

A satellite view of Earth's clouds, showing a vast expanse of white and grey cloud formations over a dark blue ocean. The clouds are dense and cover most of the visible area, with some darker patches of water visible between the cloud clusters.

**Numerical Weather Prediction**  
**Parametrization of Subgrid Physical Processes**  
**Clouds (2)**  
**Ice and Mixed-Phase Microphysics**

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(with thanks to Adrian Tompkins  
and Christian Jakob)

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# Microphysics Parametrization

Which quantities (hydrometeor types) to represent ?

- Water vapour

## Warm-phase:

- Cloud water droplets
- Rain drops

## Cold-phase

- Pristine ice crystals
- Snow particles
- Graupel pellets
- Hailstones

Note for ice phase particles:

- Additional latent heat.
- Terminal fall speed of ice hydrometeors significantly less.
- Optical properties are different (important for radiation).

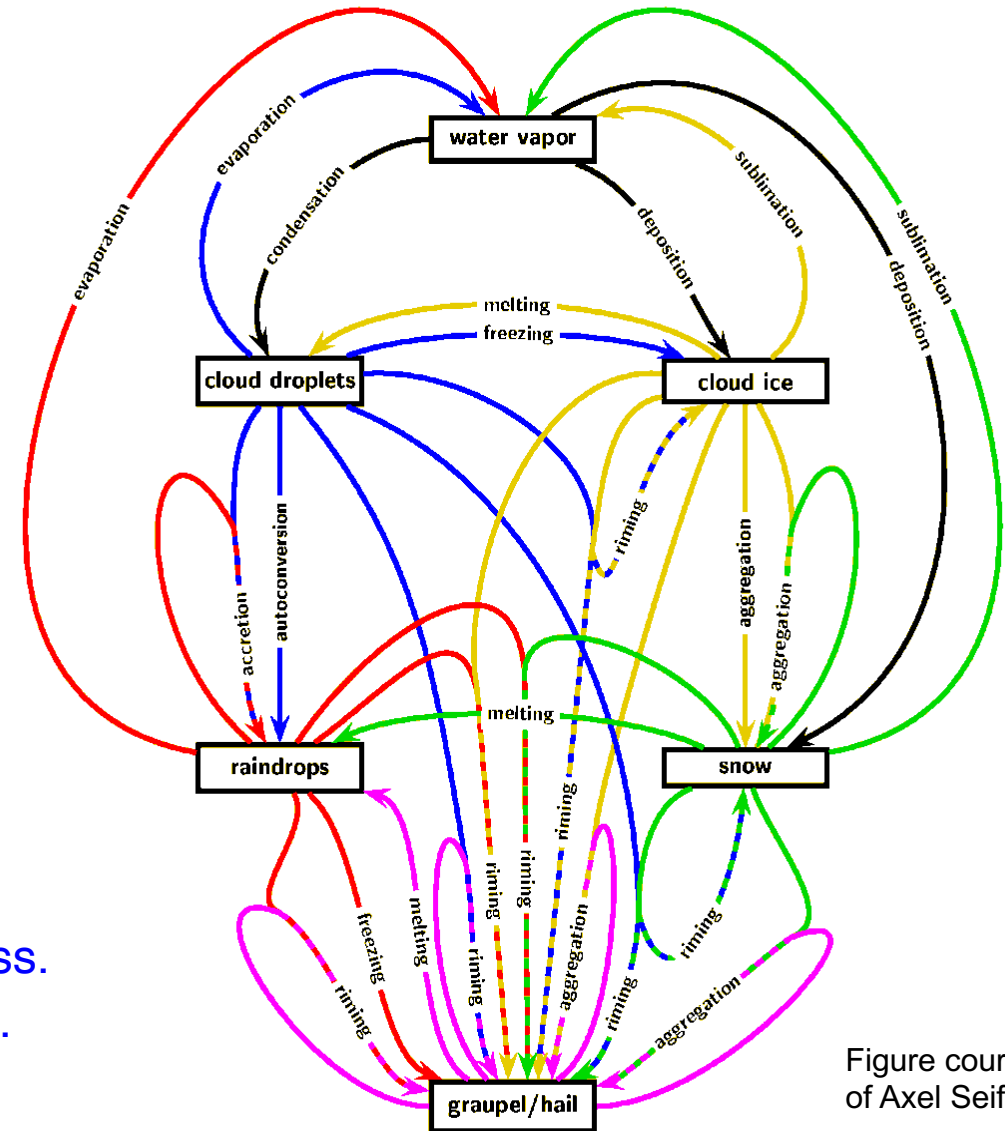


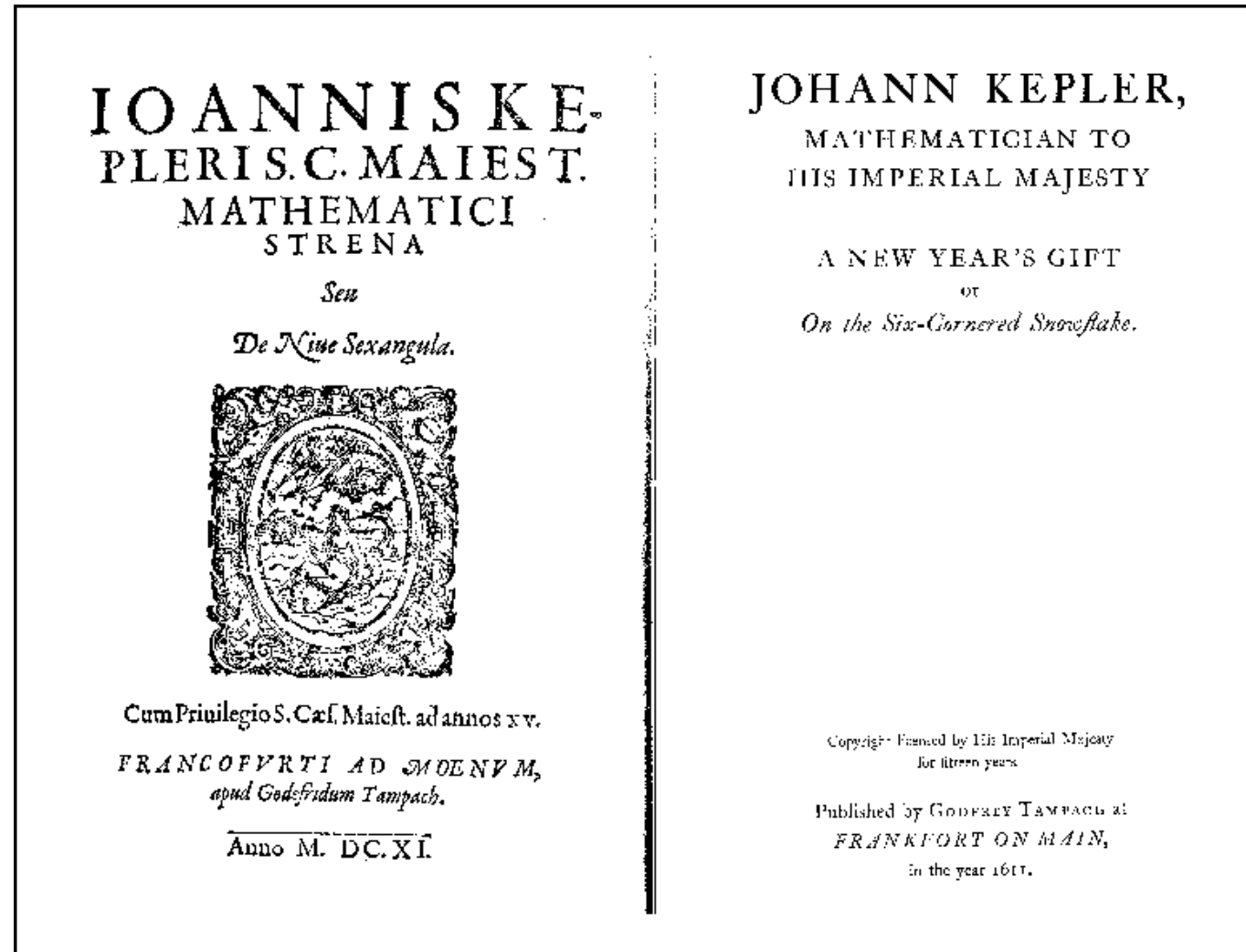
Figure courtesy of Axel Seifert

# Ice and mixed-phase microphysical processes and their parametrization

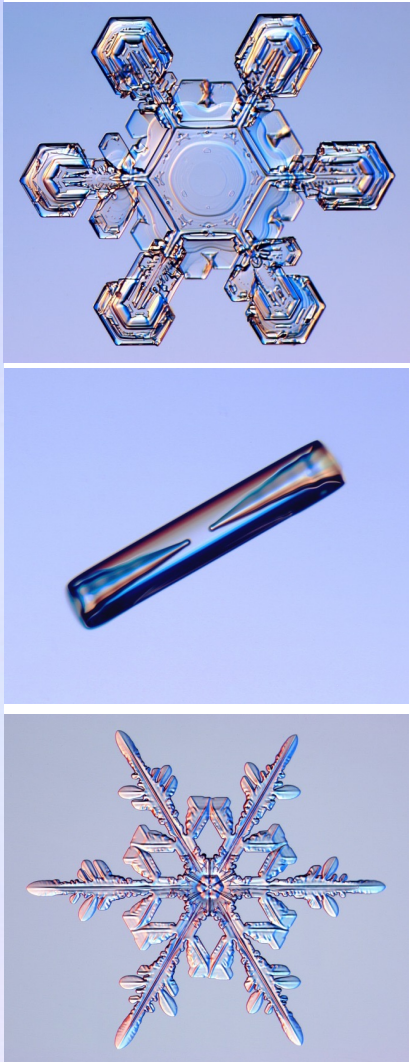
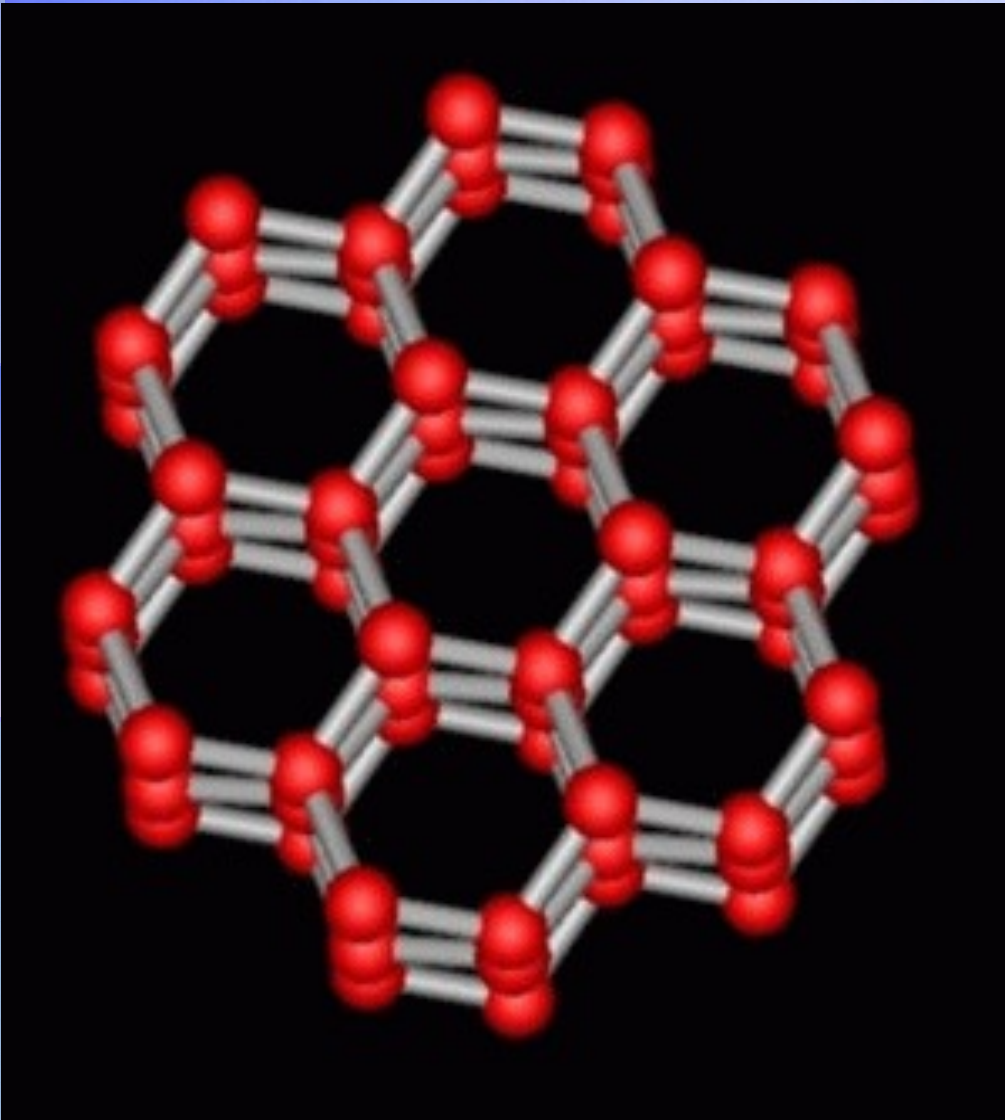
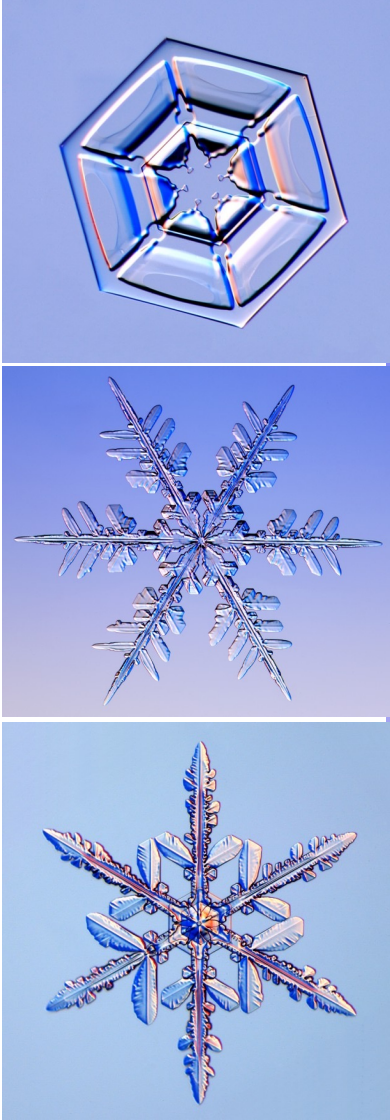
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- To describe ice-phase cloud and precipitation processes in our models we need to represent:
  - **Nucleation** of ice crystals
  - **Diffusional growth/sublimation** of ice crystals
  - **Collection processes** for ice crystals (aggregation), for ice and liquid droplets (riming)
  - **Breakup processes** for ice crystals (splintering, Hallett-Mossop)
  - The **advection** and **sedimentation** (falling) of particles
  - **Melting** and **freezing** processes

