

Model error in data assimilation

Patrick Laloyaux

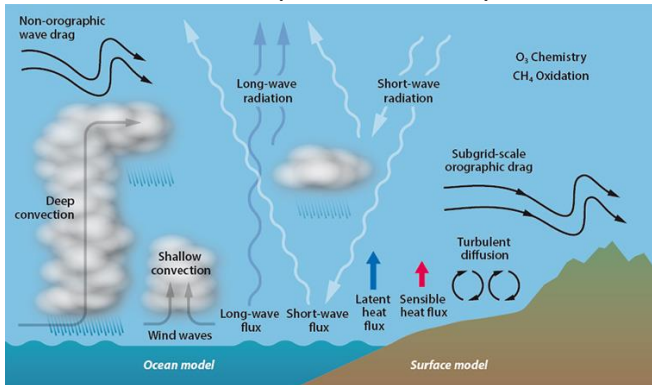
Learn about the typical issues that occur in 4D-Var

Identify systematic errors in observations and model (biases)

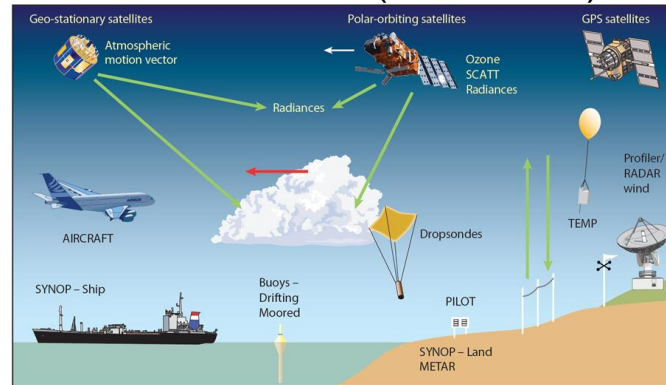
Develop bias correction methods

What you have seen so far on data assimilation

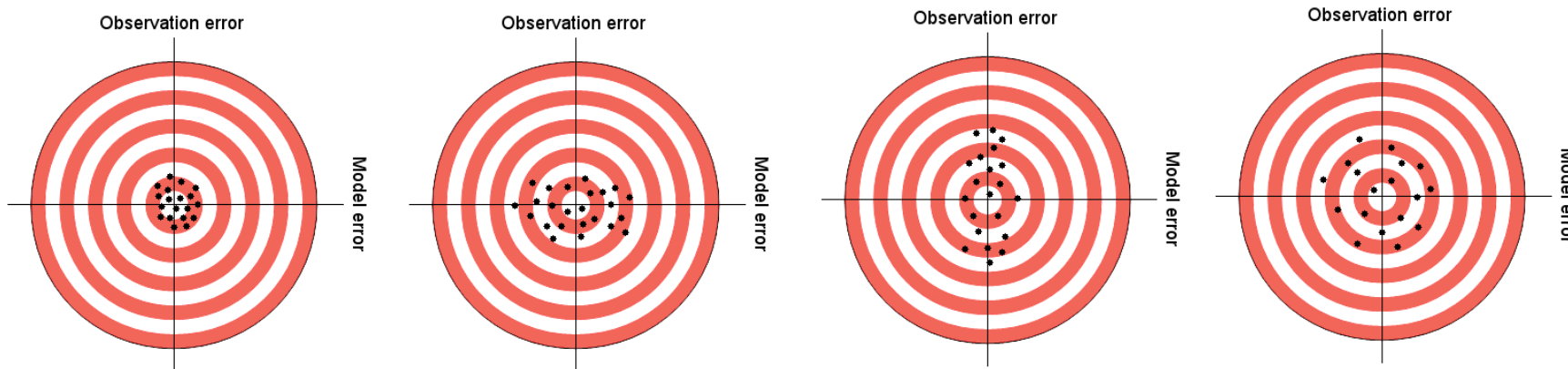
Model (with errors)



Observations (with errors)



If you are lucky, model and observations are not biased



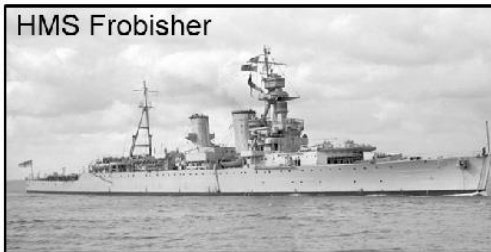
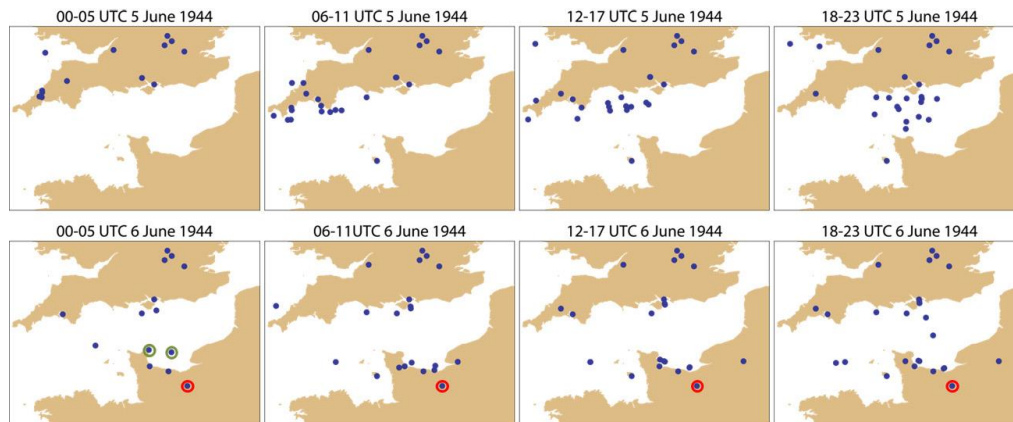
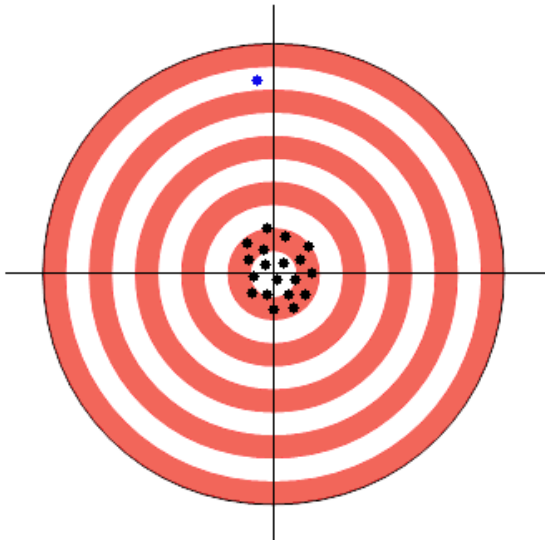
$$J(x_0) = \frac{1}{2}(x_0 - x_b)^T \mathbf{B}^{-1}(x_0 - x_b) + \frac{1}{2} \sum_{k=0}^K [y_k - \mathcal{H}(x_k)]^T \mathbf{R}_k^{-1} [y_k - \mathcal{H}(x_k)]$$

What's happening if you are unlucky?

Data assimilation and gross (obvious) errors

Observation error

Model error



HMS Frobisher	Pressure	1010.5hPa (mb)
	Temperature	285.95K (55°F)
	Dew point (wet bulb)	284.95K (54°F)
	Wind direction	225° (SW)
	Wind speed	6.7ms ⁻¹ (Force 4)
	(Weather/) Visibility	Code 97 (c/7)
	Sea temperature	285.35K (54°F)



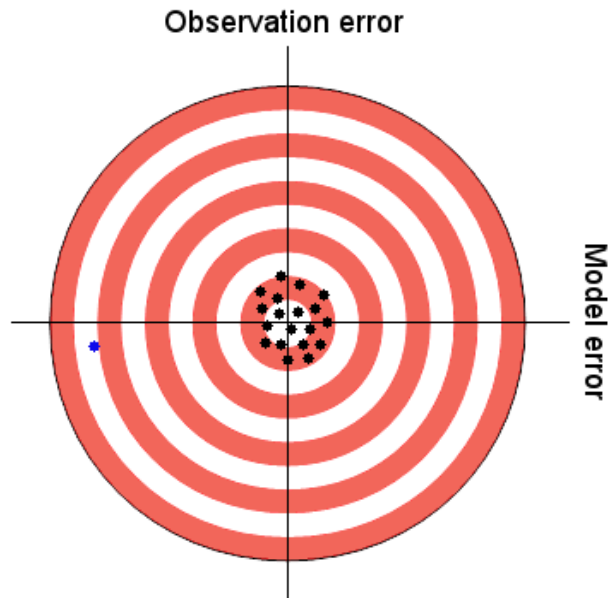
HMS Hawkins	Pressure	1014.8hPa (mb)
	Temperature	285.35K (54°F)
	Dew point (wet bulb)	283.35K (52°F)
	Wind direction	270° (W)
	Wind speed	6.7ms ⁻¹ (Force 4)
	(Weather/) Visibility	Code 96 (c/6)
	Sea temperature	284.25K (52°F)

H.M.S. " FROBISHER " TO JUESDAY THE 6 th day of JUNE . 19 44.												
From GREENOCK				To QUISTREHAM				LEAVE GRANTED TO SHIP'S COMPANY				
Time	Loc (Strand type)	Distance Run through the Water (Miles)	True Course	Miles Revolutions per minute	Wind Force (true)	Direction (true)	Corrected Barometer in Millibars	Temperature (°C)			REMARKS	
								Dry Bulb	Wet Bulb	Sea		
1100	943.46	12	3	1000	157.9							
1200	975.83	12	6	115	161.5							
1300	988.79	12	0	170	167.5							
1400	1001.49	12	7	1000	162.5	SW	4	1010.5	55	54.39		0400 action ship
1500	1013.31	12	0	1000	152.4							0515 stopped in bombardment zone
1600	1016.43	5	7	1000	107.9							0547 ground fire & salvoes on ship's battery
1700	1020.84	7	2	1000	90.1							0611 depth 4400' 0.2 speed as per manual
1800	1026.43	7	7	1000	97.5	W	4	1011.10	54.59	54		0645 ground from base. 0702 salvo
1900	1031.47	6	6	1000	82.5							0711 enemy search direct hit on h.c. 1 in 100 yds
2000	1035.38	6	0	1000	75.0							0712 search fire & salvoes on ship's battery
2100	1036.08	2	0	1000	28.1							0713 search fire & salvoes on ship's battery
2200	1037.77	2	0	1000	28.7	W	3	1011.6	55.54	56		0714 search fire & salvoes on ship's battery
ANCHOR BEARINGS												
Position through the Water		0900	49 20 N	0 15 W	land fire							2112 } Longitude Ch. 197°
Zero Time kept at 0000		1200	49 25 N	0 13 W	land fire							2113 } Lat. Ch. 197°
Zero Time kept at 0000		1500	49 32 N	0 15 W	land fire							2114 } Longitude Ch. 195°
Number on SICK LIST												
19												
1300	1037.77	2	0	1000	57.2							
1400	1042.32	5	0	1000	64.2							1430h L1(6) 5% abo-pidewith six casualties on board
1500	1042.32	1	5	1000	22.9							1500 1 Rating casualty died & was buried
1600	1042.32	2	2	1000	57.1	W	3	1012.4	57.57	57		1500 ship's casualty died & was buried

