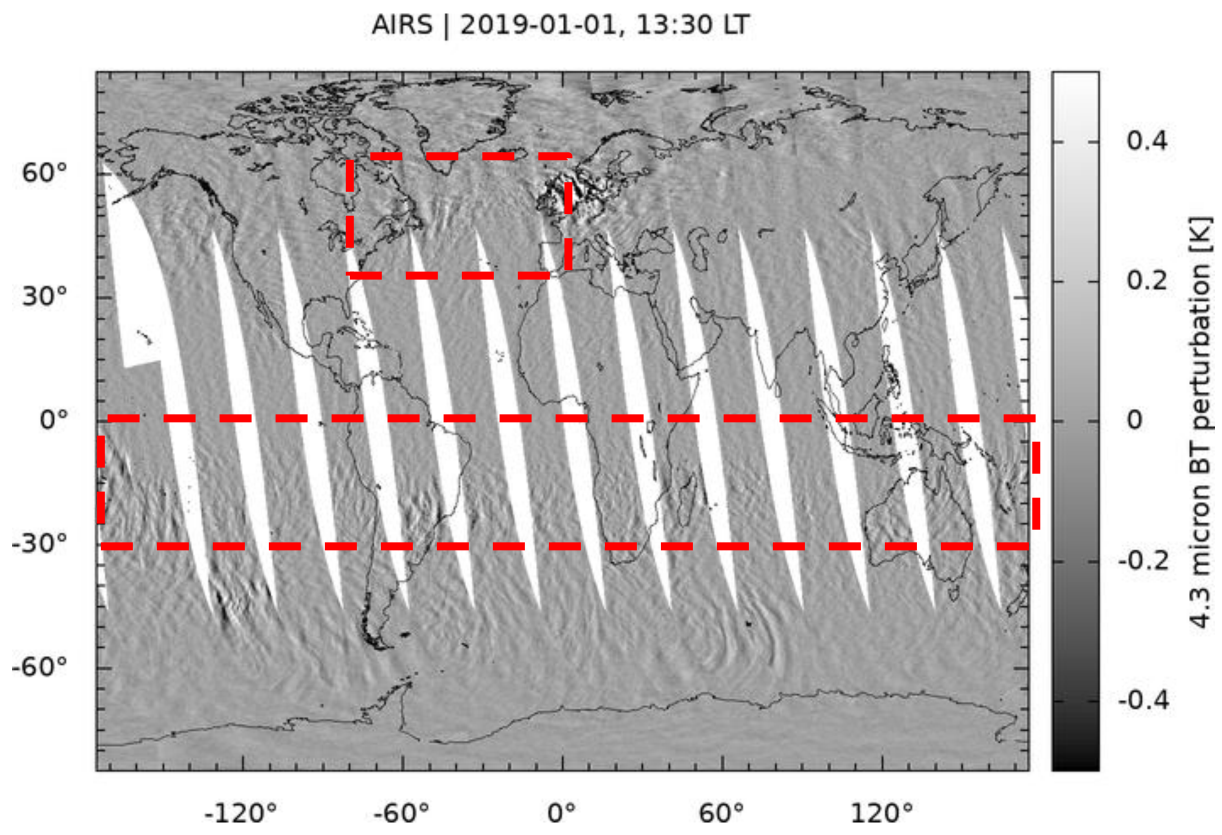
Non-orographic gravity wave drag

Brightness Temperature Perturbations from AIRS satellite at ~ 40 km ASL



https://datapub.fz-juelich.de/slcs/airs/gravity_waves/

‘Non-orographic’ gravity waves are all gravity waves not generated by mountains

They can be generated from:

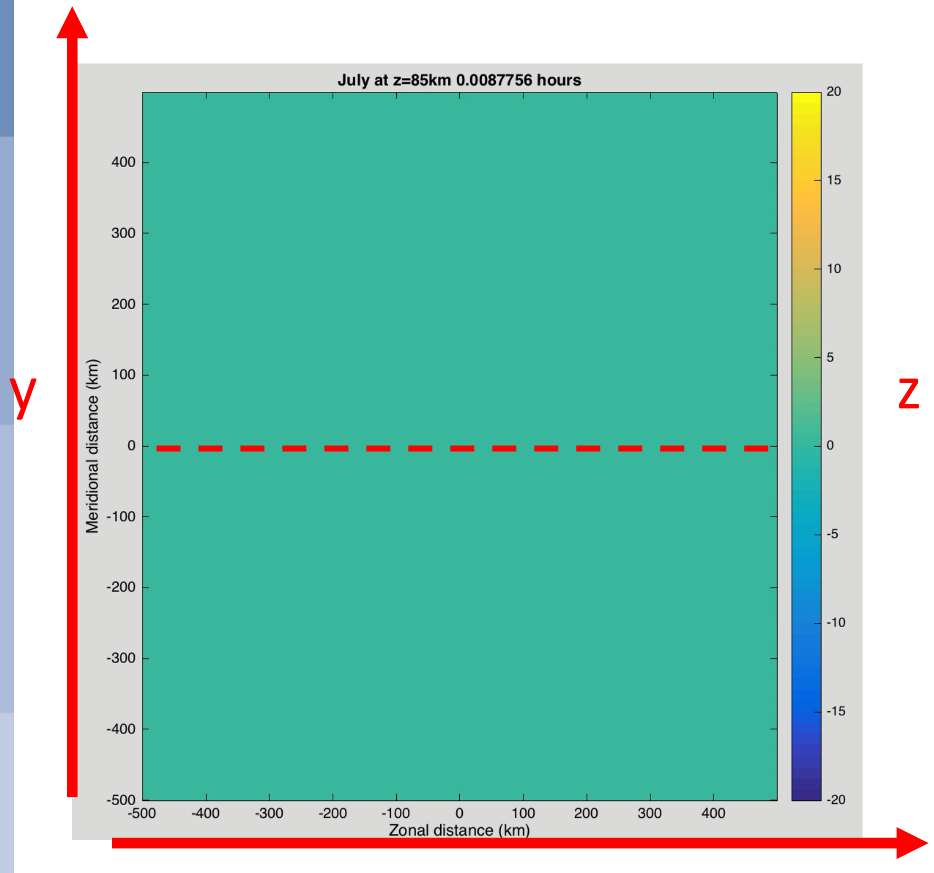
- front\jet instabilities
- convection
- secondary gravity wave breaking

