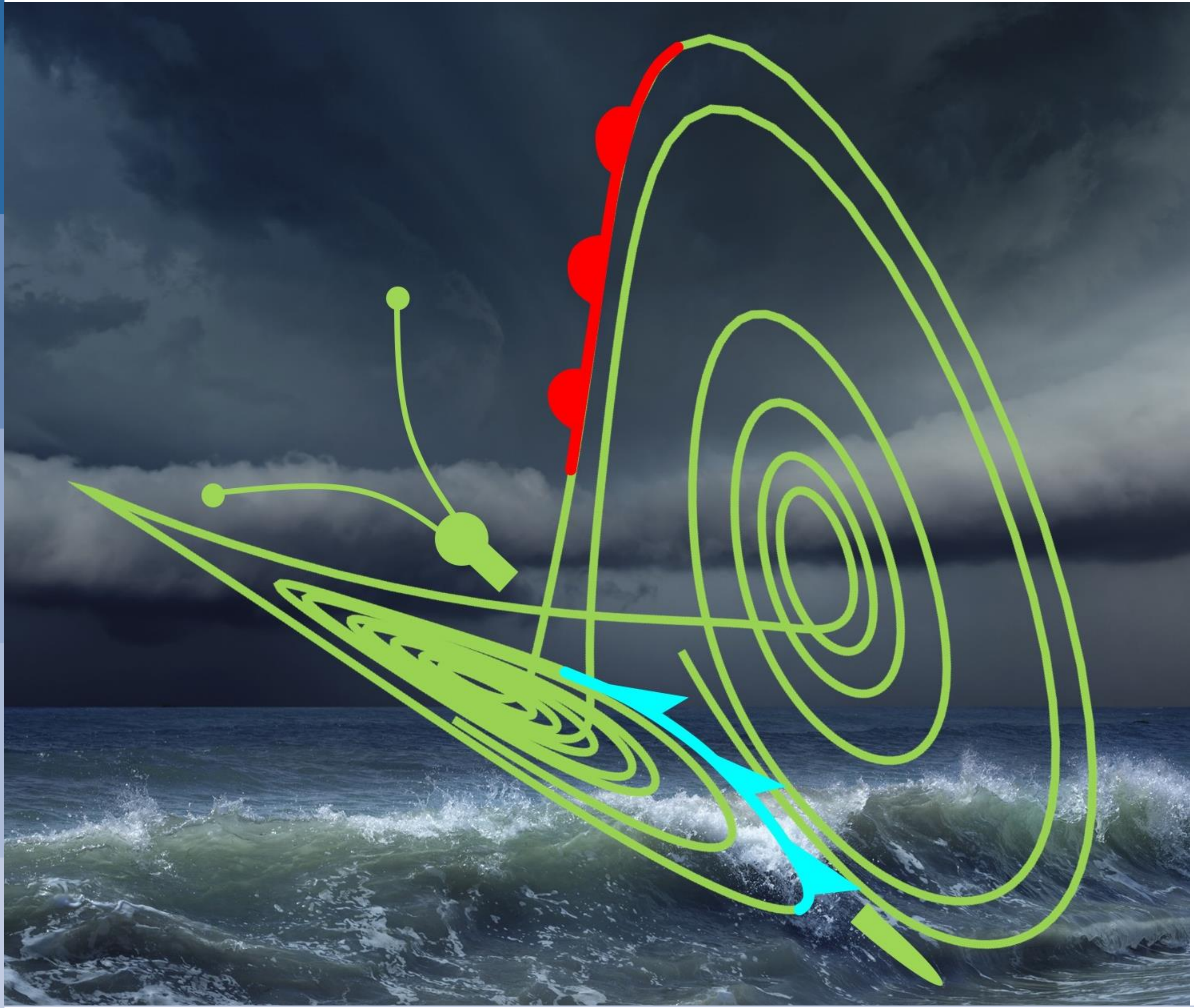

Diagnostics 1

Mark Rodwell

Training Course on Predictability

27 November 2023, ECMWF Reading

Outline

- Tropical waves, teleconnections, and the propagation of errors
- Identifying the root-causes of forecast biases and assessing models