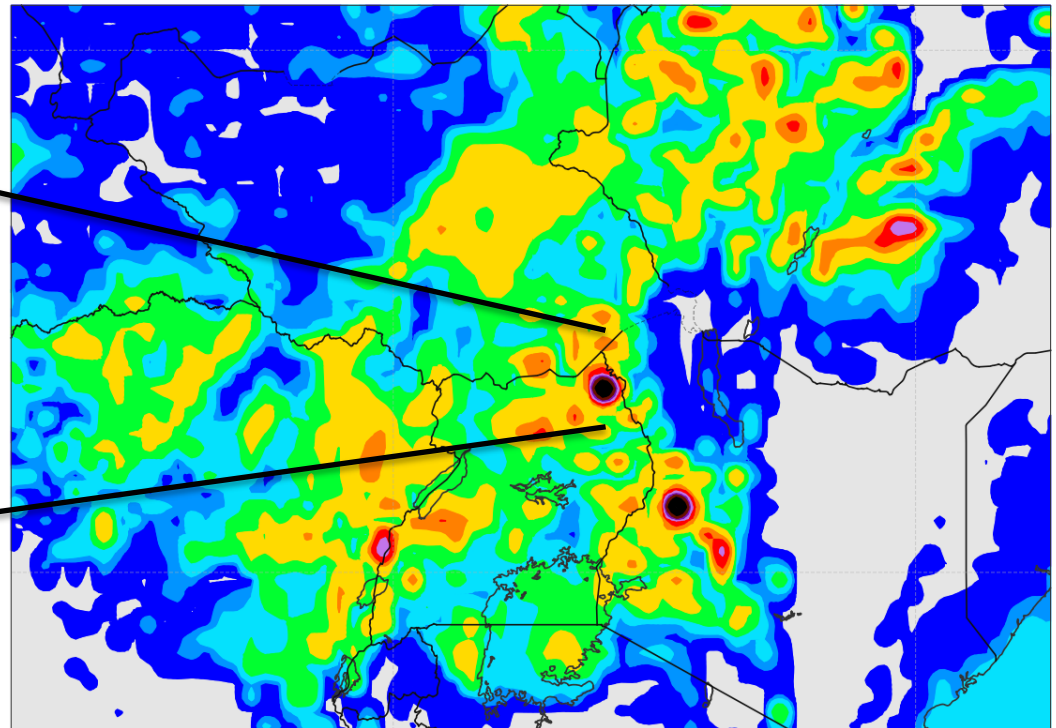
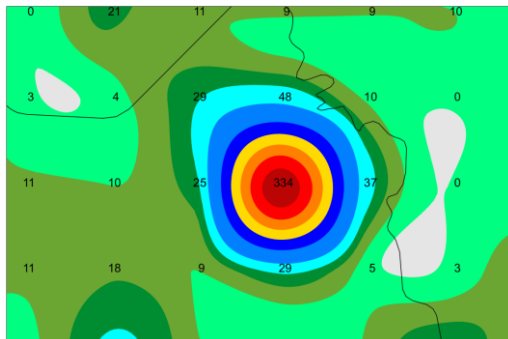
→ Preliminary analysis (blacklist,...)

→ Online Quality Control

Data assimilation and gross (obvious) errors

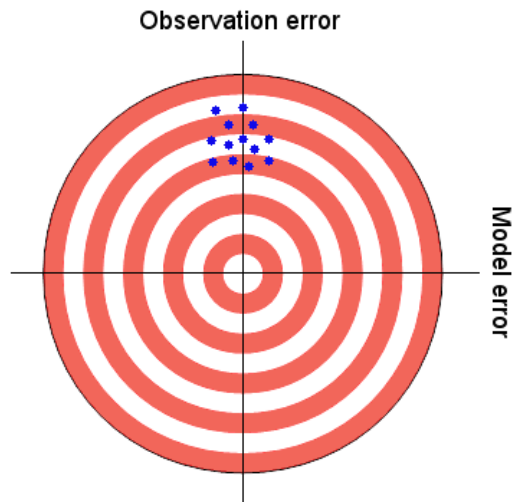


Total precipitation on 07 June 2019
(accumulated over 6 hours)



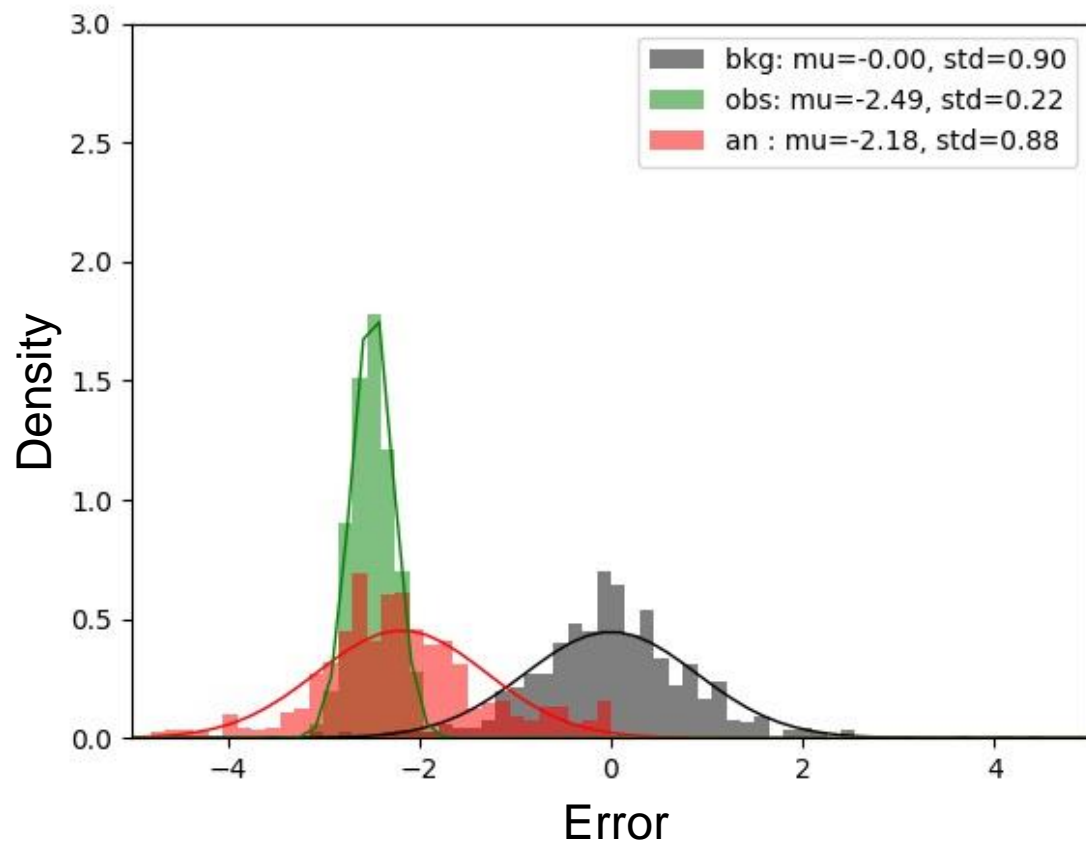
- Continuous monitoring
- Keep improving the model

Data assimilation and biased observations



If standard 4D-Var is used to assimilate biased observations (systematic errors), the resulting analysis will be biased.

In this case the background is more accurate than the analysis!



Data assimilation and biased observations

The USS Jeannette (1879, Arctic, 33 crew members)

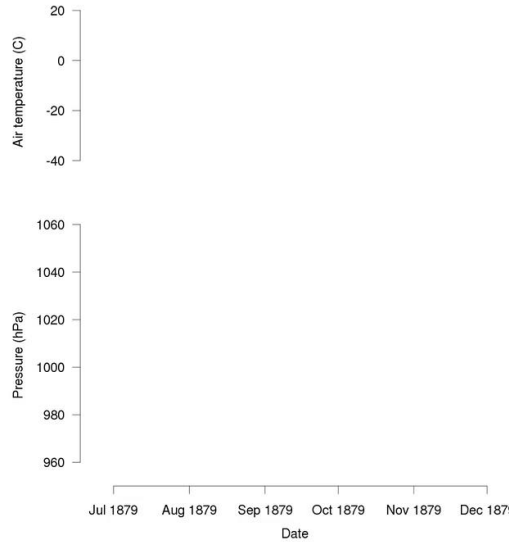
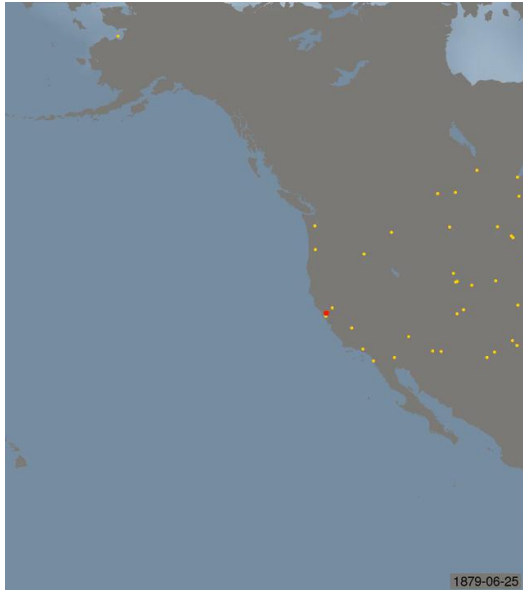


Photo # NH 52000 Steamer Jeannette sinking after being crushed by Arctic ice, June 1881



THE SINKING OF THE JEANNETTE.

