Planetary Boundary Layer 2

Fundamental concepts

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Contents

- Local eddy diffusion (k-profile)
- Surface layer similarity theory
- Roughness length
- Outer layer local eddy diffusion
- Non-local mass flux in convective PBLs

Set of equations to solve in the model

Momentum

$$\frac{\partial \bar{u}}{\partial t} = -\frac{1}{\rho}\left[\nabla \cdot (\rho \bar{u} \bar{\boldsymbol{u}}) + \frac{\partial \rho \overline{u'w'}}{dz}\right] + fv - \frac{1}{\rho}\frac{\partial p}{\partial x}$$

$$\frac{\partial \bar{v}}{\partial t} = -\frac{1}{\rho}\left[\nabla \cdot (\rho \bar{v} \bar{\boldsymbol{u}}) + \frac{\partial \rho \overline{v'w'}}{dz}\right] - f\mathrm{u} - \frac{1}{\rho}\frac{\partial p}{\partial y}$$

$$\frac{\partial \bar{p}}{\partial z} = -\rho g$$

Thermodynamics

$$\frac{\partial \bar{\theta}}{\partial t} = -\frac{1}{\rho}\left[\nabla \cdot (\rho \bar{\theta} \bar{\boldsymbol{u}}) + \frac{\partial \rho \overline{\theta'w'}}{dz}\right] + S_\theta$$

Moisture

$$\frac{\partial \bar{q}}{\partial t} = -\frac{1}{\rho}\left[\nabla \cdot (\rho \bar{q} \bar{\boldsymbol{u}}) + \frac{\partial \rho \overline{q'w'}}{dz}\right] + S_q$$

Large scale terms – resolved by model

Small scale turbulent fluxes – must be parametrized

Sources and sinks (e.g. heating and cooling from radiation)

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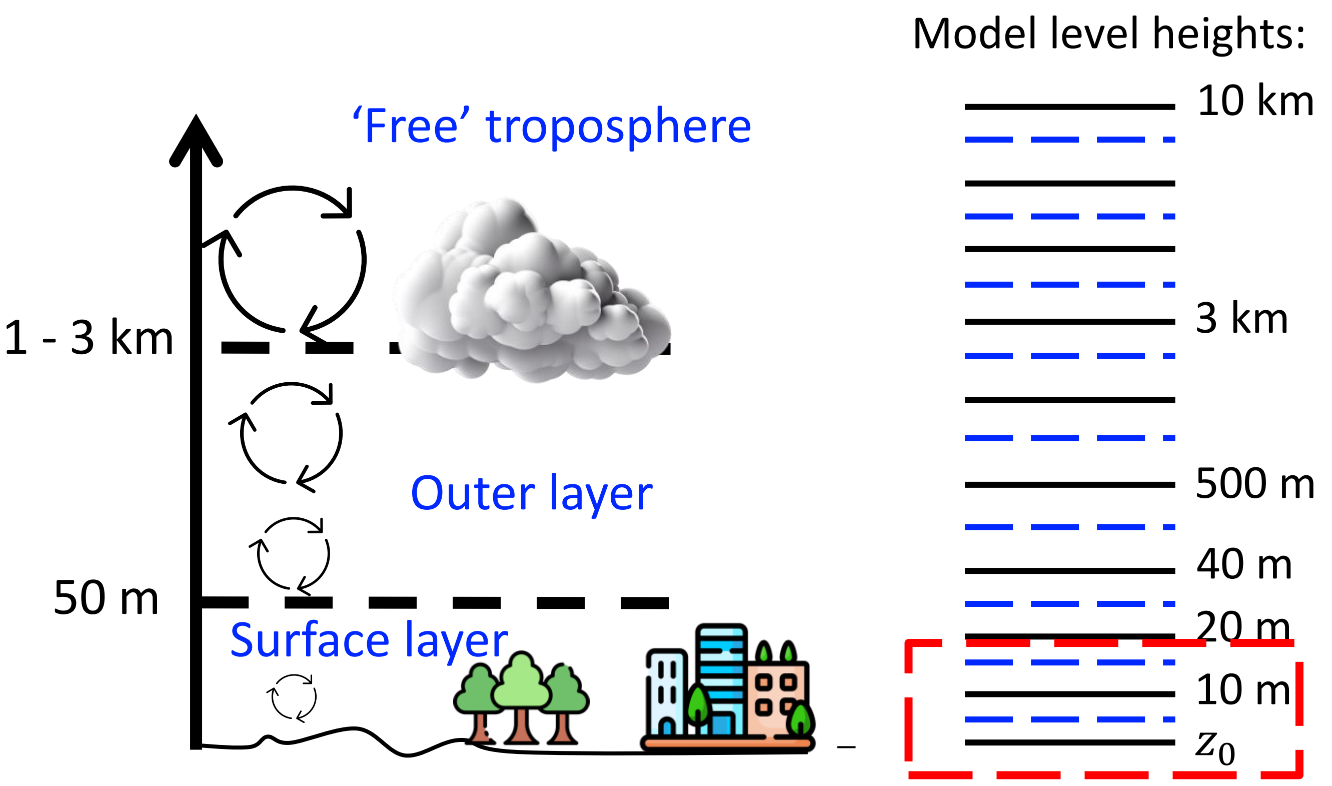
What do we need from a BL turbulence parametrization?

- Provide turbulent fluxes of heat, momentum, moisture (and tracers) between the surface and the upper atmosphere

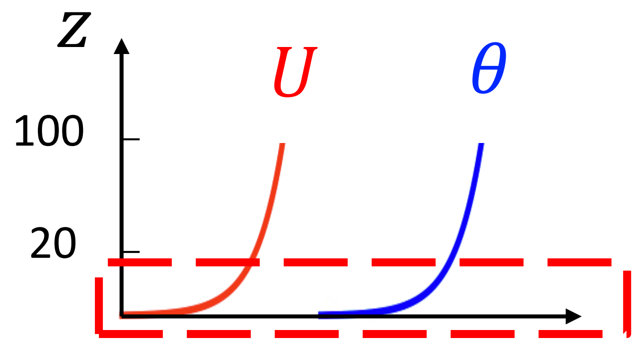
- Provide turbulent mixing throughout the entire atmosphere – the mixed layer, the cloud layer and the stratosphere

- Account for differences in stability, surface properties and clouds

- Provide profiles of winds and temperatures at the surface, where the model does not resolve in the vertical



- Model does not resolve surface layer
- There are strong gradients and is where people live
- Requires diagnosis of profiles below 10m



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‘Local’ turbulence closure: eddy diffusion (K-profile)

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Momentum

$$\frac{\partial \overline{u}}{\partial t} = -\frac{1}{\rho}\left[\frac{\partial \rho \overline{u'w'}}{dz}\right]$$

Thermodynamics

$$\frac{\partial \overline{\theta}}{\partial t} = -\frac{1}{\rho}\left[\frac{\partial \rho \overline{\theta' w'}}{dz}\right]$$

Moisture

$$\frac{\partial \overline{q}}{\partial t} = -\frac{1}{\rho}\left[\frac{\partial \rho \overline{q' w'}}{dz}\right]$$

‘Local’ turbulence closure: eddy diffusion (K-profile)

Momentum

$$\frac{\partial \overline{u}}{\partial t} = -\frac{1}{\rho}\left[\frac{\partial \rho \overline{u'w'}}{dz}\right] \sim -\frac{1}{\rho}\frac{\partial}{dz}\left(-\rho K_M \frac{\partial \overline{u}}{\partial z}\right)$$

K_M, K_H and K_q are the exchange coefficients of momentum, heat and moisture

Their magnitude determines the transfer of these conserved quantities by turbulent eddies

Thermodynamics

$$\frac{\partial \overline{\theta}}{\partial t} = -\frac{1}{\rho}\left[\frac{\partial \rho \overline{\theta'w'}}{dz}\right] \sim -\frac{1}{\rho}\frac{\partial}{dz}\left(-\rho K_H \frac{\partial \overline{\theta}}{\partial z}\right)$$

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Generally assumed that diffusion of heat == diffusion of moisture

$$K_H = K_q$$

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Thermodynamics

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Their magnitude determines the transfer of these conserved quantities by turbulent eddies

'Local' turbulence closure: eddy diffusion (K-profile)

Momentum

$$\overline{u'w'} \sim -K_M \frac{\partial \overline{u}}{\partial z}$$

Thermodynamics

$$\overline{\theta'w'} \sim -K_H \frac{\partial \overline{\theta}}{\partial z}$$

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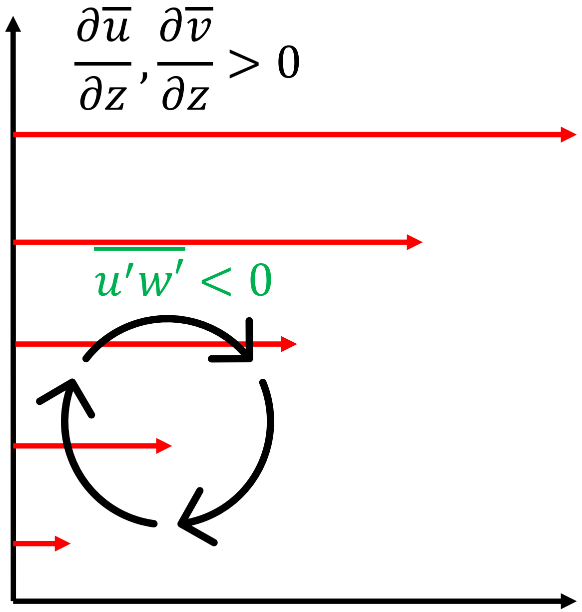
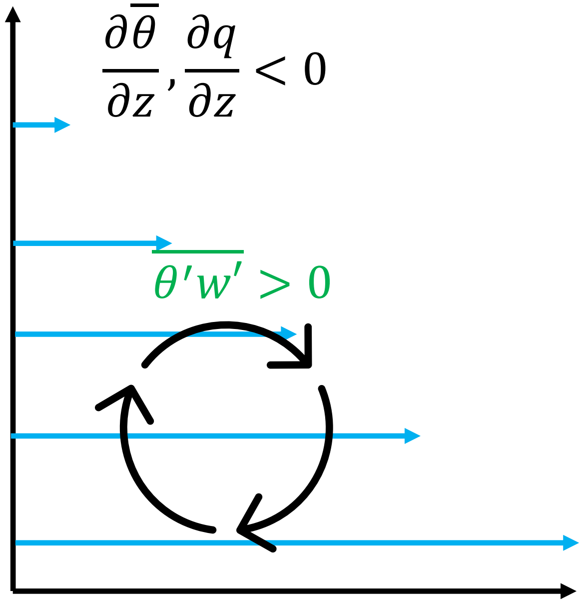


Wind / temperature gradient with turbulent eddies will generate mixing



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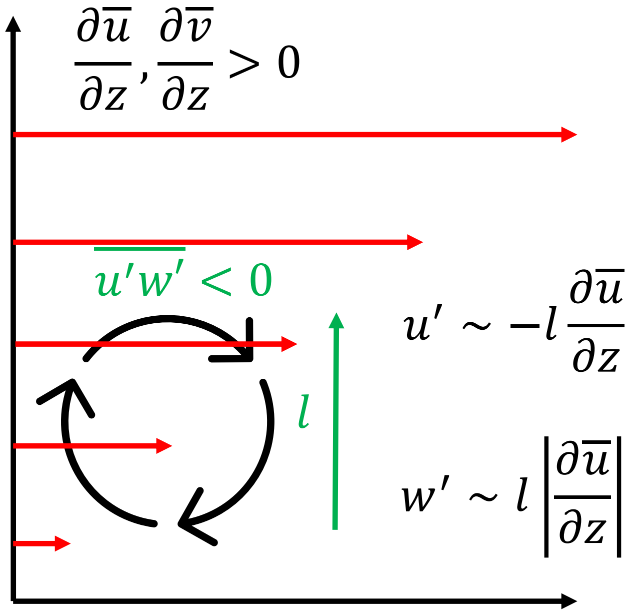
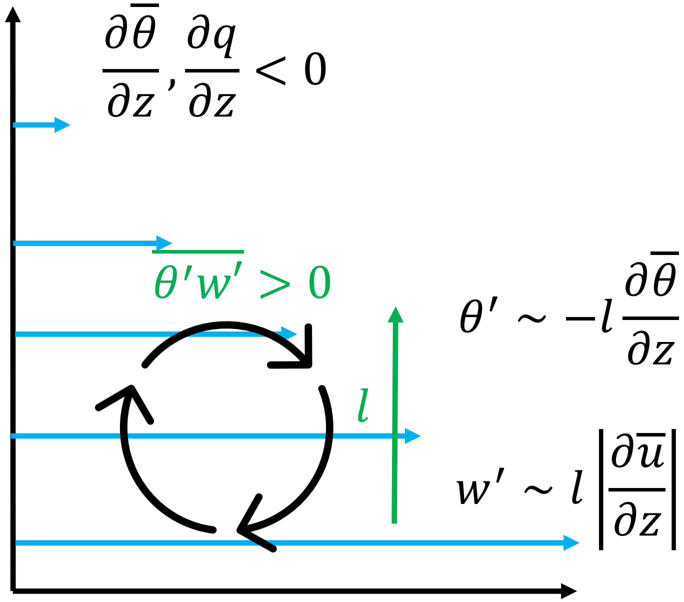


Wind / temperature gradient with turbulent eddies will generate mixing

Mixing occurs over a certain lengthscale l, related to size of eddies



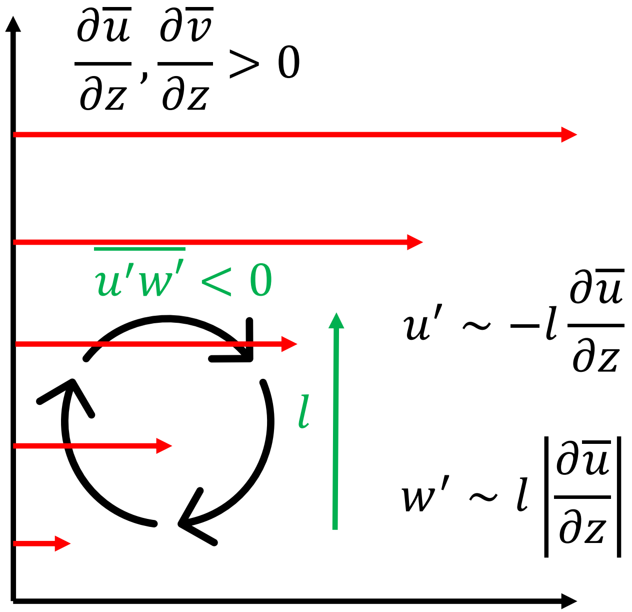
‘Local’ turbulence closure: eddy diffusion (K-profile)

Momentum

$$\overline{u'w'} \sim -K_M \frac{\partial \overline{u}}{\partial z} = -l^2 \left|\frac{\partial \overline{u}}{\partial z}\right| \frac{\partial \overline{u}}{\partial z}$$

Thermodynamics

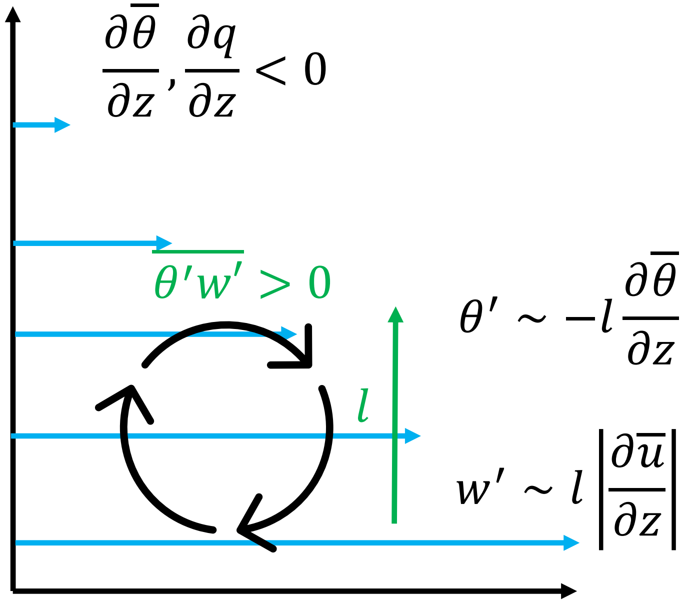
$$\overline{\theta' w'} \sim -K_H \frac{\partial \overline{\theta}}{\partial z} = -l^2 \left|\frac{\partial \overline{u}}{\partial z}\right| \frac{\partial \overline{\theta}}{\partial z}$$



Wind / temperature gradient with turbulent eddies will generate mixing

Mixing occurs over a certain lengthscale l, related to size of eddies

This lengthscale can be used to determine the exchange coefficients



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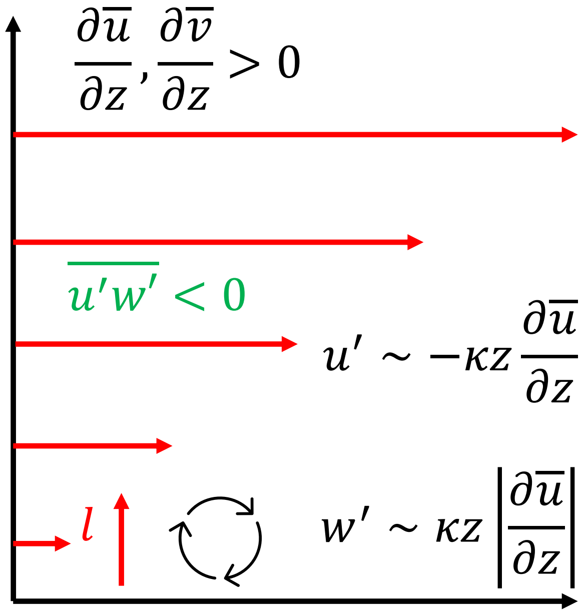
‘Local’ turbulence closure at the surface

What is l at the surface?