# The first significant recorded mention of the “six-cornered snowflake” - Kepler (1611)



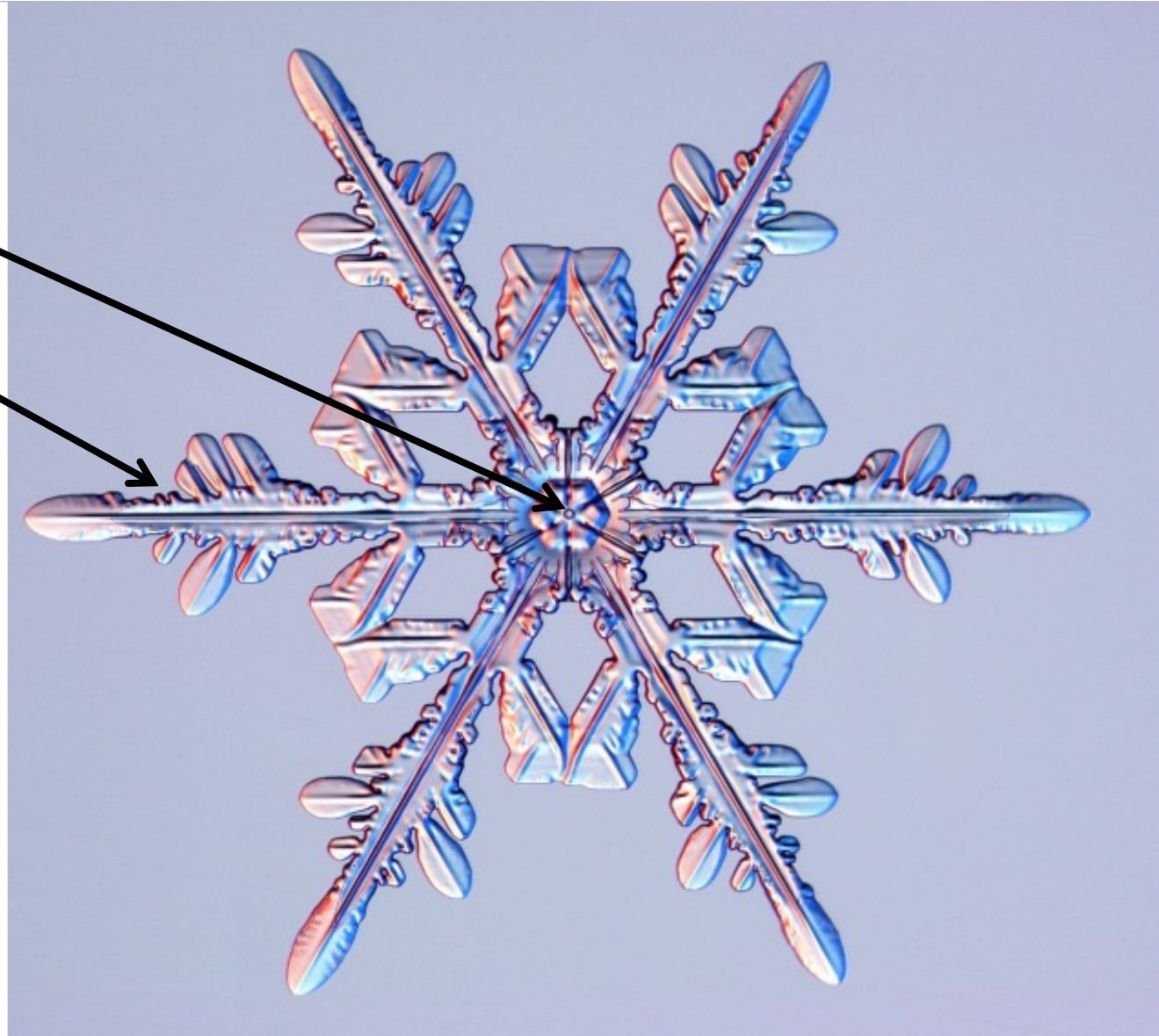
# “The Six-Cornered Snowflake”



[www.snowcrystals.com](http://www.snowcrystals.com)  
Ken Libbricht

# Ice microphysical processes

- Ice nucleation
- Depositional growth (and sublimation)
- Collection (aggregation/riming)
- Splintering
- Melting



The background of the slide is a clear blue sky filled with numerous white contrails from an airplane. The contrails are scattered across the frame, some forming straight lines, others in curved paths, and some intersecting. A semi-transparent light blue rectangular box is centered in the middle of the image, containing the text "1) Nucleation".

# 1) Nucleation

- The formation of an ice particle is called ice nucleation
- Cloud water droplets do not freeze at 0°C !
- Ice nucleation processes can be split into **homogeneous** and **heterogeneous** processes
- **Homogeneous nucleation** - no preferential nucleation sites (i.e. pure water or solution drop)
- **Heterogeneous nucleation** – foreign substance (ice nuclei) initiates freezing



# Homogeneous ice nucleation

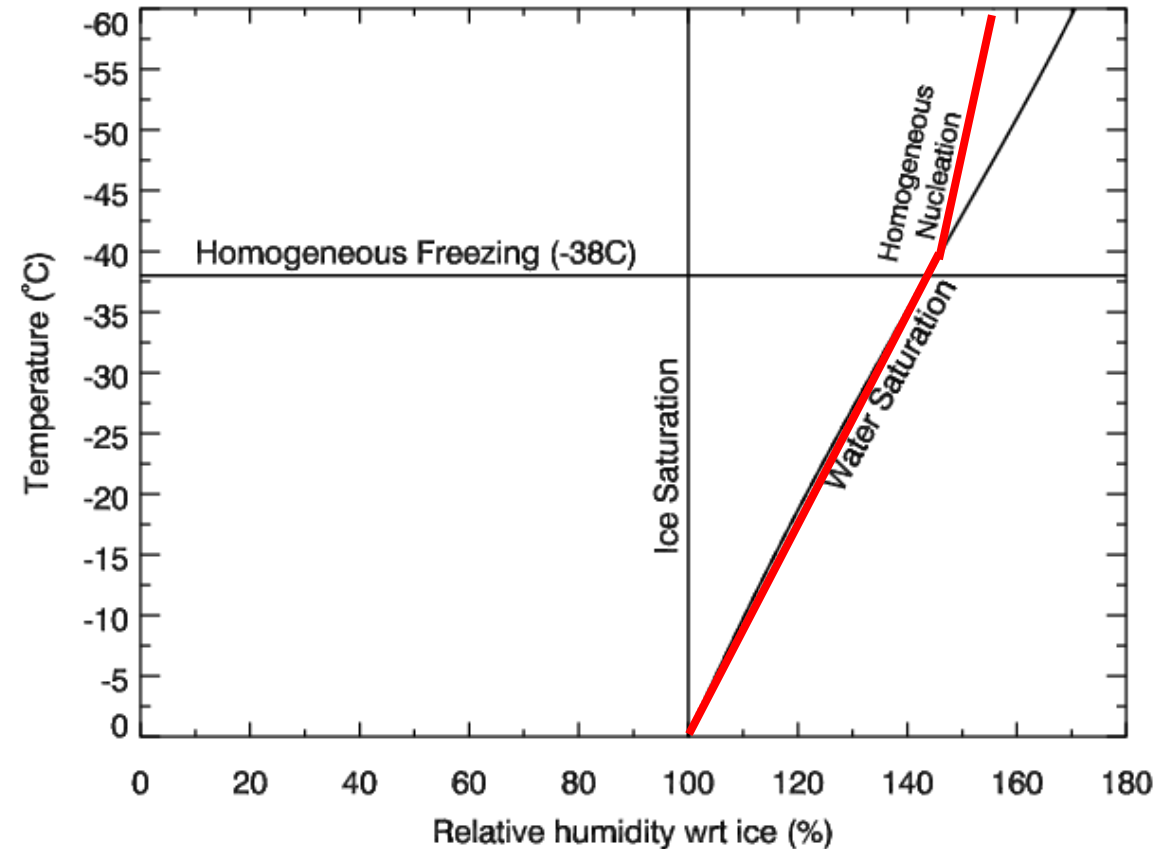
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- No preferential nucleation sites (i.e. pure water or solution drop). Random alignment of water molecules will initiate freezing of the whole particle.
- The smaller the droplet, the less likely homogeneous nucleation will occur and the colder it can survive as supercooled water
- Typical cloud droplets (10  $\mu\text{m}$ ) can stay as supercooled water down to around  $-38^\circ\text{C}$  before homogeneously freezing (for example, water droplets carried upward by deep convective updrafts)
- Small aqueous solution particles (0.1  $\mu\text{m}$ ) can be supercooled at even colder temperatures. The freezing is dependent on a critical relative humidity above saturation (supersaturation threshold increases with decreasing temperature) (Koop et al. 2000)
- An important consequence of this is that the air can be significantly supersaturated with respect to ice before cloud ice particles form (in contrast to supersaturation with respect to water)
- Observations of clear air ice supersaturation are common, long-lasting contrails...