Photo # NH 52002 Jeannette's crewmen drag their boats over the Arctic ice, June-August 1881



DRAGGING THE BOATS OVER THE ICE

Photo # NH 92142 LCdr. DeLong and his party wading ashore on the Lena Delta, Siberia, 17 Sept. 1881

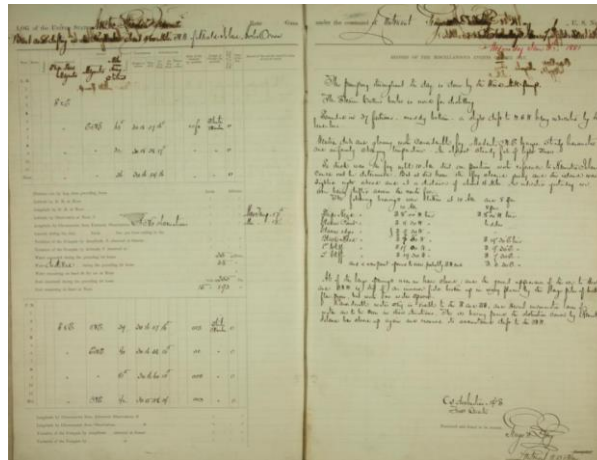


WADING ASHORE.

SST measurements from standard buckets have a cold bias ($\sim 0.4\text{C}$)



(Photos: David Parker)

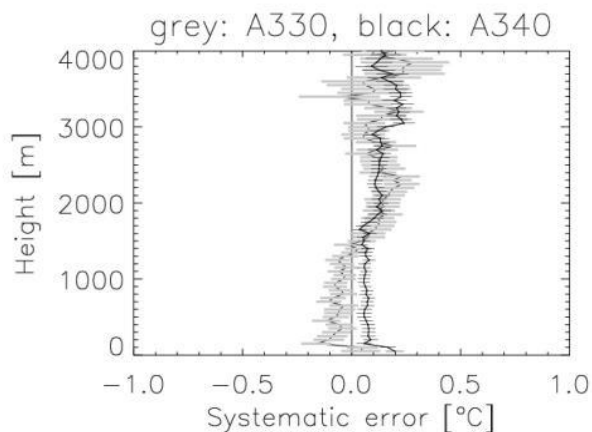
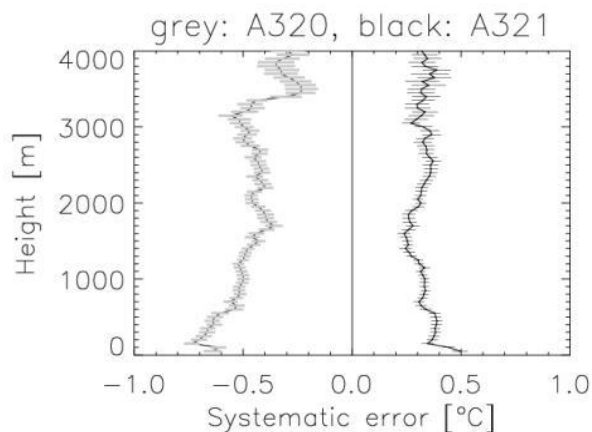
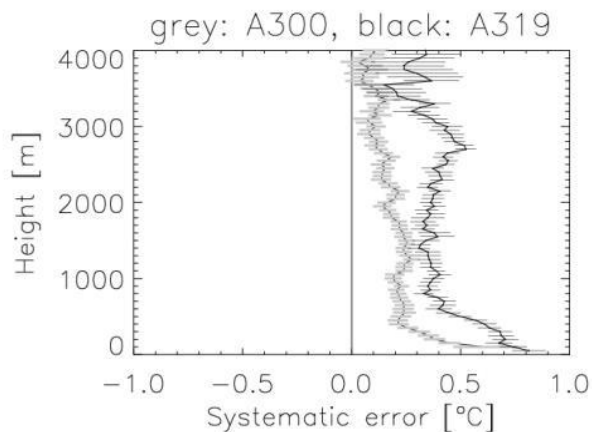
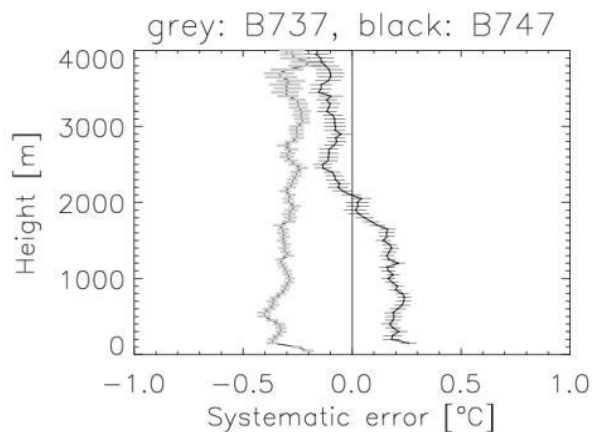


Observation bias is estimated using other (more accurate) instruments

Data assimilation and biased observations

One year of measurements from aircrafts landing at Frankfurt

No external reference, observation bias is estimated using the hourly mean of all measured profiles



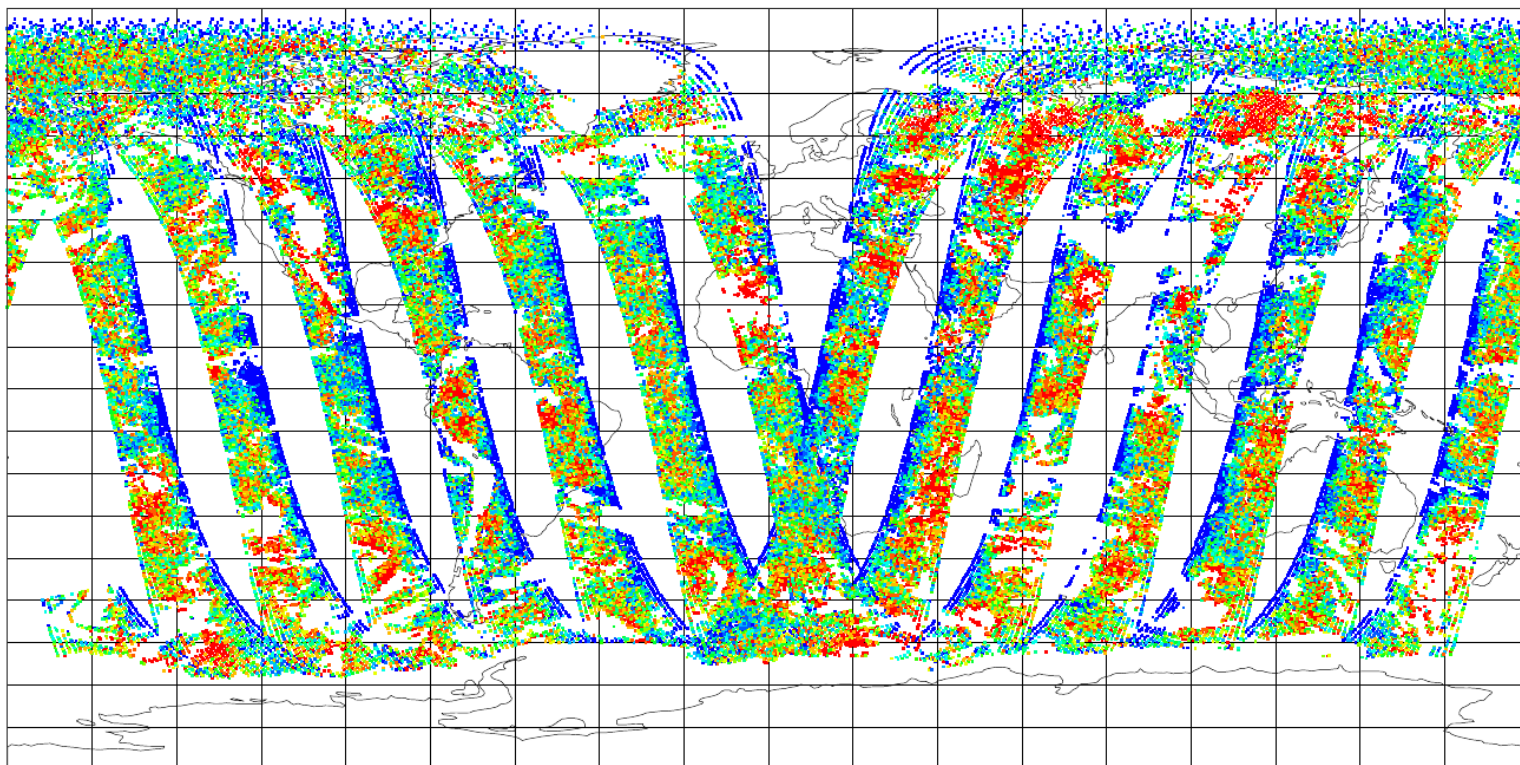
Data assimilation and biased observations