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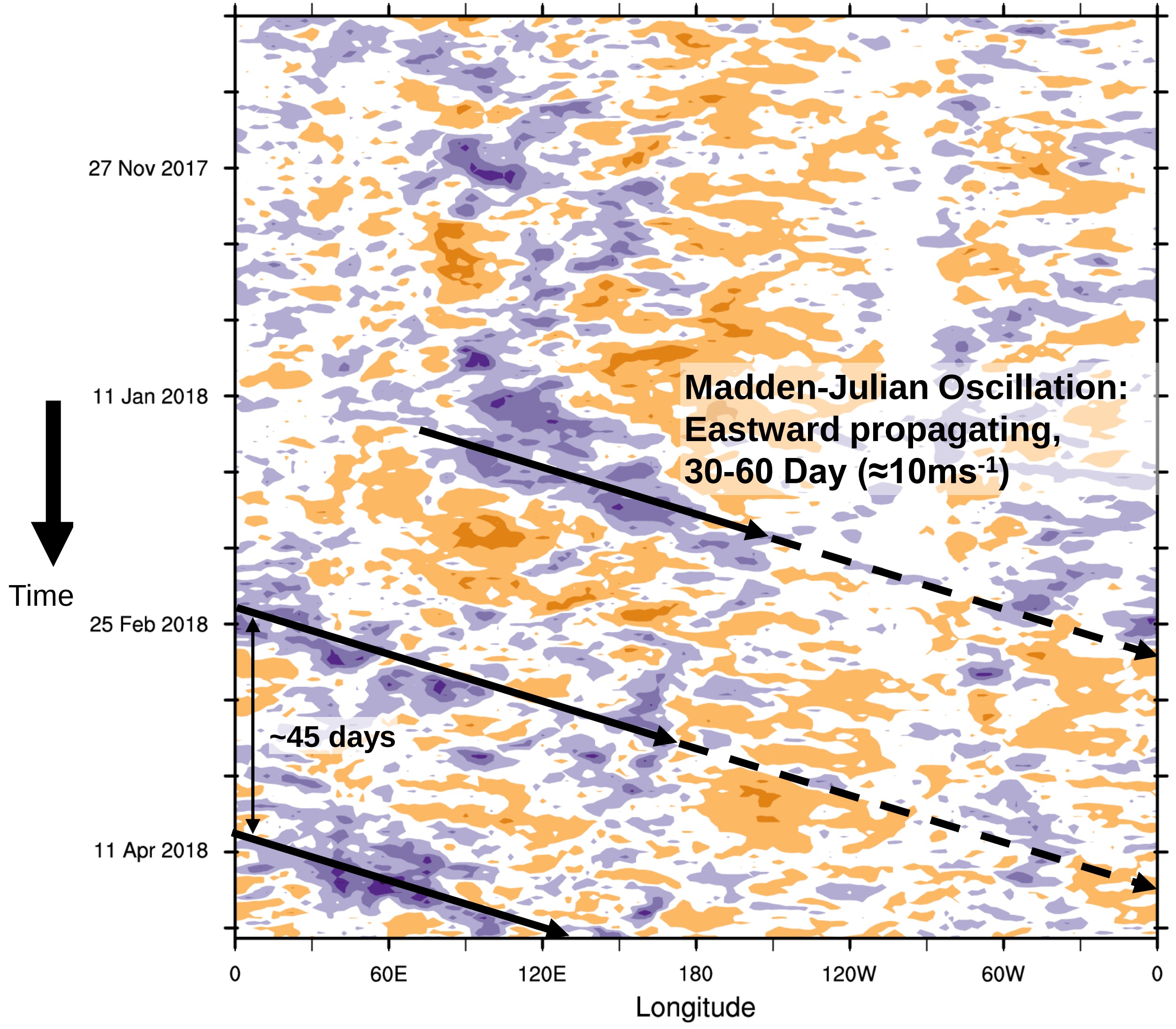
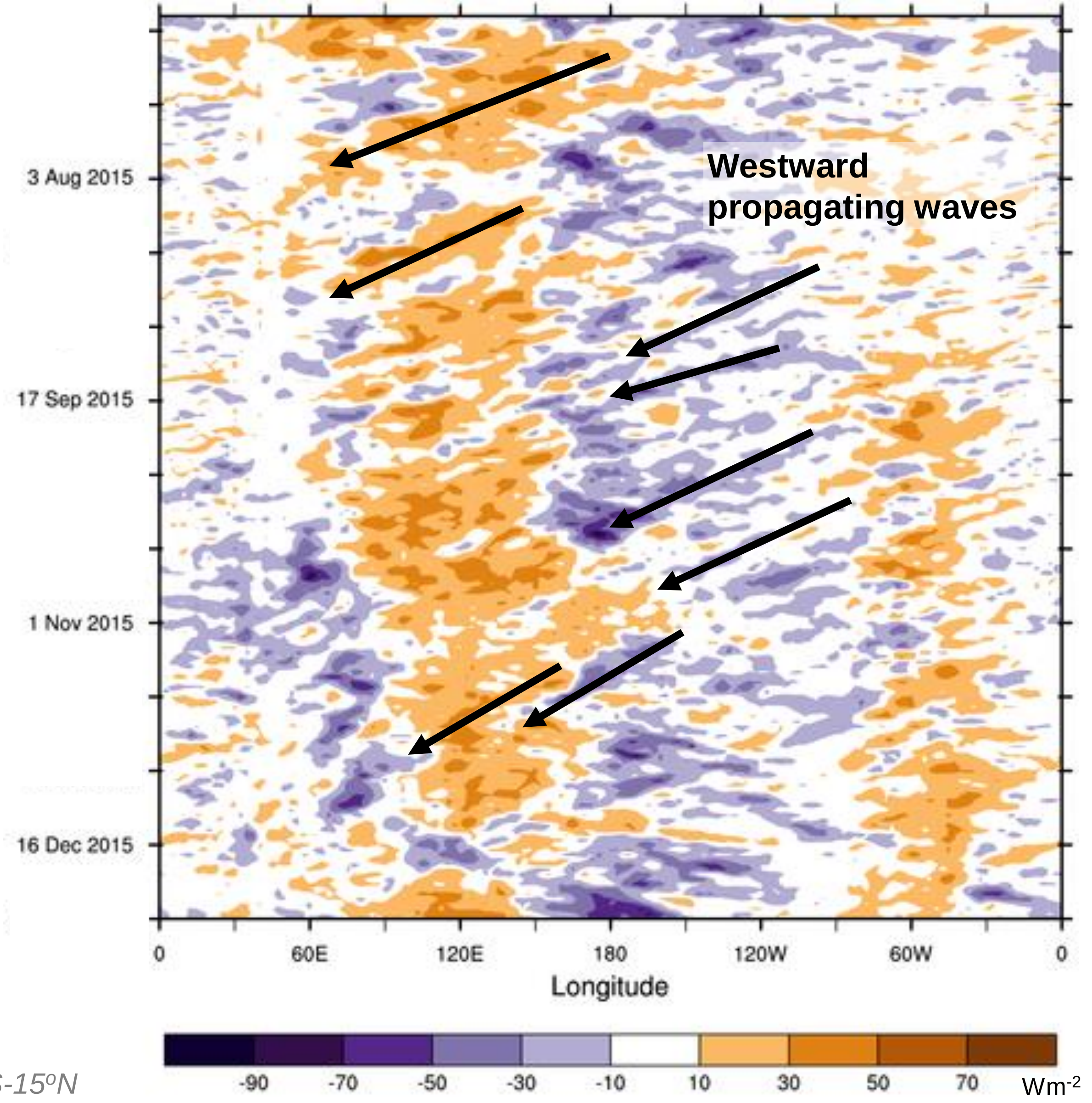
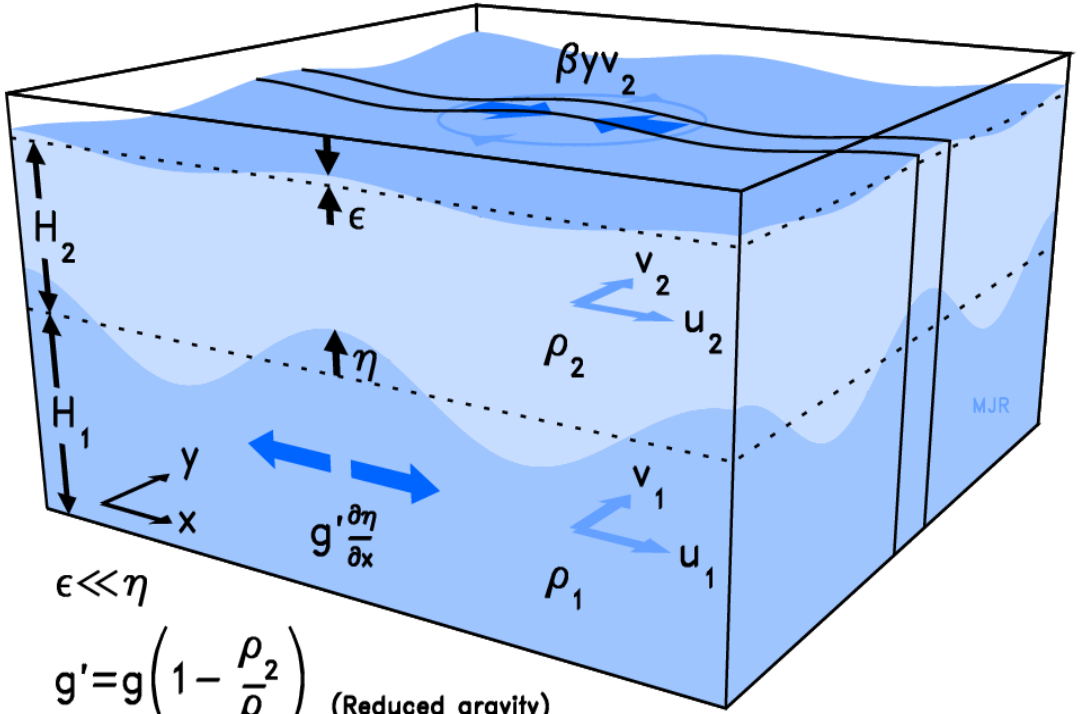
Tropical Waves (longitude-time “Hovmöller” diagram)





Plots from Bureau of Meteorology. Based on Outgoing Longwave Radiation (OLR), 15ºS-15ºN

Equatorial wave theory – the model



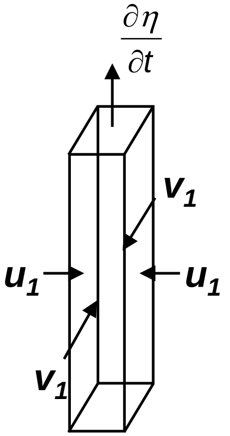
$\epsilon \ll \eta$

$g' = g\left(1 - \frac{\rho_2}{\rho_1}\right)$ (Reduced gravity)

$u \equiv u_1 - u_2 \quad v \equiv v_1 - v_2$ (Baroclinic mode)

$$c_e^2 \equiv g' \frac{H_1 H_2}{H_1 + H_2} \equiv gH_e \qquad c_e \approx 20 \text{ to } 80\ \text{ms}^{-2}$$

c_e is the propagation speed of a barotropic gravity wave in a single layer of depth H_e



Momentum:

$$\frac{\partial u}{\partial t} - \beta y v + g' \frac{\partial \eta}{\partial x} \approx 0$$

$$\frac{\partial v}{\partial t} + \beta y u + g' \frac{\partial \eta}{\partial y} \approx 0$$

(1)

Continuity:

$$\frac{\partial \eta}{\partial t} + \frac{c_e^2}{g'}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) \approx 0$$

(2)

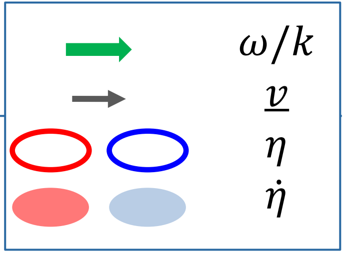
Solving for v:

$$\frac{\partial}{\partial t}\left\{\frac{\partial^2 v}{\partial t^2} + \beta^2 y^2 v - c_e^2\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)\right\} - c_e^2 \beta \frac{\partial v}{\partial x} = 0$$

(3)

Use of the shallow water equations on the β-plane (f=βy) for understanding tropical atmospheric waves. Note: No coupling with convection in this model