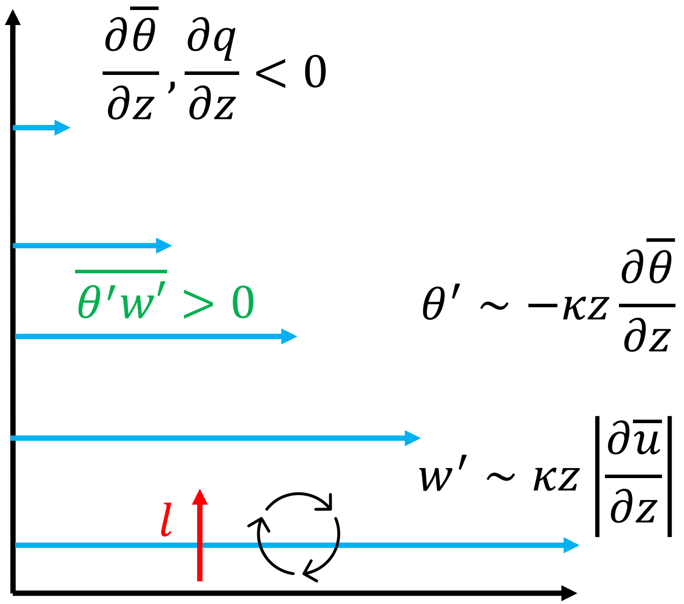
Momentum

$$\overline{u'w'} \sim -K_M \frac{\partial \bar{u}}{\partial z} = -l^2 \left|\frac{\partial \bar{u}}{\partial z}\right| \frac{\partial \bar{u}}{\partial z}$$



Thermodynamics

$$\overline{\theta'w'} \sim -K_H \frac{\partial \bar{\theta}}{\partial z} = -l^2 \left|\frac{\partial \bar{u}}{\partial z}\right| \frac{\partial \bar{\theta}}{\partial z}$$



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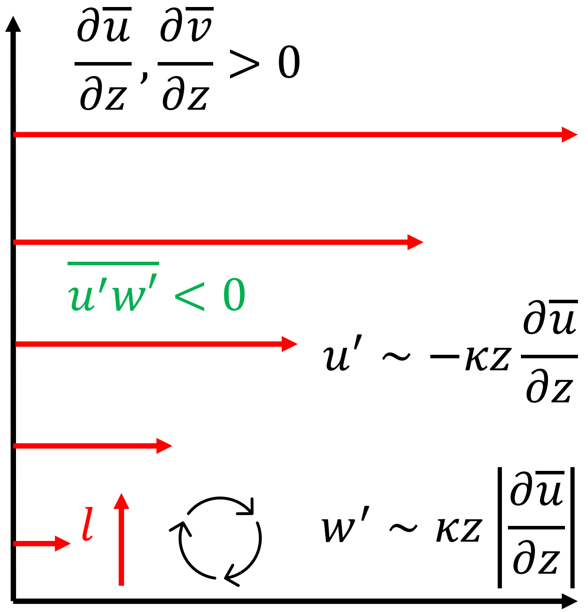

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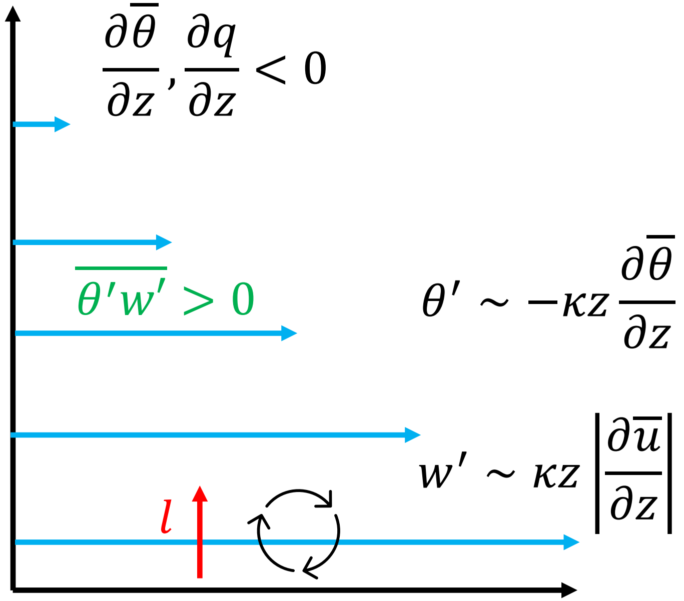


Size of eddies are constrained by the surface itself:

$$l \sim \kappa z,$$

$\kappa \sim 0.4$

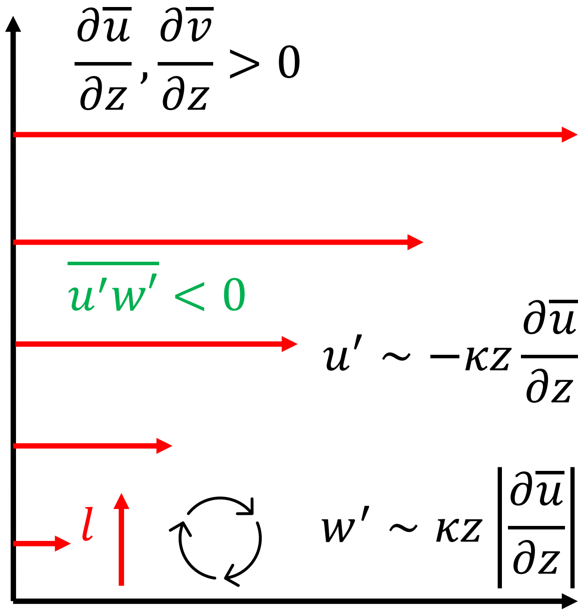
von-Karman constant

– determined from observations

What is l at the surface?

Momentum

$$\overline{u'w'} \sim -K_M \frac{\partial \overline{u}}{\partial z} = -\kappa^2 z^2 \left|\frac{\partial \overline{u}}{\partial z}\right| \frac{\partial \overline{u}}{\partial z}$$



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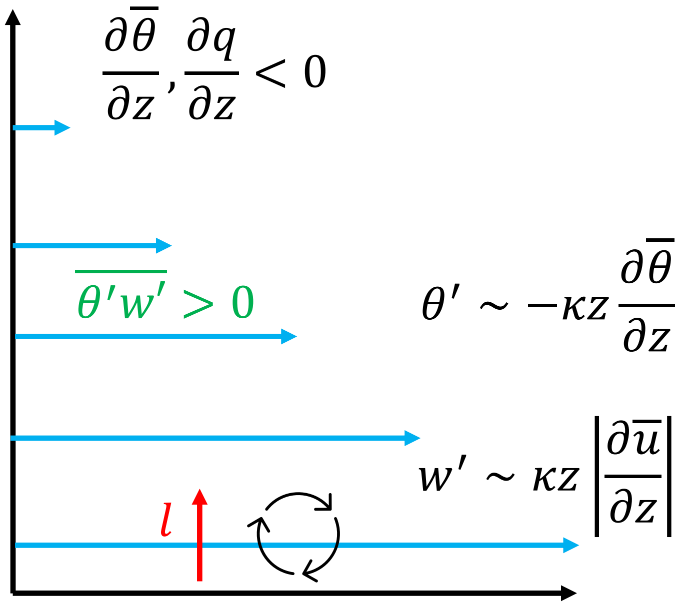

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Assume that fluxes are constant with height

Momentum

$$\overline{u'w'} \sim -K_M \frac{\partial \overline{u}}{\partial z} = -\kappa^2 z^2 \left|\frac{\partial \overline{u}}{\partial z}\right| \frac{\partial \overline{u}}{\partial z}$$

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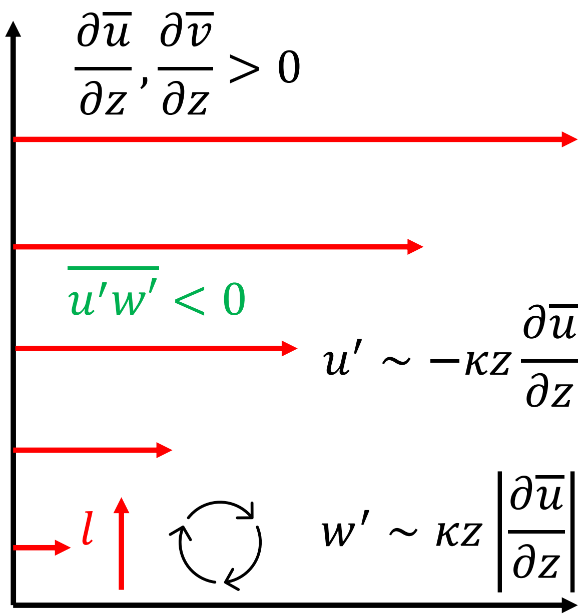

Near surface, fluxes are assumed constant with height $\left(\overline{u'w'}\right)_z = \left(\overline{u'w'}\right)_s$:

Assume that fluxes are constant with height

Momentum

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Thermodynamics

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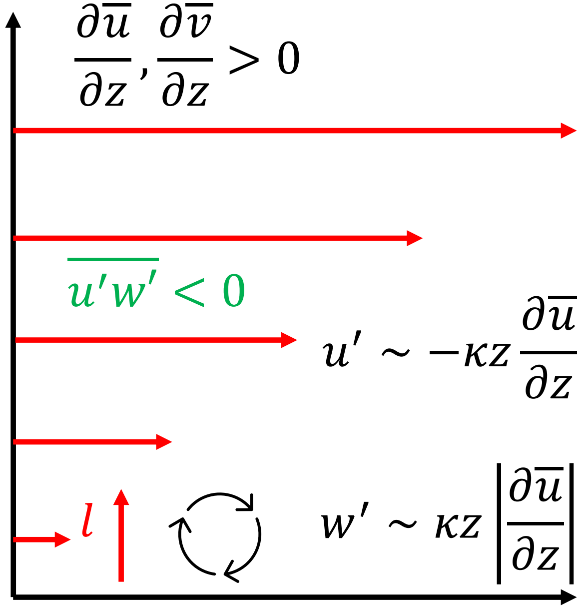

Near surface, fluxes are assumed constant with height $\left(\overline{u'w'}\right)_z = \left(\overline{u'w'}\right)_s$:

$$\left(\overline{u'w'}\right)_z = -\kappa^2 z^2 \left|\frac{\partial \overline{u}}{\partial z}\right| \frac{\partial \overline{u}}{\partial z} = u_*^2$$

$$\left(\overline{\theta' w'}\right)_z = -\kappa^2 z^2 \left|\frac{\partial \overline{u}}{\partial z}\right| \frac{\partial \overline{\theta}}{\partial z} = u_* \theta_*$$

Where $u_* = \sqrt{\left(\overline{u'w'}\right)_s} = \kappa z \left|\frac{\partial \overline{u}}{\partial z}\right|$ is the surface frictional velocity

θ_* is the temperature scaling, similarly, q_* is the moisture scaling

This means we can get profiles of $\overline{u}$ and $\overline{\theta}$ from flux

Momentum

$$\overline{u'w'} = u_*^2 = -\kappa^2 z^2 \left|\frac{\partial \overline{u}}{\partial z}\right| \frac{\partial \overline{u}}{\partial z}$$

Thermodynamics

$$\overline{\theta' w'} = \theta_* u_* = -\kappa^2 z^2 \left|\frac{\partial \overline{u}}{\partial z}\right| \frac{\partial \overline{\theta}}{\partial z}$$

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Thermodynamics

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Integrate:

$$\overline{u}_z - \overline{u}_s = \frac{u_*}{\kappa} \int_{z_0}^{z+z_0} \frac{1}{z} dz$$

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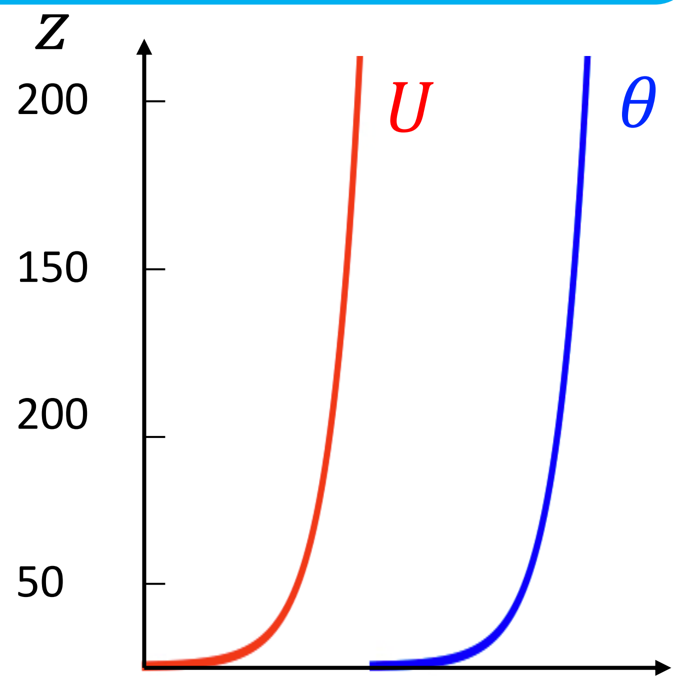
Gives log profile for winds and potential temperature



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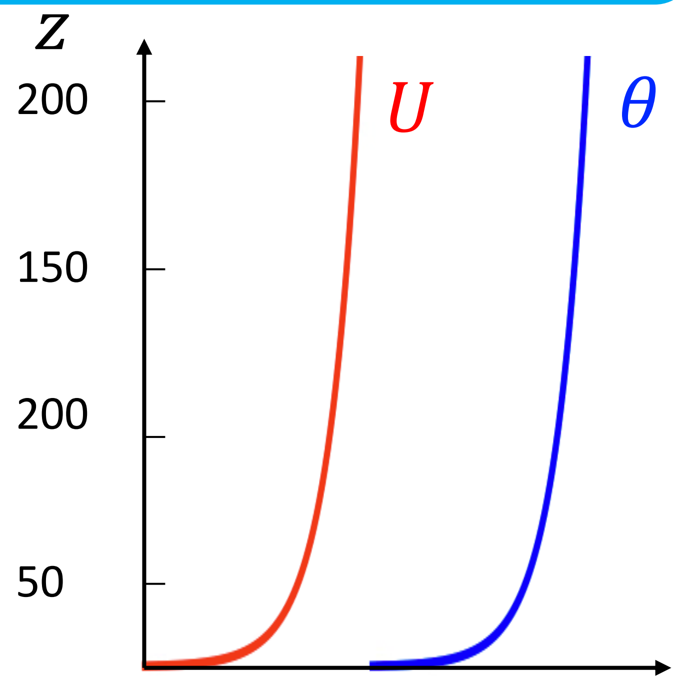

‘Local’ turbulence closure at the surface – adding stability dependence

Adding stability dependence

Momentum

$$\kappa z \frac{\partial \overline{u}}{\partial z} = u_*$$

Integrate:

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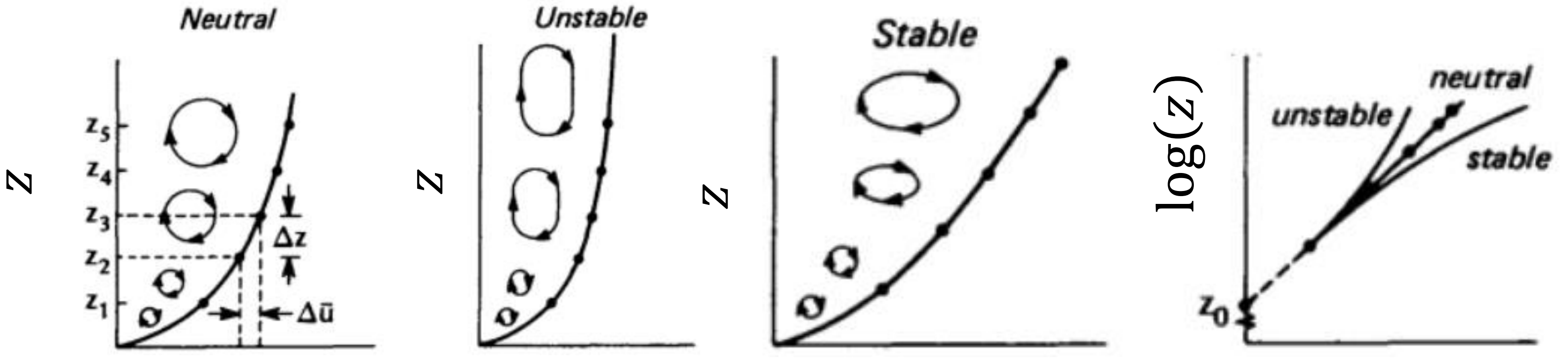

Wind Speed $\overline{U}$

Log profiles are only valid in neutral flow conditions….

Adding stability dependence

Momentum

$$\kappa z \frac{\partial \overline{u}}{\partial z} = u_*$$

Integrate:

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Thermodynamics

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Integrate:

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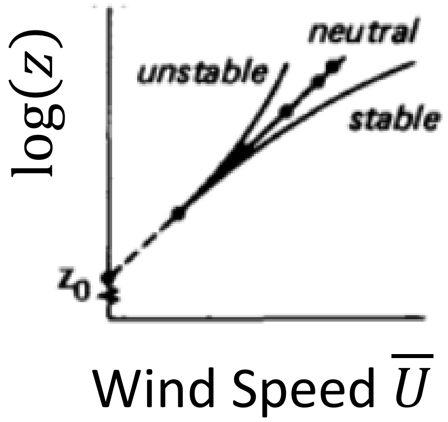
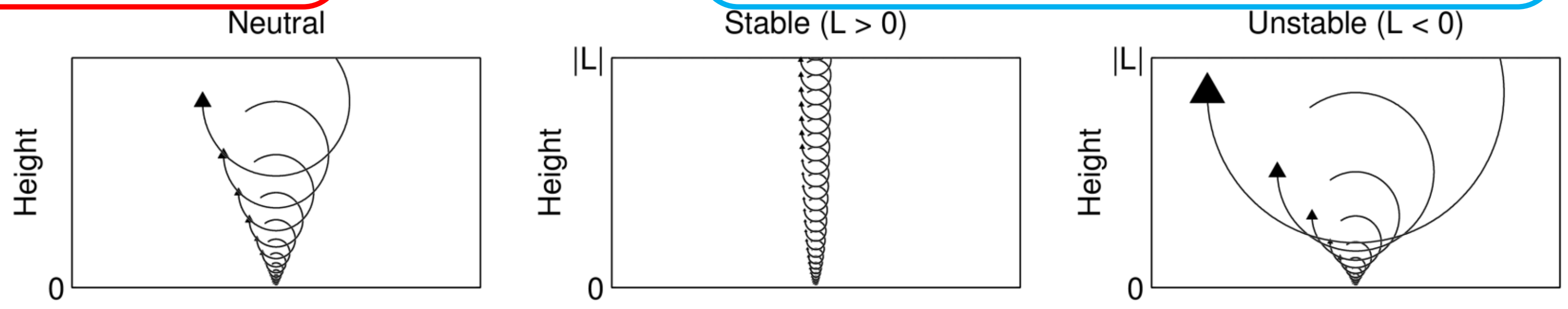




Mixing length modified to account for stability using a function ϕ :

$$l = \frac{\kappa z}{\phi(stability)}$$

This means we can get profiles of $\overline{u}$ and $\overline{\theta}$ from flux

Momentum

$$\frac{\kappa z}{\phi_M}\frac{\partial \overline{u}}{\partial z} = u_*$$

Integrate:

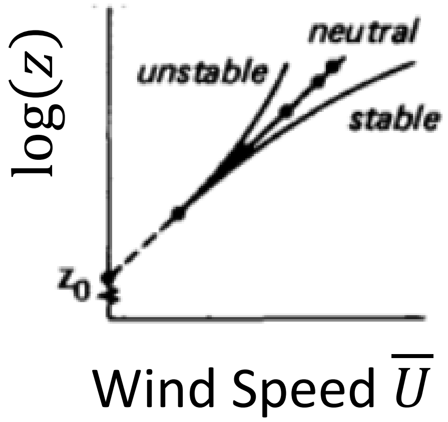
$$\overline{u}_z - \overline{u}_s = \frac{u_*}{\kappa}\int_{z_0}^{z+z_0} \frac{1}{z}\phi_M dz$$