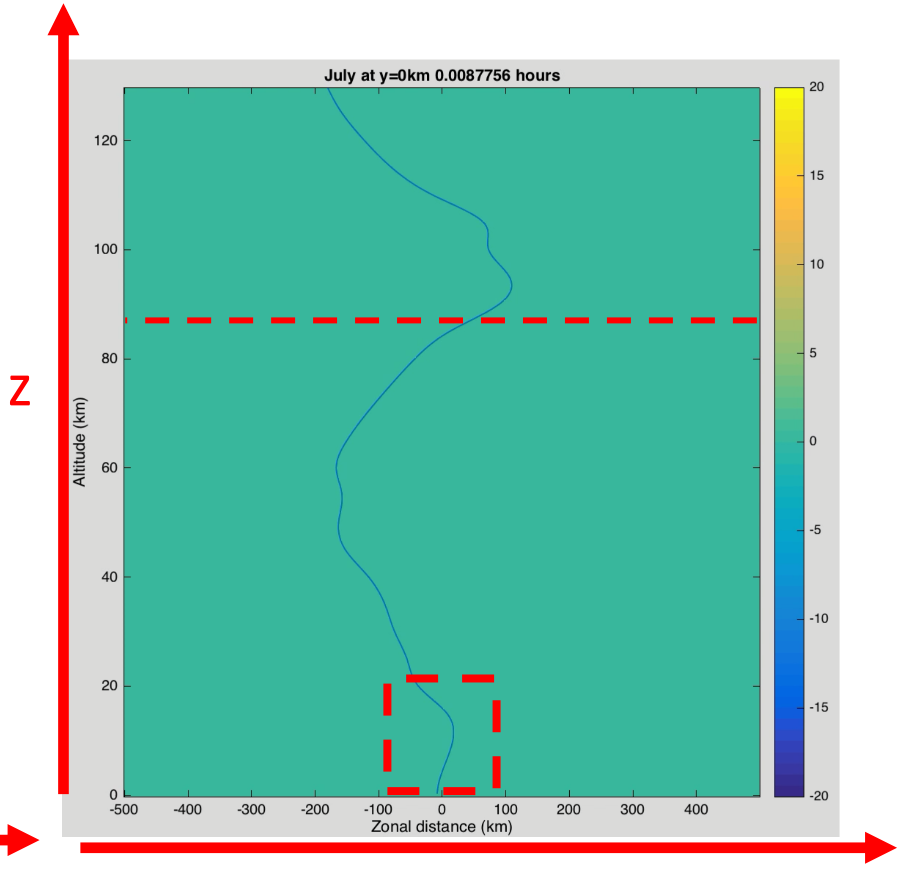
They are typically smaller amplitude and, therefore, can reach very high up in the atmosphere before breaking

They are not ‘steady’ (as with mountain waves) and so their phase varies in space and time

Non-orographic gravity wave drag - convection

Example of idealized convectively generated gravity wave





Heating is imposed near the surface → leads to vertical displacement

In stable atmosphere, this generates a wave, much like flow over mountains

Some of the waves begin to break and generate turbulence where their speed == the background wind speed (thin blue line)

This is a 'critical line' where wave 'drags' the flow

Derivation for non-orographic gravity wave drag

Momentum

$$\frac{\partial u'}{\partial t} + U\frac{\partial u'}{\partial x} + V\frac{\partial u'}{\partial y} + w'\frac{\partial U}{\partial z} = -\frac{1}{\rho}\frac{\partial p'}{\partial x}$$

$$\frac{\partial v'}{\partial t} + U\frac{\partial v'}{\partial x} + V\frac{\partial v'}{\partial y} + w'\frac{\partial V}{\partial z} = -\frac{1}{\rho}\frac{\partial p'}{\partial y}$$

$$\frac{\partial p'}{\partial z} = -\rho g$$

Mass Continuity

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0$$

Thermodynamics

$$\frac{\partial \theta'}{\partial t} + U\frac{\partial \theta'}{\partial x} + V\frac{\partial \theta'}{\partial y} + w'\frac{\partial \Theta}{\partial z} = 0$$

Following approximations are made:

Cartesian coordinates
Shallow atmosphere
No rotation
Adiabatic + incompressible
Hydrostatic
Not steady state

Linearised :

$$u = U(z) + u'(x,y,z,t), u'u' \sim 0$$

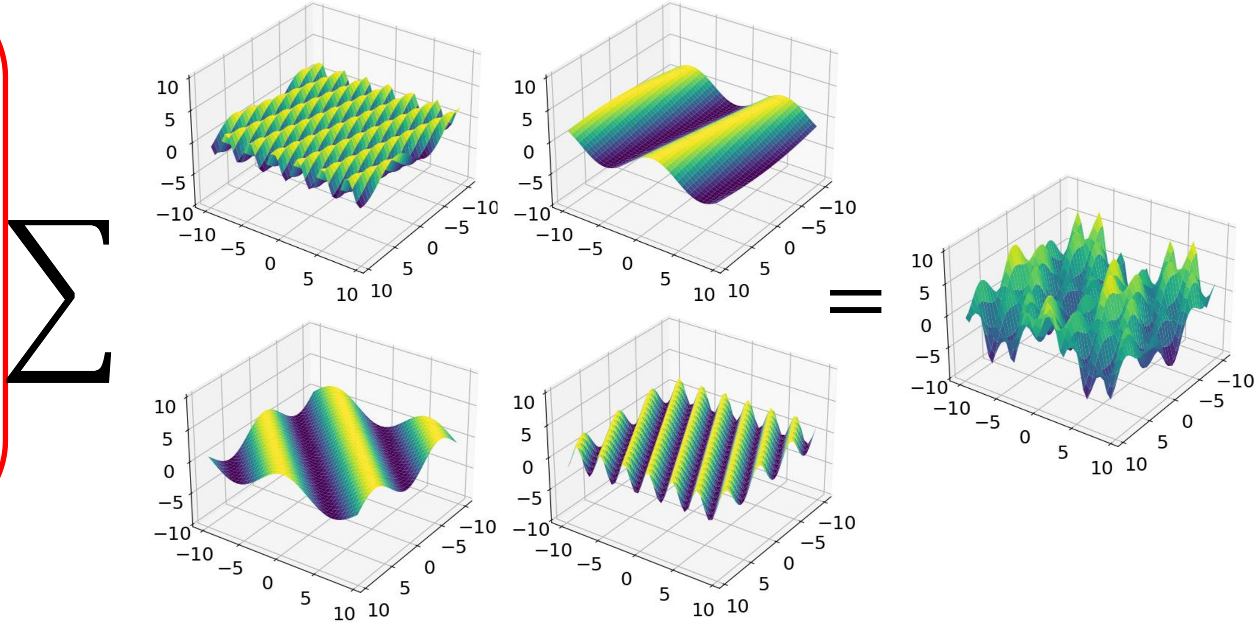
Derivation for non-orographic gravity wave drag