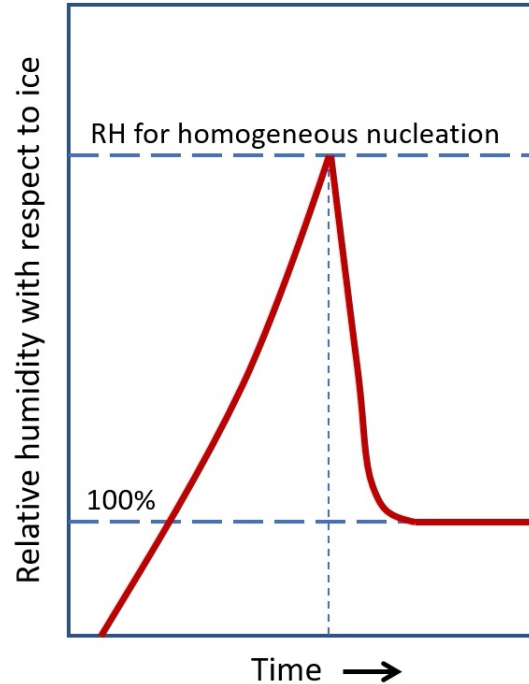
# Ice supersaturation and homogeneous nucleation

- What is the maximum ice supersaturation that can occur?
- Classical theory and laboratory experiments document the critical vapour saturation mixing ratio with respect to ice at which homogeneous nucleation initiates from aqueous solution drops (Pruppacher and Klett, 1997; Koop et al., 2000).
- Leads to a supersaturated relative humidity threshold  $S_{crit}$  as a function of temperature (Koop et al., 2000, Kärcher and Lohmann, 2002), up to 160% RH<sub>i</sub> at -60°C.
- This threshold is used in the ECMWF model to allow ice supersaturation in clear sky.

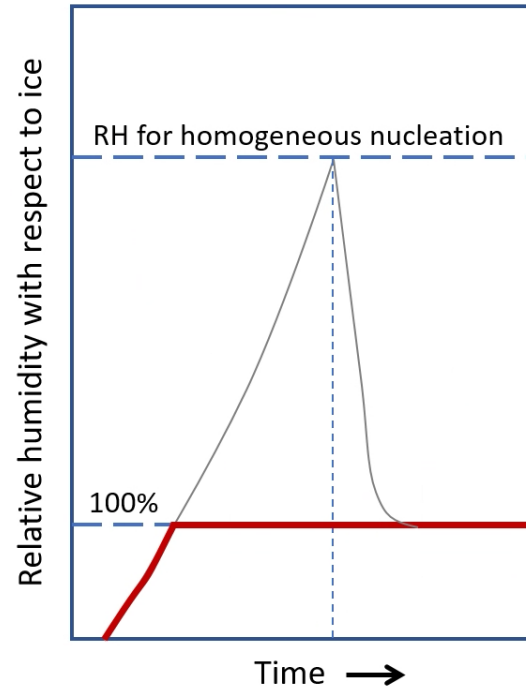


# Ice supersaturation and homogeneous nucleation - parametrization

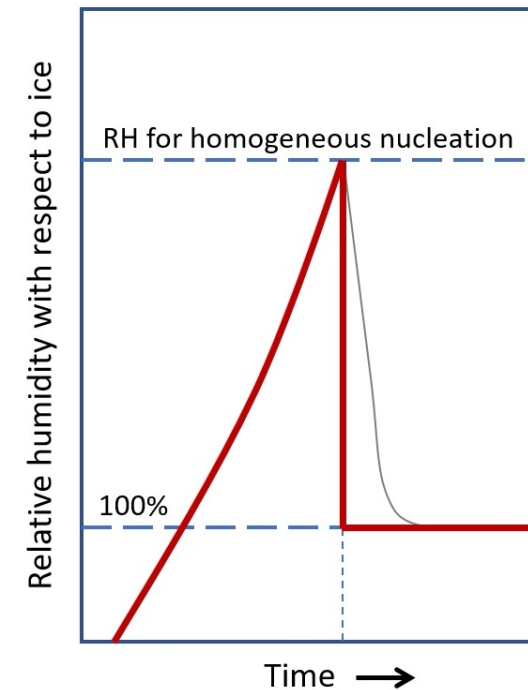
Evolution of an air parcel subjected to adiabatic cooling at low temperatures



Evolution of an air parcel subject to adiabatic cooling at low temperatures



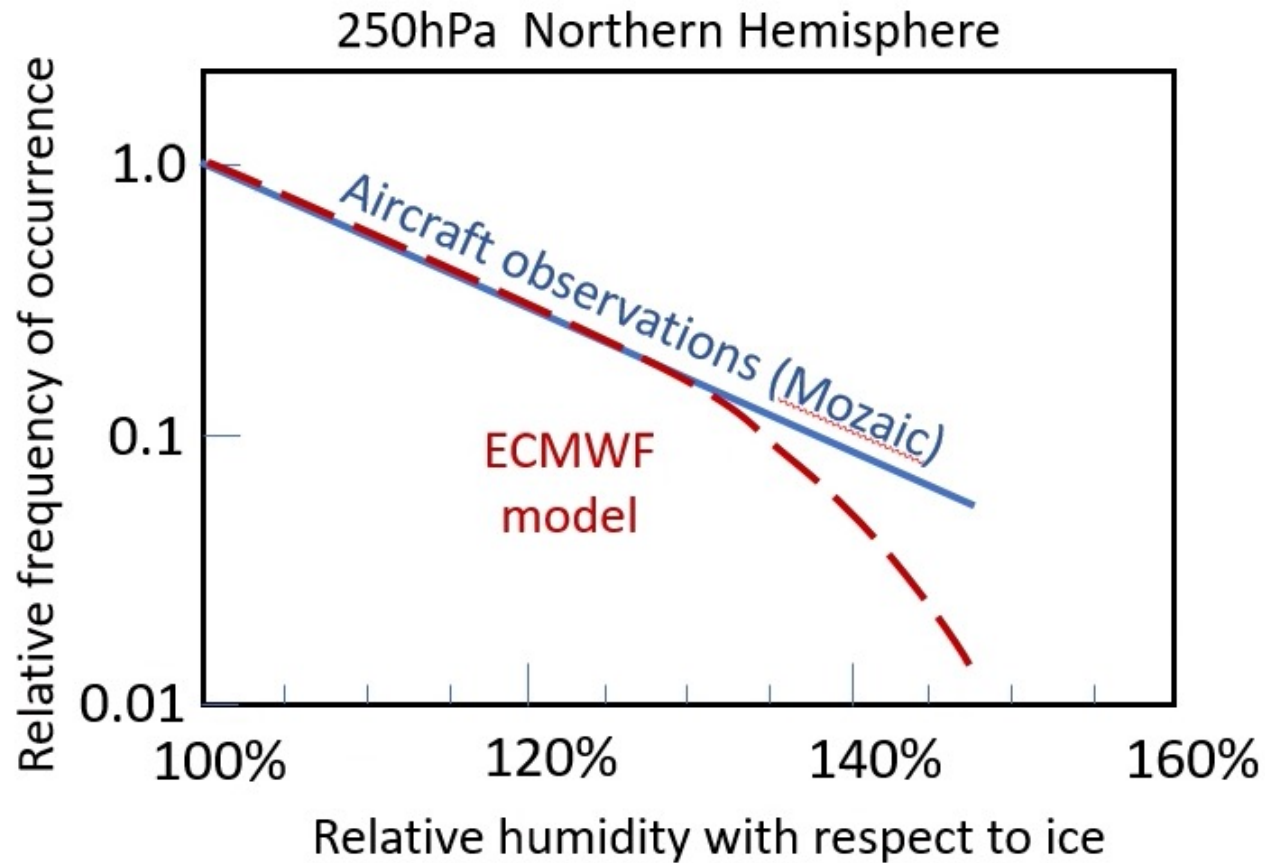
Red line: Evolution if **no ice supersaturation** allowed (many models)



Red line: Evolution if ice supersaturation is allowed **until reaches  $S_{crit}$**  then all supersaturation converted to ice (ECMWF model)

From Tompkins et al. (2007) adapted from Kärcher and Lohmann (2002)

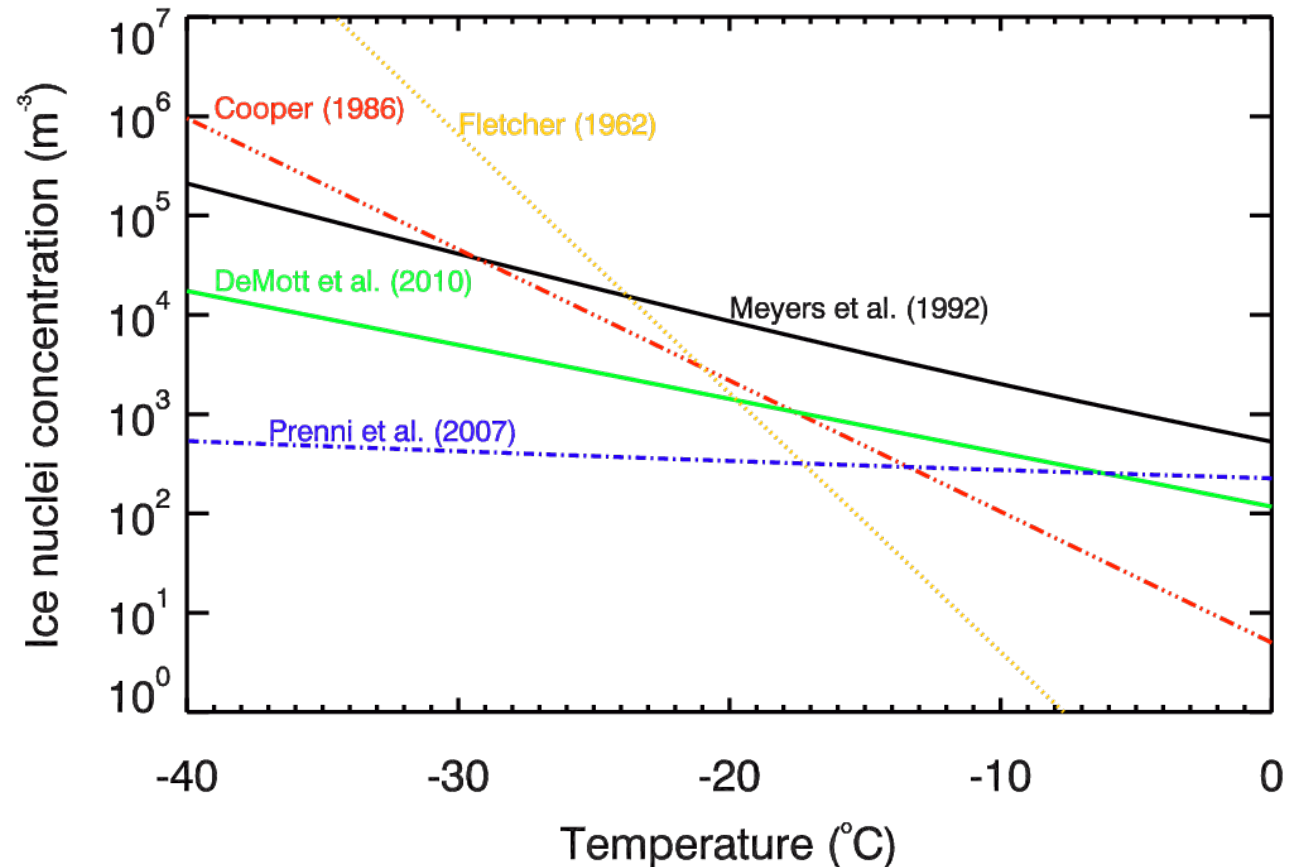
# Ice supersaturation: evaluation of the ECMWF model with aircraft obs