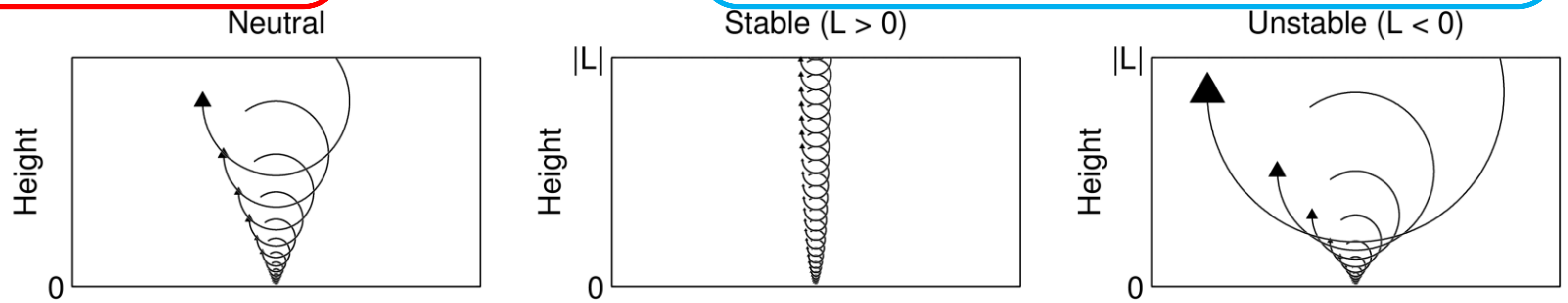
Thermodynamics

$$\frac{\kappa z}{\phi_H}\frac{\partial \overline{\theta}}{\partial z} = \theta_*$$

Integrate:

$$\overline{\theta}_z - \overline{\theta}_s = \frac{\theta_*}{\kappa}\int_{z_0}^{z+z_0} \frac{1}{z}\phi_H dz$$





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Thermodynamics

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Integrate:

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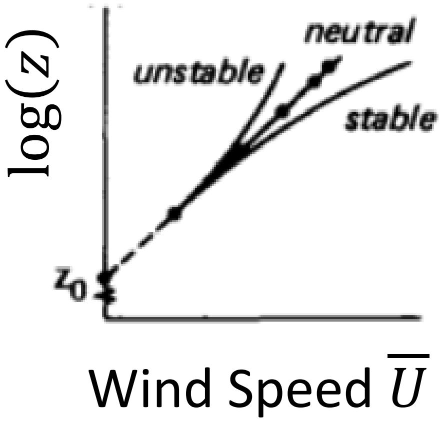

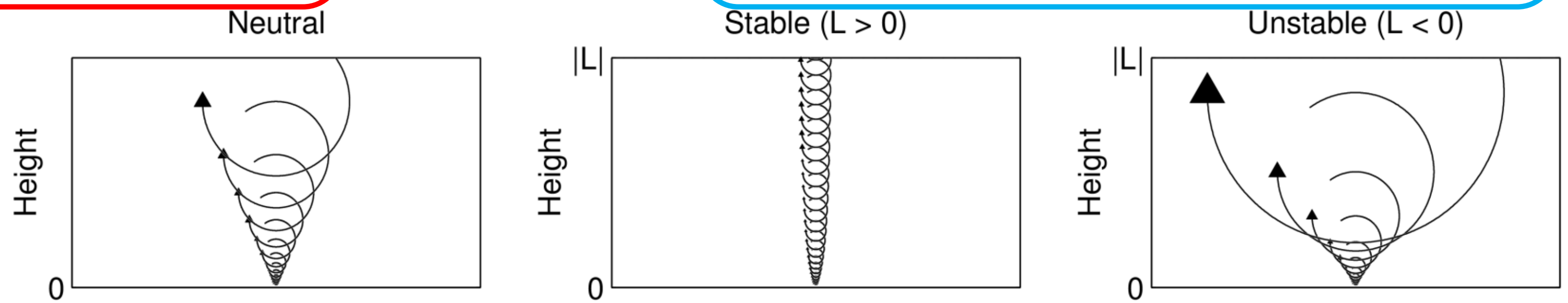

Mixing length modified to account for stability using a function ϕ :

$$l = \frac{\kappa z}{\phi(\zeta)}, \qquad \zeta = \frac{z}{L}$$

L = Obukhov length (will come back to this)

This means we can get profiles of $\overline{u}$ and $\overline{\theta}$ from flux

Momentum

$$\frac{\kappa z}{\phi_M}\frac{\partial \overline{u}}{\partial z} = u_*$$

Integrate:

$$\overline{u}_z - \overline{u}_s = \frac{u_*}{\kappa}\int_{z_0}^{z+z_0} \frac{1}{z}\phi_M dz$$

Thermodynamics

$$\frac{\kappa z}{\phi_H}\frac{\partial \overline{\theta}}{\partial z} = \theta_*$$

Integrate:

$$\overline{\theta}_z - \overline{\theta}_s = \frac{\theta_*}{\kappa}\int_{z_0}^{z+z_0} \frac{1}{z}\phi_H dz$$

Relationship between $\phi_M(\zeta)$, $\phi_H(\zeta)$ and ζ measured empirically and then integrated vertically

To do this, requires a change of variable:

$$d\zeta = \frac{1}{L}dz$$

This means we can get profiles of $\overline{u}$ and $\overline{\theta}$ from flux

Momentum

$$\frac{\kappa z}{\phi_M}\frac{\partial \overline{u}}{\partial z} = u_*$$

Integrate:

$$\overline{u}_z - \overline{u}_s = \frac{u_*}{\kappa}\int_{\zeta(z_0)}^{\zeta(z+z_0)} \frac{1}{\zeta}\phi_M(\zeta)\, d\zeta$$

Thermodynamics

$$\frac{\kappa z}{\phi_H}\frac{\partial \overline{\theta}}{\partial z} = \theta_*$$

Integrate:

$$\overline{\theta}_z - \overline{\theta}_s = \frac{\theta_*}{\kappa}\int_{\zeta(z_0)}^{\zeta(z+z_0)} \frac{1}{\zeta}\phi_H(\zeta)\, d\zeta$$

Relationship between $\phi_M(\zeta)$, $\phi_H(\zeta)$ and ζ measured empirically and then integrated vertically

To do this, requires a change of variable:

$$d\zeta = \frac{1}{L}dz$$

This means we can get profiles of $\overline{u}$ and $\overline{\theta}$ from flux

Momentum

$$\frac{\kappa z}{\phi_M}\frac{\partial \overline{u}}{\partial z} = u_*$$

Integrate:

$$\overline{u}_z = \frac{u_*}{\kappa}\left[\log\left(\frac{z + z_{0m}}{z_{0m}}\right) - \Psi_{\mathrm{M}}\left(\frac{z + z_{0m}}{L}\right)\right]$$

Thermodynamics

$$\frac{\kappa z}{\phi_H}\frac{\partial \overline{\theta}}{\partial z} = \theta_*$$

Integrate:

$$\overline{\theta}_z - \overline{\theta}_s = \frac{\theta_*}{\kappa}\left[\log\left(\frac{z+z_{0m}}{z_{0H}}\right) - \Psi_{\mathrm{H}}\left(\frac{z+z_{0M}}{L}\right)\right]$$

Relationship between $\phi_M(\zeta)$, $\phi_H(\zeta)$ and ζ measured empirically and then integrated vertically

Ψ_{H}, Ψ_{M} are integrals of $\phi_M(\zeta)$

What is the Obukhov-length?

- Derived from scaling arguments - Reduces degrees of freedom so that ‘universal’ relations (they work for all situations) can be derived

$\zeta > 0$ Stable

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- Height above the surface at which:

buoyant production > shear production of turbulence

Buoyancy production : $\frac{g}{\theta}\overline{\theta' w'} = \frac{g}{\theta}\theta_* u_*$

$\div$

Shear production: $-\overline{u' w'}\frac{\partial u}{\partial z} = u_*^2 \frac{\partial u}{\partial z}$

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$$\zeta = \frac{z}{L} = \frac{g}{\theta}\frac{\theta_* u_*}{u_*^2 \frac{\partial u}{\partial z}} = -\frac{\frac{g}{\theta}\theta_* \kappa z}{u_*^2}$$

Made use of : $\frac{\partial u}{\partial z} = \frac{u_*}{\kappa z}$

This means we can get profiles of $\overline{u}$ and $\overline{\theta}$ from flux

Momentum

$$\frac{\kappa z}{\phi_M}\frac{\partial \overline{u}}{\partial z} = u_*$$

Integrate:

$$\overline{u}_z = \frac{u_*}{\kappa}\left[\log\left(\frac{z + z_{0m}}{z_{0m}}\right) - \Psi_\mathrm{M}\left(\frac{z + z_{0m}}{L}\right)\right]$$

Thermodynamics

$$\frac{\kappa z}{\phi_H}\frac{\partial \overline{\theta}}{\partial z} = \theta_*$$

Integrate:

$$\overline{\theta}_z - \overline{\theta}_s = \frac{\theta_*}{\kappa}\left[\log\left(\frac{z+z_{0m}}{z_{0H}}\right) - \Psi_\mathrm{H}\left(\frac{z+z_{0M}}{L}\right)\right]$$

Recall that:

$$\overline{u'w'} = u_*^2$$

$$\overline{\theta'w'} = \theta_* u_*$$

Relationship between $\phi_M(\zeta)$, $\phi_H(\zeta)$ and ζ measured empirically and then integrated vertically

Ψ_H, Ψ_M are integrals of $\phi_M(\zeta)$

This means we can get surface fluxes

Momentum

$$\rho\overline{u'w'} = \rho u_*^2 = \rho C_M |\overline{u_z}|^2$$

Thermodynamics

$$\rho\overline{\theta'w'} = \rho u_* \theta_* = \rho C_H (\overline{\theta_z} - \overline{\theta_s})\, |\overline{u_z}|$$

Surface exchange coefficient for heat:

$$\mathrm{C_H} = \frac{\kappa^2}{\left[\log\left(\frac{z+z_{0m}}{z_{0m}}\right) - \Psi_\mathrm{M}\left(\frac{z+z_{0m}}{L}\right)\right]\left[\log\left(\frac{z+z_{0m}}{z_{0H}}\right) - \Psi_\mathrm{H}\left(\frac{z+z_{0m}}{L}\right)\right]}$$

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This means we can get surface fluxes **but…**

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Exchange coefficients depend on $\zeta = \frac{z+z_{0m}}{L}$, which itself depends on surface fluxes

How can we determine z/L and the surface fluxes?

1. Start with relationship between bulk Richardson number and z/L:

$$Ri_b = \frac{g}{\overline{\theta_z}} \frac{(\overline{\theta_z} - \overline{\theta_s})z}{|\overline{u_z}|^2} = z \frac{g}{\overline{\theta_z}} \frac{\theta_*}{u_*^2} \frac{C_M^{\frac{3}{2}}}{C_H} = \frac{z}{L} \frac{\left[\log\left(\frac{z+z_{0m}}{z_{0h}}\right) - \Psi_H\left(\frac{z+z_{0m}}{L}\right)\right]}{\left[\log\left(\frac{z+z_{0m}}{z_{0m}}\right) - \Psi_M\left(\frac{z+z_{0m}}{L}\right)\right]^2}$$

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2. Compute Ri_b from model fields and solve for $\frac{z}{L}$ by either:

- Iteration
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5. AND we can determine profiles of winds, temperature and humidity near the surface

Summary of Monin-Obukhov surface layer similarity theory

- The Obukhov-length is a measure of surface layer stability and can be thought of as the ratio of buoyancy / shear production of turbulence
- It is assumed that turbulent fluxes do not vary across the surface layer
- ‘Universal’ functions that relate the Obukhov length (stability) to the vertical profiles of conserved quantities (e.g. wind and temperature) in the surface layer can be derived from observations
- This is useful because we can relate Richardson number to z/L and get profiles and surface fluxes

What is the roughness length z_0?

What is the roughness length z_0?

Momentum

$$\kappa z \frac{\partial \overline{u}}{\partial z} = u_*$$

Integrate:

$$\overline{u}_z - \overline{u}_s = \frac{u_*}{\kappa} \log\left(\frac{z + z_0}{z_0}\right)$$

Where $u_* = \sqrt{\left(\overline{u'w'}\right)_s} = \kappa z \left|\frac{\partial \overline{u}}{\partial z}\right|$ is the surface frictional velocity

θ_* is the temperature scaling, similarly, q_* is the moisture scaling

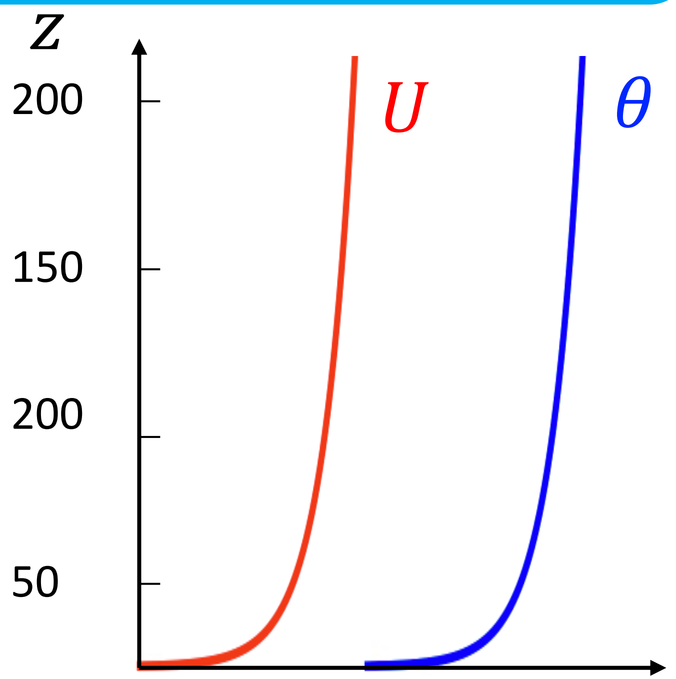
Thermodynamics

$$\kappa z \frac{\partial \overline{\theta}}{\partial z} = \theta_*$$

Integrate:

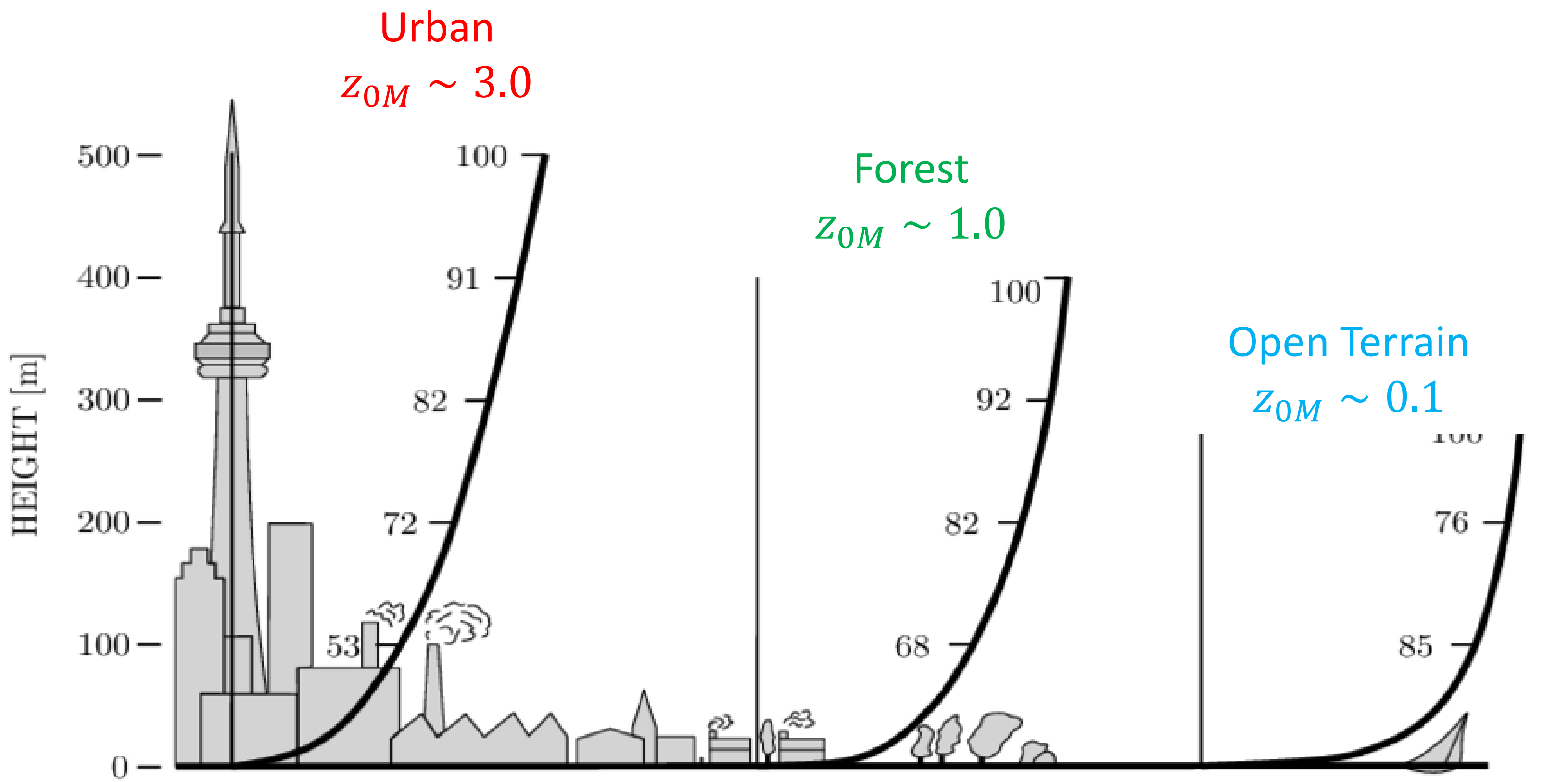
$$\overline{\theta}_z - \overline{\theta}_s = \frac{\theta_*}{\kappa} \log\left(\frac{z + z_0}{z_0}\right)$$

Gives log profile for winds and potential temperature



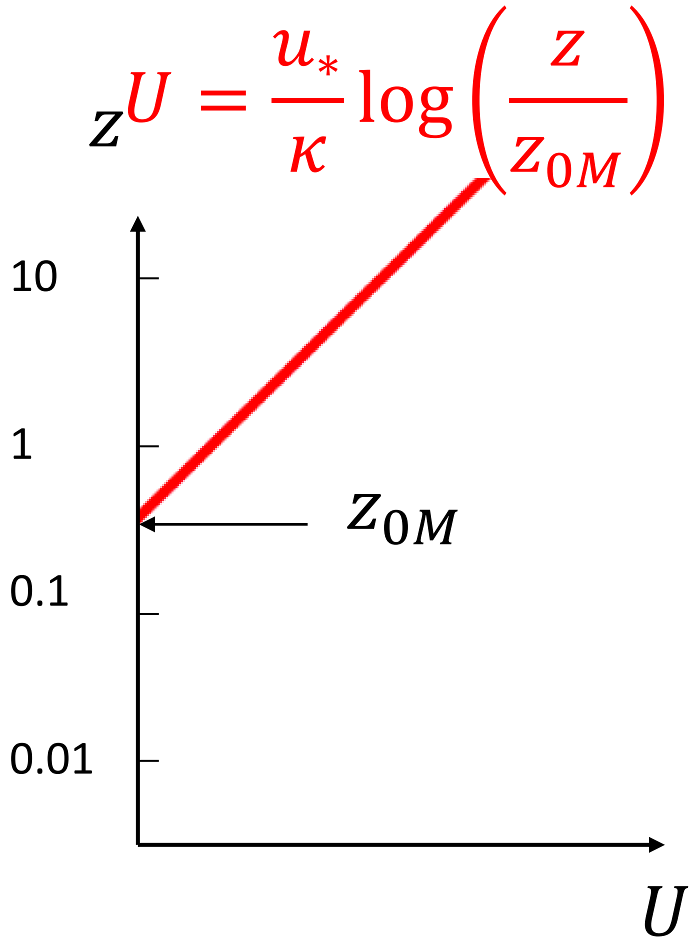
What is the roughness length z_0?