Momentum

$$-\hat{u}i\omega + U\,\hat{u}ik + V\,\hat{u}il + \hat{w}\frac{\partial U}{\partial z} = -\frac{1}{\rho}\,\hat{p}ik$$

$$-\hat{v}i\omega + U\,\hat{v}ik + V\,\hat{v}il + \hat{w}\frac{\partial V}{\partial z} = -\frac{1}{\rho}\,\hat{p}il$$

$$\frac{\partial\,\hat{p}}{\partial z} = -\rho g$$



Mass Continuity

$$\hat{u}ik + \hat{v}il + \frac{\partial \hat{w}}{\partial z} = 0$$

Thermodynamics

$$-\hat{\theta}i\omega + U\,\hat{\theta}ik + V\,\hat{\theta}il + \hat{w}\frac{\partial \Theta}{\partial z} = 0$$

Transform to spectral space:

$$w' \sim \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \hat{w}\exp\big(i(kx + ly - \omega t)\big)\,dk\,dl\,d\omega$$

…

ECMWF

Derivation for non-orographic gravity wave drag

Momentum

$$-\hat{u}i\omega + U\,\hat{u}ik + V\,\hat{u}il + \hat{w}\frac{\partial U}{\partial z} = -\frac{1}{\rho}\,\hat{p}ik$$

$$-\hat{v}i\omega + U\,\hat{v}ik + V\,\hat{v}il + \hat{w}\frac{\partial V}{\partial z} = -\frac{1}{\rho}\,\hat{p}il$$

$$\frac{\partial\,\hat{p}}{\partial z} = -\rho g$$

Mass Continuity

$$\hat{u}ik + \hat{v}il + \frac{\partial \hat{w}}{\partial z} = 0$$

Thermodynamics

$$-\hat{\theta}i\omega + U\,\hat{\theta}ik + V\,\hat{\theta}il + \hat{w}\frac{\partial \Theta}{\partial z} = 0$$

Combine equations:

$$\frac{\partial^2\,\hat{w}}{\partial z^2} + \left[\frac{N^2(k^2+l^2)}{(\omega - Uk + Vl)^2}\right]\hat{w} = 0$$

Solution:

$$\hat{w} = \hat{w}_0 \exp(imz)\,, m^2 = \left[\frac{N^2(k^2+l^2)}{(\omega - Uk + Vl)^2}\right]$$

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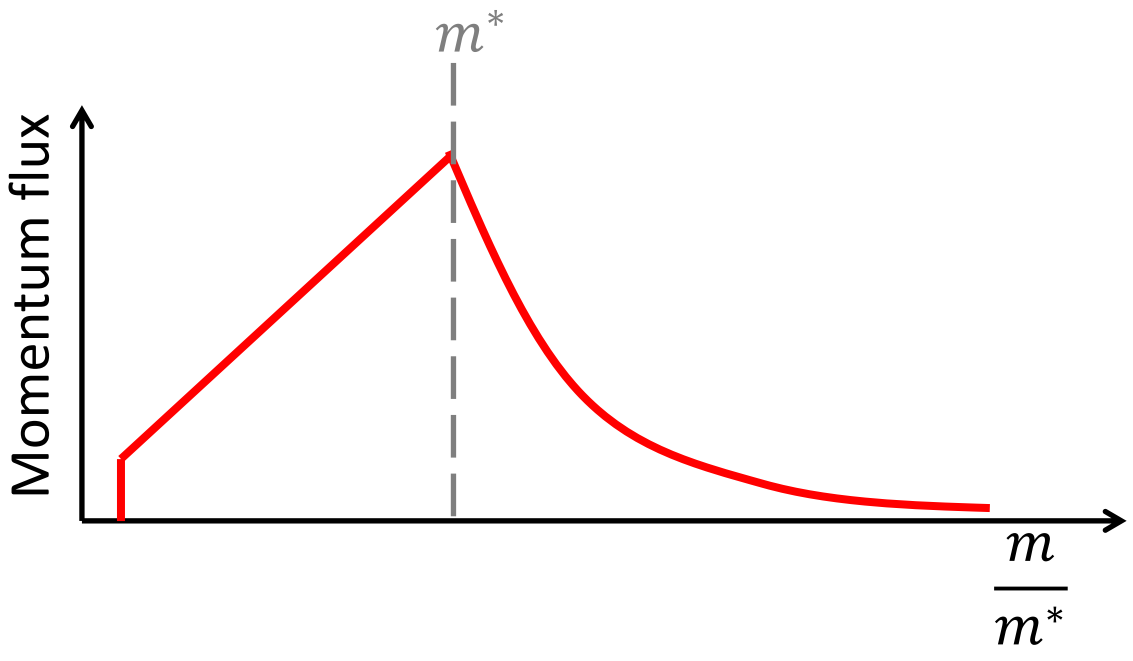
There is not a simple surface boundary condition (as with mountains) for this problem

We do not know the nature of the sources well enough

Parametrizing non-orographic gravity wave drag



Empirical relationship between the momentum fluxes and vertical wavenumber is assumed