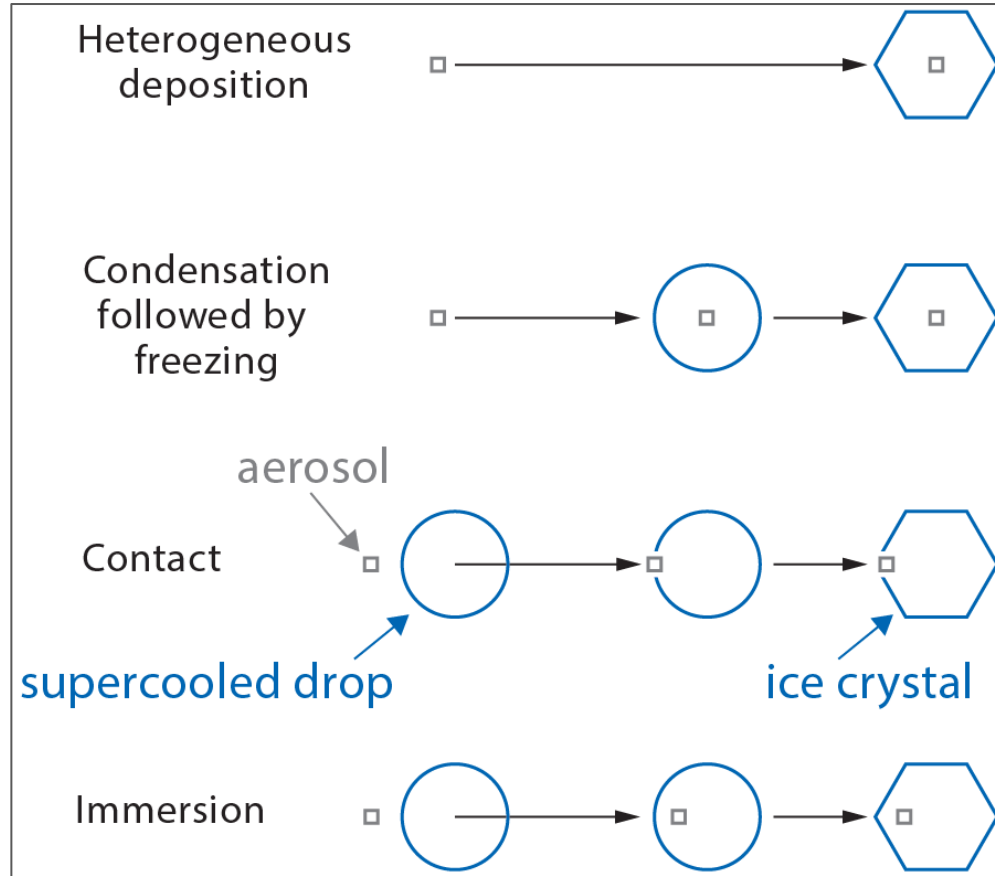
Relative humidity (wrt ice) clear sky, over 1 month, 250hPa, Region: 60S-60N, 180W-180E  
Observations from the MOZAIC campaign (based on Tompkins et al. 2007)

# Heterogeneous ice nucleation

- Preferential sites for nucleation (interaction with solid aerosol particles – ice nuclei)
- Frequent observation of ice between 0°C and colder temperatures indicates heterogeneous processes are active.
- Number of **activated ice nuclei increases with decreasing temperature** so heterogeneous nucleation more likely with increasing altitude, e.g. Fletcher (1962); Cooper (1986), Meyers (1991); Prenni et al. (2007) DeMott et al (2010).
- Lots of uncertainty!



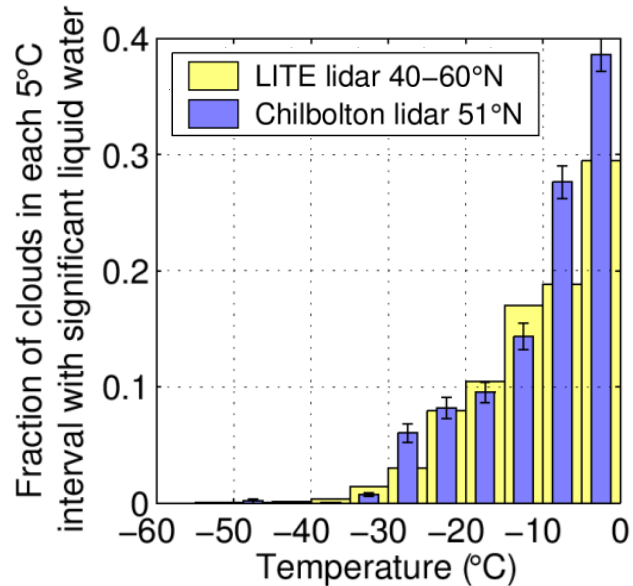
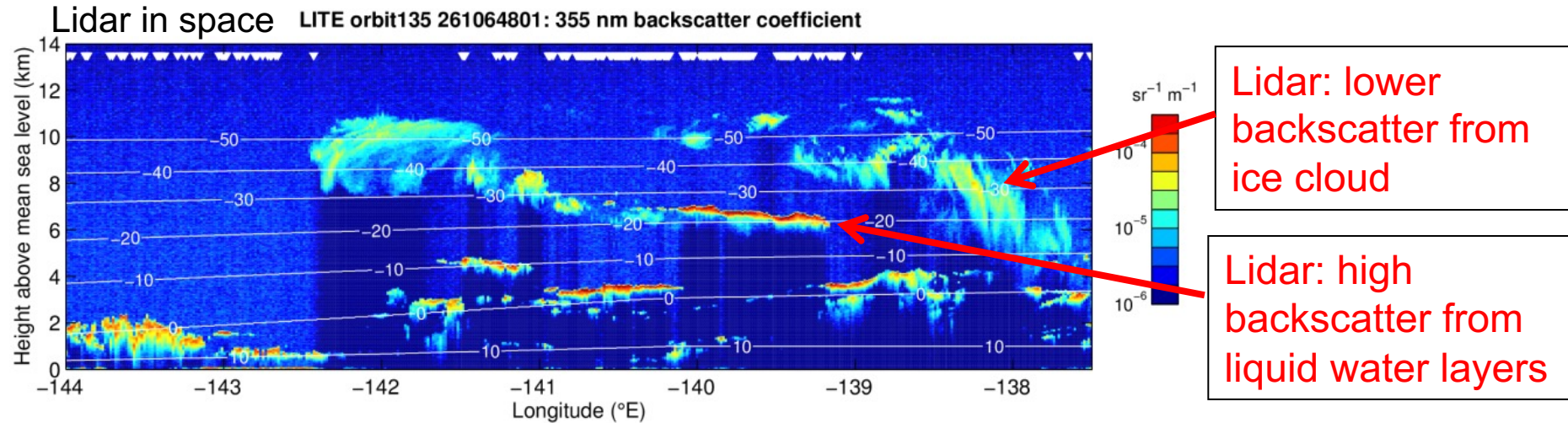
# Heterogeneous ice nucleation



Still many uncertainties in heterogeneous ice nucleation processes in the atmosphere and their impacts!

**Schematic of heterogeneous ice nucleation mechanisms**  
(after Rogers and Yau, 1996)

# Observed supercooled liquid water occurrence



(Hogan et al., GRL, 2004)

## Observations:

- Colder than -38°C, no supercooled liquid water.
- Supercooled liquid water increasingly common as approach 0°C.
- Often in shallow layers at cloud top, or in strong updraughts associated with convection
- Often mixed-phase cloud – liquid and ice present
- Convective clouds with tops warmer than -5°C rarely have ice.

The background of the slide is a clear blue sky filled with numerous white contrails from an airplane. The contrails are scattered across the frame, some forming straight lines while others are more curved or overlapping, creating a complex pattern that visually represents the concept of diffusion growth.

## 2) Diffusion growth



# Diffusional growth of ice crystals: Deposition and sublimation

Equation for the rate of change of mass for an ice particle of diameter  $D$  due to deposition (diffusional growth), or sublimation if subsaturated air:

$$\frac{dm(D)}{dt} = \frac{4\pi s C F}{\left(\frac{L_s}{RT} - 1\right) \frac{L_s}{k_a T} + \frac{RT}{\chi e_{si}}}$$

$$\propto s C F$$

$s$ , the supersaturation (or subsaturation)

$C$ , the particle shape (habit) (*plate, column, aggregate*)

$F$ , the ventilation factor (*particle falling through air*)

Integrate over assumed particle size spectrum  $N(D)$  to get total ice mass growth rate:

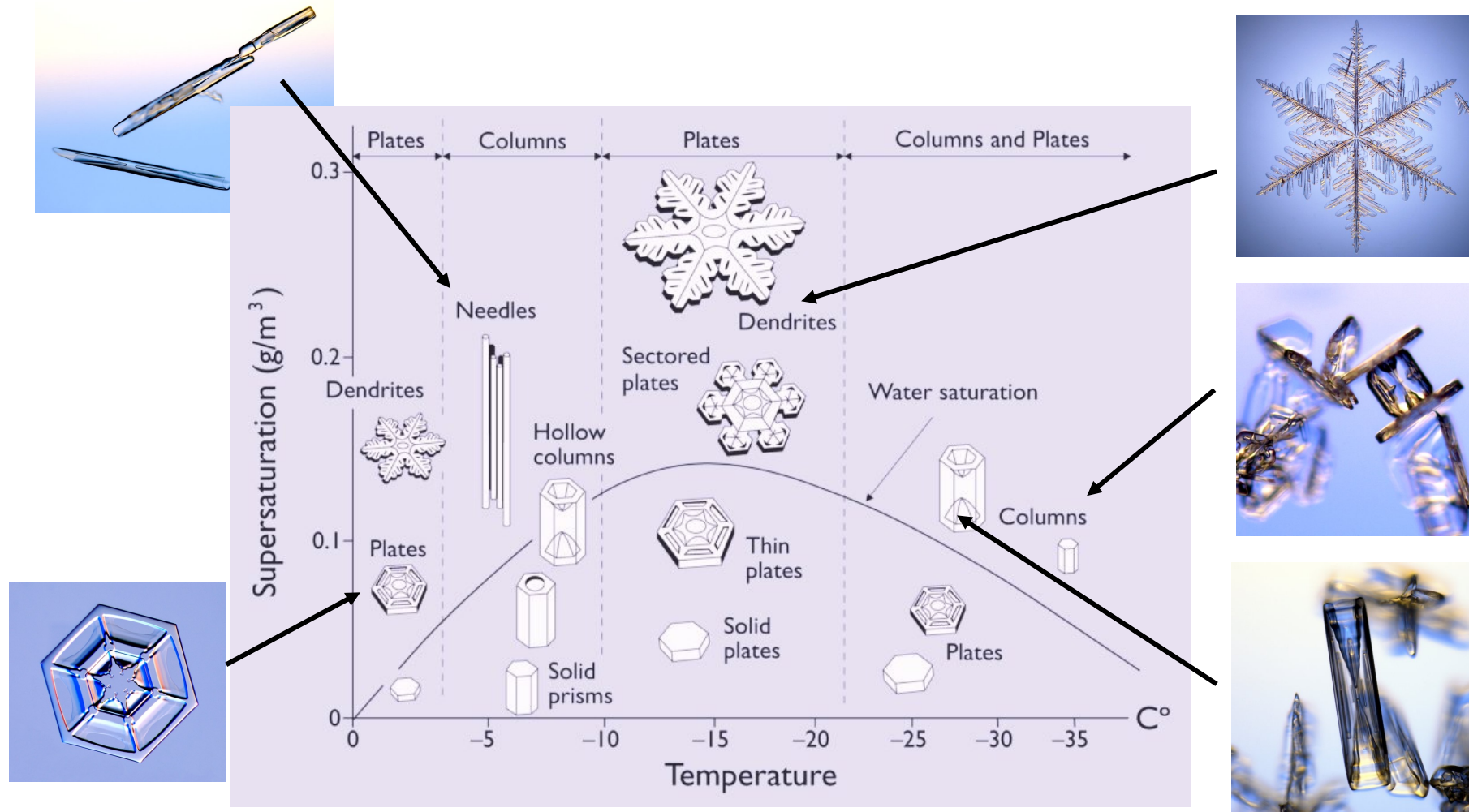
$$P_{depsub} = \int_0^{\infty} \frac{dm(D)}{dt} N(D) dD$$

Particle size distribution (number of particles of each size  $D$ ) for ice/snow often represented as an exponential function:

$$N(D) = N_0 \exp(-\Lambda D)$$

# Diffusional growth of ice crystals: Ice habits

- The mode of growth (edge growth vs corner growth) is sensitive to the temperature and ice supersaturation.
- Ice habits can be complex: influences fall speeds and radiative properties