Equatorial wave theory – limiting solutions

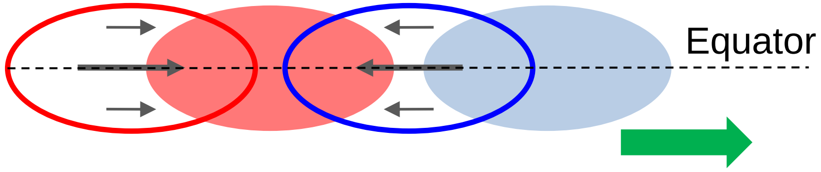


Kelvin waves: $v \equiv 0$

(K1) $\dfrac{\partial u}{\partial t} = -g'\dfrac{\partial \eta}{\partial x}$

(K2) $\beta y u = -g'\dfrac{\partial \eta}{\partial y}$ $\Rightarrow$ u, η in phase

(K3) $\dfrac{\partial \eta}{\partial t} = -\dfrac{c_e^2}{g'}\dfrac{\partial u}{\partial x}$ $\Rightarrow$ Eastward propagation →



$\dfrac{\partial^2 \eta}{\partial t^2} = c_e^2 \dfrac{\partial^2 \eta}{\partial x^2}$ (K1 & K3)

$\eta \propto \sin(kx - \omega t)\hat{\eta}(y)$

phase speed $= \dfrac{\omega}{k} = c_e$

(non−dispersive)

k = zonal wavenumber
ω = frequency
$\hat{\eta}(y)$= meridional structure

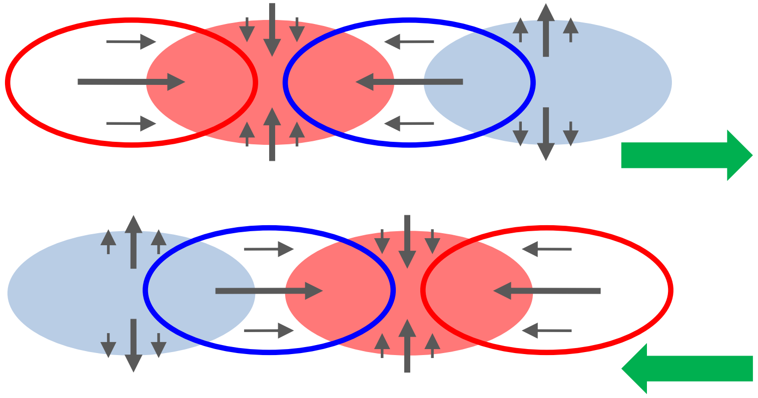
$\hat{\eta}(y) = e^{-\frac{\beta}{2c_e}y^2}$ (K2 & K3) Decays away from equator

Gravity waves: Fast; pressure gradient force dominates

(G1) $\dfrac{\partial u}{\partial t} = -g'\dfrac{\partial \eta}{\partial x}$ $\Rightarrow$ u, η in phase:→ out of phase:←

(G2) $\dfrac{\partial v}{\partial t} = -g'\dfrac{\partial \eta}{\partial y}$ $\Rightarrow$ v in quadrature with η and u

(G3) $\dfrac{\partial \eta}{\partial t} = -\dfrac{c_e^2}{g'}\left(\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y}\right)$



$\dfrac{\partial^2 \eta}{\partial t^2} = c_e^2\left(\dfrac{\partial^2 \eta}{\partial x^2} + \dfrac{\partial^2 \eta}{\partial y^2}\right)$ (G1, G2 & G3)

$\eta \propto \sin(kx - \omega t)\hat{\eta}(y)$

phase speed $= \dfrac{\omega}{k} = \pm c_e\left(1 - \dfrac{1}{k^2\hat{\eta}}\dfrac{\partial^2 \hat{\eta}}{\partial y^2}\right)^{\frac{1}{2}}$

$\hat{\eta}$ and $\partial^2\hat{\eta}/\partial y^2$ have opposite sign

(larger ⇒ faster, $\dfrac{\omega}{k} \to \pm c_e$ as $|k| \to \infty$)

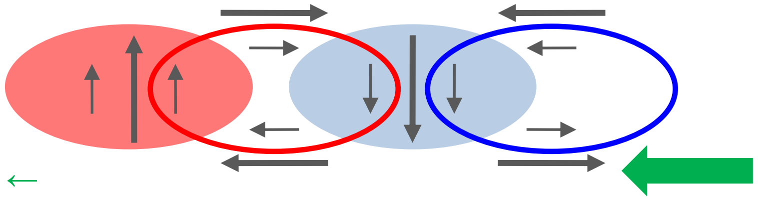
$\hat{\eta}$ multi-node ($v = 0$)

Rossby waves: Slow; Coriolis affect important, less convergence

Curl of (1):

(R1) $\dfrac{\partial \xi}{\partial t} = -\beta v - \beta y\left(\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y}\right)$
$\approx -\beta v$

Vorticity anomaly strengthened to west: ←



$\dfrac{\partial \nabla^2 \psi}{\partial t} = -\beta \dfrac{\partial \psi}{\partial x}$

$\psi \propto \sin(kx - \omega t)\hat{\psi}(y)$

phase speed $= \dfrac{\omega}{k} = -\beta\left(k^2 - \dfrac{1}{\hat{\psi}}\dfrac{\partial^2 \hat{\psi}}{\partial y^2}\right)^{-1}$

(larger ⇒ faster)

ψ = streamfunction
$\xi = \nabla^2\psi$, $v = \dfrac{\partial \psi}{\partial x}$

$\hat{\psi}(y)$ = structure
$\hat{\psi}$ and $\partial^2\hat{\psi}/\partial y^2$ have opposite sign
Multi-node ($v = 0$)

Free Equatorial Waves

Not discussed in lecture

V=0:

$$u = u_0 e^{-y^2/2} e^{ik(x-c_e t)}$$

East propagating **Kelvin Wave**
- Non-dispersive
- In geostrophic balance

V≠0:

$$v = \hat{v}(y) e^{i(kx-\omega t)}$$

Substitute into equation for *v*

Structures

(Meridional structures are solutions to Schrodinger's simple harmonic oscillator)

$$\hat{v}(y) = \begin{bmatrix} 1 \\ 2y \\ 4y^2 - 1 \\ 8y^3 - 12y \\ \vdots \\ H_n(y) \end{bmatrix} e^{-y^2/2}$$

Hermite Polynomials: $H_n(y)$
- Each successive polynomial (n=0,1,2,…) has one more node
- Modes alternate asymmetric / symmetric about equator

Dispersion

(How phase speed is related to spatial scale)

$$\left(\frac{\omega^2}{c_e^2} - k^2 - \frac{\beta k}{\omega}\right) = (2n+1)\frac{\beta}{c_e}$$