- Roughness length for momentum z_{0M} is not the same as for heat z_{0H}
- z_{0M} and z_{0H} determines the shape of the wind and temperature profiles
- They are a property of the underlying surface and are (assumed) to be a function of the height of the roughness elements



What is the roughness length z_0?

- Surface aerodynamic roughness length is defined from the logarithmic wind profile
- The roughness length is the height at which the winds become zero
- In the model, the displacement height is used to obtain $U = 0$ at $z = 0$. This



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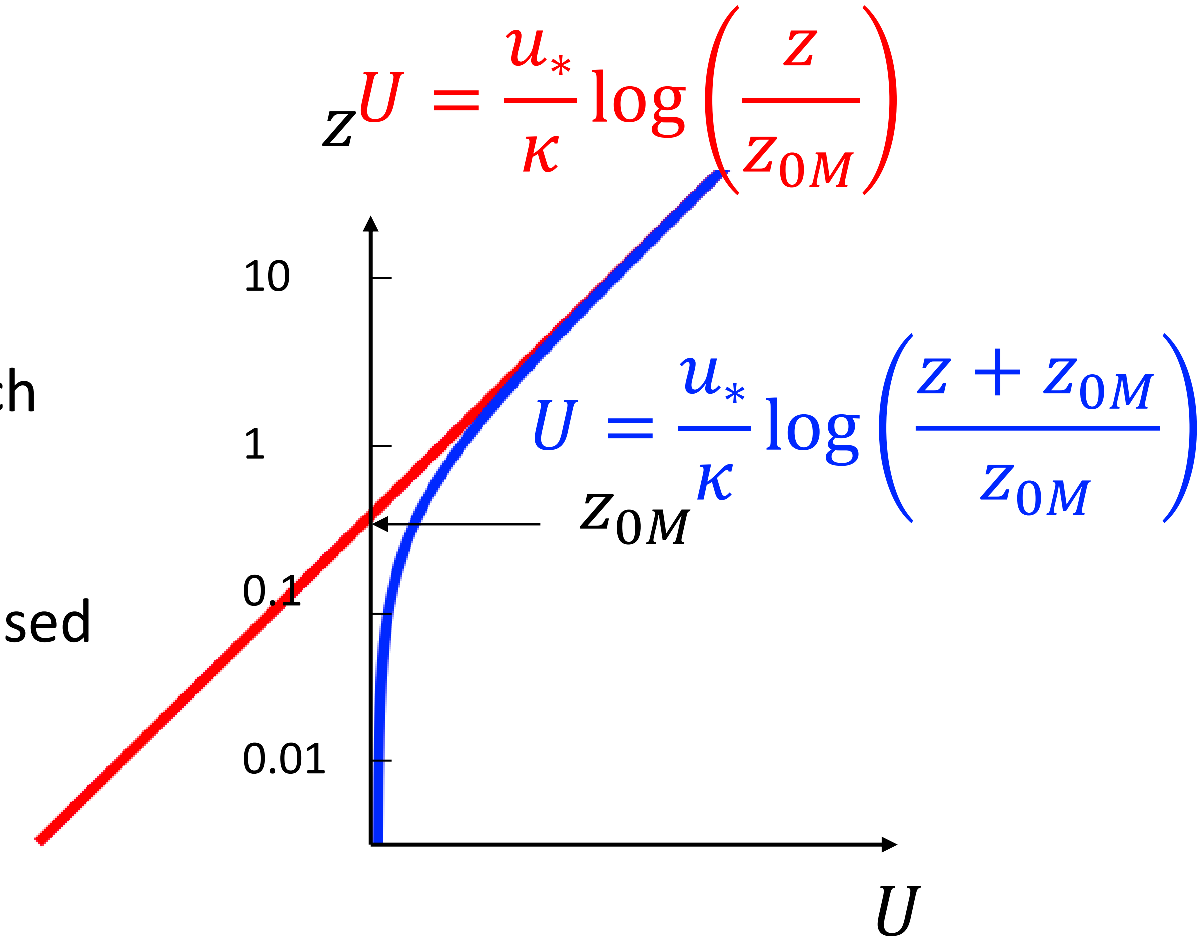
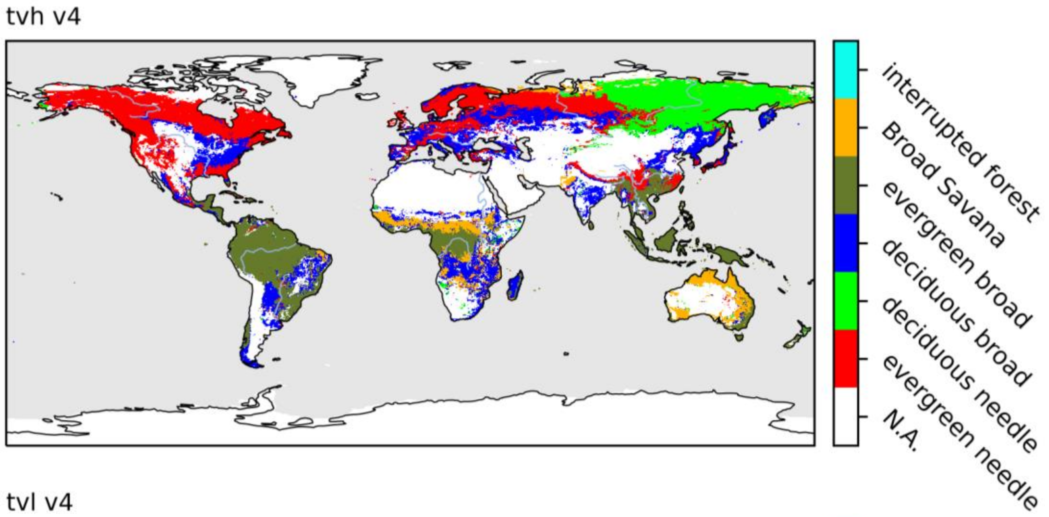
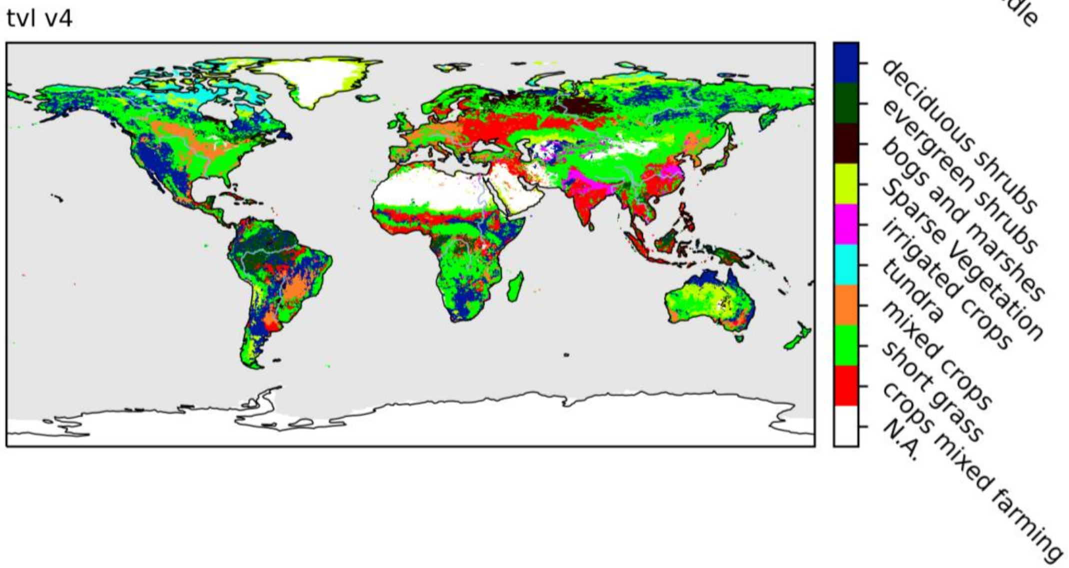


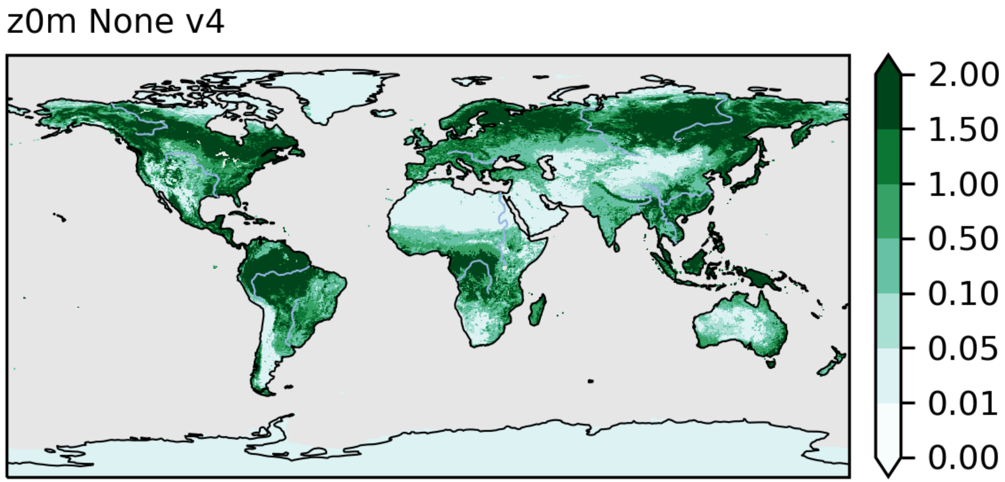
What is the roughness length z_0 over land?

Note that $z_{0H} = \frac{z_{0M}}{10}$

Index	Vegetation type	H/L veg	z_{0m}	z_{0h}
1	Crops, mixed farming	L	0.25	$0.25\ 10^{-2}$
2	Short grass	L	0.1	$0.1\ 10^{-2}$
3	Evergreen needleleaf trees	H	2.0	2.0
4	Deciduous needleleaf trees	H	2.0	2.0
5	Deciduous broadleaf trees	H	2.0	2.0
6	Evergreen broadleaf trees	H	2.0	2.0
7	Tall grass	L	0.47	$0.47\ 10^{-2}$
8	Desert	–	0.013	$0.013\ 10^{-2}$
9	Tundra	L	0.034	$0.034\ 10^{-2}$
10	Irrigated crops	L	0.5	$0.5\ 10^{-2}$
11	Semidesert	L	0.17	$0.17\ 10^{-2}$
12	Ice caps and glaciers	–	$1.3\ 10^{-3}$	$1.3\ 10^{-4}$
13	Bogs and marshes	L	0.5	$0.5\ 10^{-2}$
14	Inland water	–	–	–
15	Ocean	–	–	–
16	Evergreen shrubs	L	0.100	$0.1\ 10^{-2}$
17	Deciduous shrubs	L	0.25	$0.25\ 10^{-2}$
18	Mixed forest/woodland	H	2.0	2.0
19	Interrupted forest	H	1.1	1.1
20	Water and land mixtures	L	–	–







What is the roughness length z_0 over ocean?

$$z_{0M} = \alpha_M \frac{\nu}{u_*} + \alpha_{Ch} \frac{u_*^2}{g}$$

$$z_{0H} = \alpha_H \frac{\nu}{u_*}$$

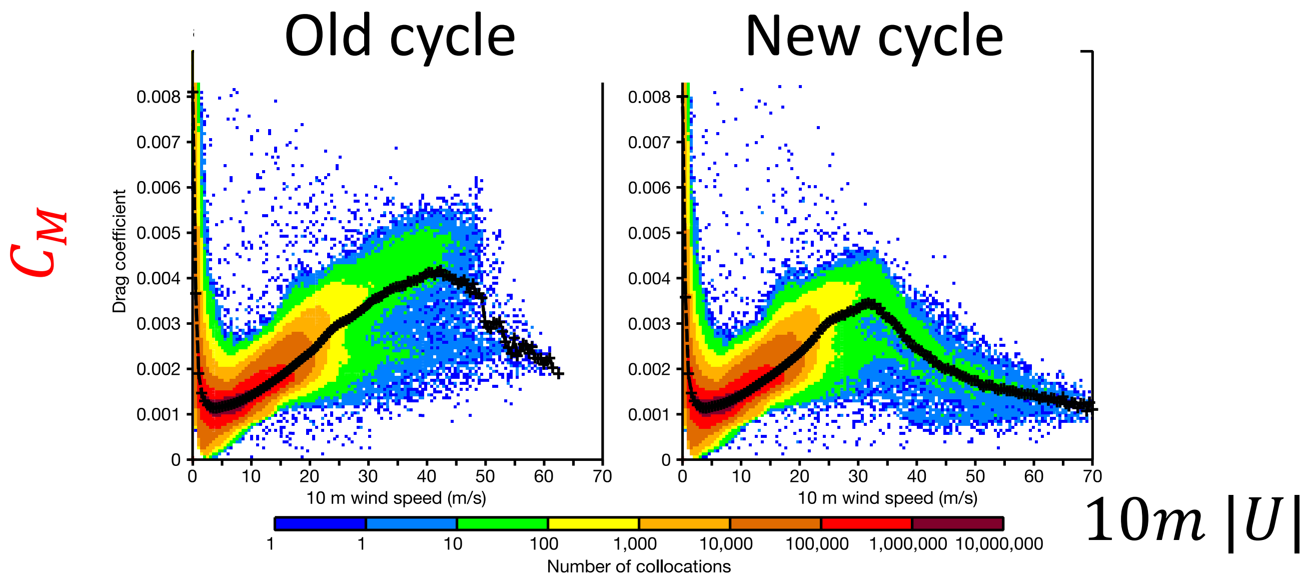
$$z_{0Q} = \alpha_Q \frac{\nu}{u_*}$$

ν = kinematic viscocity

$$u_* = C_M^{\frac{1}{2}} |U_n|$$

$\alpha_M, \alpha_H, \alpha_Q$ = constants

α_{Ch}= Charnock coefficient, provided by the wave model



$$C_M = \frac{\kappa^2}{\left[\log\left(\frac{z + z_{0m}}{z_{0m}}\right) - \Psi_M\left(\frac{z + z_{0m}}{L}\right)\right]^2}$$

Bidlot et al, 2020

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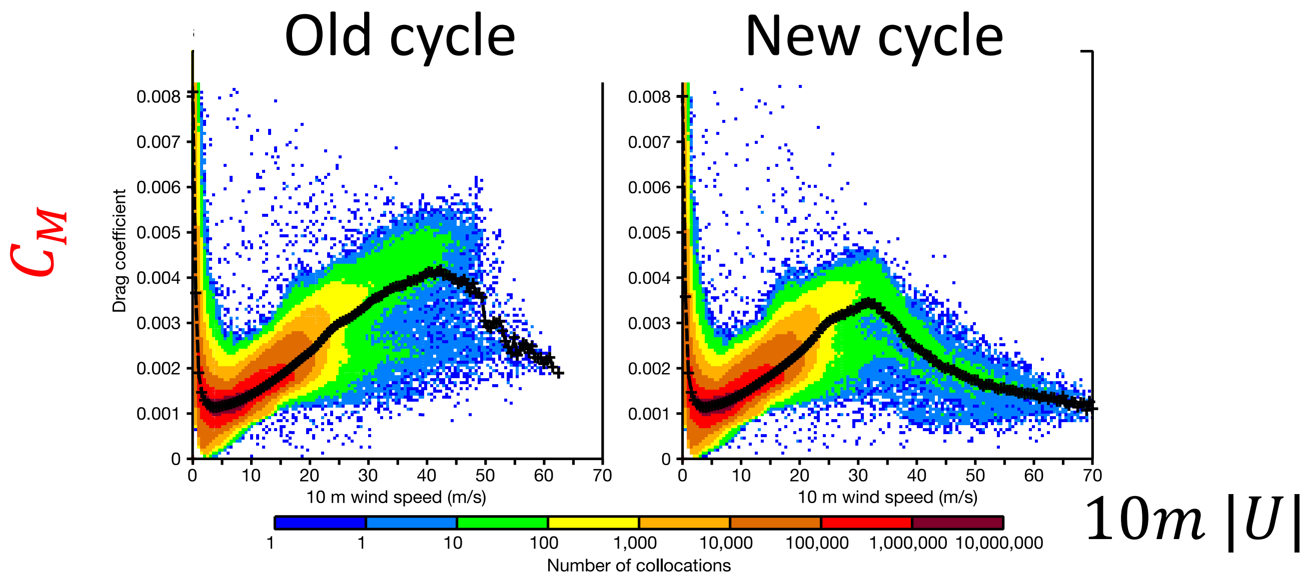
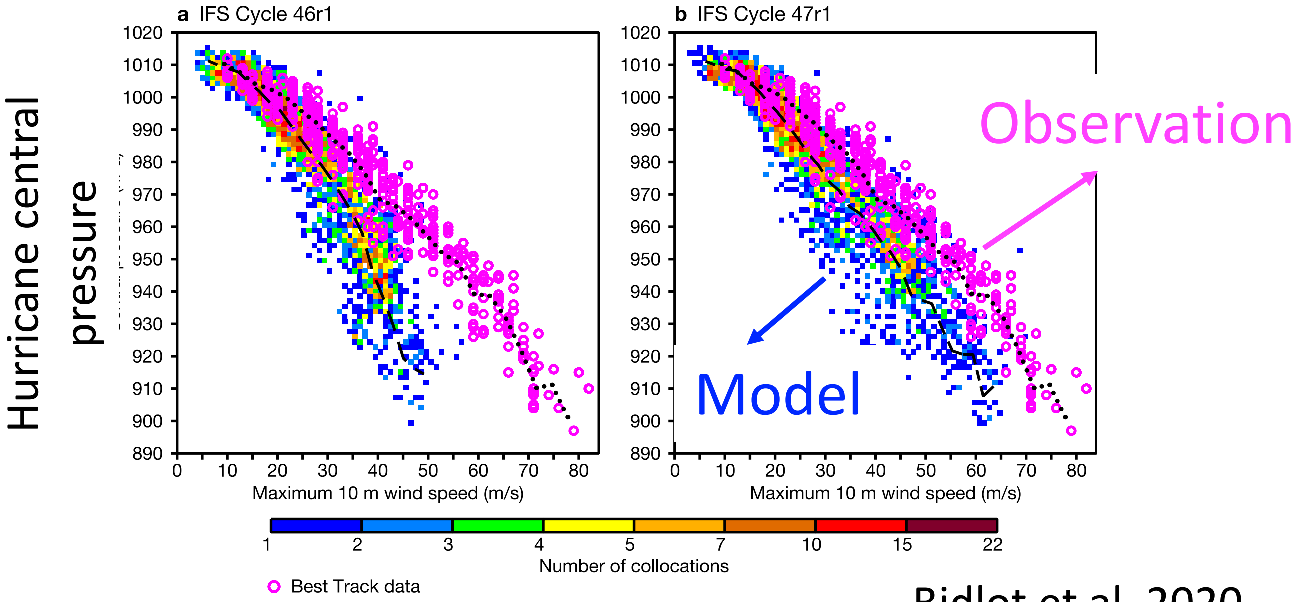




Bidlot et al, 2020

ECMWF

What is the roughness length z_0 over sea-ice?