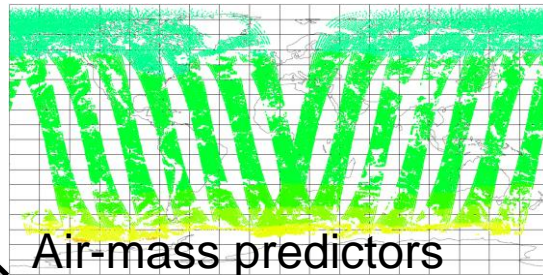
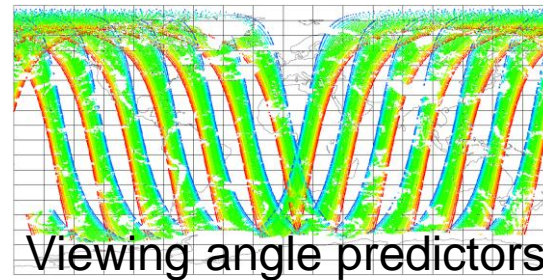
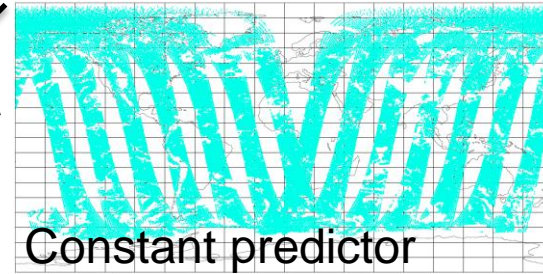
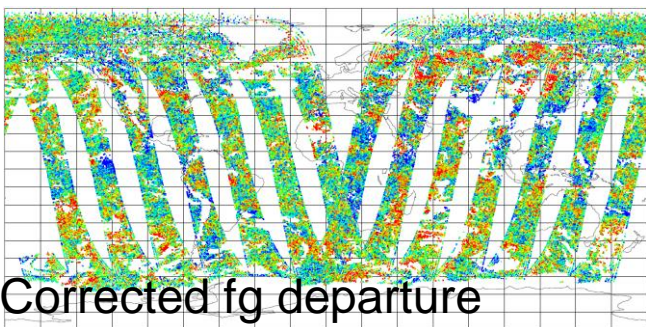
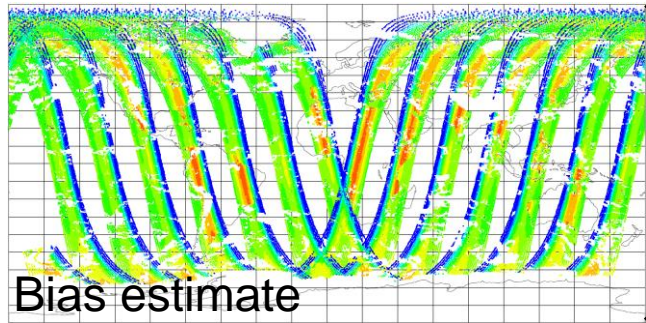
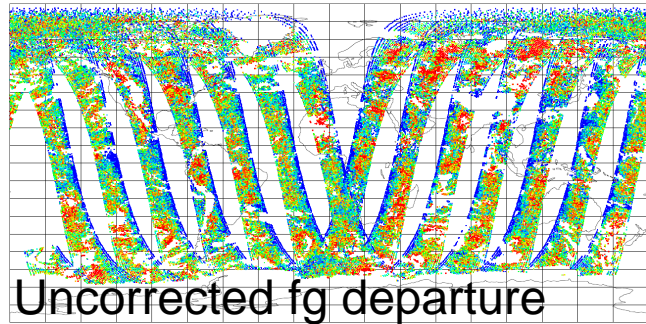
ECMWF is assimilating polar-orbiting Metop-C satellite (launched on 7 November 2018)

Observation bias is estimated inside 4D-Var
→ comparing measurements with model
→ specifying the structure of the model bias

Metop-C AMSUA-A Channel 5 (obs-model)



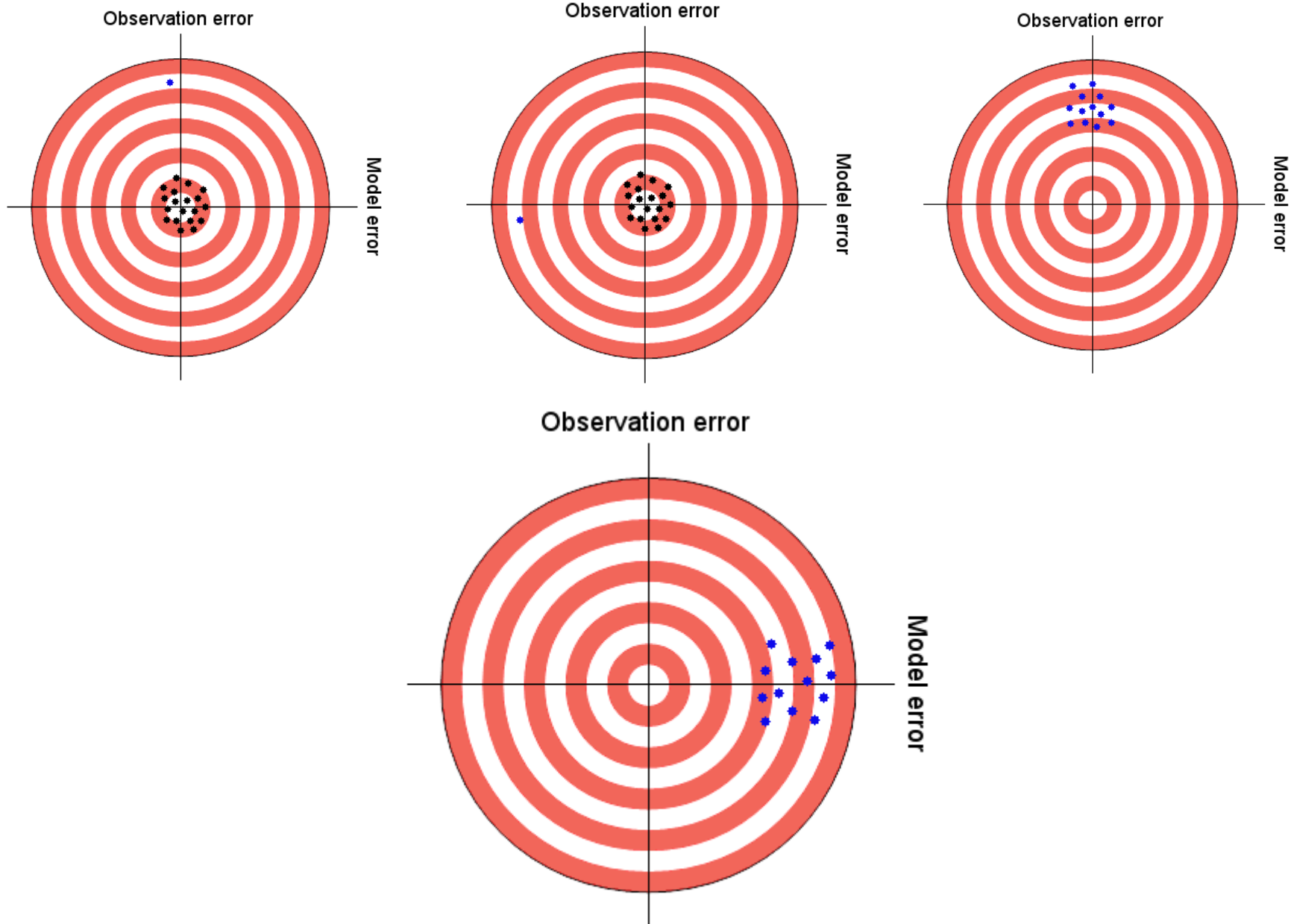
Data assimilation and biased observations



$$\begin{aligned}
 J(x_0, \beta) &= \frac{1}{2}(x_0 - x_b)^T \mathbf{B}^{-1}(x_0 - x_b) \\
 &+ \frac{1}{2} \sum_{k=0}^K [y_k - \mathcal{H}(x_k) - b(x_k, \beta)]^T \mathbf{R}_k^{-1} [y_k - \mathcal{H}(x_k) - b(x_k, \beta)] \\
 &+ \frac{1}{2} (\beta - \beta_b)^T \mathbf{B}_\beta^{-1} (\beta - \beta_b)
 \end{aligned}$$

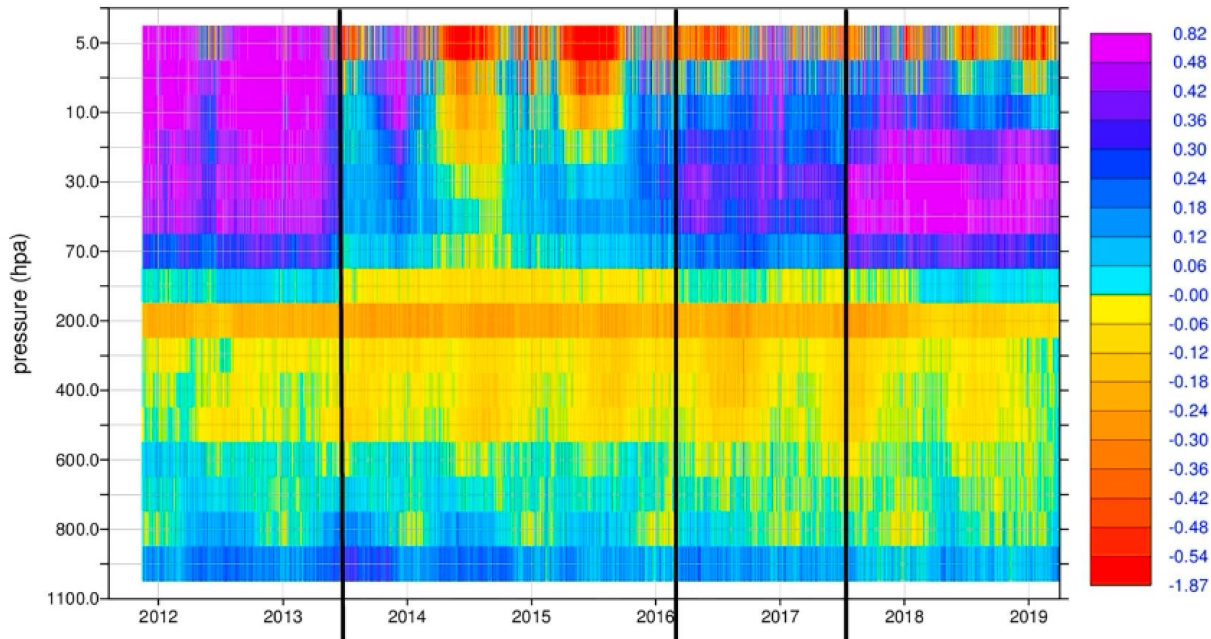
We choose the predictors and 4D-Var estimate the parameters β

Data assimilation and biased model



How to estimate model biases

The first-guess trajectory of the model can be compared to accurate observations



Difference between
radiosonde temperature
observations and the IFS
first-guess trajectory (O-B)



