Predictability and Ensemble Forecasting with Lorenz-96 systems

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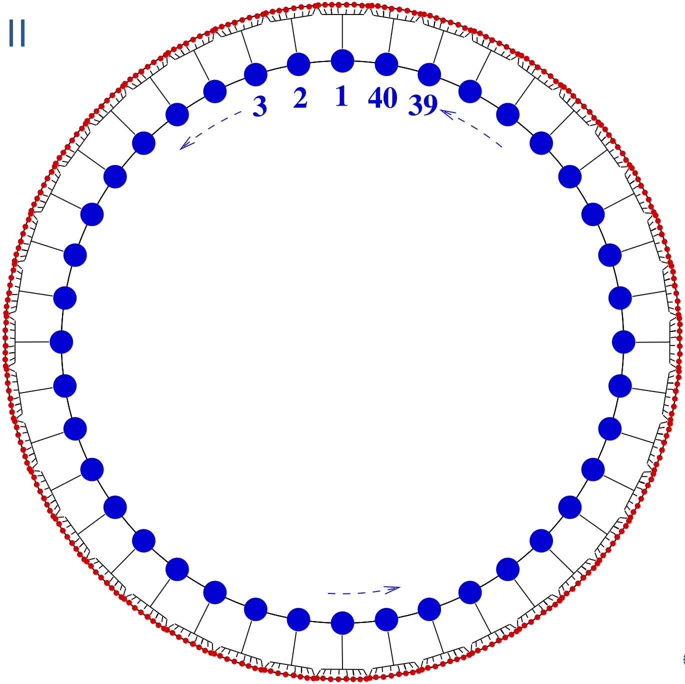
Training Course 2023

- Part 1: The L96 model, chaos, error growth, ensemble forecast experiments (with a perfect model)
- Part 2: How to experiment with L96 using scilab, proposed activities
- **Part 3:** An imperfect forecast model and the representation of model uncertainties
- Part 4: Proposed activities with an imperfect model

System II



System and model equations

The **system** comprises slow variables x_k and fast variables y_j

$$\frac{\mathrm{d}x_k}{\mathrm{d}t} = -x_{k-1}\left(x_{k-2} - x_{k+1}\right) - x_k + F - \frac{hc}{b} \sum_{j=J(k-1)+1}^{Jk} y_j \qquad (1)$$

$$\frac{\mathrm{d}y_j}{\mathrm{d}t} = -cby_{j+1}\left(y_{j+2} - y_{j-1}\right) - cy_j + \frac{c}{b}F_y + \frac{hc}{b} x_{1+\lfloor \frac{j-1}{J} \rfloor} \qquad (2)$$

with $k = 1, \ldots, K$ and $j = 1, \ldots, JK$. Here, $K = 40, J = 8$.

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with $k = 1, \ldots, K$ and $j = 1, \ldots, JK$. Here, $K = 40, J = 8$.
The **forecast model** is given by

$$\frac{\mathrm{d}x_k}{\mathrm{d}t} = -x_{k-1}\left(x_{k-2} - x_{k+1}\right) - x_k + F - g_U(x_k) + \eta_k(t). \tag{3}$$

ECMWF

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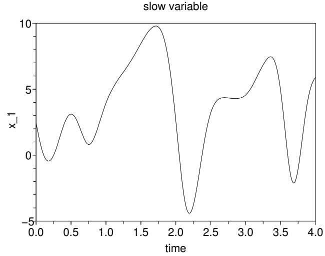
- g_U represents a **deterministic parameterization** of the net effect of the fast variables on the slow variables, see slide 11.
- η_k is a **stochastic forcing term** which represents the uncertainty due to the forcing of the fast variables, see slide 12.