# Diffusional growth of ice crystals: Animation of crystal growth

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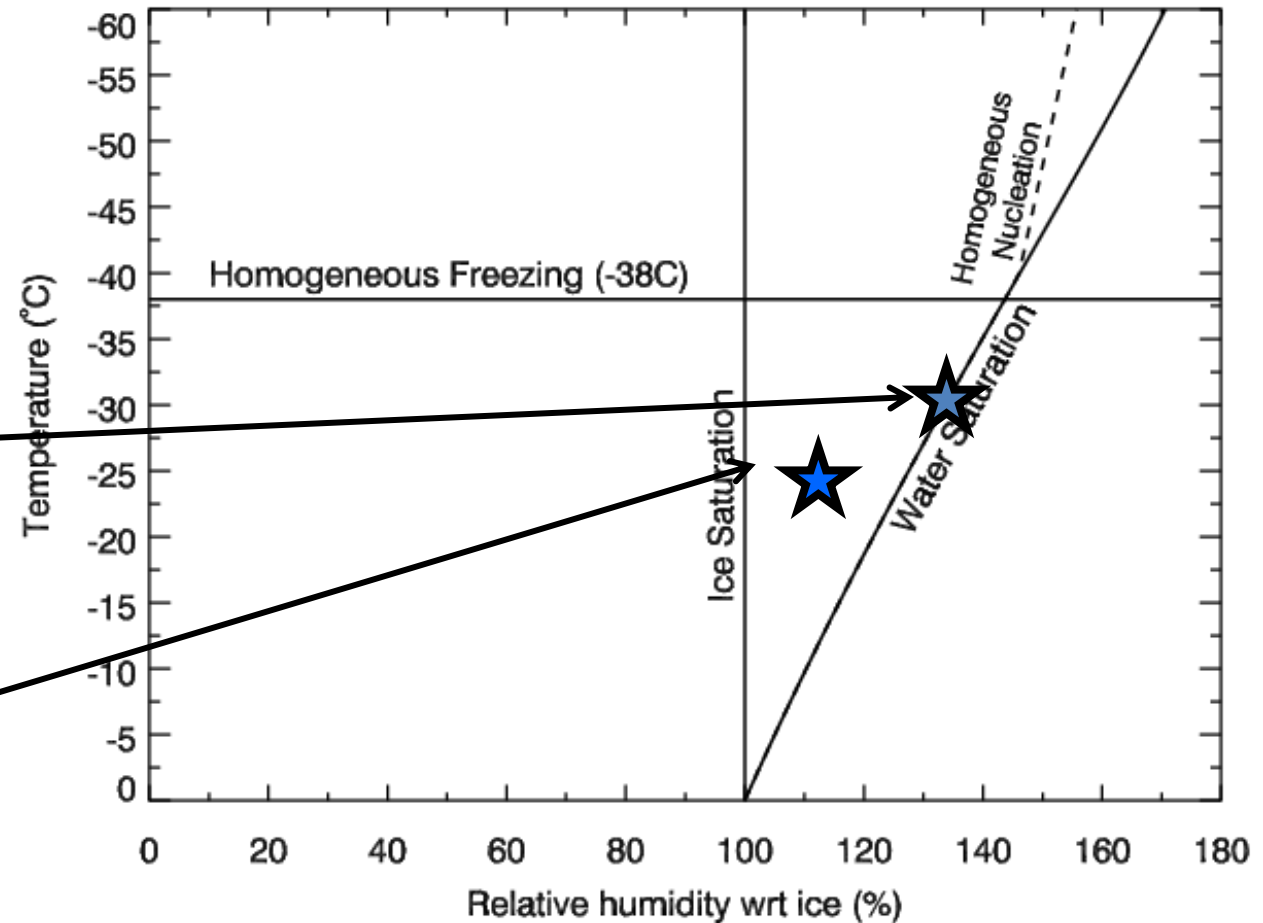
# Diffusional growth of ice crystals: Mixed-phase clouds

## Wegener-Bergeron-Findeisen (WBF) process (1)

The saturation vapour pressure with respect to ice is smaller than with respect to water.  
This means that:

1) A cloud which is **saturated** with respect to water is **supersaturated** with respect to ice.

2) A cloud which is **sub-saturated** with respect to water can be **supersaturated** with respect to ice.



# Diffusional growth of ice crystals: Mixed-phase clouds

## Wegener-Bergeron-Findeisen (WBF) process (2)

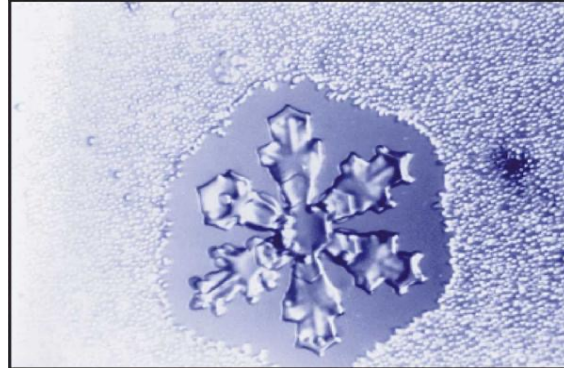
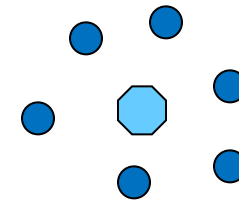


Photo by R. P. Tier

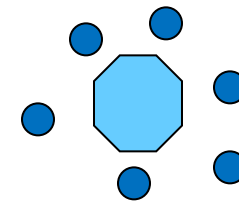
Ice particles grow at the expense of water droplets

Ice particle enters water cloud



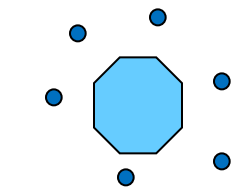
Cloud is supersaturated with respect to ice

Diffusion of water vapour onto ice particle



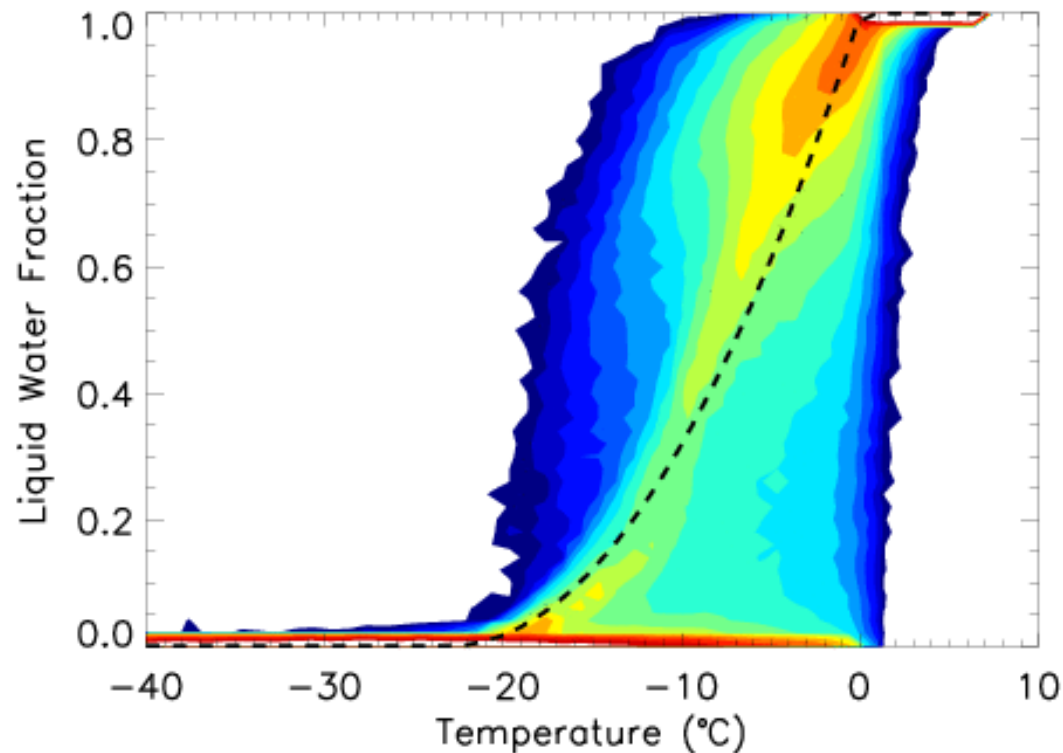
Cloud will become sub-saturated with respect to water

Water droplets evaporate to increase water vapour



# Parametrizing cloud phase: diagnostic vs prognostic

- Many (global) models with a single condensate prognostic parametrize ice/liquid phase as a **diagnostic function of temperature** (see dashed line for ECMWF model pre-2010 below).
- Models with separate prognostic variables for liquid water and ice, parametrize deposition allowing a **wide range of supercooled liquid water/ice fraction** for a given temperature (see shading in example below).



$$\frac{\partial m}{\partial t} = \frac{4\pi sCF}{\left(\frac{L_s}{RT} - 1\right) \frac{L_s}{k_a T} + \frac{RT}{\chi e_{si}}}$$

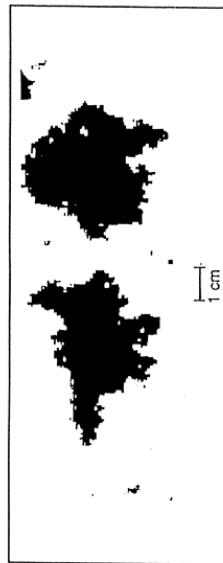
PDF of liquid water fraction of cloud for a diagnostic mixed phase scheme (dashed line) and prognostic ice/liquid scheme (shading)



## 3) Collection processes

# Collection processes: Ice crystal aggregation

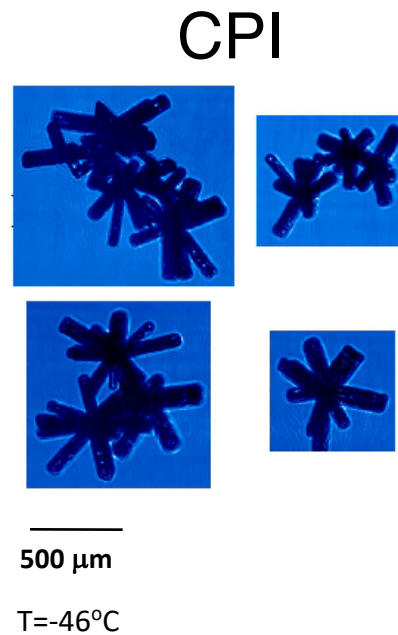
- Ice crystals can aggregate together to form “snow”
- “Sticking” efficiency increases at temperatures warmer than  $-5^{\circ}\text{C}$
- Irregular crystals are most commonly observed in the atmosphere (e.g. Korolev et al. 1999, Heymsfield 2003)