Errors in models are often systematic rather than random, zero-mean

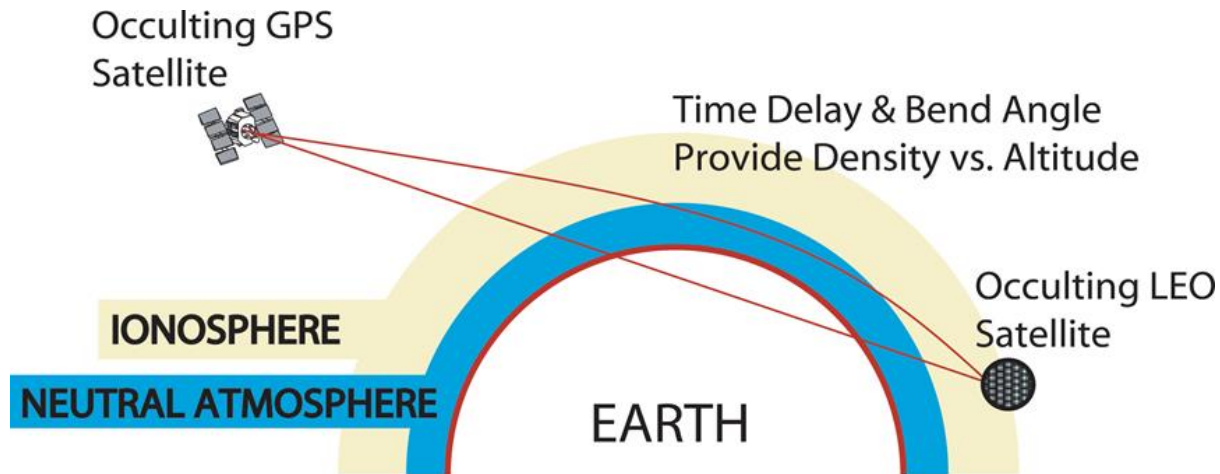
→ Largest bias in the stratosphere

→ Model has a temperature cold bias in the lower/mid stratosphere

→ Model has a warm bias in the upper stratosphere

How to estimate model biases

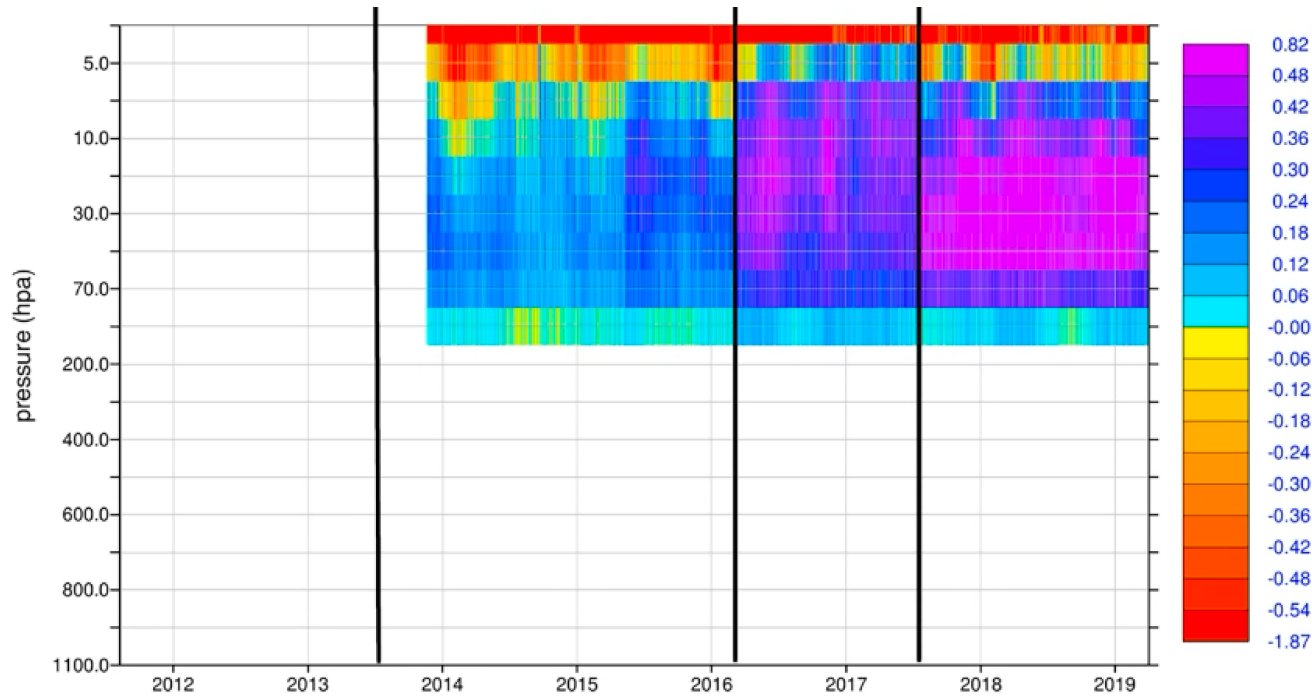
The GPS satellites are used for positioning and navigation. GPS-RO (Radio Occultation) is based on analysing the bending caused by the atmosphere along paths between a GPS satellite and a receiver placed on a low-earth-orbiting satellite.



- As the LEO moves behind the earth, we obtain a profile of bending angles
- Temperature profiles can then be derived
- GPS-RO can be assimilated without bias correction. They are good for highlighting errors/biases

How to estimate model biases

The first-guess trajectory of the model can be compared to accurate observations



Difference between
GPS-RO temperature
retrievals and the IFS
first-guess trajectory
(O-B)



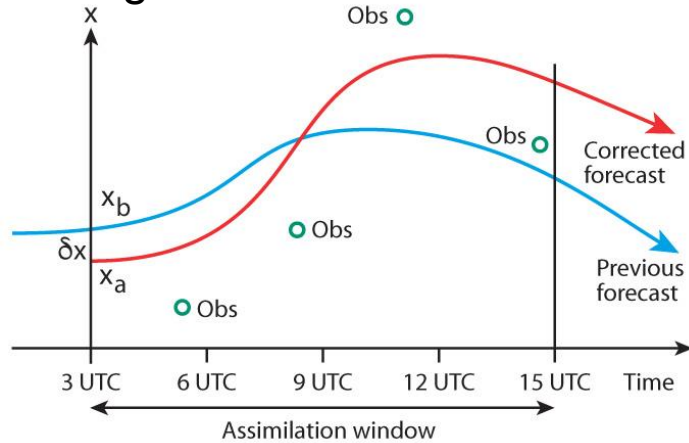
Errors in models are often systematic rather than random, zero-mean

→ Model has a temperature cold bias in the lower/mid stratosphere

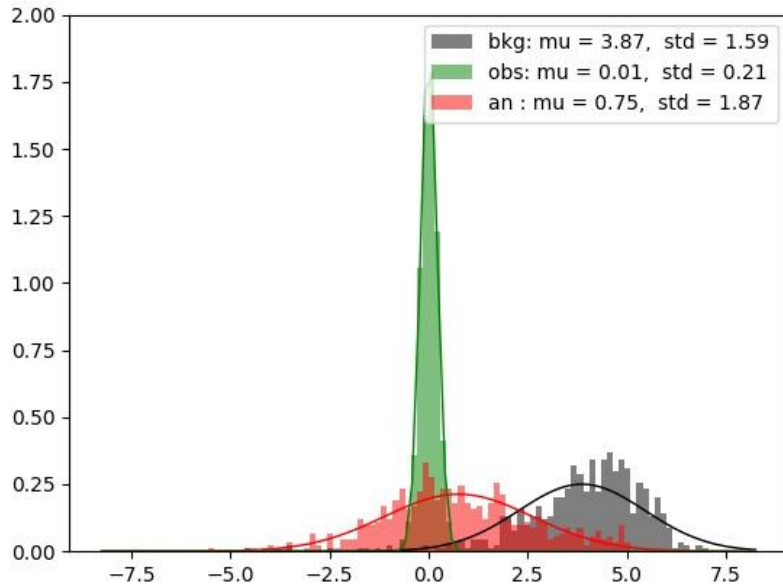
→ Model has a warm bias in the upper stratosphere

How to deal with model biases in data assimilation

Strong constraint 4D-Var

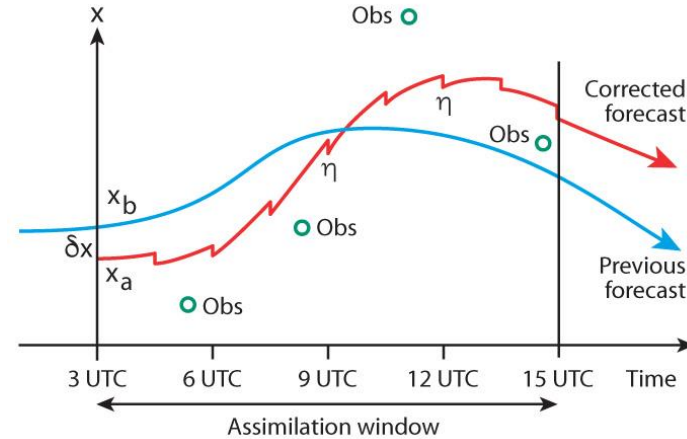


$$x_k = \mathcal{M}_k(x_{k-1})$$

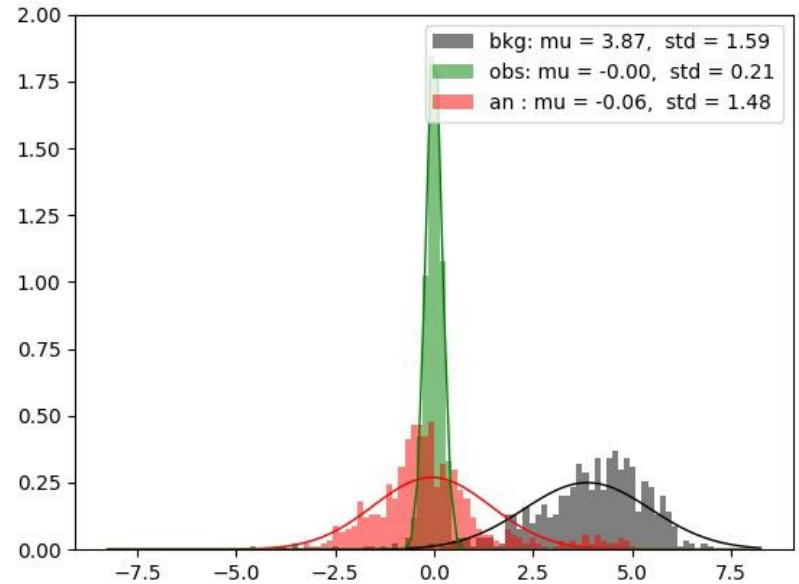


→ Large bias and standard deviation in the analysis

Weak constraint 4D-Var



$$x_k = \mathcal{M}_k(x_{k-1}) + \eta \quad \text{for } k = 1, 2, \dots, K$$



→ Bias in the analysis has been reduced, standard deviation as well

Weak constraint 4D-Var


We assume that the model is not perfect, adding an error term η in the model equation

$$x_k = \mathcal{M}_k(x_{k-1}) + \eta \quad \text{for } k = 1, 2, \dots, K$$

The model error estimate η contains 3 physical fields

- temperature
- vorticity
- divergence

Constant model error forcing over the assimilation window to correct the model bias


$$\begin{aligned} J(x_0, \beta, \eta) &= \frac{1}{2}(x_0 - x_b)^T \mathbf{B}^{-1}(x_0 - x_b) \\ &+ \frac{1}{2} \sum_{k=0}^K [y_k - \mathcal{H}(x_k) - b(x_k, \beta)]^T \mathbf{R}_k^{-1} [y_k - \mathcal{H}(x_k) - b(x_k, \beta)] \\ &+ \frac{1}{2}(\beta - \beta_b)^T \mathbf{B}_\beta^{-1}(\beta - \beta_b) \\ &+ \frac{1}{2}(\eta - \eta_b)^T \mathbf{Q}^{-1}(\eta - \eta_b) \end{aligned}$$