$$(n = 0,1,2,\ldots)$$

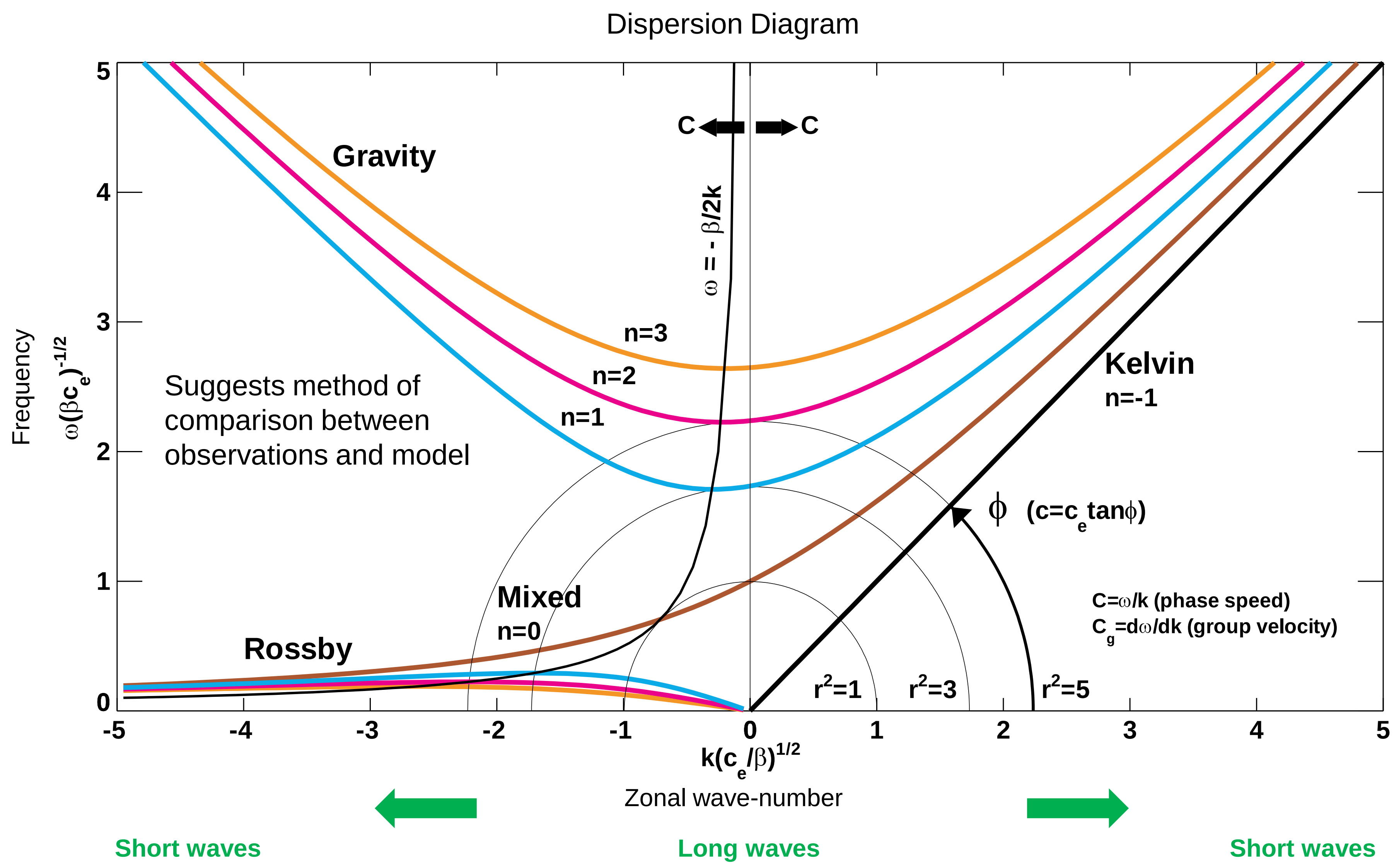
For *n ≠0: 3* values of *ω* for each *k*
- West propagating **Rossby Wave**
- E & W propagating **Gravity Wave**

For *n=0: 2* values of *ω* for each *k*
- E & W prop. **Mixed Rossby-Gravity**

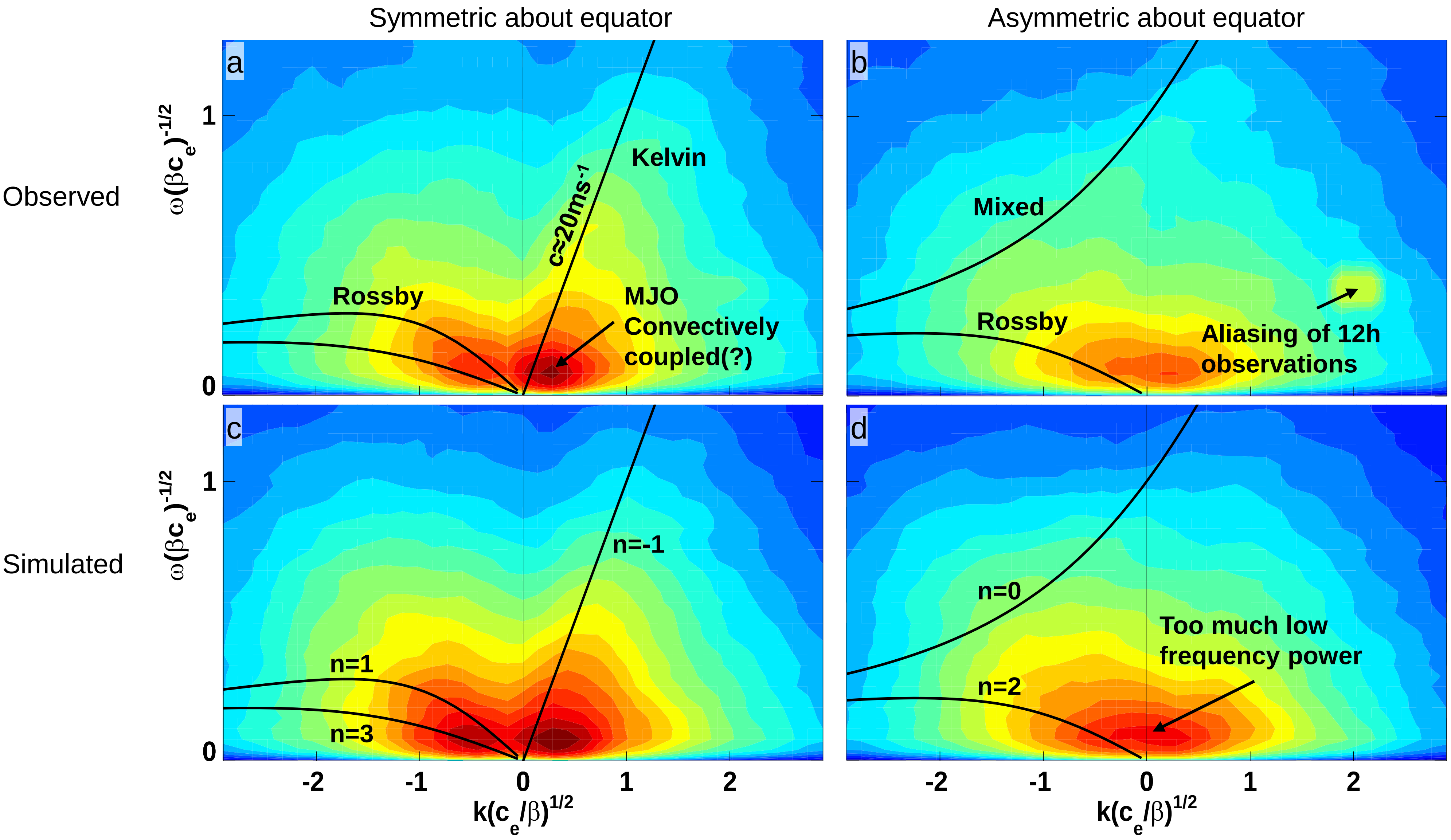
Note: y has been non-dimensionalised by the factor $(\beta/c_e)^{1/2}$

In dispersion relation, gravity waves mainly associated with first two terms on lhs, Rossby waves with last two terms on lhs, mixed Rossby-gravity waves with all three terms