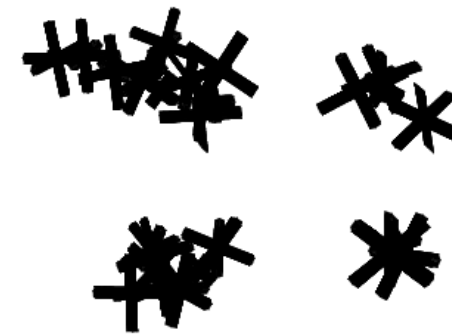
Lawson, JAS'99



Field & Heymsfield '03

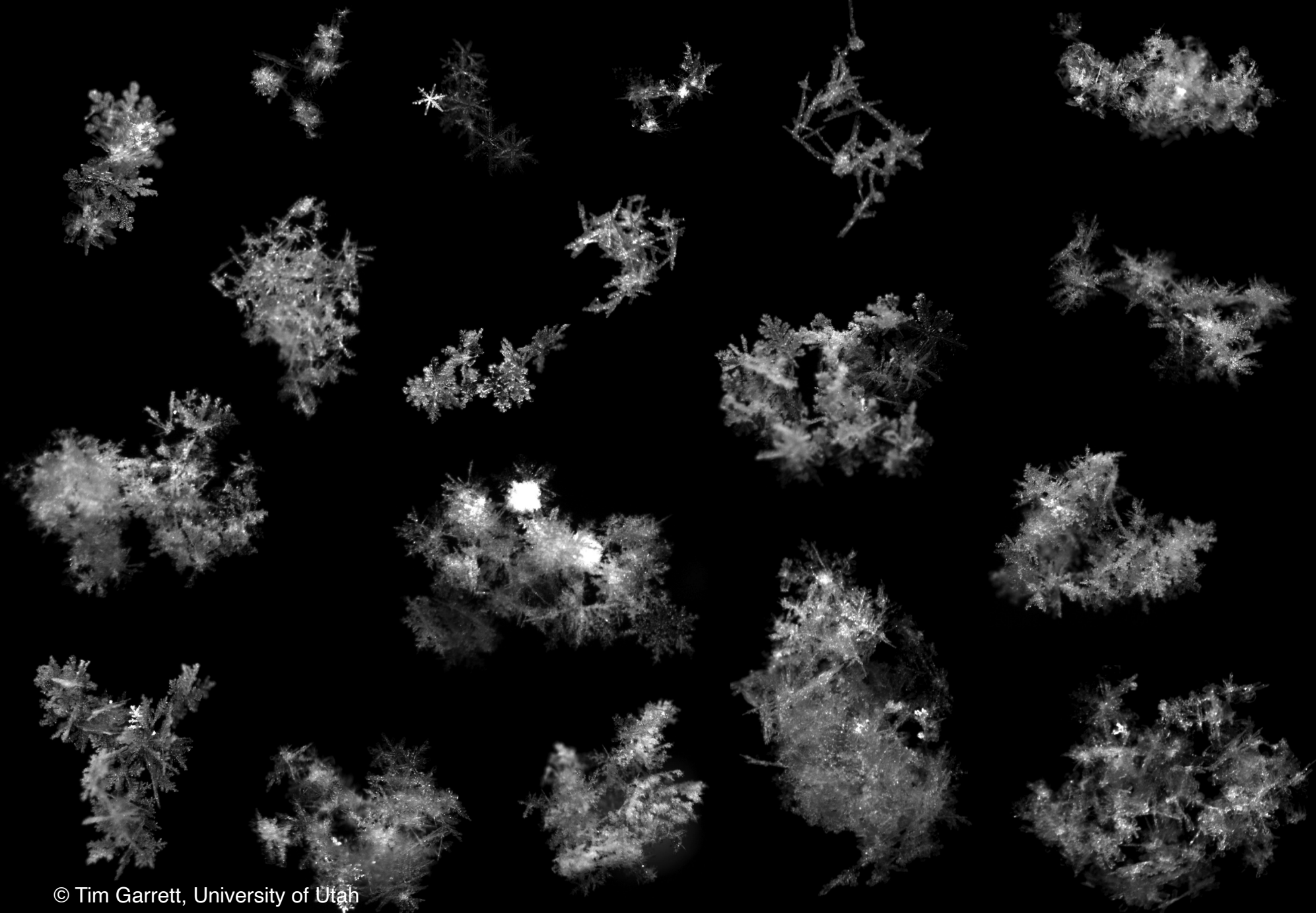


Model



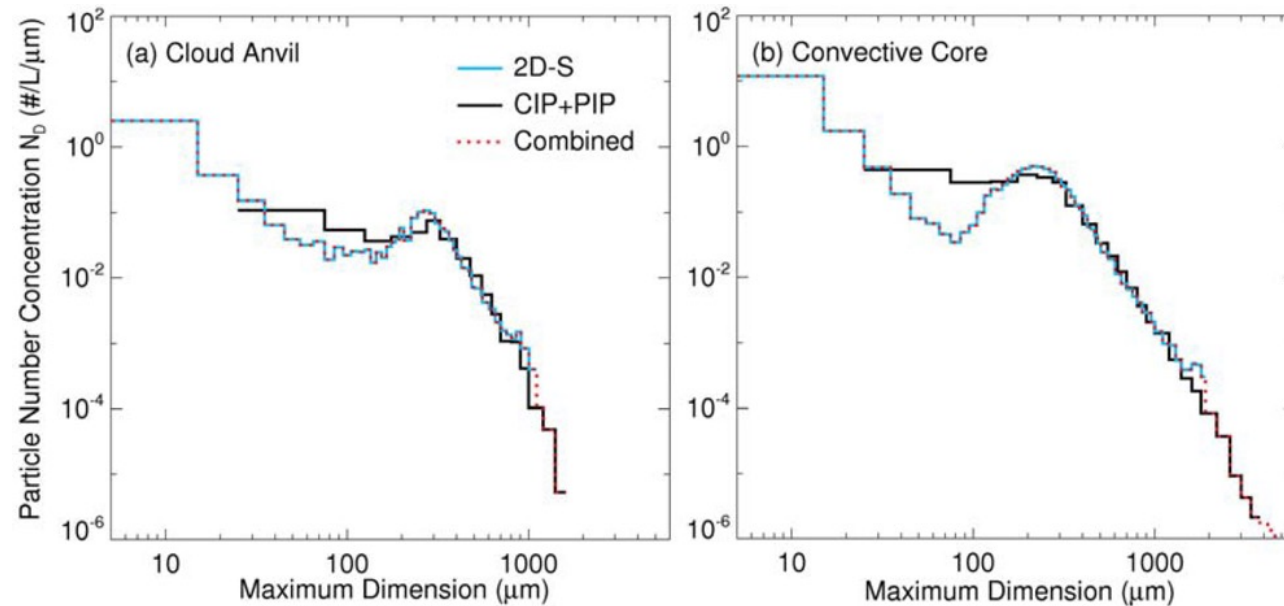
Westbrook et al. (2008)





# Parametrization of aggregation

- Many models still have separate variables for ice and snow with a parametrization for **aggregation**, represented as an autoconversion.
- But any separation in the particle size spectra between ice and snow is much less clear than for cloud droplets and rain, e.g. as seen in PSDs from aircraft observations (Minnis et al. 2012):

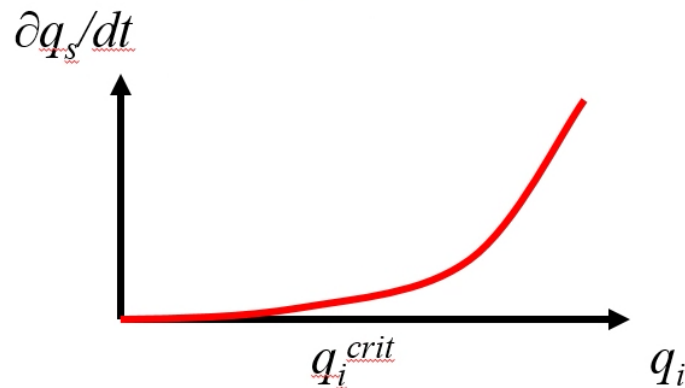


- Some schemes represent aggregation as an evolving particle size distribution, either prognostic number concentration (i.e.  $q_i$ ,  $N_i$ ) or as a diagnostic function (e.g.  $fn(q_i, T)$ ).

# Parametrization of aggregation: “ice” to “snow” autoconversion

Representing **aggregation** in the ice phase with separate ice and snow variables (conversion ice-to-snow) in the ECMWF model analogous to warm-phase autoconversion but function of temperature (increases for warmer T)

Sundquist-type ice-to-snow autoconversion



$$\frac{\partial q_s}{\partial t} = c_0(T) q_i \left( 1 - e^{-\left( q_i / q_i^{crit} \right)^2} \right)$$

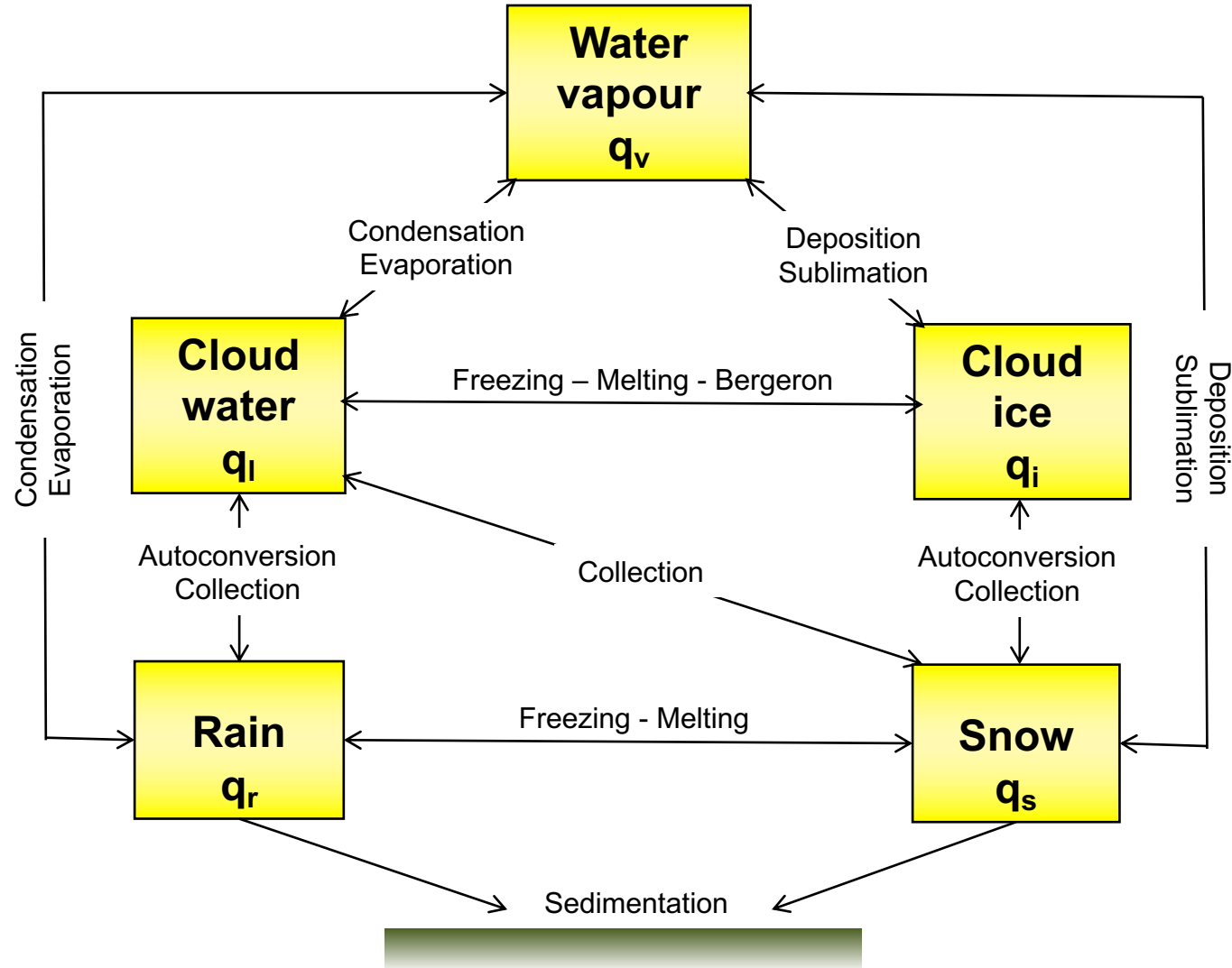
Rate of conversion of ice (small particles) to snow (large particles) increases as the temperature increases.