Relationship is assumed to hold for every k and ω

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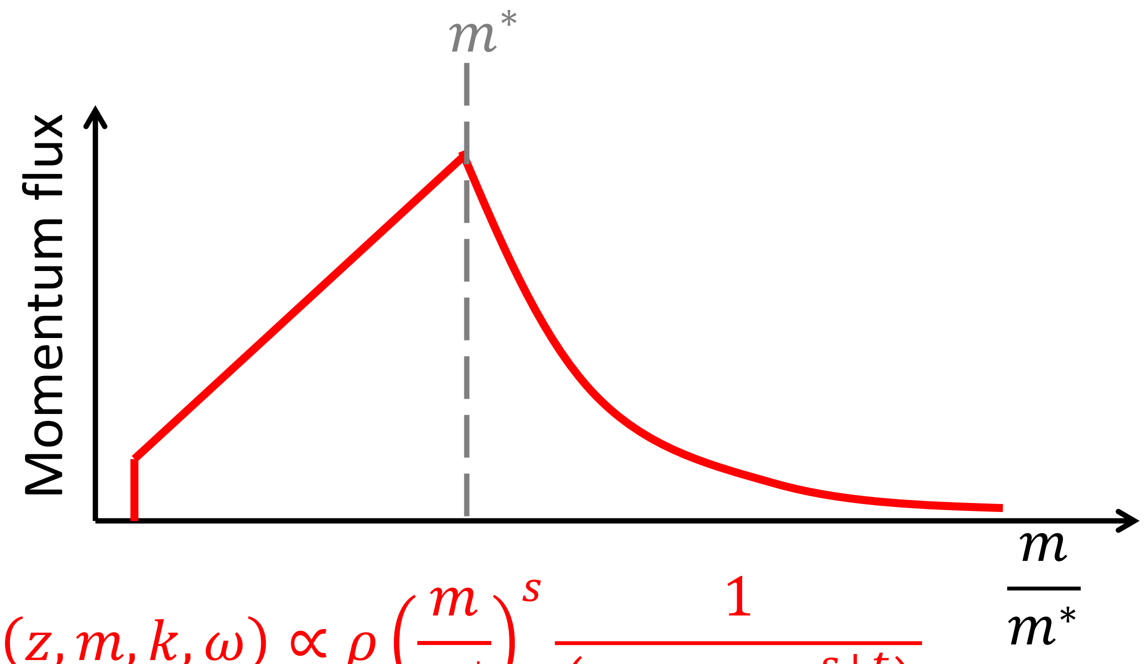

$$\tau(z,m,k,\omega) \propto \rho \left(\frac{m}{m^*}\right)^s \frac{1}{\left(1+\left(\frac{m}{m^*}\right)^{s+t}\right)}$$

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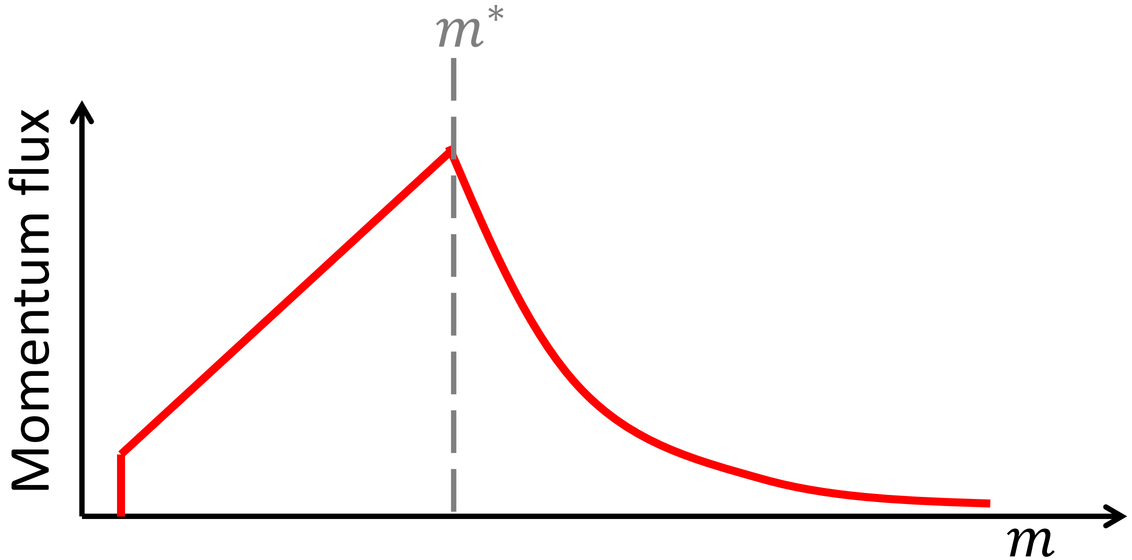