Interpretation of Free Equatorial Waves



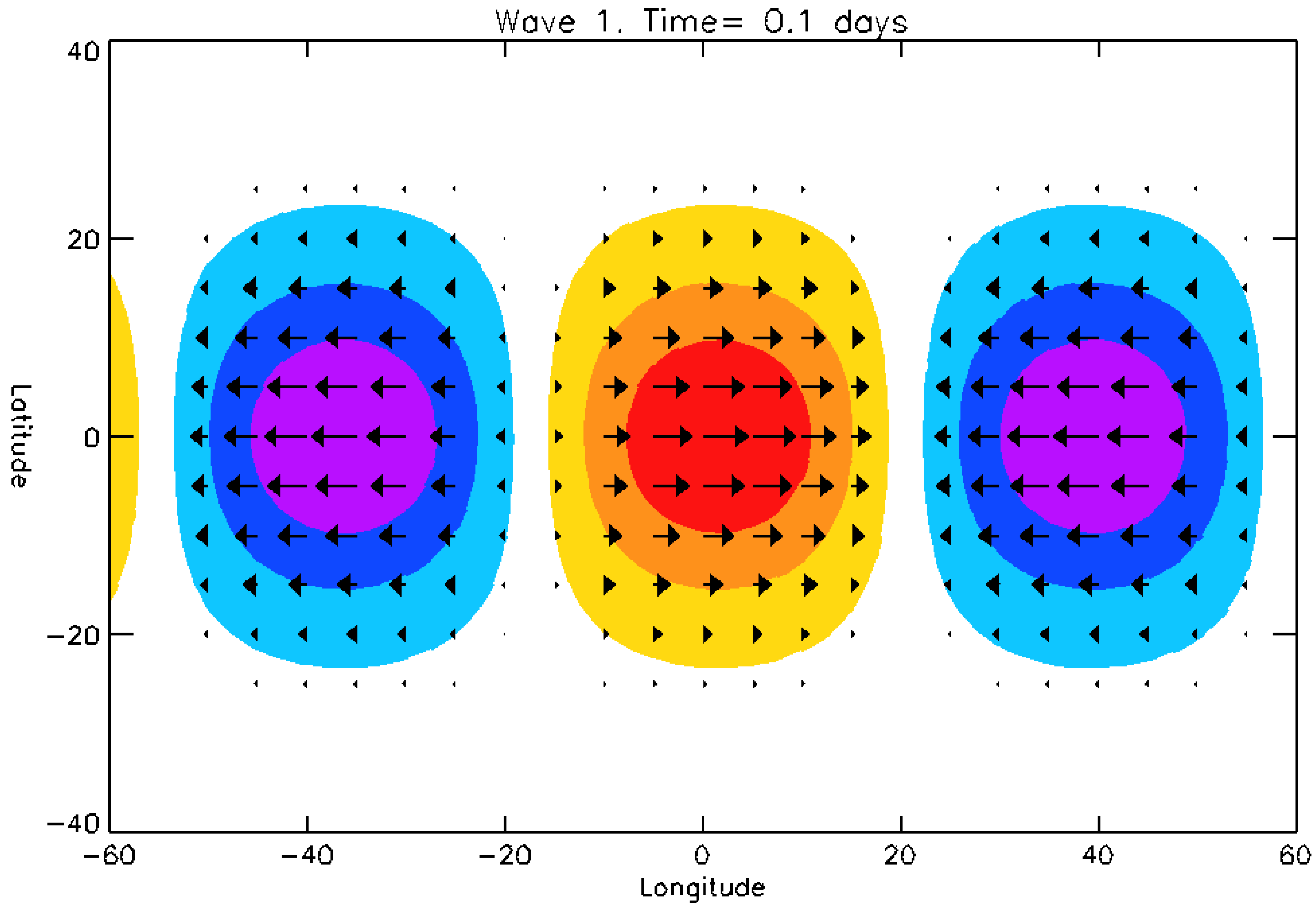
Wave power for OLR, with dispersion relation overlaid



Agreement with shallow water theory if OLR is a 'slave' to the free waves, linearity, etc.

Based on data from NOAA & IFS cycle 32R3 for DJF 1990-2005

Wave Spotting! – How information (and errors) can propagate within the Tropics

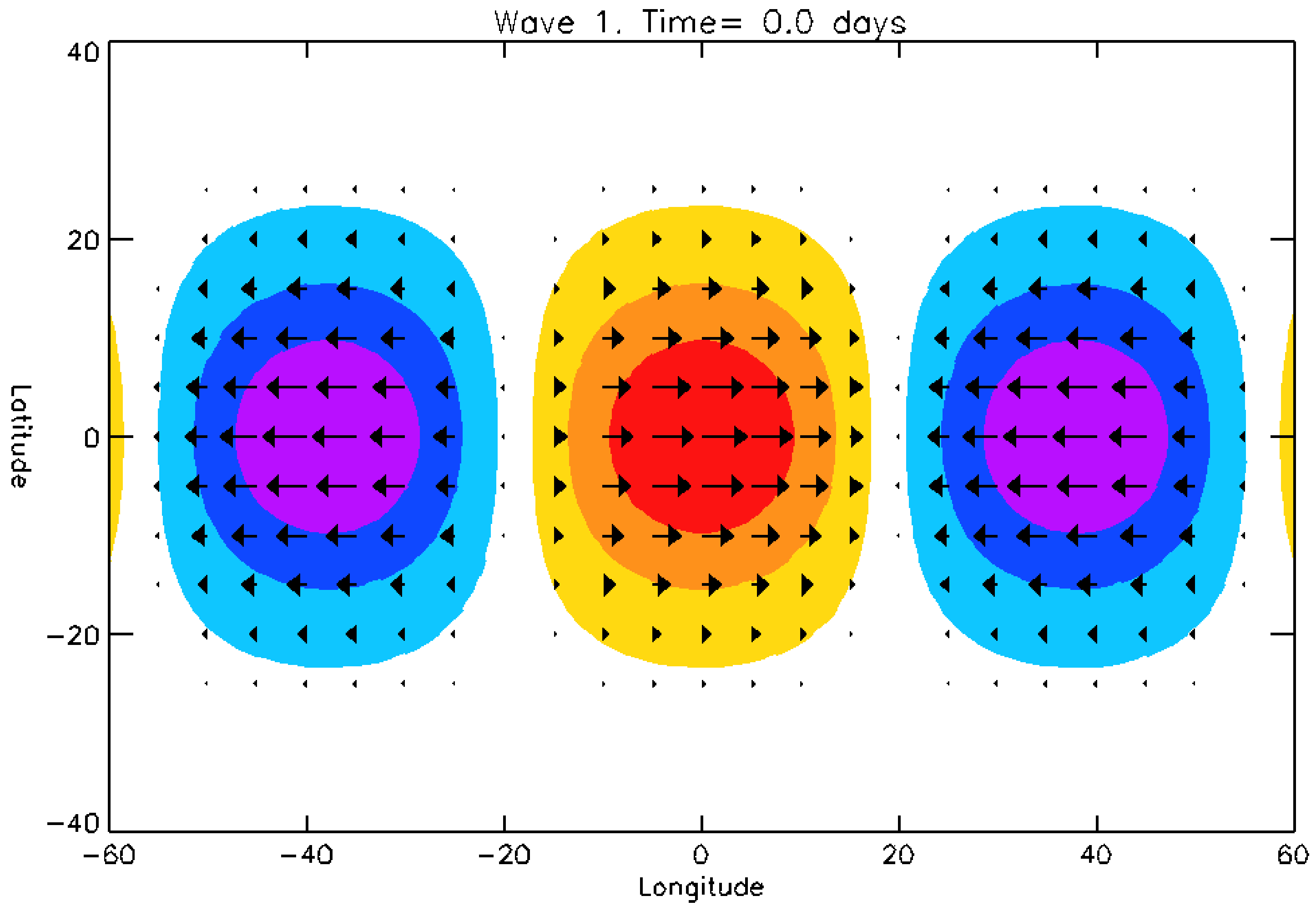


What is the wave?