$$c_0(T) = 10^{-3} e^{0.025(T - 273.15)} \text{ s}^{-1}$$

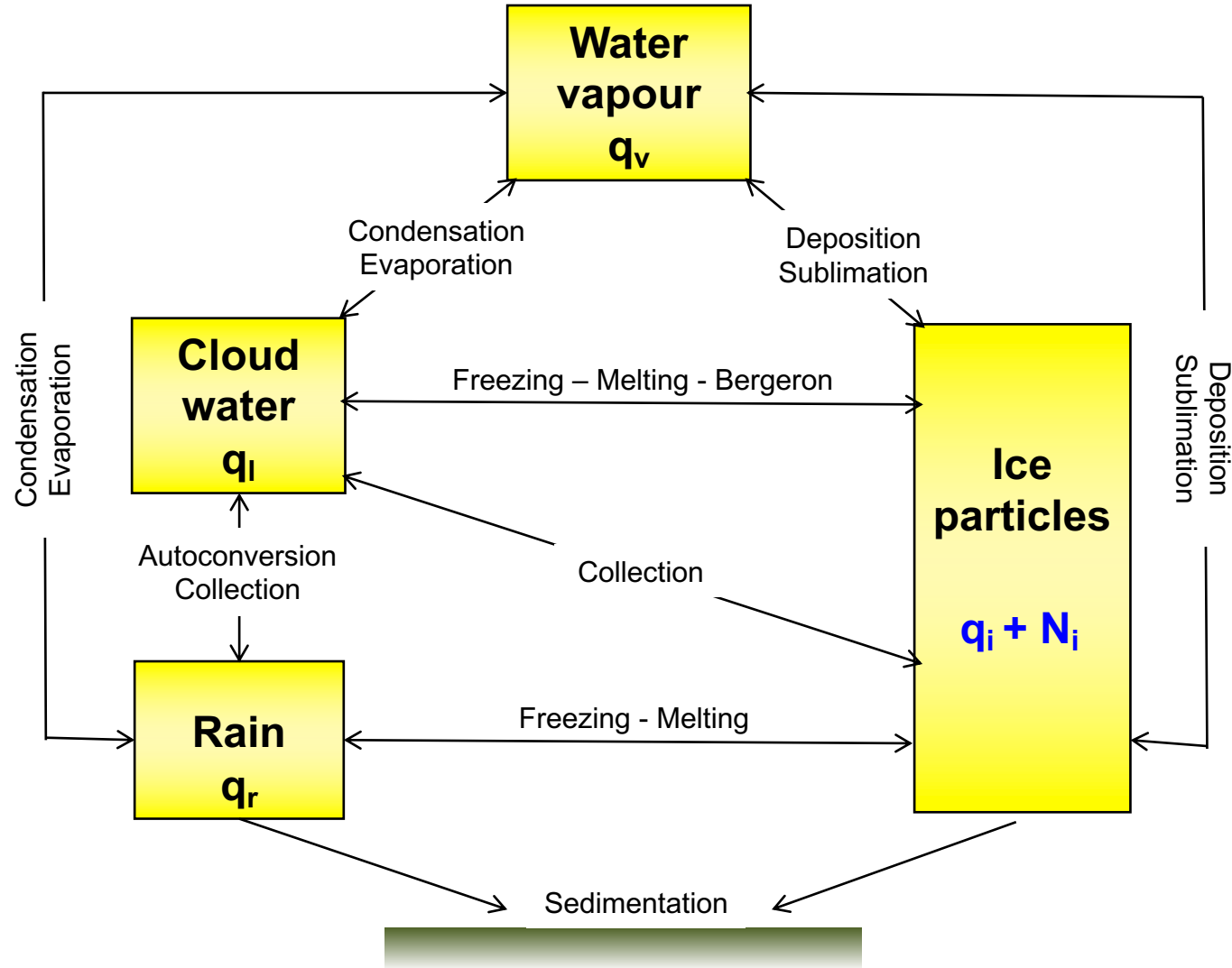
$$q_i^{crit} = 3 \cdot 10^{-5} \text{ kg kg}^{-1}$$

# Microphysics parametrization: Separate “cloud ice” and “snow” mass

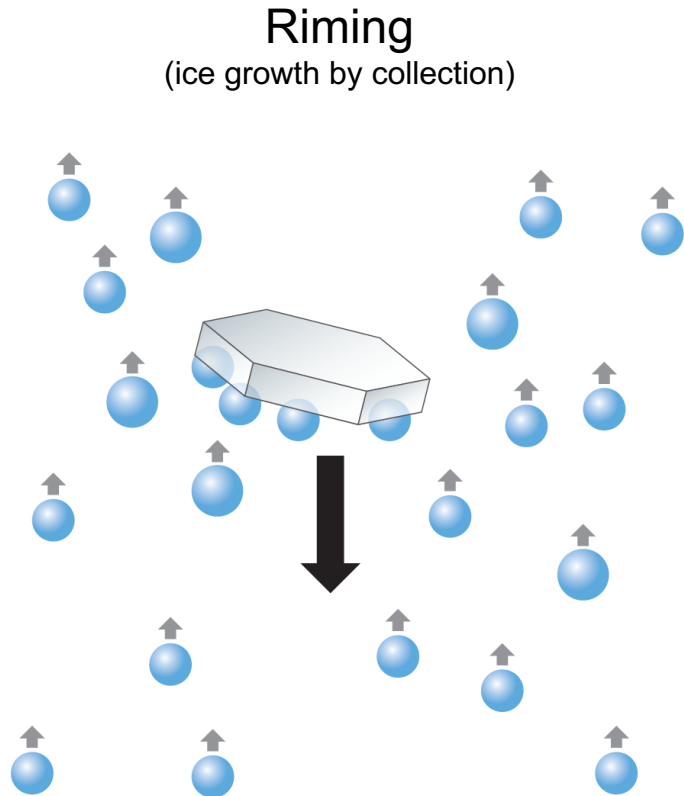
Current ECMWF model



# Microphysics parametrization: Ice particle mass and number concentration



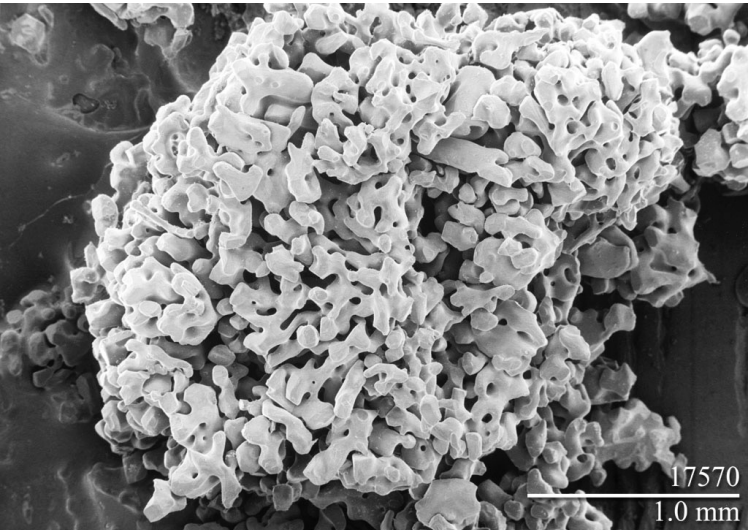
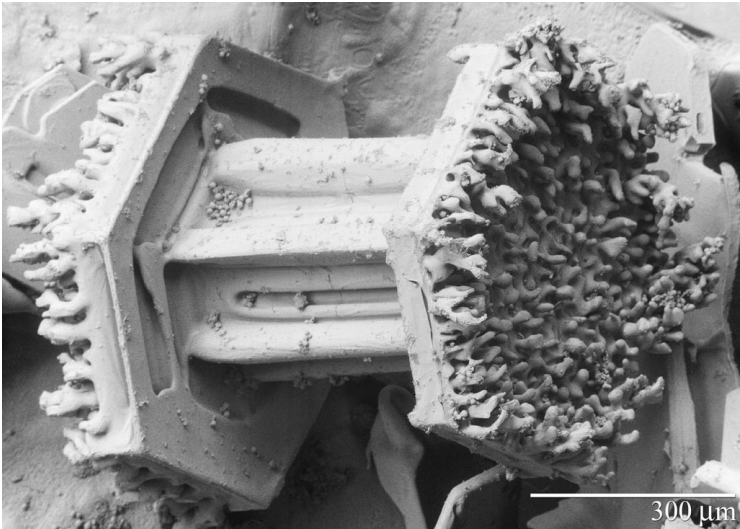
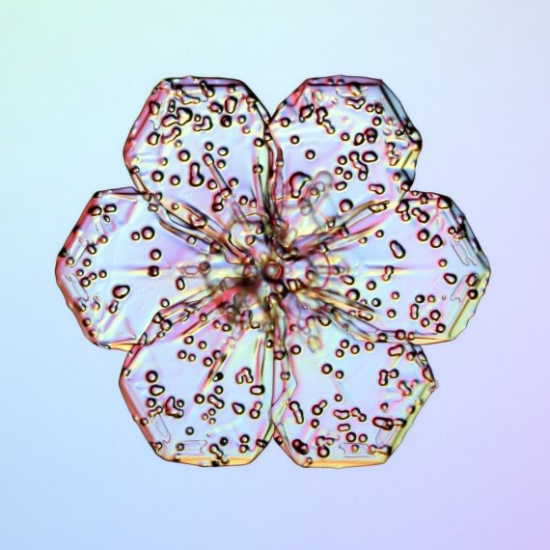
# Collection processes: Riming – capture of water drops by ice particles



- **Graupel** formed by collecting liquid water drops in mixed phased clouds (“riming”)
- Occurs when the cloud is at water saturation in strong updraughts (convection).
- Results in rounded ice crystals with higher densities and fall speeds than snow dendrites.

- **Hail** forms if particle temperature is close to 0°C or above
- The liquid water at the surface “spreads out” before refreezing, creating a more spherical denser particle.
- Higher fall speed (up to 40 m/s!) implies hail only forms in convection with strong updraughts able to support the particle long enough for sufficient growth.

# Rimed ice crystals



<http://www.its.caltech.edu/~atomic/snowcrystals>

Electron micrographs (emu.arsusda.gov)

# Parametrization of collection processes and rimed ice particles

Collection processes (between any hydrometeor categories, e.g. snow and cloud liquid, rain and snow etc.), can be represented with the following equation for hydrometeors x and y:

Double integral over all particles x and y

$$P_{xy} = \frac{\pi E_{xy}}{4\rho} \int_{D_x=0}^{\infty} \int_{D_y=0}^{\infty} (D_x + D_y)^2 \underbrace{|V_x(D_x) - V_y(D_y)|}_{\text{Fall velocity difference}} \underbrace{N_x(D_x)m_x(D_x)}_{\text{Total mass of particle x}} \underbrace{N_y(D_y)m_y(D_y)}_{\text{Total mass of particle y}} dD_x dD_y$$

E=collection efficiency

- Need assumptions for fall velocities, particle size distributions, mass/density relationships for each hydrometeor and the collection efficiency.
- For collection of *cloud droplets* by larger particles (e.g. snow), the collection equation can be significantly simplified by assuming that cloud liquid drop size and fall speed are negligible compared to the larger particles.



# Parametrization of collection processes and rimed ice particles

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- Most GCMs with parametrized convection don't explicitly represent graupel or hail (too small scale), although they may include riming using the simplified collection equation as a sink of cloud liquid water, i.e. snow + cloud liquid → snow
- In cloud resolving models, traditional split between **ice**, **snow** and **graupel** and **hail** as prognostic variables with the collection equation for each pair, but this split is rather artificial.
- Degree of riming can be light or heavy, particle density can vary smoothly.
- Alternative approach is to have **ice particle properties** as the prognostic variables, e.g.
  - Morrison and Grabowski (2008) have 3 ice variables: **deposition mass**, **rime mass** and **number**.
  - Morrison and Milbrandt (2015) have 4 ice variables to also represent hail-type particles: **total ice mass**, **rime mass**, **rime volume** and **number**.
  - Avoids artificial thresholds between different categories.

## Other ice-phase microphysical processes

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- **Splintering** of ice crystals, Hallett-Mossop splintering through riming around  $-5^{\circ}\text{C}$ . Leads to increased numbers of smaller crystals and greater diffusion growth. Used when particle number concentrations are represented.
- **Melting** as ice particles fall into air warmer than a wet-bulb temperature of  $0^{\circ}\text{C}$ . Melting layer typically  $\sim 100\text{-}500$  metres deep. The main source of rain in mid-latitude fronts is from snow melted aloft.
- **Freezing of raindrops** – if there is a warm  $T > 0^{\circ}\text{C}$  layer above a colder sub-freezing layer, snow particles can melt. If the particle is only partially melted as it enters the sub-freezing layer, it will rapidly refreeze to an ice pellet. However, if it melts completely it can fall to the surface as a supercooled rain drop and freeze on impact with the surface – freezing rain – a major hazard!
- **Sedimentation** due to Earth's gravitational force. Fall speed depends on particle size (and habit/density for ice).