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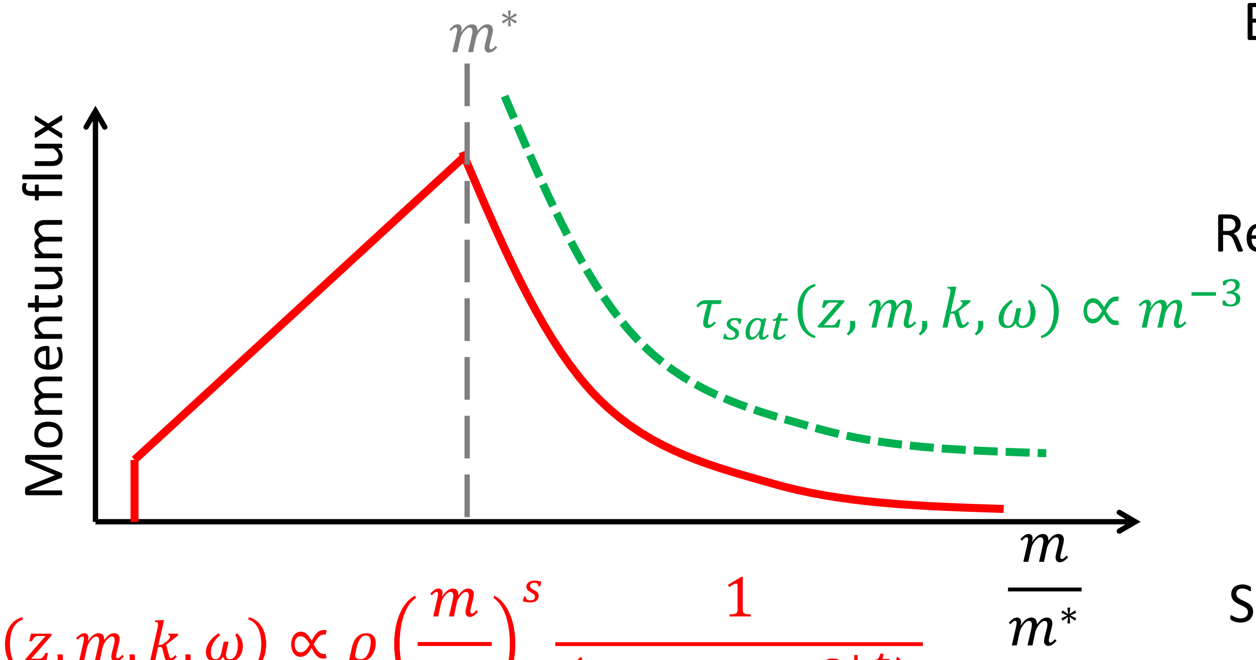
Scheme then uses discretely 'binned' values of k and ω, and solves for these individually

Parametrizing non-orographic gravity wave drag



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Waves are then saturated (only at large m) using:

$$\tau(z,m,k,\omega) < \tau_{sat}(z,m,k,\omega)$$

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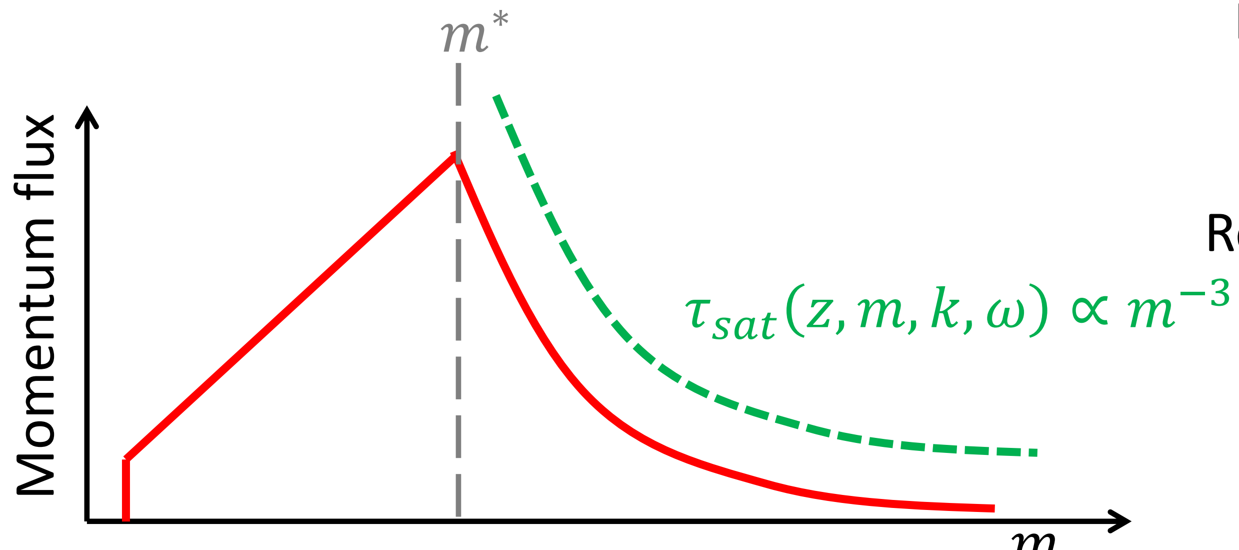
ECMWF

Parametrizing non-orographic gravity wave drag



$$\tau(z,m,k,\omega) \propto \rho \left(\frac{m}{m^*}\right)^s \frac{1}{\left(1+\left(\frac{m}{m^*}\right)^{s+t}\right)}$$

Total drag is given by the sum of fluxes over bins:

$$\frac{d\,|U|}{dt} = -\frac{1}{\rho}\frac{\partial}{\partial z}\left(\sum_{\omega}\sum_{-k}\tau(z,m,k,\omega)\right)$$

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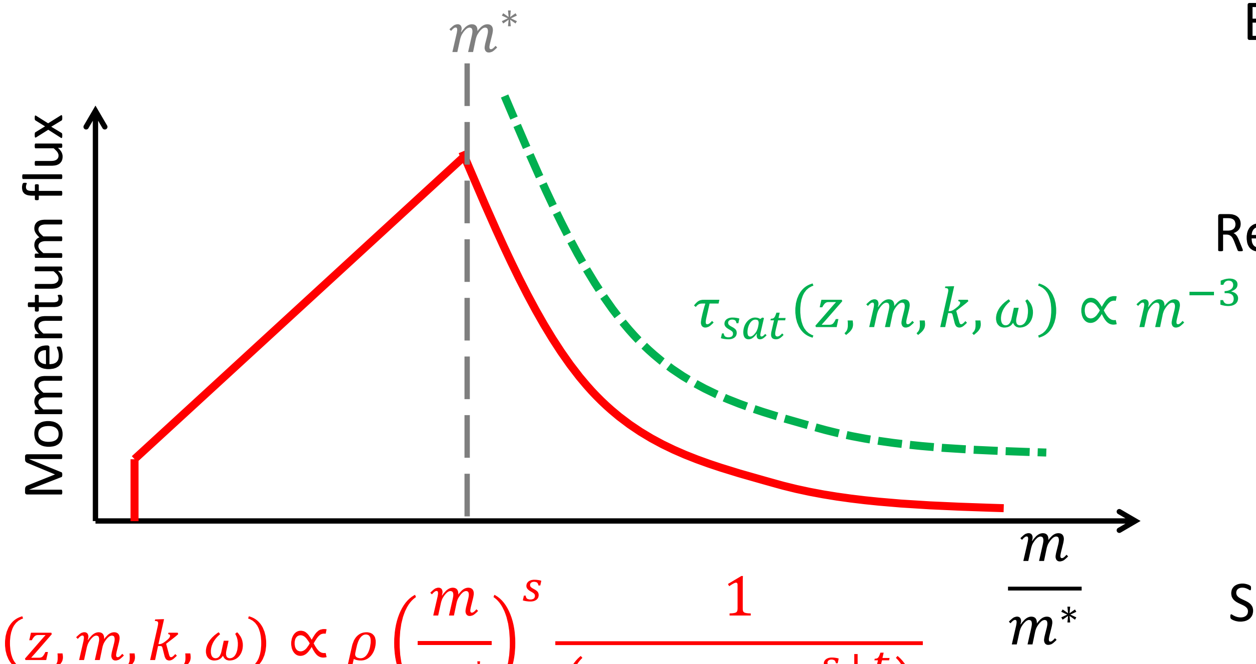