1. Kelvin
2. Mixed Rossby-Gravity
3. Rossby
4. Westward Gravity
5. Eastward Gravity

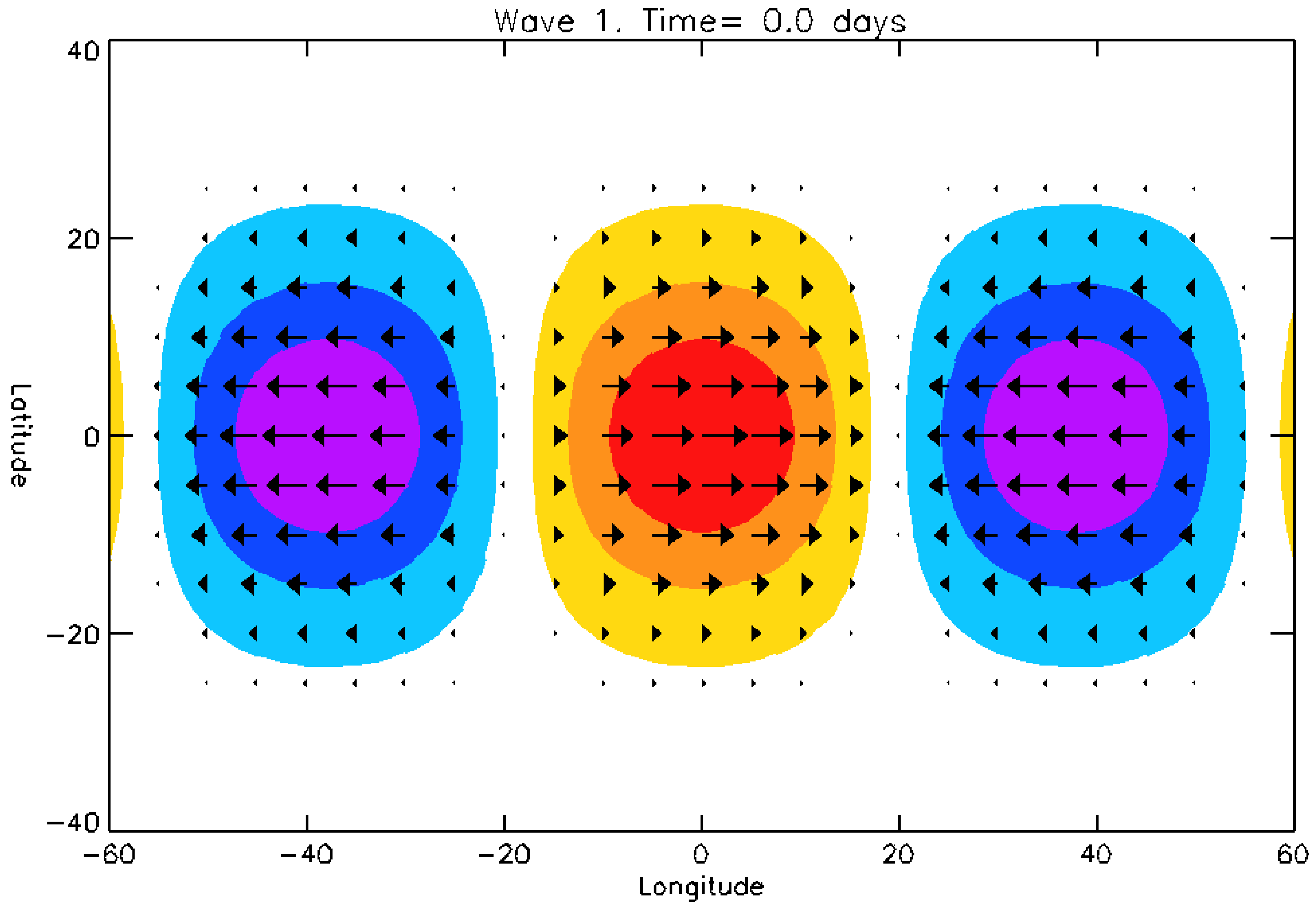
Colours show height perturbation (red positive, blue negative), arrows show lower-level winds

Wave Spotting



Colours show height perturbation (red positive, blue negative), arrows show lower-level winds

Wave Spotting Answers



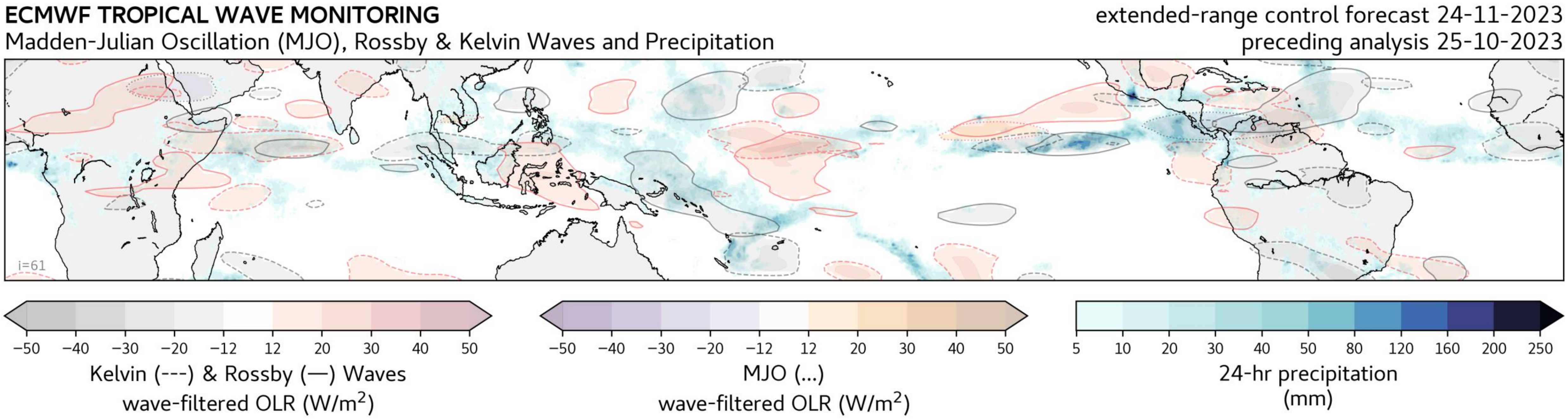
Colours show height perturbation (red positive, blue negative), arrows show lower-level winds

Wave spotting: Your Answers

Wave	Kelvin	Mixed Rossby-Gravity	Rossby	Eastward Gravity	Westward Gravity
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					
12					

Animation of tropical waves – analysed then forecast

Thanks: Rebecca Emerton



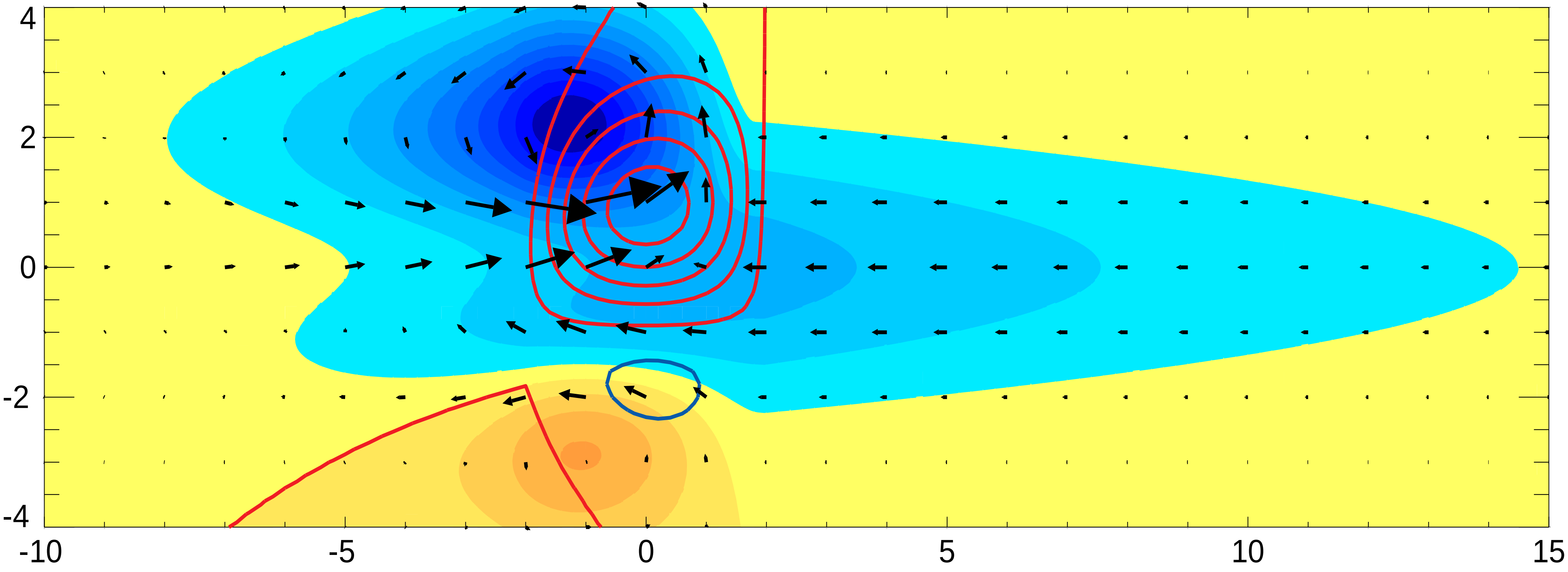
Tropical waves identified using wavenumber-frequency power of OLR, as in the previous dispersion diagram

Gill's steady solution to monsoon heating

Following Gill (1980). See also Matsuno (1966)

Damping/heating terms take the place of the time derivatives. Explicitly solve for the x-dependence

Obtain a Kelvin wave solution to the east (zero meridional flow) and a Rossby wave response to the west (super-position of two Rossby waves: one symmetric and one anti-symmetric about the equator)