→ Introduce additional controls to target an unbiased analysis

→ The model error covariance matrix \mathbf{Q} constrains the model error field

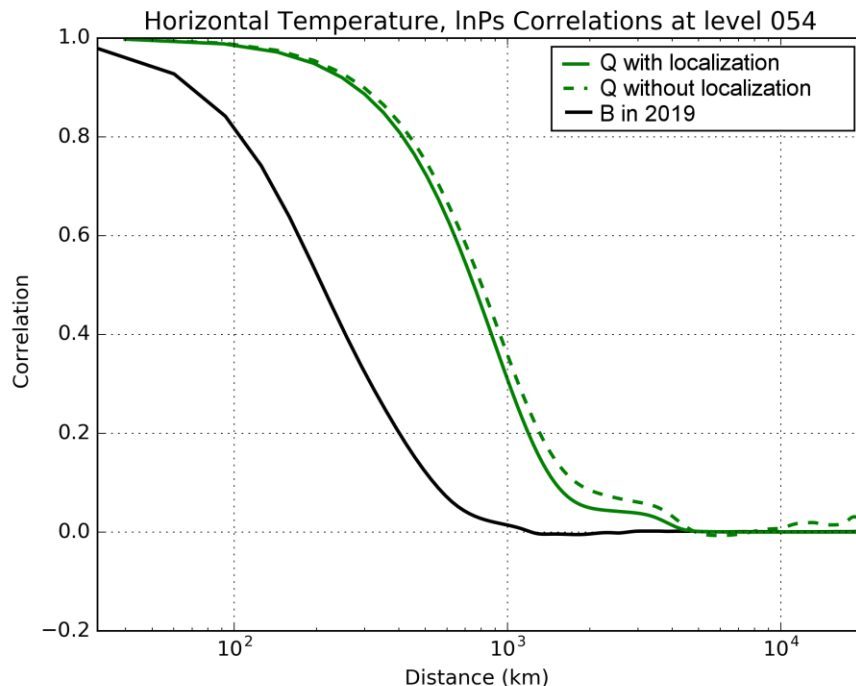
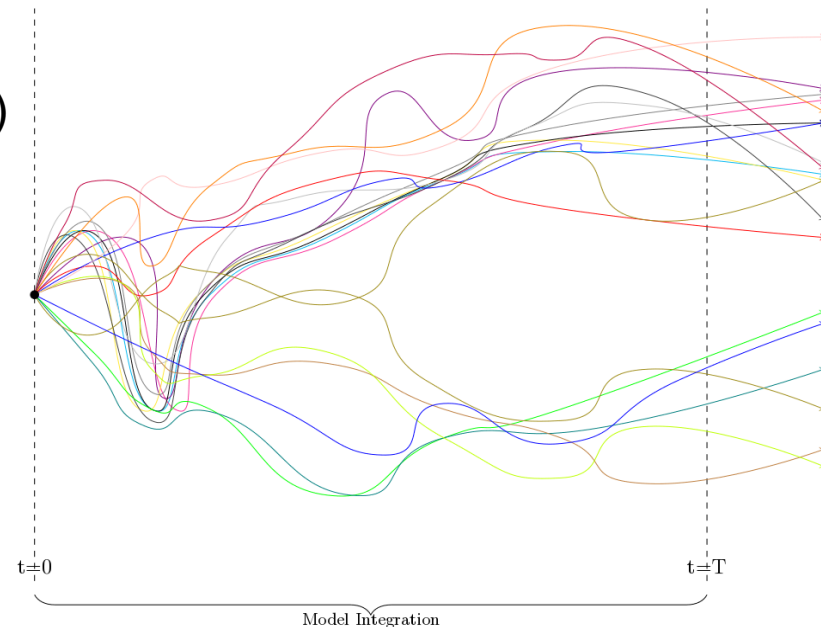
How to estimate the model error covariance matrix (Q)

Estimate the model error covariance matrix

→ run the ensemble forecasting system (ENS) with perturbed physics (51 members with the same initial condition for different days)

→ differences after 12 hours are used to compute Q

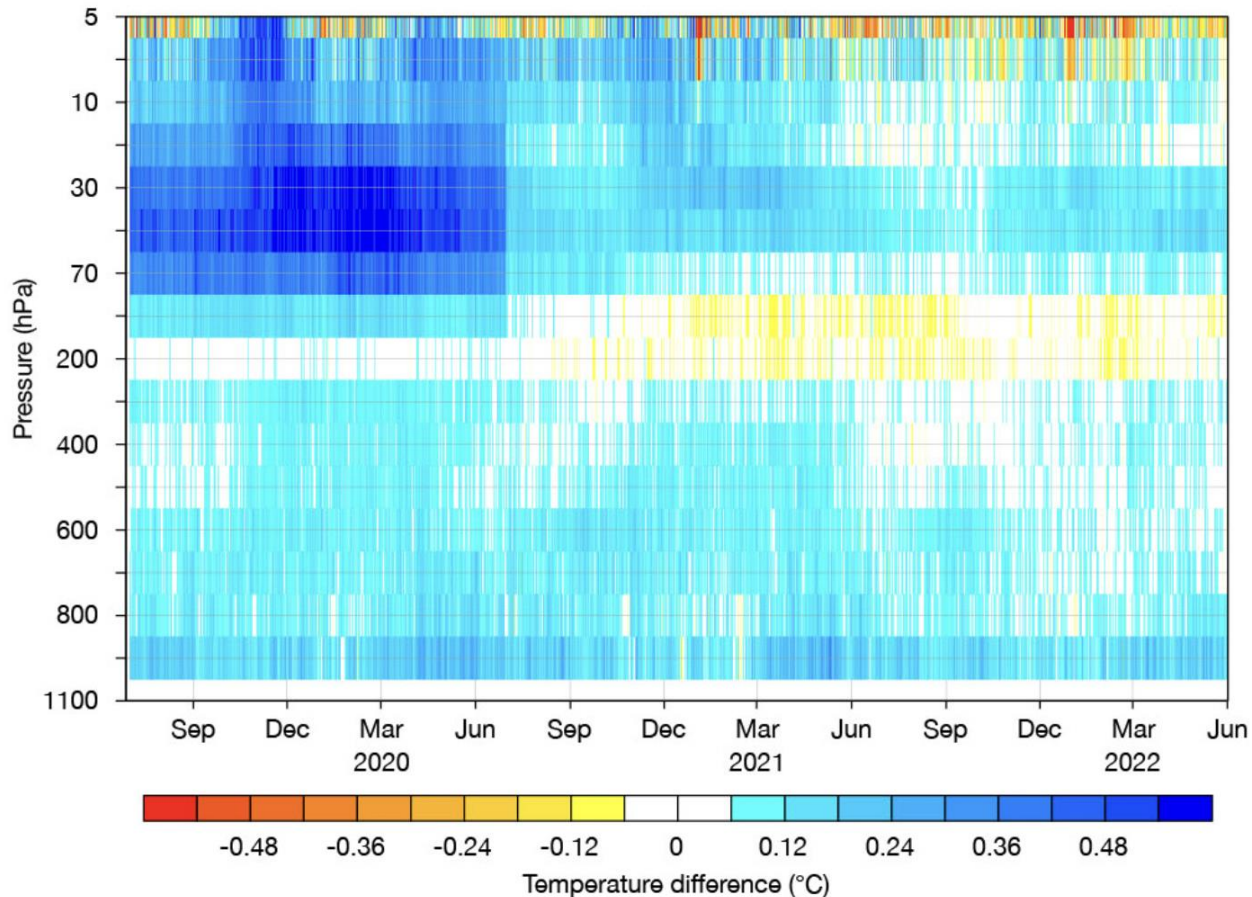
$$Q_f = \frac{1}{N-1} \sum_{i=1}^N (f_i^{12} - f_{i+1}^{12}) (f_i^{12} - f_{i+1}^{12})^T$$



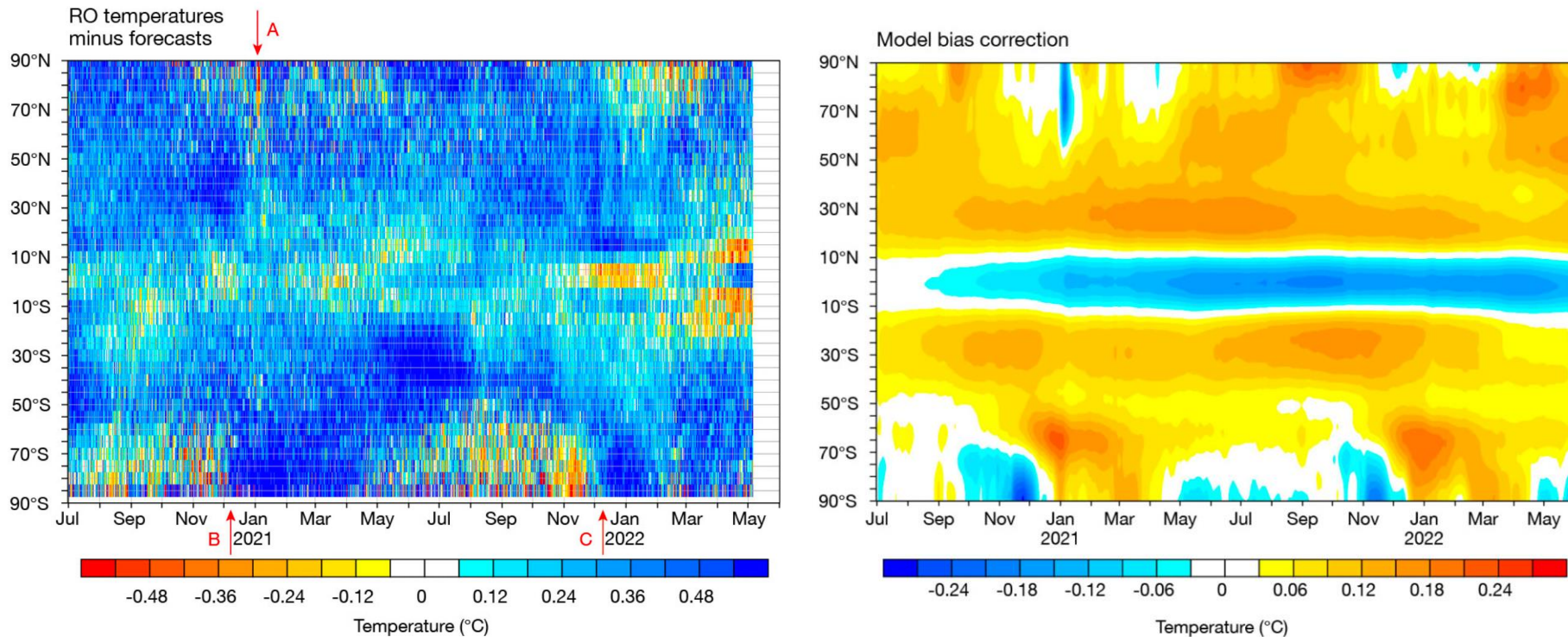
4D-Var corrects small scale errors (background errors) by changing the initial condition and large scale errors (model errors) by changing the model forcing