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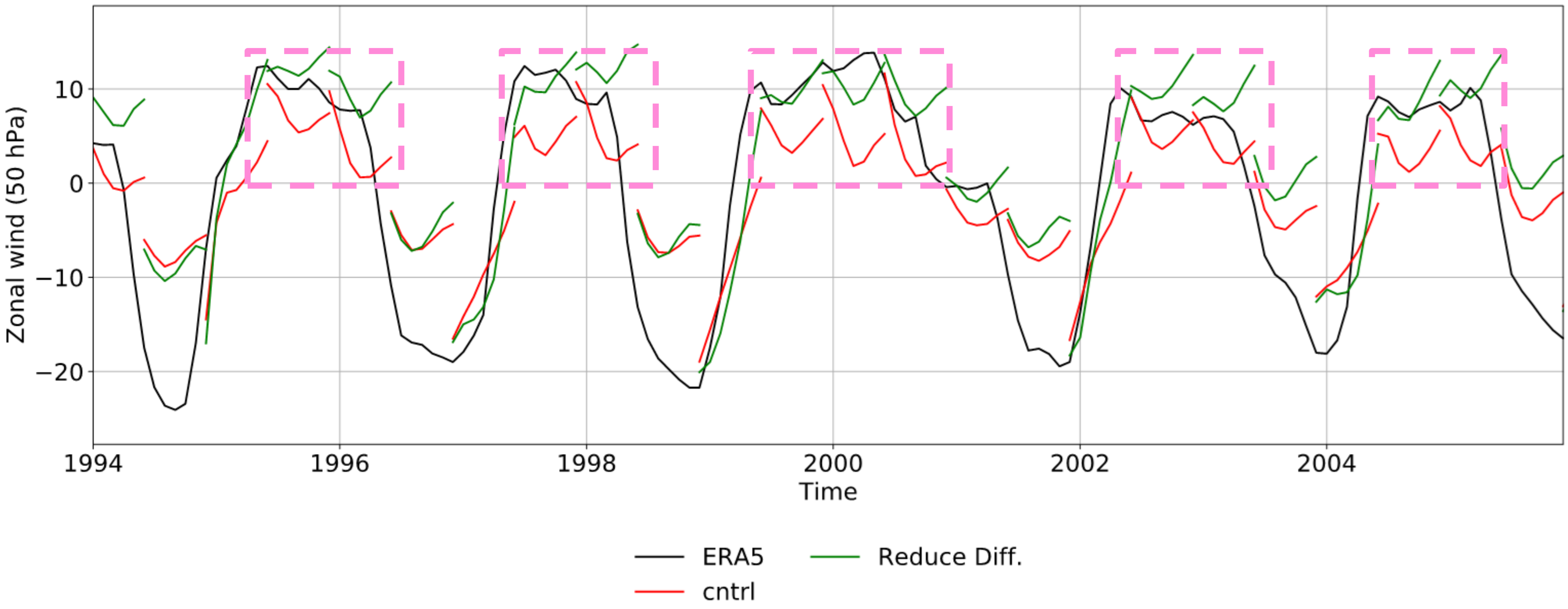
ECMWF