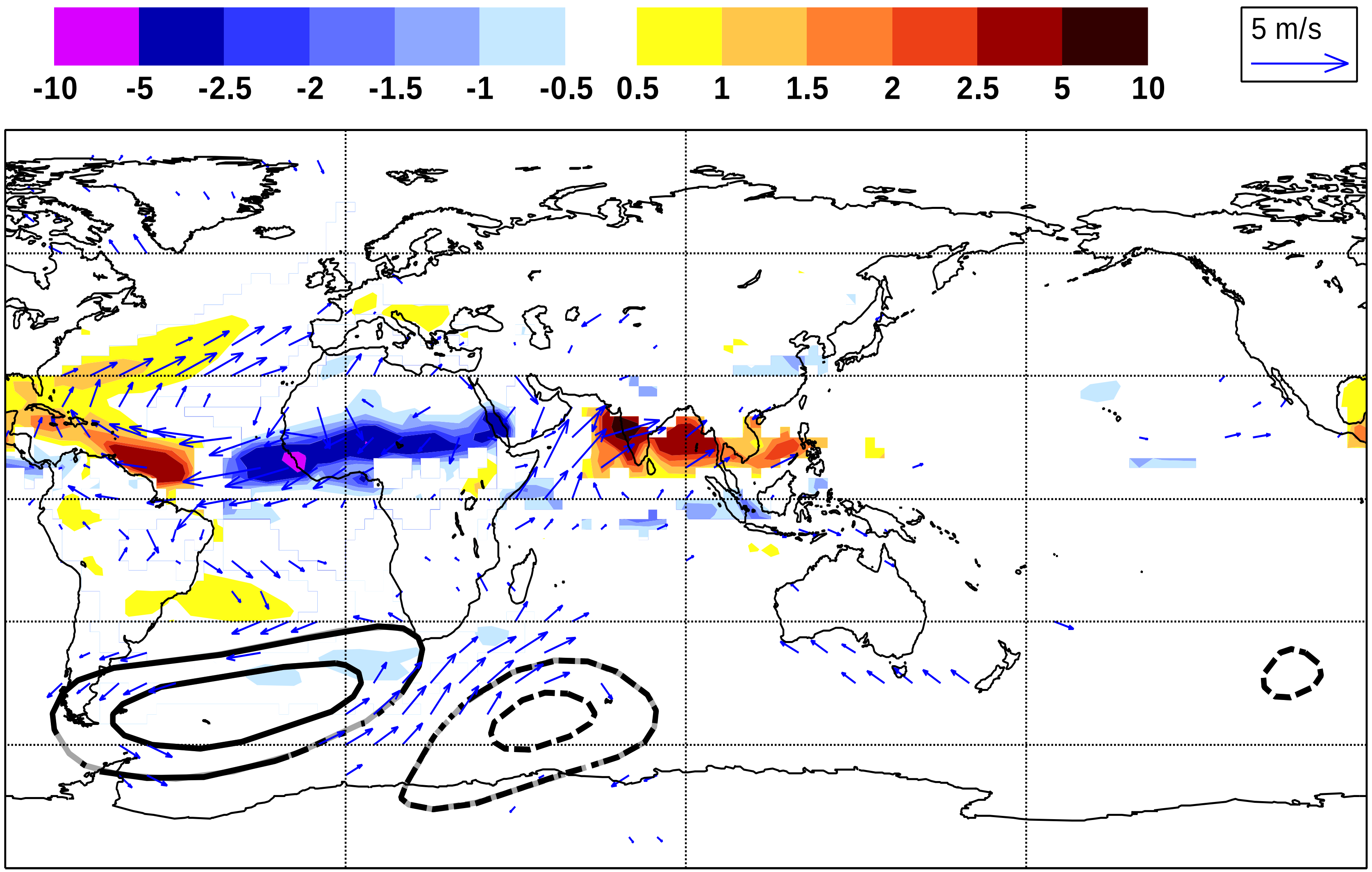


Colours show perturbation pressure, vectors show velocity field for lower level, contours show vertical motion (blue = -0.1, red = 0.0,0.3,0.6,…)

Model climate response to a change in aerosol climatology

Change in JJA precipitation (shaded), v925 (vectors) and Z500 (contours)

mm day^{-1}. 10% Sig.



Gill's steady solution helps explain the model's climate response in the tropics and subtropics, but could we have identified aerosol as a problem *a priori*?

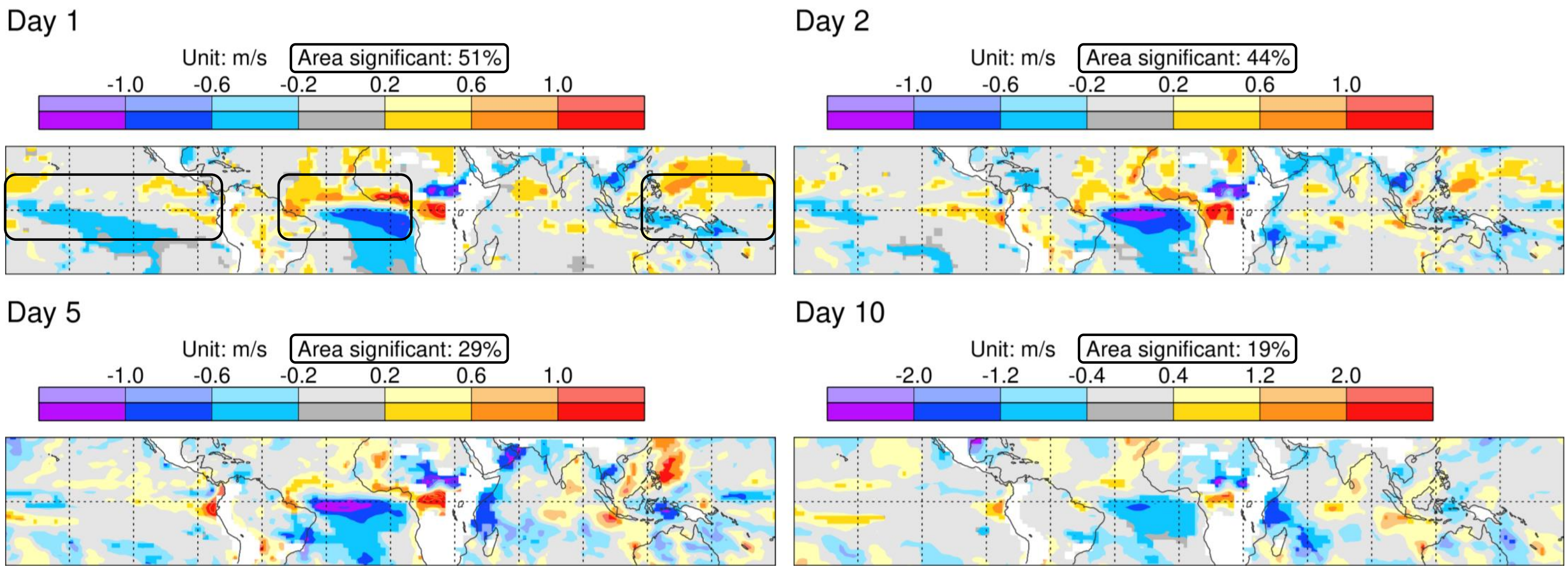
The extratropical response will be discussed later

Outline

- Tropical waves, teleconnections, and the propagation of errors
- Identifying the root-causes of forecast errors and assessing models

v925 mean forecast error (forecast minus analysis) for different lead-times

In general, at short lead-times, mean errors are more coherent, statistically significant and linked to model deficiencies. Here they indicate a lack of convergence into the Hadley Circulation. Note this is only ~0.6ms^{-1}. Can we be sure it is a model error and not an analysis error?

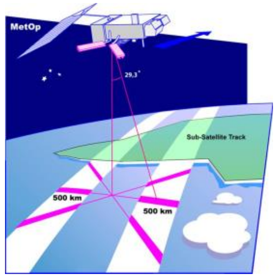


December 2022 – February 2023. Saturated colours highlight mean errors which are statistically significant at the 5% level (~5% of points would pass the test by chance)

At longer lead-times, errors are more associated with lack of predictability (of equatorial waves, etc.)