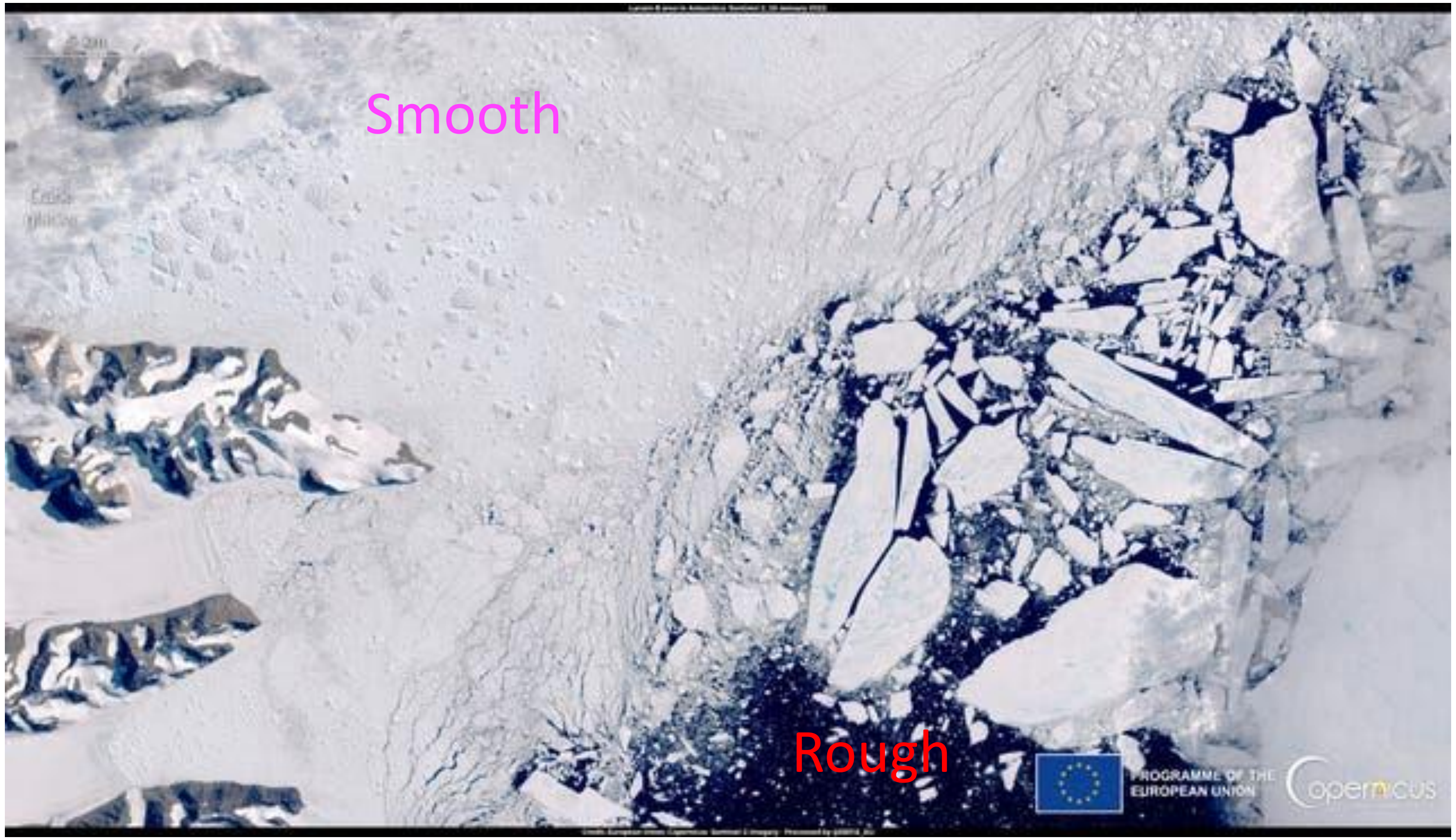
$$z_{0M} = \max(10^{-3}, f(c_i))$$

$$z_{0H} = 10^{-3}$$

$$z_{0Q} = 10^{-3}$$

c_i = sea ice concentration

$f(c_i)$: The dependence on sea-ice concentration reflects observation that partial ice-cover leads to more broken up sea ice and therefore increased drag



ECMWF

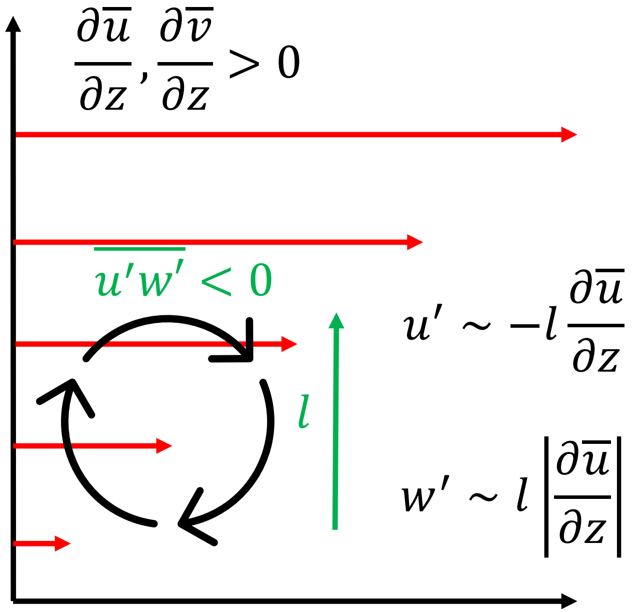
‘Local’ turbulence closure: eddy diffusion above the surface

ECMWF

‘Local’ turbulence closure: eddy diffusion above the surface

Momentum

$$\overline{u'w'} \sim -K_M \frac{\partial \overline{u}}{\partial z} = -l^2 \left|\frac{\partial \overline{u}}{\partial z}\right| \frac{\partial \overline{u}}{\partial z}$$



Thermodynamics

$$\overline{\theta' w'} \sim -K_H \frac{\partial \overline{\theta}}{\partial z} = -l^2 \left|\frac{\partial \overline{u}}{\partial z}\right| \frac{\partial \overline{\theta}}{\partial z}$$

‘Local’ turbulence closure: eddy diffusion above the surface

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Thermodynamics

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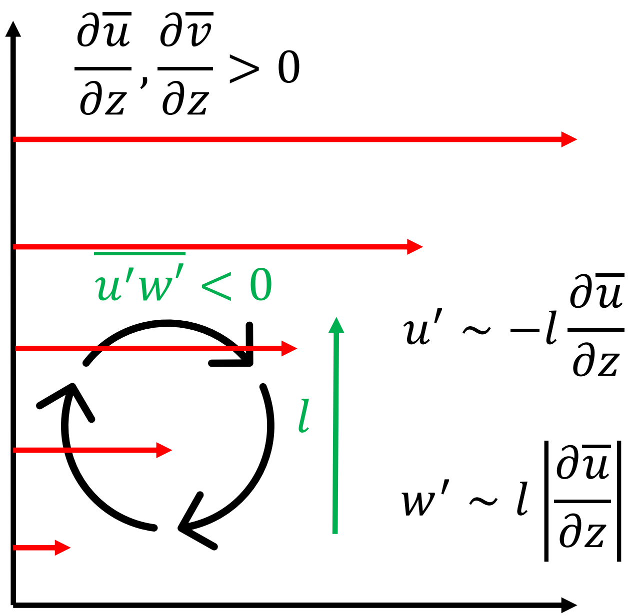
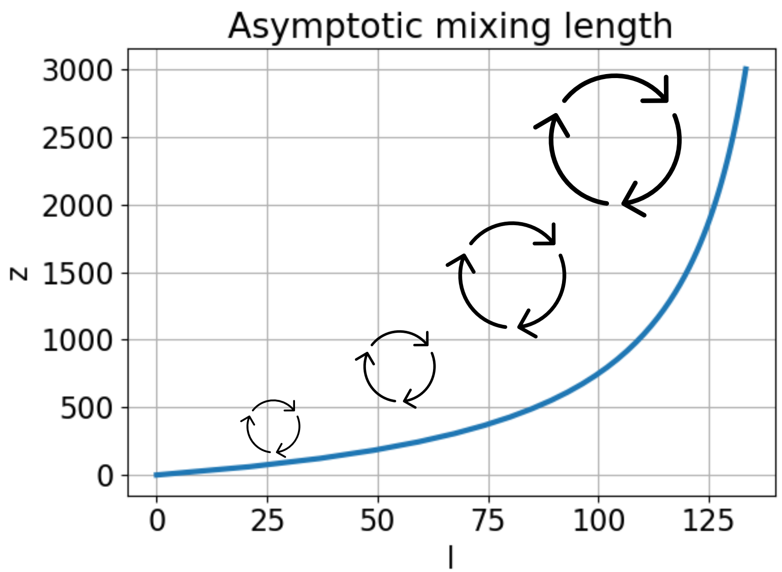


Size of eddies get larger further away from the surface:

$$l \sim \frac{\kappa z \lambda}{\kappa z + \lambda}$$

κ =von-Karman constant

λ =asymptotic mixing length (150 m)



‘Local’ turbulence closure: eddy diffusion above the surface

Momentum

$$\overline{u'w'} \sim -K_M \frac{\partial \overline{u}}{\partial z} = -l^2 \left|\frac{\partial \overline{u}}{\partial z}\right| f_M(Ri) \frac{\partial \overline{u}}{\partial z}$$

Thermodynamics

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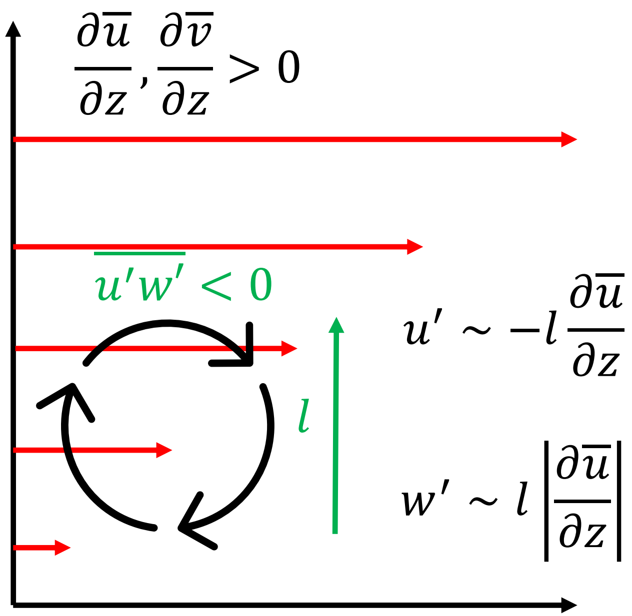

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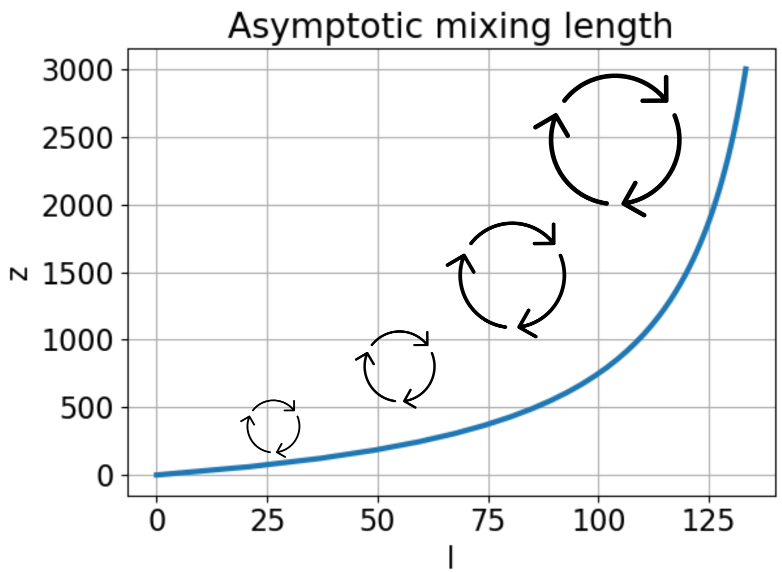

$f_M(Ri), f_H(Ri)$ determined empirically and depend on $Ri(z)$, since we are away from the surface

Local similarity theory in the outer layer

Momentum

$$\overline{u'w'} \sim -K_M \frac{\partial \overline{u}}{\partial z} = -l^2 \left| \frac{\partial \overline{u}}{\partial z} \right| f_M(Ri) \frac{\partial \overline{u}}{\partial z}$$

Thermodynamics

$$\overline{\theta'w'} \sim -K_H \frac{\partial \overline{\theta}}{\partial z} = -l^2 \left| \frac{\partial \overline{u}}{\partial z} \right| f_H(Ri) \frac{\partial \overline{\theta}}{\partial z}$$

- In stable conditions, the mid and upper boundary layer may not be in equilibrium with the surface fluxes
- Local fluxes and stability (Ri) dominate

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- In stable conditions, the mid and upper boundary layer may not be in equilibrium with the surface fluxes
- Local fluxes and stability (Ri) dominate
- Local similarity states that the surface layer functions can be used in the outer layer:

$$K_H = \frac{l^2}{\phi_H(\zeta)\phi_M(\zeta)} \left|\frac{\partial \overline{u}}{\partial z}\right|$$

$$K_M = \frac{l^2}{\phi_M^2(\zeta)} \left|\frac{\partial \overline{u}}{\partial z}\right|$$

Use the relation

$$Ri = \zeta \frac{\phi_H(\zeta)}{\phi_M^2(\zeta)}$$

to convert $\zeta = \frac{z}{L}$ to the gradient Richardson number in the outer layer

ECMWF

‘Non-Local’ turbulence: eddy-diffusivity mass-flux (EDMF)

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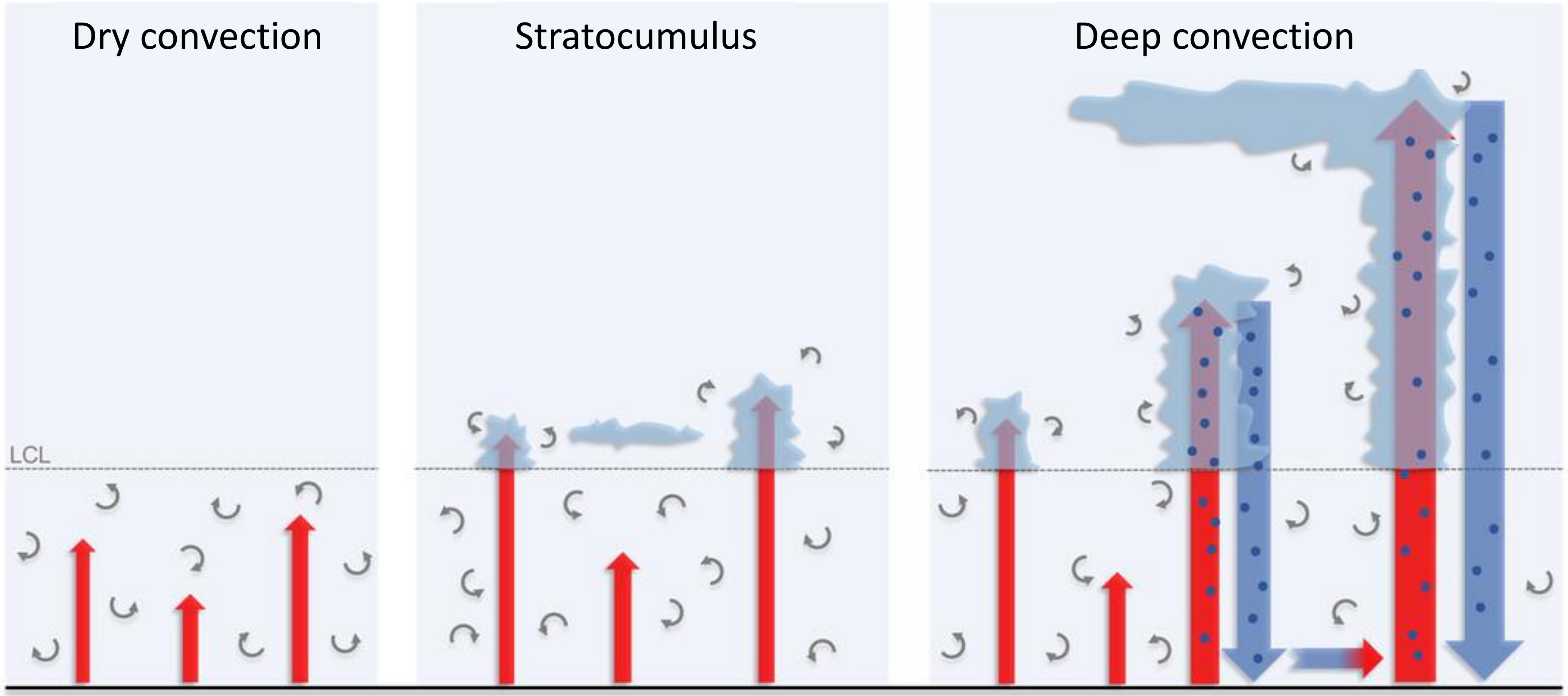
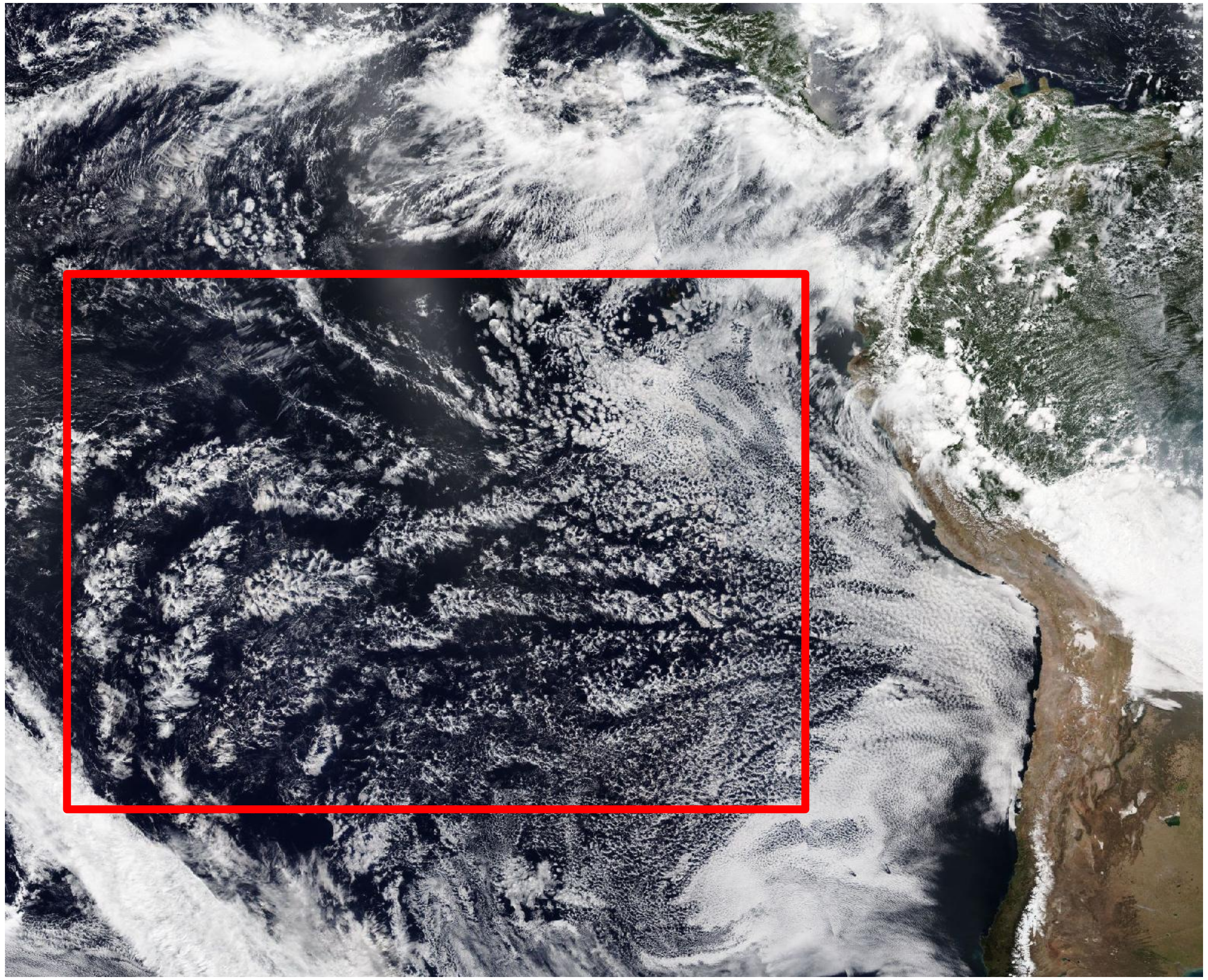
Local turbulent diffusion fails in convective boundary layers because it yields unrealistic zero flux in an environment with small gradients