Getting the QBO right

Reduced diffusion improves model winds in the QBO positive phase

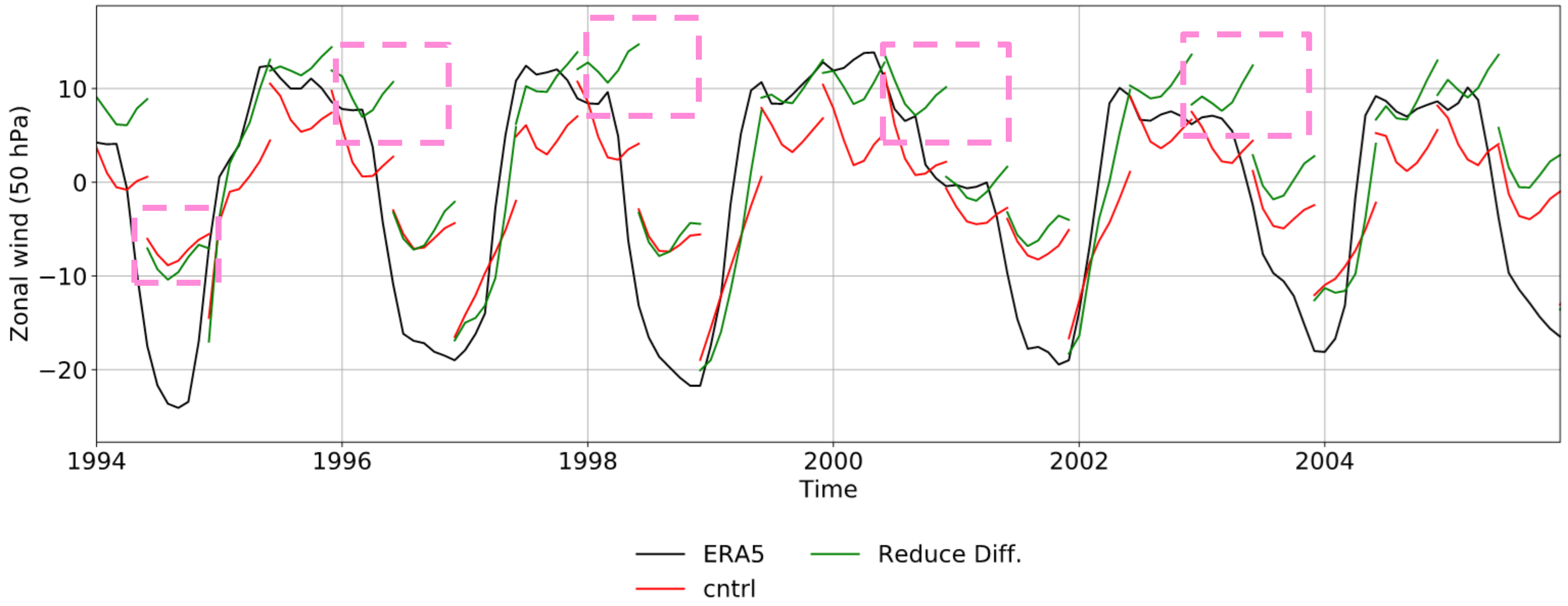


Plot shows 50 hPa zonal winds averaged between 5S – 5N

Seasonal hindcasts run with the ECMWF IFS, 7 months long

Getting the QBO right

Reduced diffusion improves model winds in the QBO positive phase but does not make things better at the longer range



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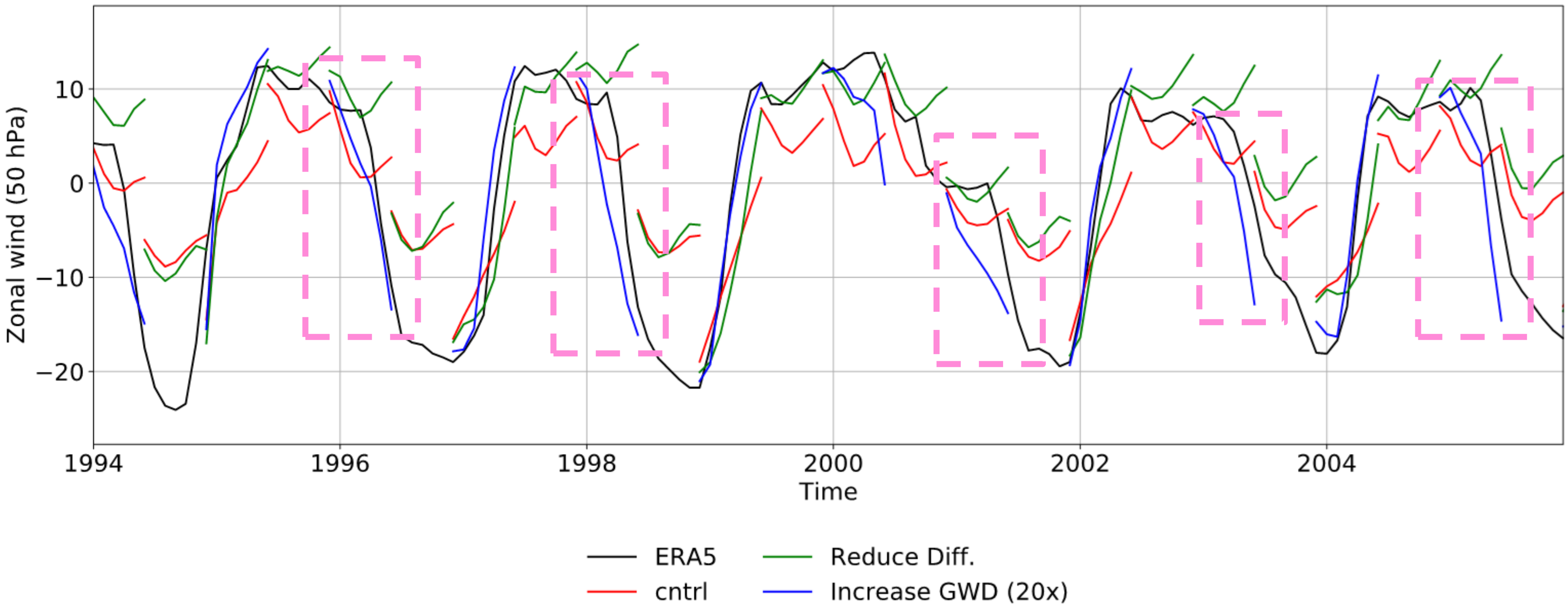
ECMWF