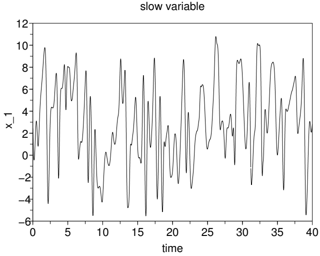
ECMWF

Time series for system II

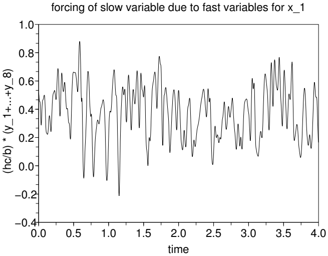
x_1



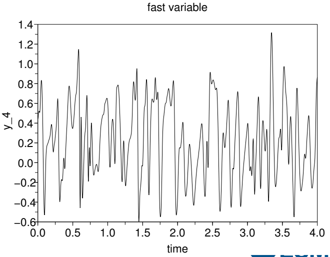
x_1



$hc/b \times \sum_1^8 y_k$



y_4



(system II, version T2)

Four versions of System II

- There are four different version of System II available in the tutorial
- The coupling between slow and fast variables differs
- We refer to them by T1, T2, T5 and T10
- It may be sufficient for you to focus on one of them, say T5

System II: Truth integrations

System constants F and h and integration time step $\mathrm{d}t$ for the five different systems

name	T0	T1	T2	T5	T10
h	0	0.1	0.2	0.5	1.0
F	8.0	8.2	8.4	9.0	10.0
$10^3\,\Delta t$	25	2.5	2.5	2.5	2.5

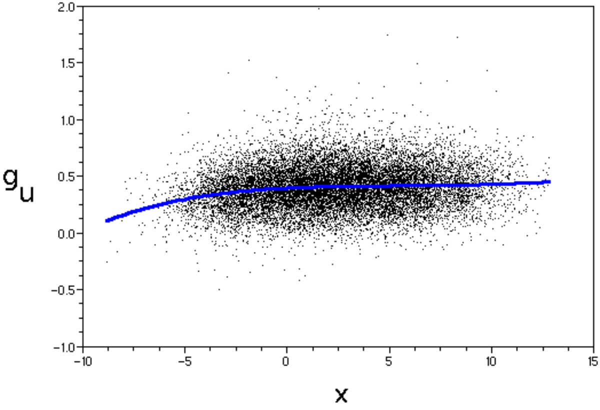
The other variables are set to

$b = 10$	**amplitude ratio** between slow variables and fast variables
$c = 10$	**time-scale ratio** between slow and fast variables
$F_y = F$	**forcing amplitude**

For all systems, the climatological mean of the slow variables is about 2.4 and their climatological standard deviation is about 3.5.

Deterministic parameterisation of unresolved scales

The unresolved scales (fast y-variables of system II) have a net effect on the resolved scales (slow x-variables). A term $g_U(x)$ is subtracted from the RHS of model I to account for the net effect of the unresolved scales.

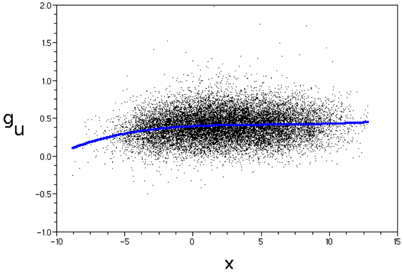


Deterministic parameterisation g_U (blue curve) of the net effect of the fast y-variables in system T2 on the slow x-variables.

Black dots represent the actual forcing due to y-variables

Representation of model uncertainty

In order to represent the scatter of the slow-variable tendencies due to the fast variables, i.e. the deviation of the black dots from the blue curve, a stochastic forcing term η_k can be activated in the ensemble forecasts.



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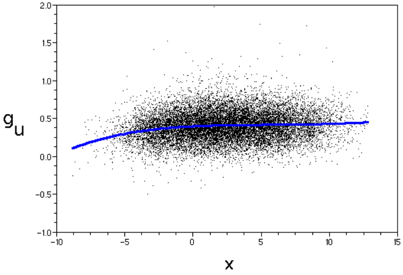

- It is represented by independent AR(1)-processes for each slow variable

$$\eta_k(t + \Delta t) = \phi \eta_k(t) + \sigma_e (1 - \phi^2)^{1/2} z_k(t), \quad (4)$$

  where the $z_k(t)$ are drawn from a Gaussian distribution $N(0, 1)$.

- the standard deviation σ_e and lag-1 autocorrelation ϕ over one integration time-step Δt can be set in menu 3 (EPS configuration: `sigma_e` and `phi`);
- see Wilks (2005) for further details.