‘Non-Local’ turbulence: eddy-diffusivity mass-flux (EDMF)

In convective boundary layers, the area of strongest updraft is typically much smaller than the environment



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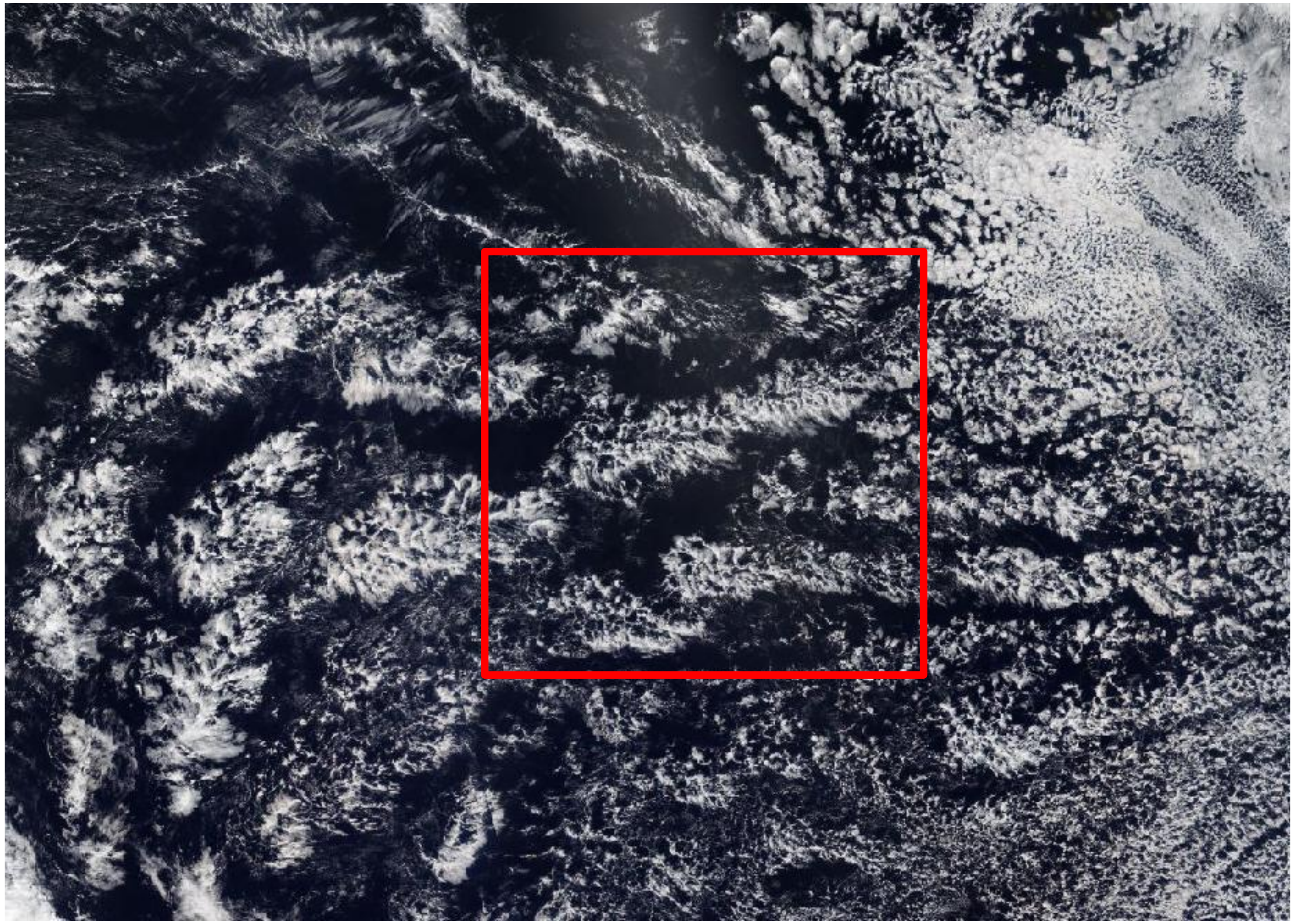

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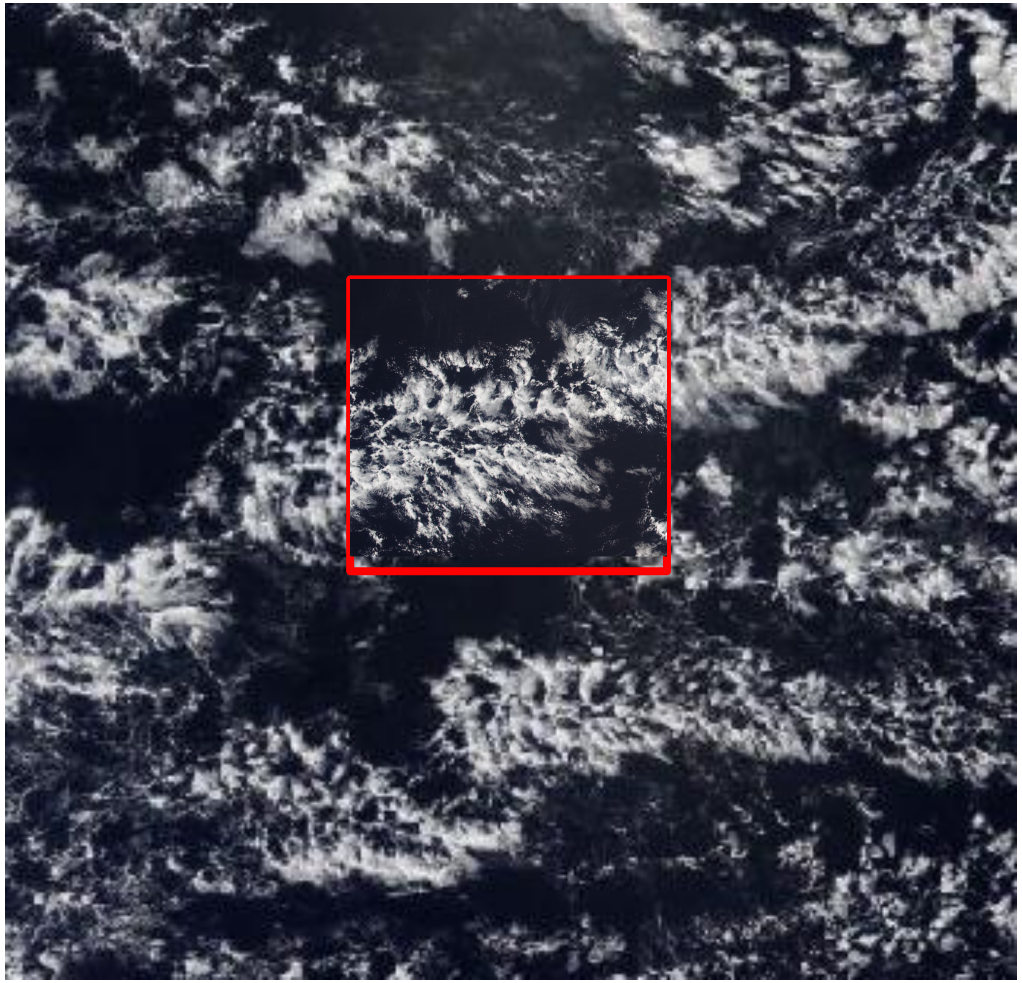

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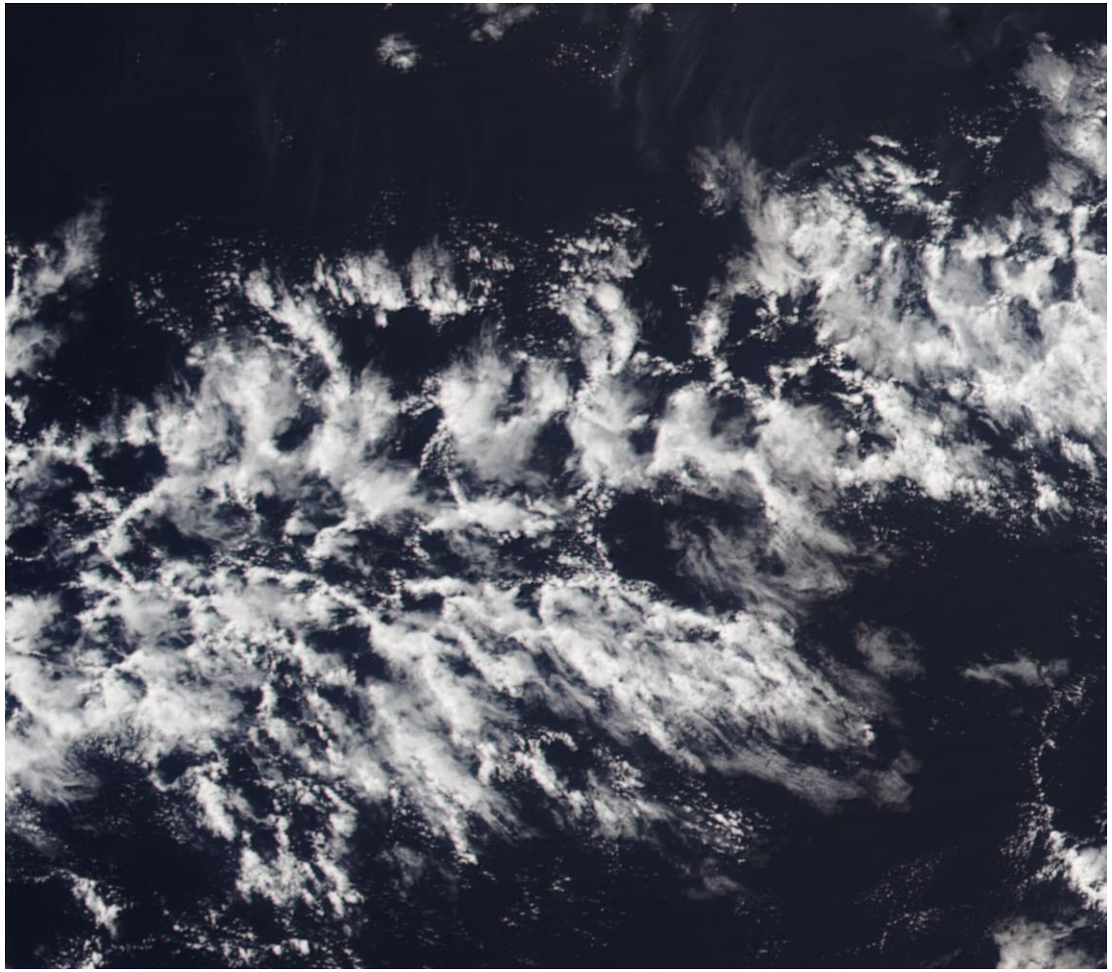
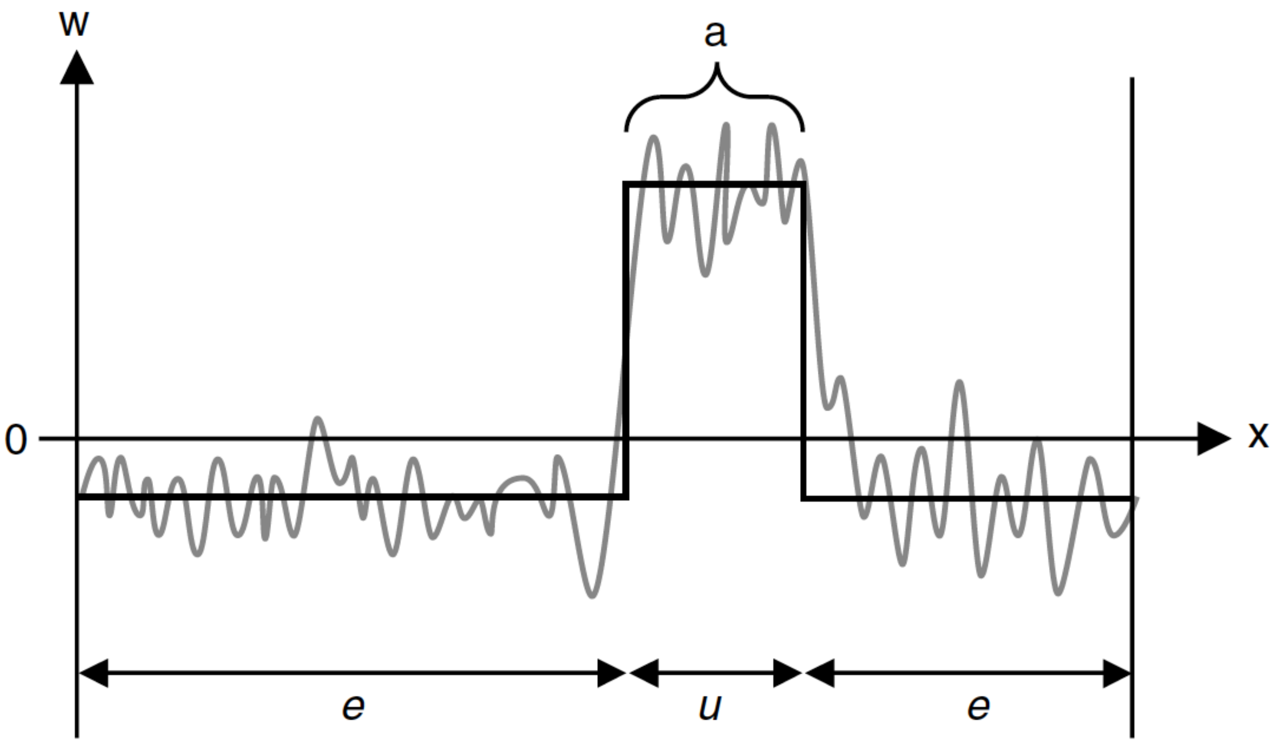


ECMWF

'Non-Local' turbulence: eddy-diffusivity mass-flux (EDMF)

Total turbulent flux of ϕ :

$$\overline{\phi' w'} = a\overline{\phi'_u w'} + (1-a)\overline{\phi'_e w'} + a(\overline{w}^u - \overline{w})(\overline{\phi}^u - \overline{\phi}^e)$$



Turbulent flux within the strong updraft region

ECMWF

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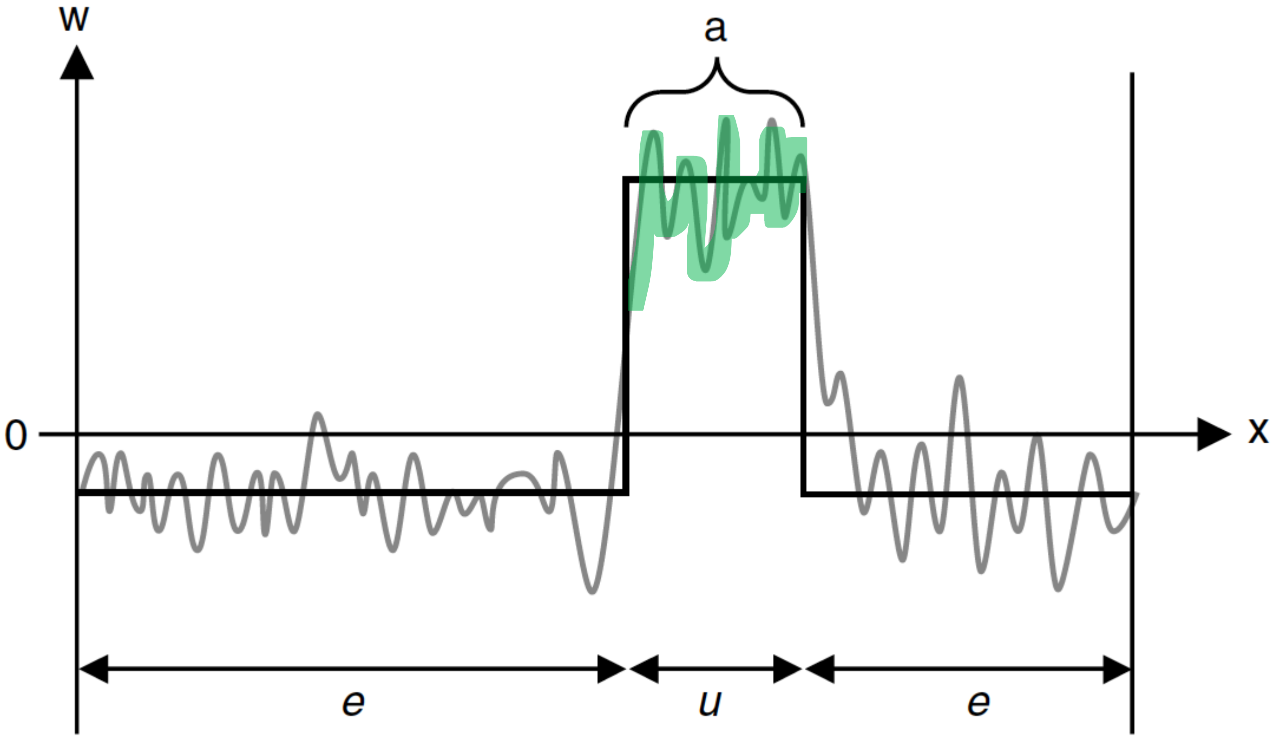
Subcore flux