Weak-constraint 4D-Var in operations for the stratosphere

Time series of the difference between radiosonde temperature observations and model first-guess (47r1 implemented on 30 June 2020)



Weak-constraint 4D-Var in operations for the stratosphere

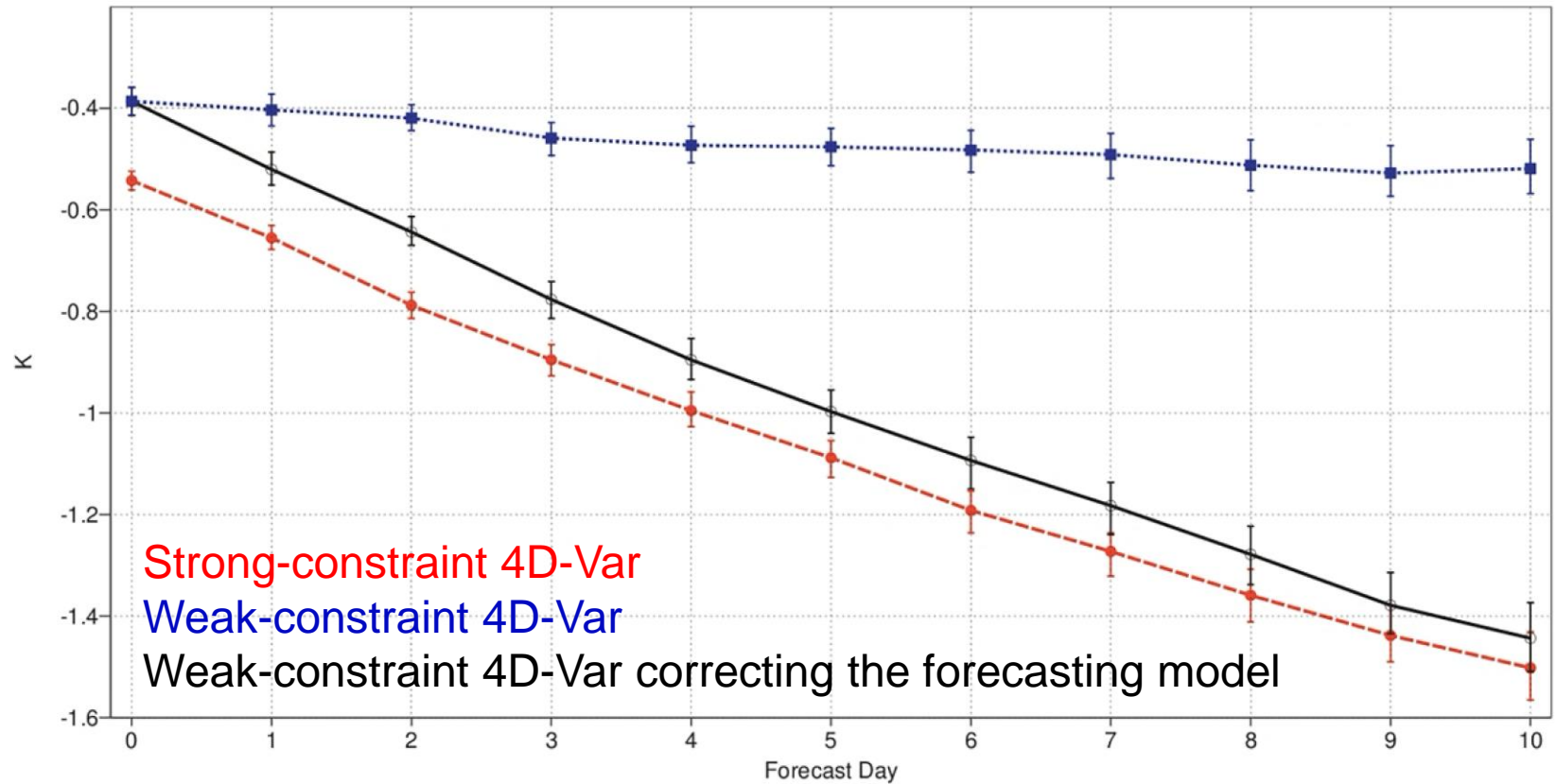


A) On 31 December 2020, a Sudden Stratospheric Warming (SSW) event started over the northern hemisphere

B&C) Clear seasonal cycle in the model bias over the southern hemisphere with a sharp transition in early December 2020 and 2021

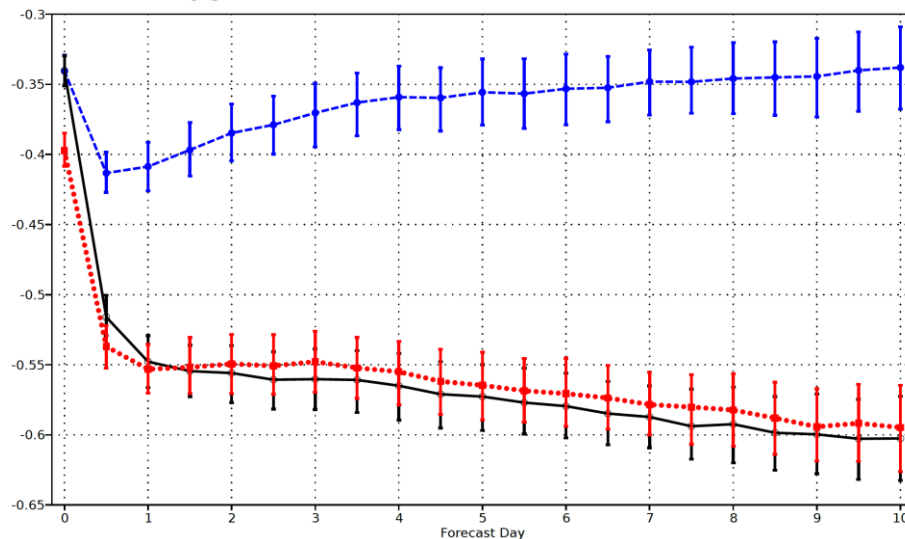
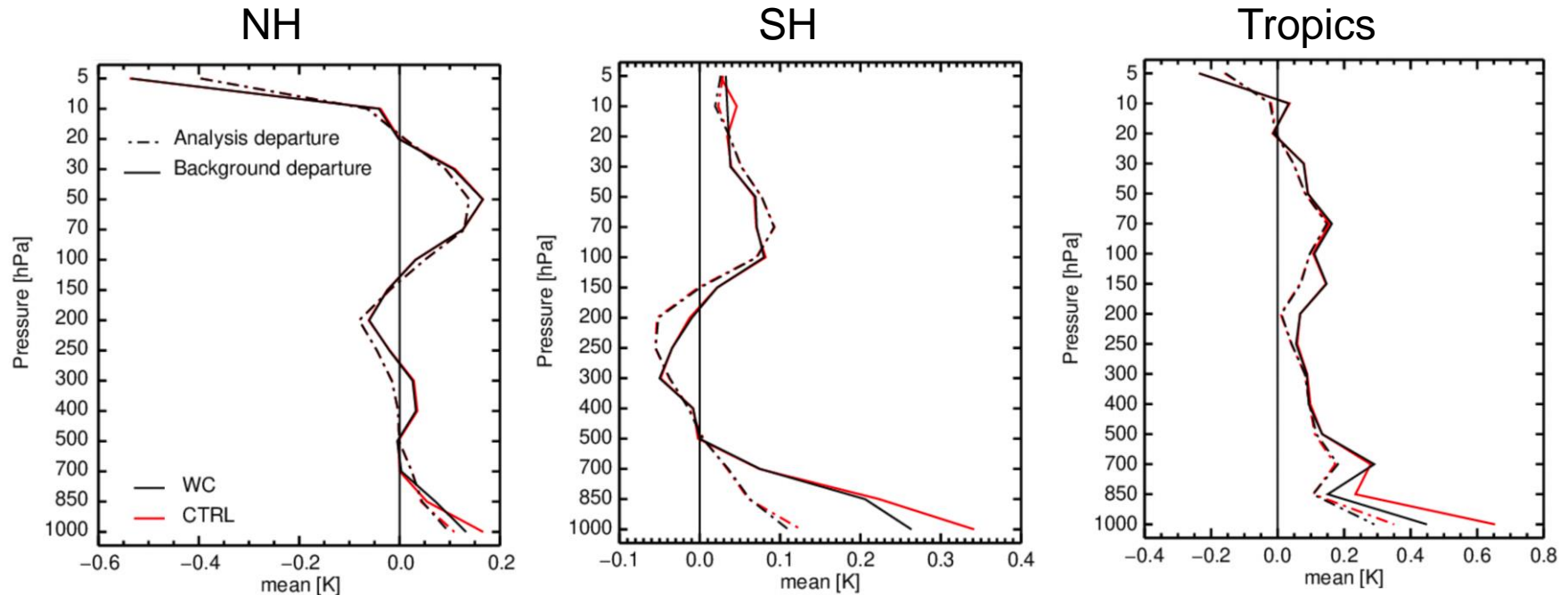
Weak-constraint 4D-Var and medium-range forecast