Tuning non-orographic gravity wave drag

Increased non-orographic gravity wave drag makes the wind evolution better



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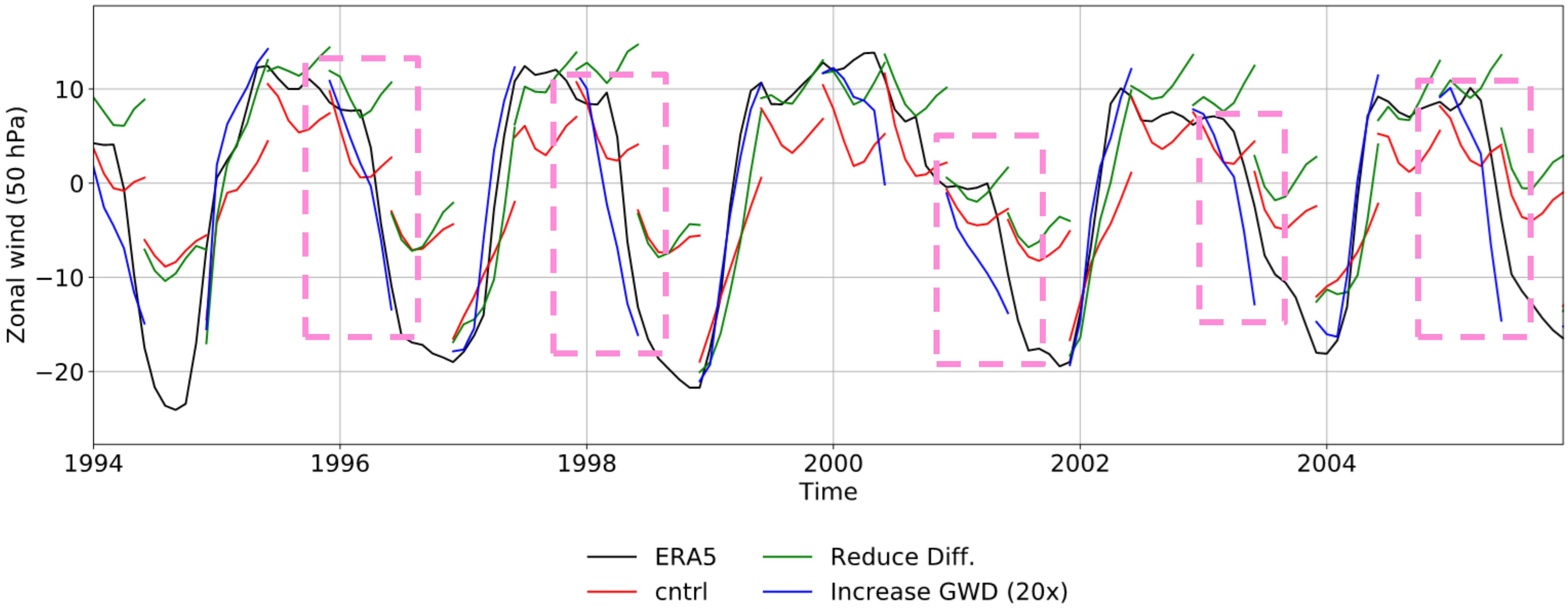
Seasonal hindcasts run with the ECMWF IFS, 7 months long

Tuning non-orographic gravity wave drag

Increased non-orographic gravity wave drag makes the wind evolution better – but the winds transition to negative too quickly



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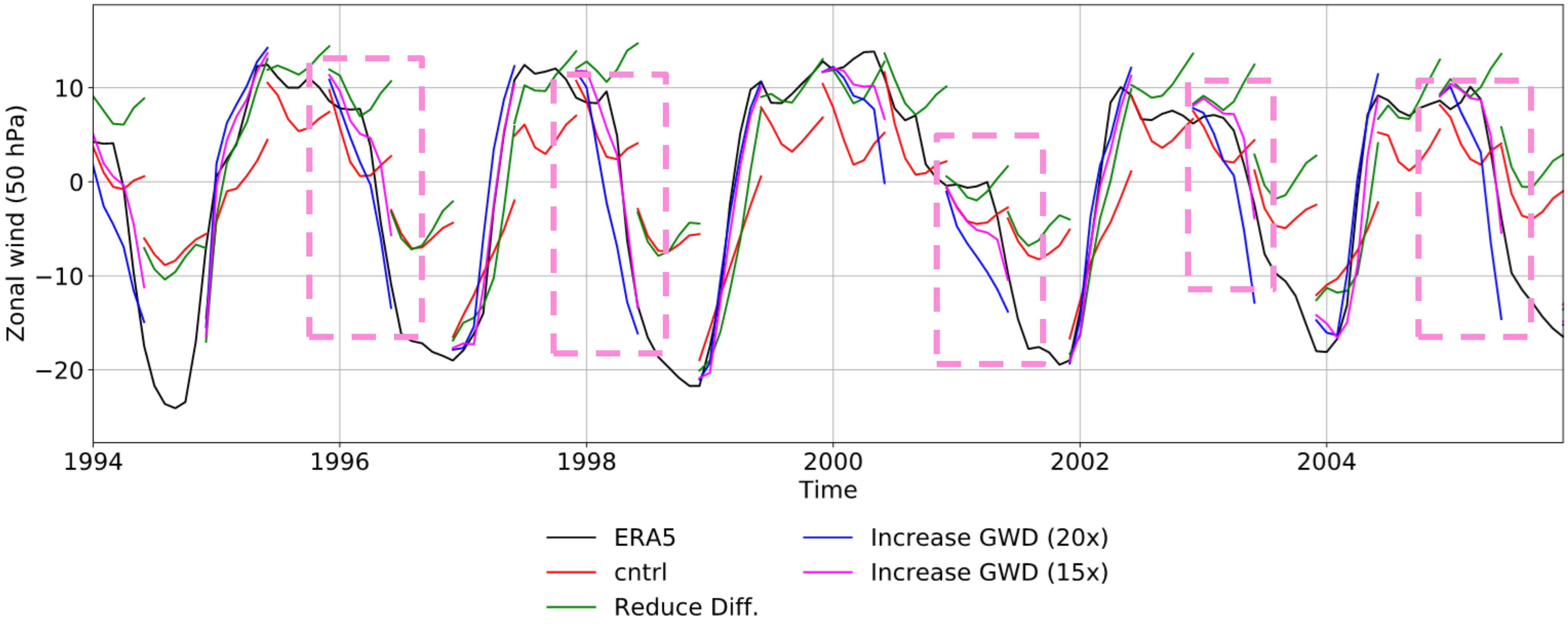
ECMWF

Tuning non-orographic gravity wave drag

Fine tuning the increased gravity wave drag gives better transition to negative QBO phase



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ECMWF

Summary of orographic drag and gravity wave drag

- **Orographic gravity wave drag:**
  - These are waves generated by flow over mountains and lead to drag in the upper atmosphere
  - In the model, the mountains are assumed to be ellipses (not good for resolution sensitivity)
- **Orographic flow blocking:**
  - Flow blocking occurs when the surface wind is weak or the stability is very high
  - This drag occurs near the surface, around the mountains
- **Turbulent orographic form drag:**
  - Occurs when there is turbulent stress near mountains that generate non-propgating waves
  - Assumed to be from small-scale mountain < 5 km wide
- **Non-orographic gravity wave drag:**
  - This is drag from all gravity wave sources that are not from mountains
  - The source of these waves are assumed to follow an empirical relationship between vertical wavenumber (m) and momentum flux