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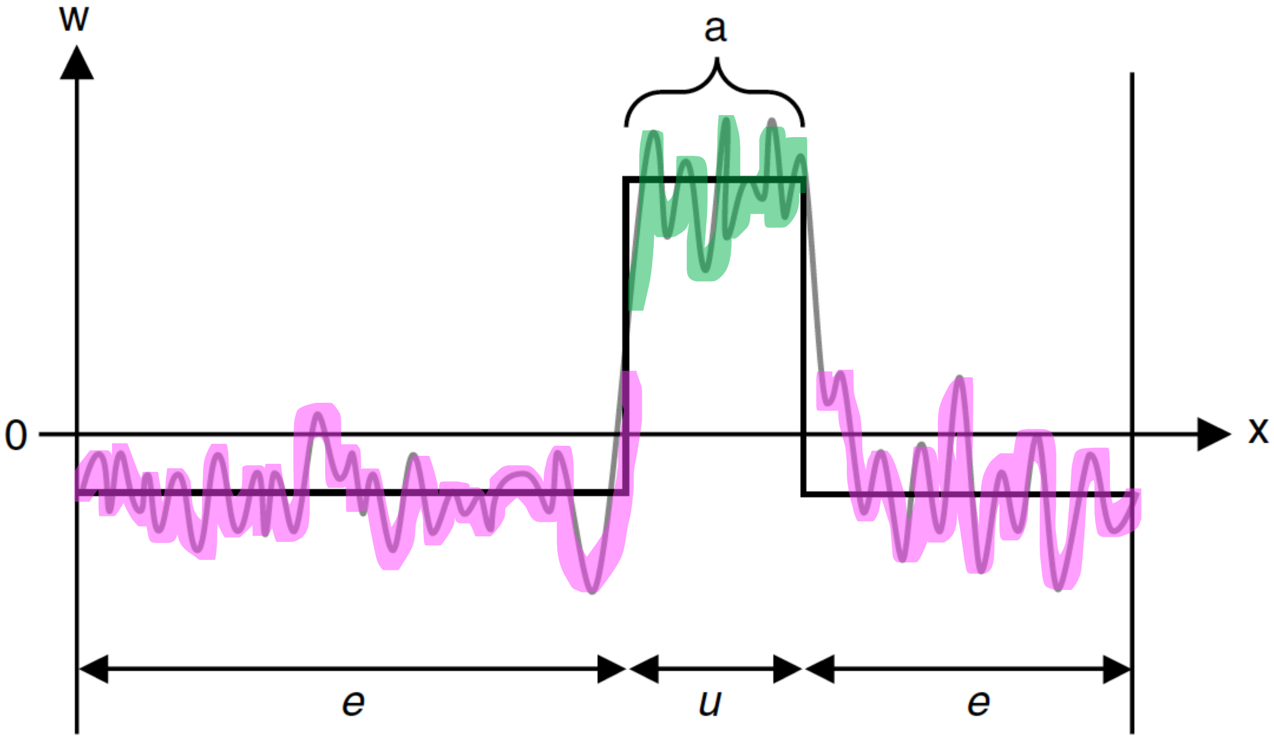
Subcore flux

Environmental flux



Turbulent flux in the environment outside the strongest updraft

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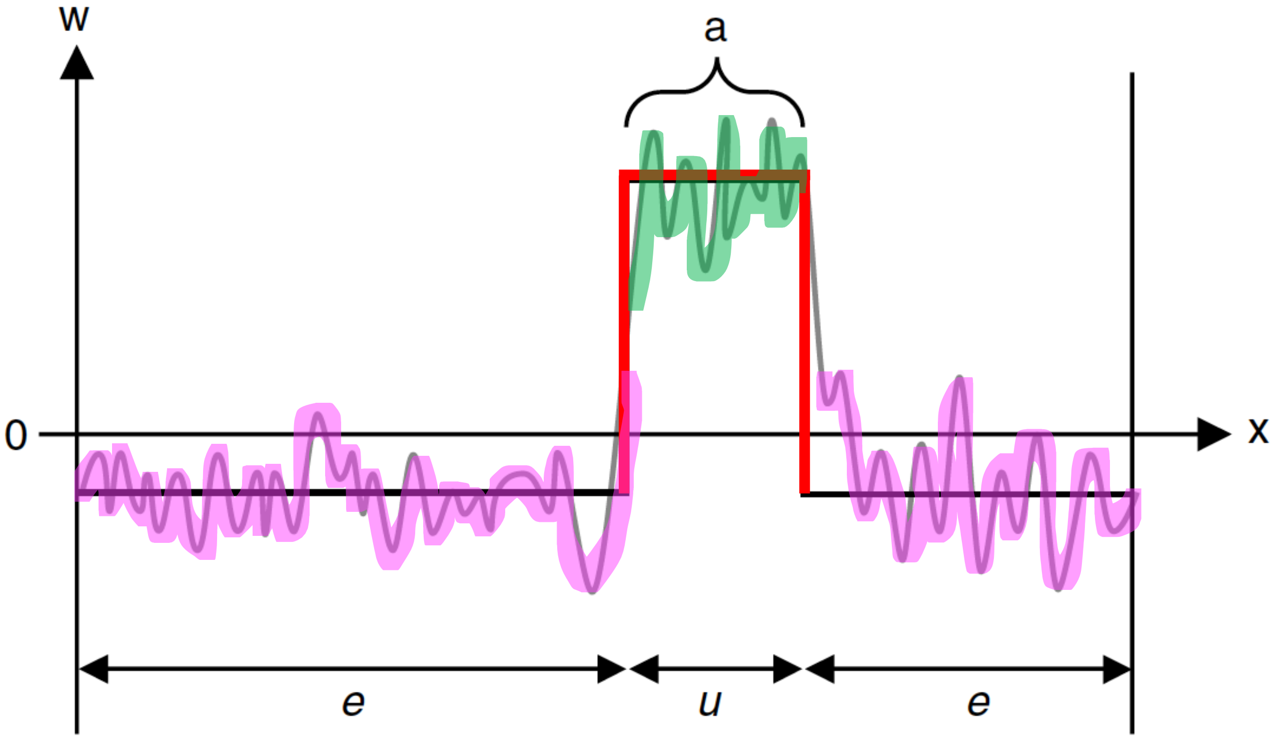
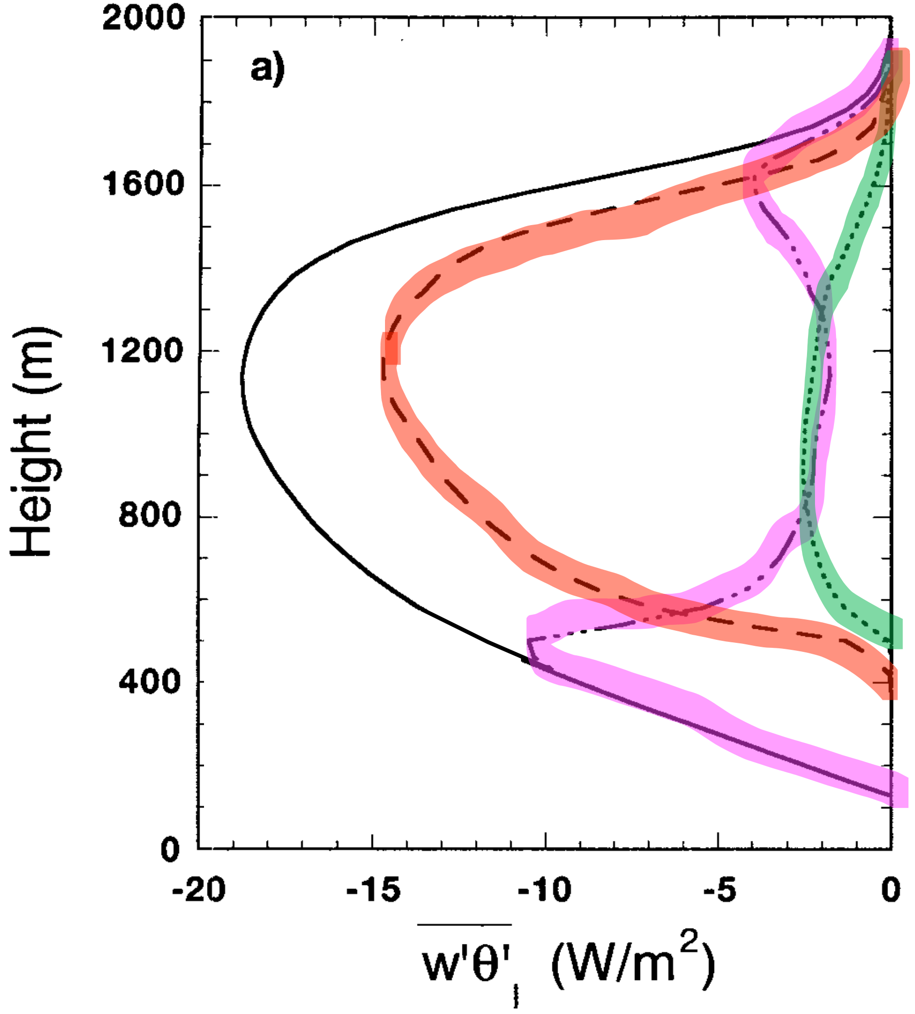
Subcore flux | Environmental flux | Mass flux



Mean flux inside the strongest updraft region

‘Non-Local’ turbulence: eddy-diffusivity mass-flux (EDMF)

Heat flux from LES



Total turbulent flux of ϕ :

$$\overline{\phi' w'} = a\overline{\phi'_u w'} +$$

Subcore flux

$$(1-a)\overline{\phi'_e w'} +$$

Environmental flux

$$a(\overline{w}^u - \overline{w})(\overline{\phi}^u - \overline{\phi}^e)$$

Mass flux

M-flux covers 80% of the flux for heat and moisture,

Siebesma & Cuijpers, 1995

less for momentum – environment plays a bigger role for momentum transport

Zhu 2015, Schlemmer et al, 2016

‘Non-Local’ turbulence: eddy-diffusivity mass-flux (EDMF)

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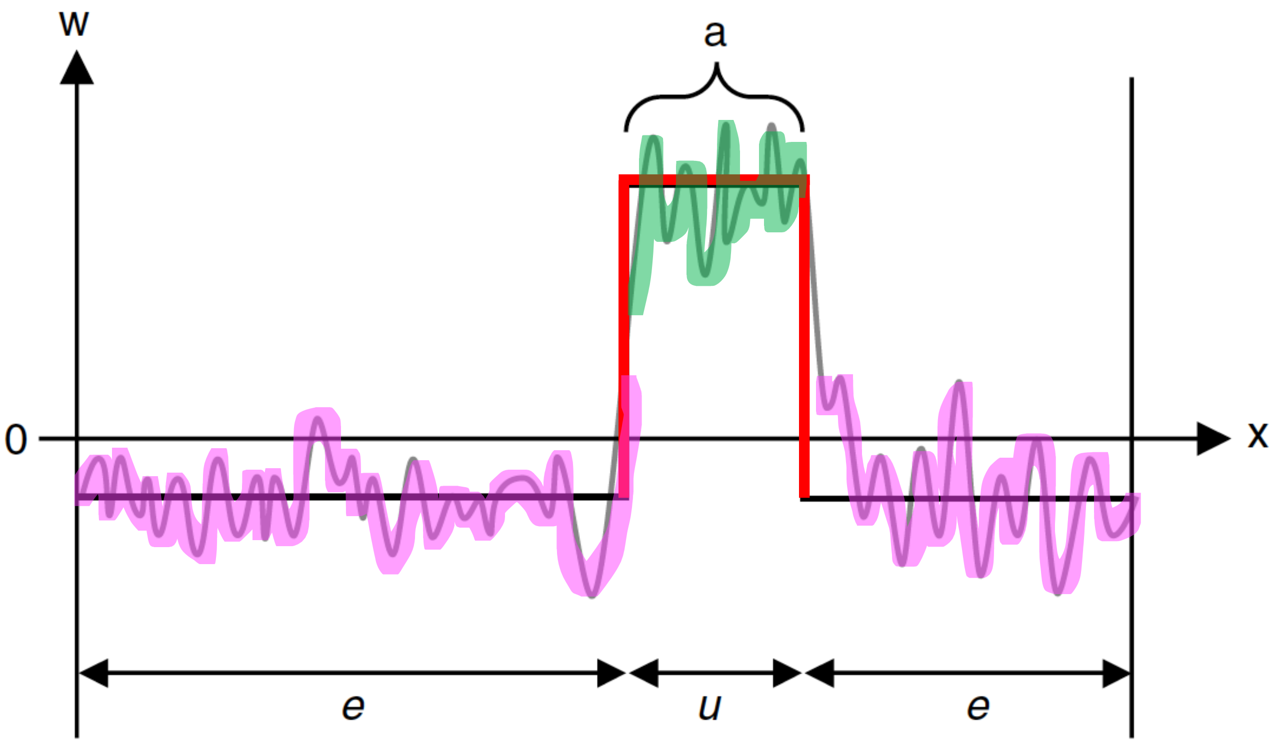
Subcore flux — Environmental flux — Mass flux



Assumptions made:

1. Area of strongest updraft is small compared with the environment (a<<1). Subcore flux is neglected

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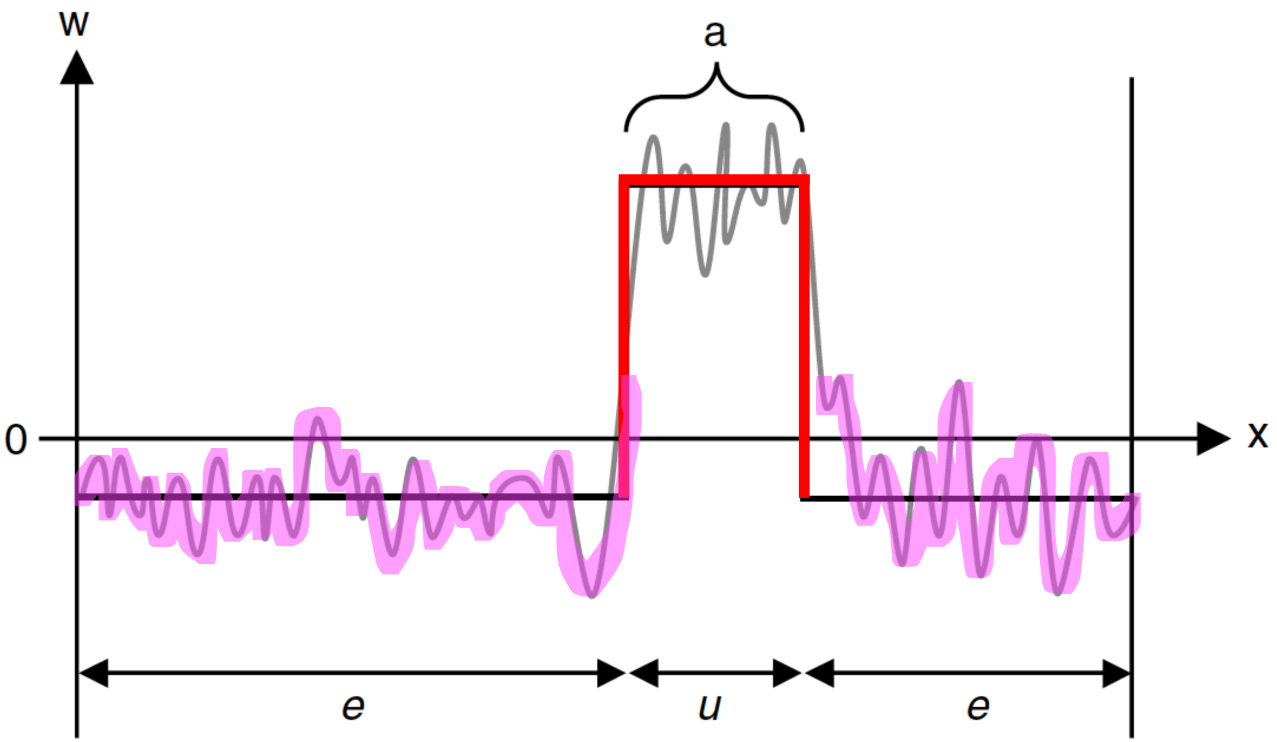
Environmental flux

Mass flux



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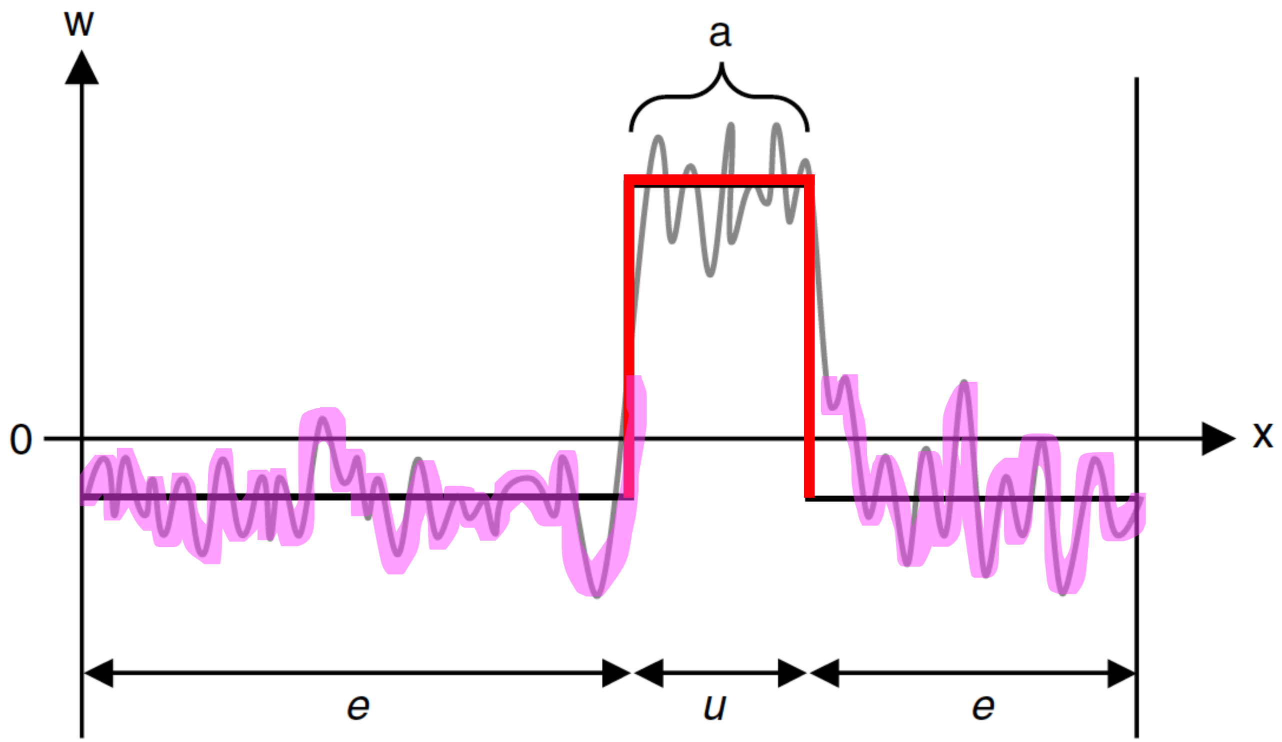

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2. Environmental flux is given by K-diffusion:

$$(1-a)\overline{\phi_e' w'} = -K_\phi \frac{\partial \phi}{\partial z}$$

‘Non-Local’ turbulence: eddy-diffusivity mass-flux (EDMF)