Mean error of the 10-day forecast at 50hPa with respect to the radiosonde observations



Weak-constraint 4D-Var in the boundary layer

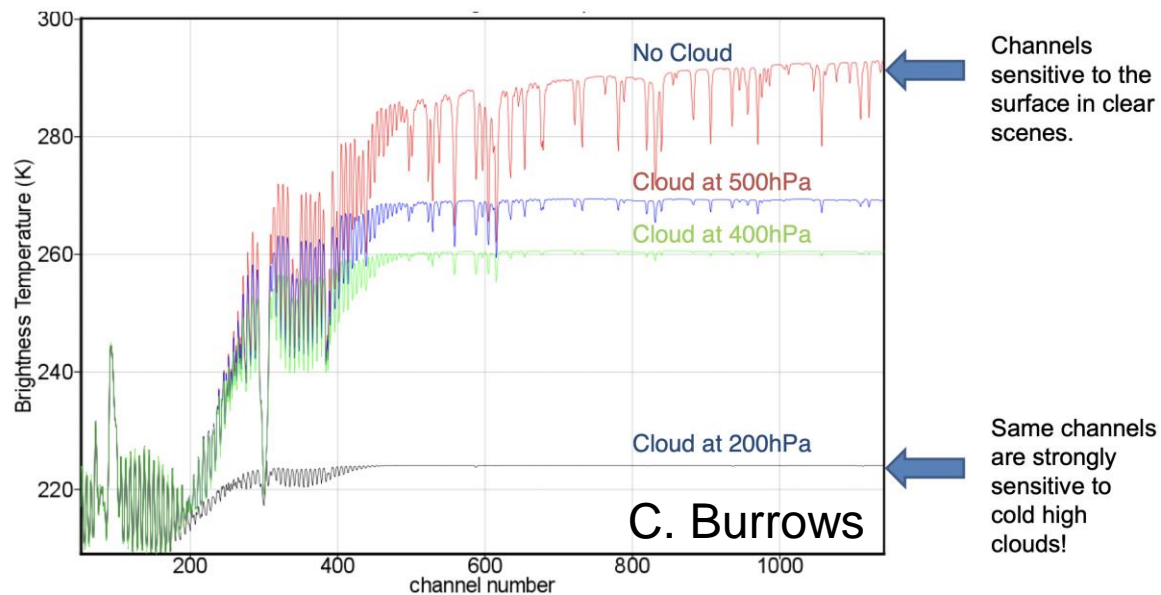
Activate WC4D-Var in the boundary layer (up to ~ 900hPa) to correct temperature



T2m mean error (Tropics)

- CTRL
- WC4D-Var correcting IC
- WC4D-Var correcting IC and forecast model

Positive feedback for satellite infrared assimilation

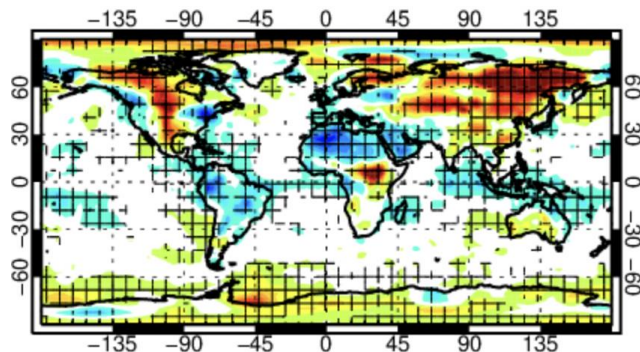


Clouds have a very strong impact on infrared radiance measurements

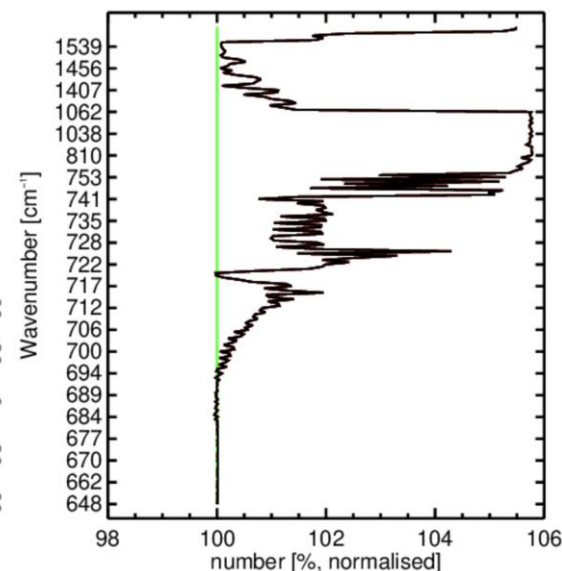
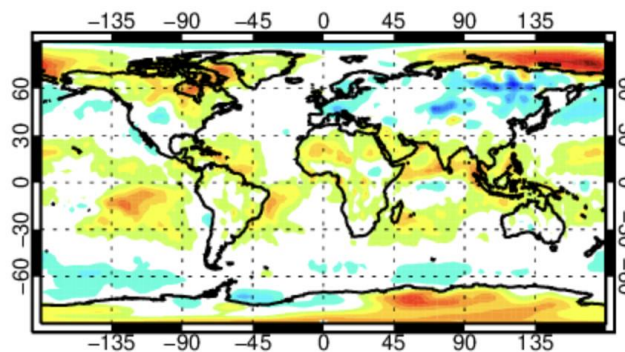
$$\Delta BT_{thresh1} < (y_{obs} - H(x_{clear})) < \Delta BT_{thresh2}$$

RT operator

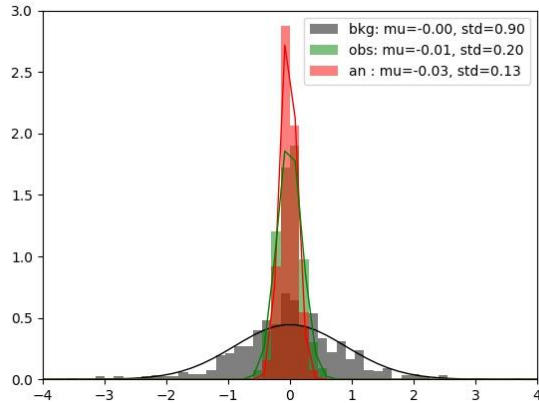
Temperature model bias at 1000hPa



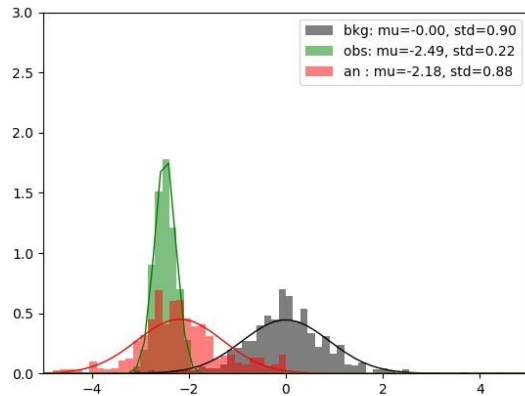
Model bias correction at 1000hPa



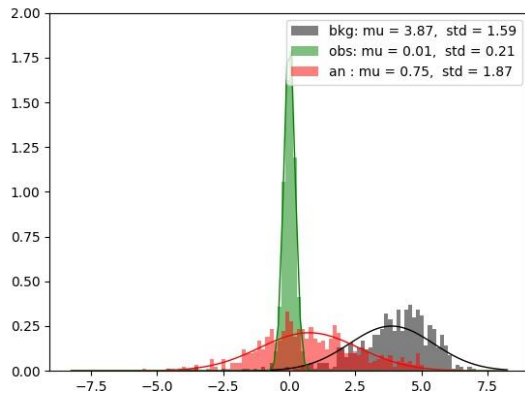
Summary 1/3



Background: unbiased (only random errors)
Observation: unbiased (only random errors)
Standard 4D-Var



Background: unbiased (only random errors)
Observation: biased
Standard 4D-Var & Variational Bias Control (VarBC)



Background: biased
Observation: unbiased (only random errors)
Weak constraint 4D-Var

Summary 2/3

How do I know if my observations are biased?

How do I know if my model is biased?

You don't know the truth, but you have to trust something

Reference observations are used



Radiosondes

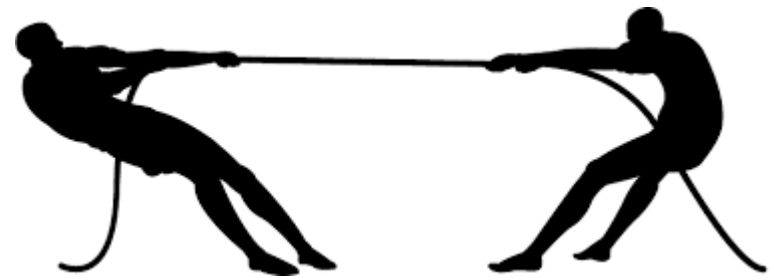


GPS-RO

Summary 3/3

From bias-blind to bias-aware data assimilation

$$\begin{aligned} J(x_0, \beta, \eta) &= \frac{1}{2} (x_0 - x_b)^T \mathbf{B}^{-1} (x_0 - x_b) \\ &+ \frac{1}{2} \sum_{k=0}^{\text{Radiosonde}} [y_k - \mathcal{H}(x_k)]^T \mathbf{R}_k^{-1} [y_k - \mathcal{H}(x_k)] \\ &+ \frac{1}{2} \sum_{k=0}^{\text{GPSRO}} [y_k - \mathcal{H}(x_k)]^T \mathbf{R}_k^{-1} [y_k - \mathcal{H}(x_k)] \\ &+ \frac{1}{2} \sum_{k=0}^{\text{Others}} [y_k - \mathcal{H}(x_k) - b(x_k, \beta)]^T \mathbf{R}_k^{-1} [y_k - \mathcal{H}(x_k) - b(x_k, \beta)] \\ &+ \frac{1}{2} (\beta - \beta_b)^T \mathbf{B}_\beta^{-1} (\beta - \beta_b) \\ &+ \frac{1}{2} (\eta - \eta_b)^T \mathbf{Q}^{-1} (\eta - \eta_b) \end{aligned}$$



Any questions? Feel free to contact me patrick.laloyaux@ecmwf.int