## 4) Sedimentation and numerics

# Numerics: Explicit vs Implicit

$$\frac{d\phi}{dt} = -P\phi$$

$\phi$  = e.g. cloud water/ice

Process = e.g. autoconversion, sedimentation

Upstream forward in time solution ( $n$  = current time level,  $n+1$  = next time level)

Explicit solution

$$\frac{\phi^{n+1} - \phi^n}{\Delta t} = -P\phi^n$$

Rearrange  $\longrightarrow$

$$\phi^{n+1} = (1 - P\Delta t)\phi^n$$

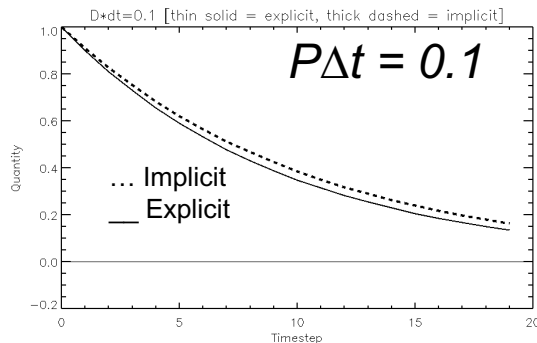
For long timesteps  $P\Delta t$  maybe  $>1$  so explicit  $\phi^{n+1}$  becomes negative!

Implicit solution

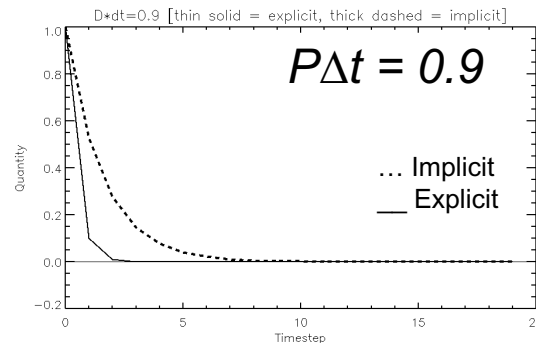
$$\frac{\phi^{n+1} - \phi^n}{\Delta t} = -P\phi^{n+1}$$

Rearrange  $\longrightarrow$

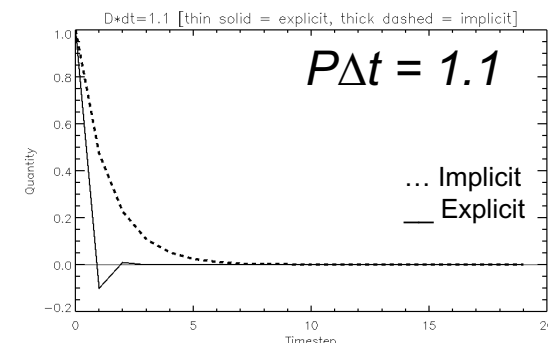
$$\phi^{n+1} = \frac{\phi^n}{(1 + P\Delta t)}$$



Explicit/implicit solutions similar



Explicit solution more accurate



Explicit unstable, implicit stable


$$\frac{d\phi}{dt} = P_{exp} + P_{imp}\phi + \frac{1}{\rho} \frac{d}{dz} (\rho v_x \phi)$$

Advected quantity (e.g. ice) Sedimentation term  
 Explicit Source/Sink Implicit Source/Sink (not required for short timesteps)

Hydrometeors can fall through more than one grid layer in a timestep.

Options for sedimentation?

- (1) semi-Lagrangian
- (2) time splitting
- (3) implicit numerics**

 what is short?  
( $P\Delta t \ll 1$ )

Implicit:

Upstream forward in time,

k = vertical level

n = time level

$\phi$  = cloud water ( $q_x$ )

$$\frac{\phi_k^{n+1} - \phi_k^n}{\Delta t} = P_{exp} + \frac{\rho_{k-1} V_{k-1} \phi_{k-1}^{n+1}}{\rho_k \Delta Z} + \left( P_{imp} - \frac{\rho_k V_k}{\rho_k \Delta Z} \right) \phi_k^{n+1}$$

Implicit solution:

$$\phi_k^{n+1} = \frac{P_{exp} \Delta t + \frac{\rho_{k-1} V_{k-1} \phi_{k-1}^{n+1}}{\rho_k \Delta Z} \Delta t + \phi_k^n}{1 - P_{imp} \Delta t + \frac{V_k \Delta t}{\Delta Z}}$$

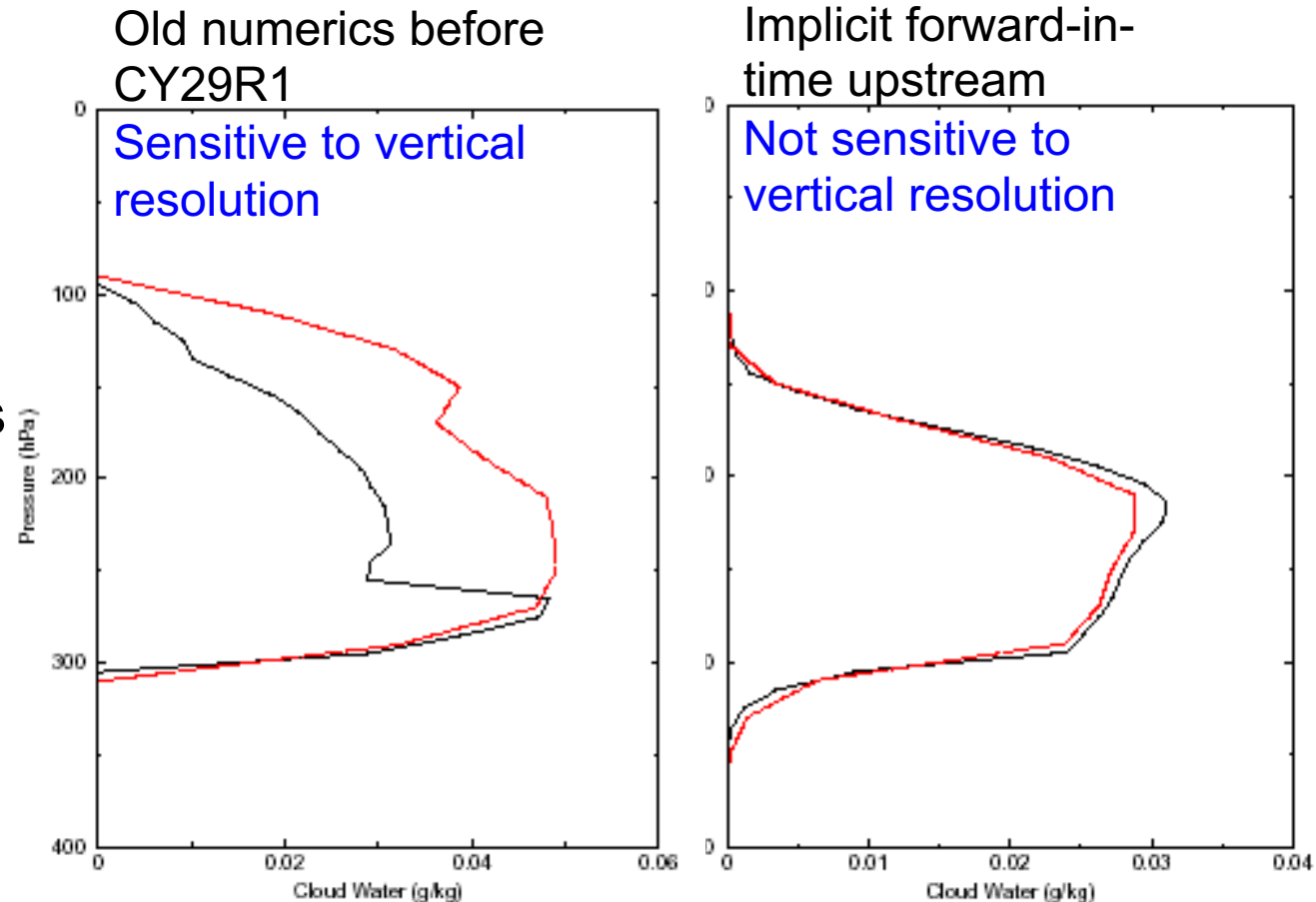
(current implementation in IFS)

# Ice sedimentation: Improved numerics in SCM cirrus case

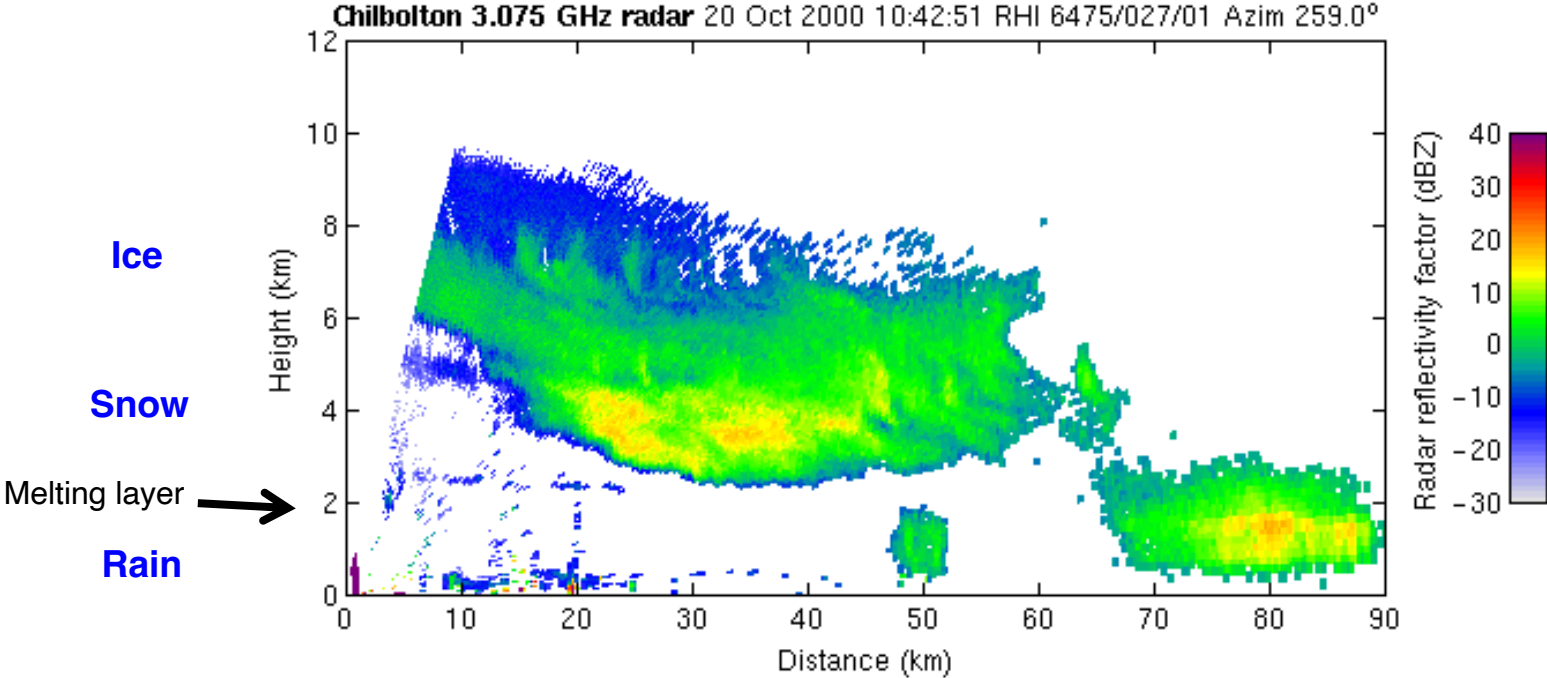
- Important to have a sedimentation scheme that is not sensitive to vertical resolution and timestep.

Vertical profile of  
ice water content

100 vertical levels  
(black) **versus**  
**50 vertical levels**  
(red)



# Falling Precipitation



Courtesy R Hogan, University of Reading, [www.met.rdg.ac.uk/radar](http://www.met.rdg.ac.uk/radar)



# Microphysics - Summary



# From global to micro-scales



Hugely complex system  
Need to simplify!

# Summary

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- Parametrization of cloud and precipitation microphysical processes:
  - Need to simplify a complex system
  - Accuracy vs. complexity vs. computational efficiency trade off
  - Appropriate for the application and no more complexity than can be constrained and understood
  - Dynamical interactions (latent heating), radiative interactions
  - Still many uncertainties (particularly ice phase)
  - Active area of research is aerosol-microphysics interactions.
  - Microphysics often driven by small scale dynamics – how do we represent this in models.....
- Next lecture: Sub-grid scale heterogeneity of cloud and condensate
  - Linking the micro-scale to the macro-scale

# References

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Reference books for cloud and precipitation microphysics:

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Mason, B. J., (1971). *The Physics of Clouds*. Oxford University Press.

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