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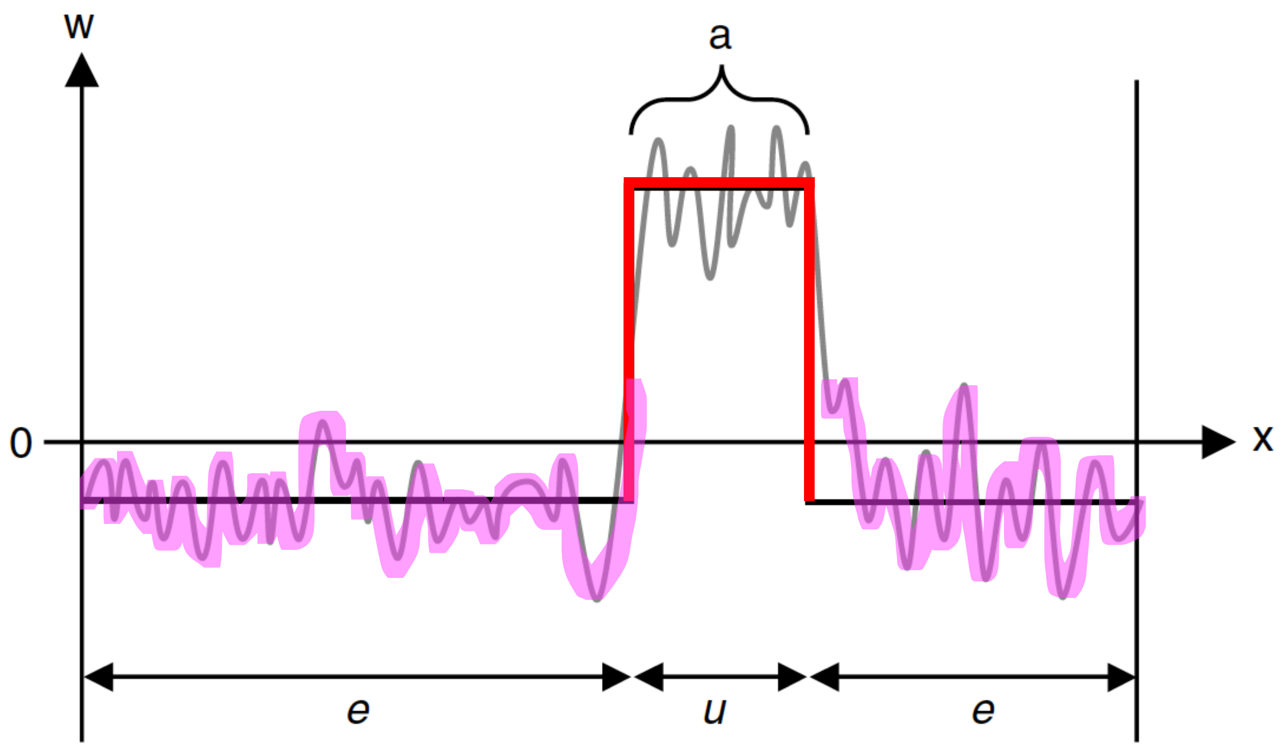
Environmental flux

Mass flux



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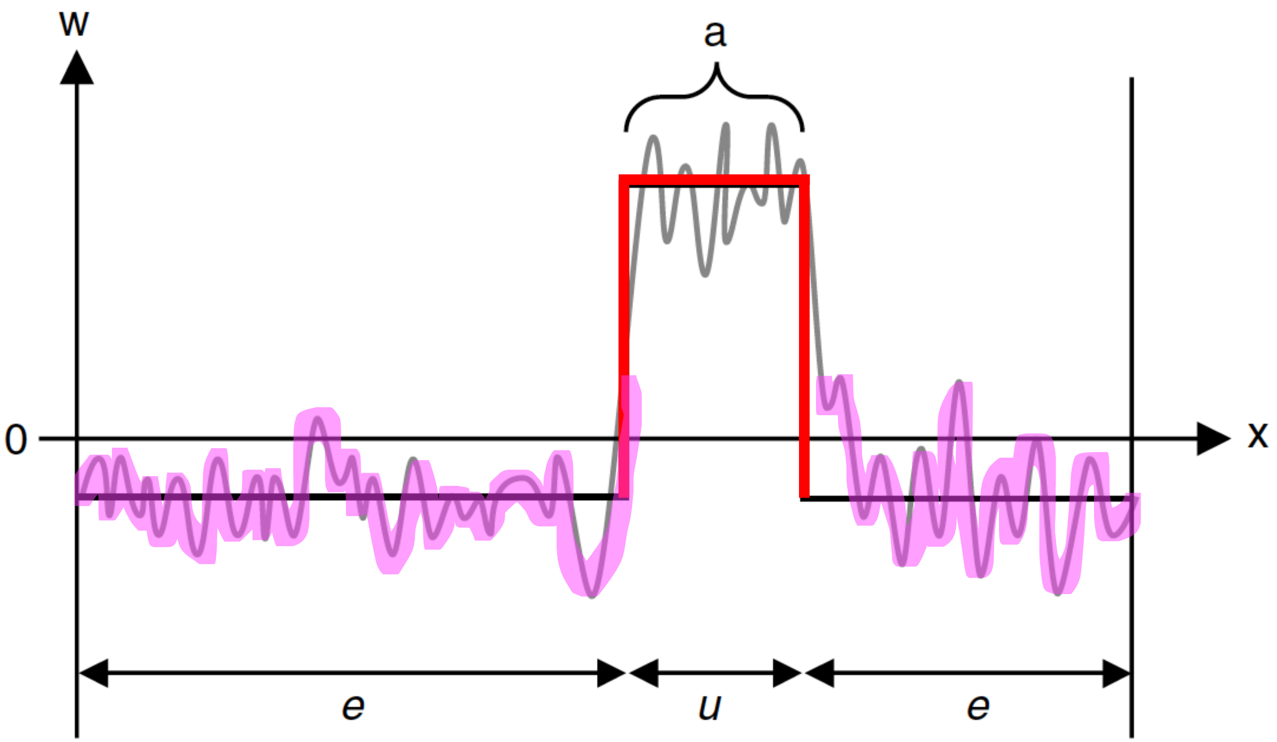
Total turbulent flux of ϕ :

$$\overline{\phi' w'} = -K_\phi \frac{\partial \phi}{\partial z} + M(\overline{\phi}^u - \overline{\phi}^e)$$

$$M = a(\overline{w}^u - \overline{w})$$

Environmental flux

Mass flux



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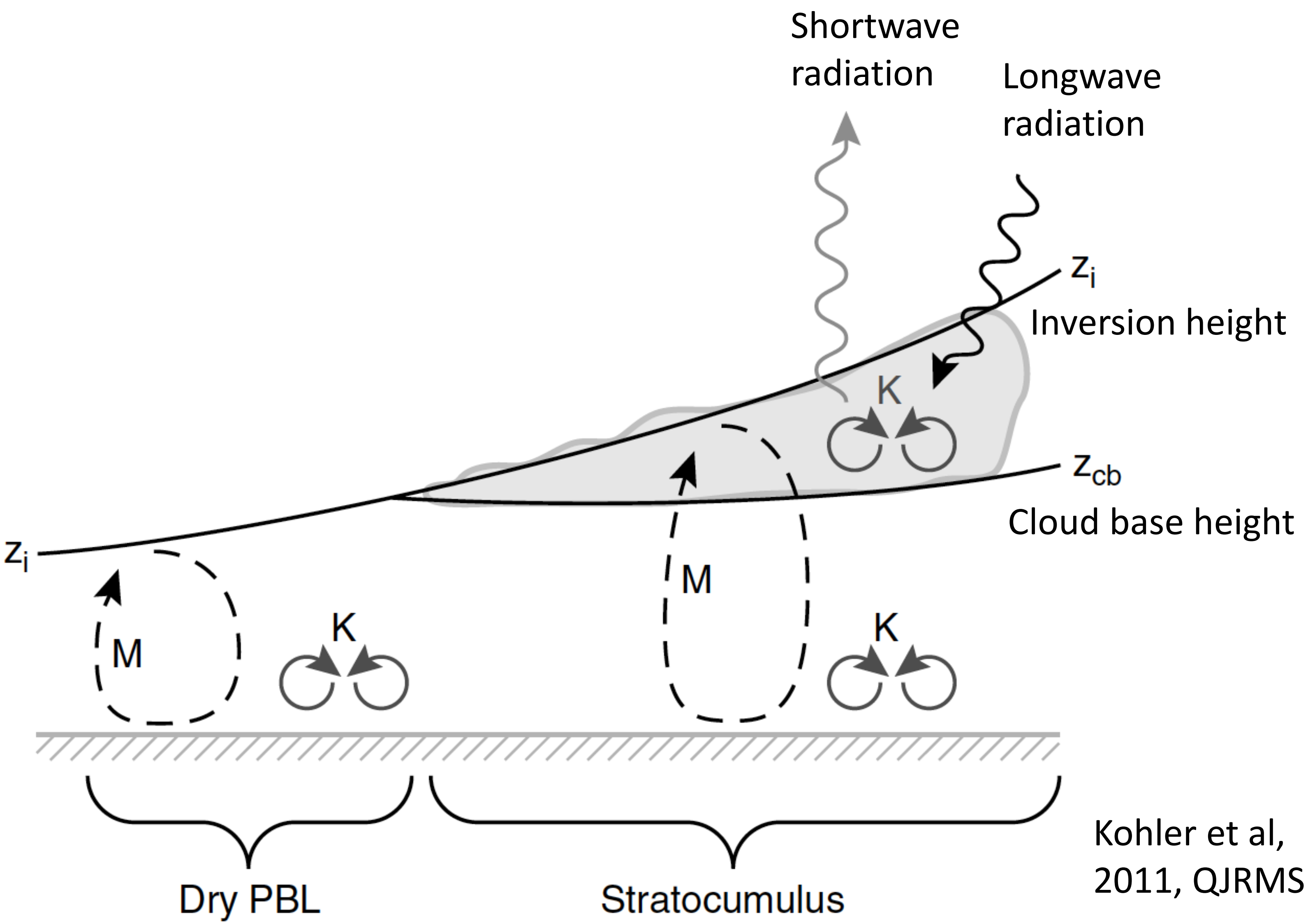
‘Non-Local’ turbulence: eddy-diffusivity mass-flux (EDMF)

The surface mass flux (M) is initialised at the first model level

The mass flux profile then depends on the inversion height (z_i) or the cloud base height (z_{cb})



Kohler et al, 2011, QJRMS

ECMWF

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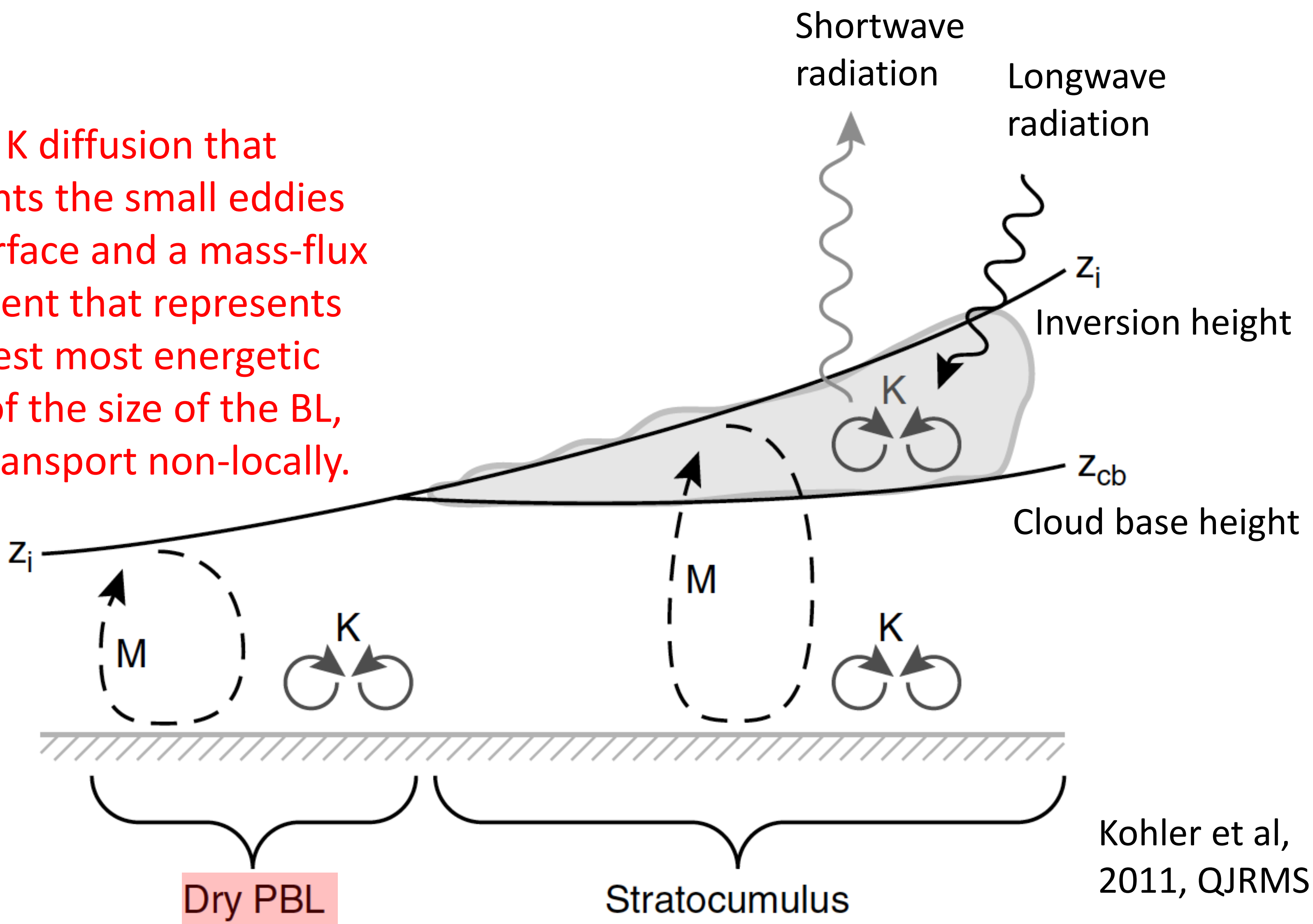
Dry BL - K diffusion that represents the small eddies from surface and a mass-flux component that represents the largest most energetic eddies of the size of the BL, which transport non-locally.



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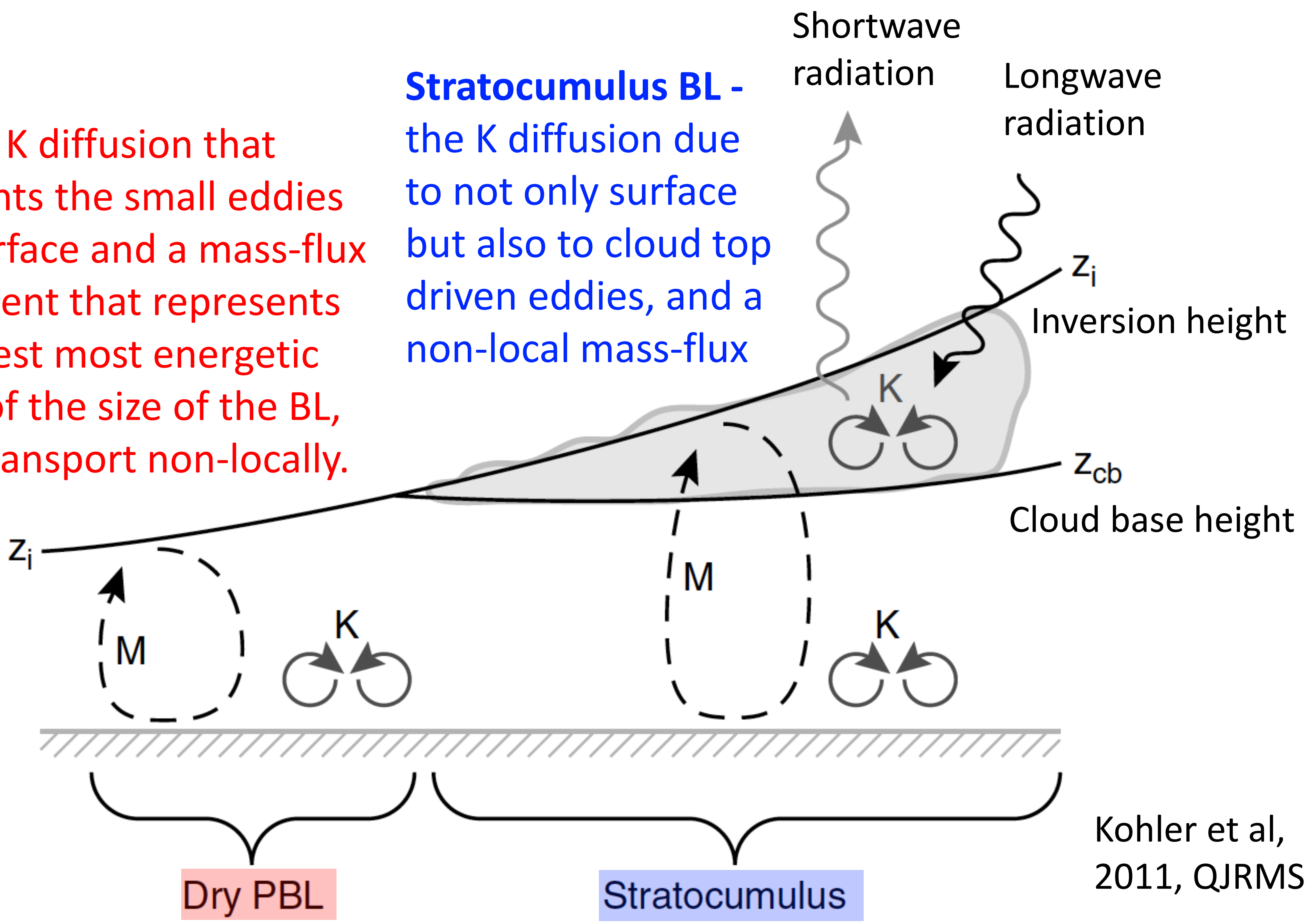
Dry BL - K diffusion that represents the small eddies from surface and a mass-flux component that represents the largest most energetic eddies of the size of the BL, which transport non-locally.

Stratocumulus BL - the K diffusion due to not only surface but also to cloud top driven eddies, and a non-local mass-flux



Kohler et al, 2011, QJRMS

Summary of fundamental concepts

- **Local turbulence closure:**
  - Assumes local turbulent fluxes can be determined by a K-profile and the background gradients
  - Concept of an eddy lengthscale is used to determine the turbulent mixing
  - Lengthscale depends on height above the surface and the stability
- **MO surface layer similarity theory:**
  - Possible to relate the surface fluxes and near-surface gradients through universal functions
  - Functions depend on the Obukhov length (measure of surface stability)
- **Roughness length:**
  - Assumed to be a property of the surface roughness elements (e.g. vegetation / wave height)
- **Non-local turbulence (EDMF):**
  - Assumes that areas of strongest updrafts are small compared with environment
  - Allows convective mixing in well-mixed environments with small gradients