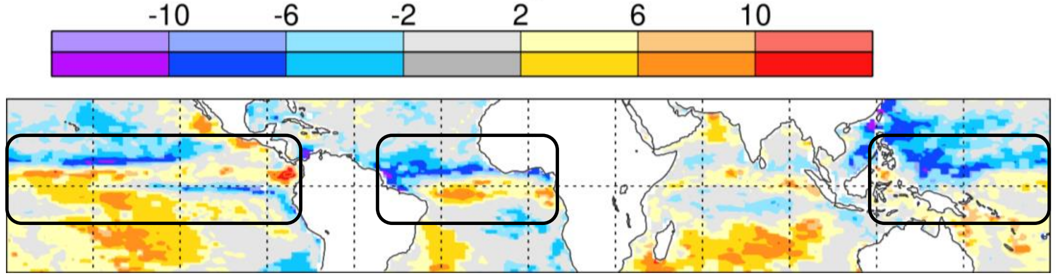
Using data assimilation to better determine the root cause of the problem

Here the Scatterometer wind ‘observations’ show a consistent story with weaker convergence in the model (background). The analysis increments correct this mean departure



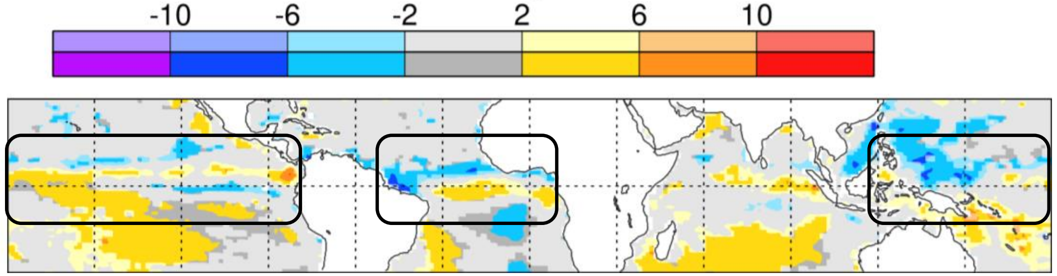
Observation – Background (mean “departure”)

Unit: $0.1ms^{-1}$ Area significant: 47%



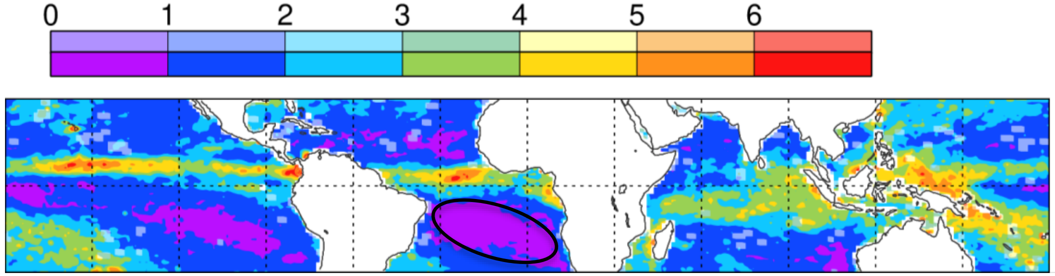
Analysis – Background (mean “increment”)

Unit: $0.1ms^{-1}$ Area significant: 49%



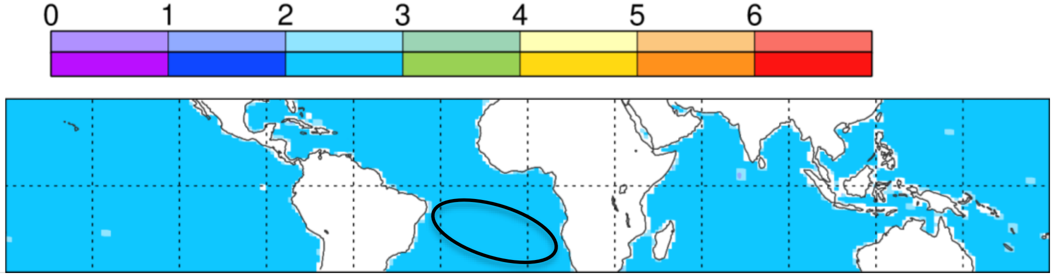
Mean-squared departure of EDA-mean

Unit: $(ms^{-1})^2$



Modelled observation error variance

Unit: $(ms^{-1})^2$



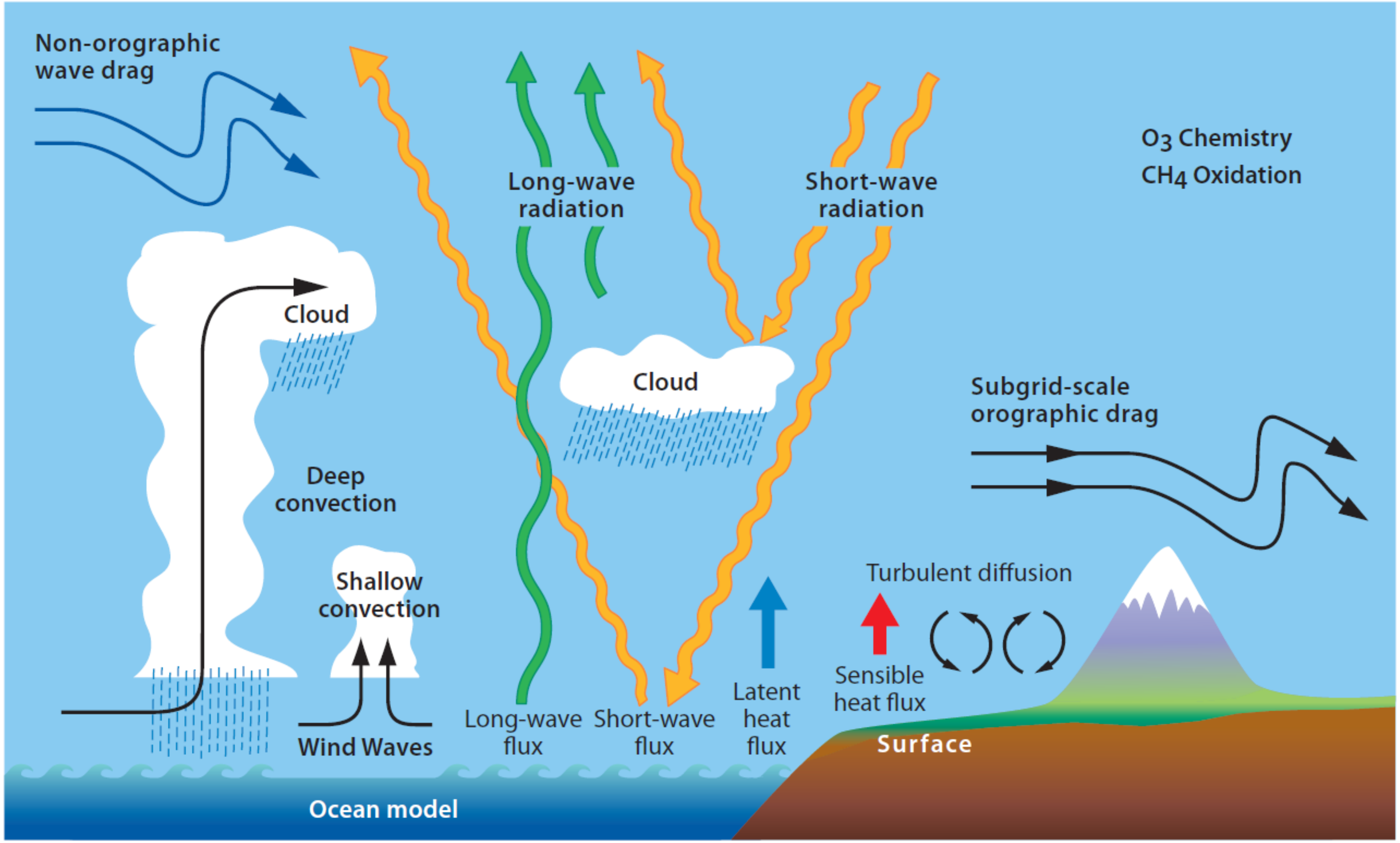
December 2022 – February 2023. Saturated colours highlight mean errors which are statistically significant at the 5% level (~5% of points would pass the test by chance)

The Ensemble of Data Assimilations (EDA) suggests we over-estimate the scatterometer errors (their variance is bigger than the squared departures in some regions) hence the mean analysis increments could be even stronger. This is evidence that the problem is with the model

The complexity of present-day model physics

Figure from Peter Bechtold

Ideally, we wish to identify deficiencies at the process level. Again, this should be easier at short timescales since interactions between physical processes and the resolved flow (including teleconnections) are minimised



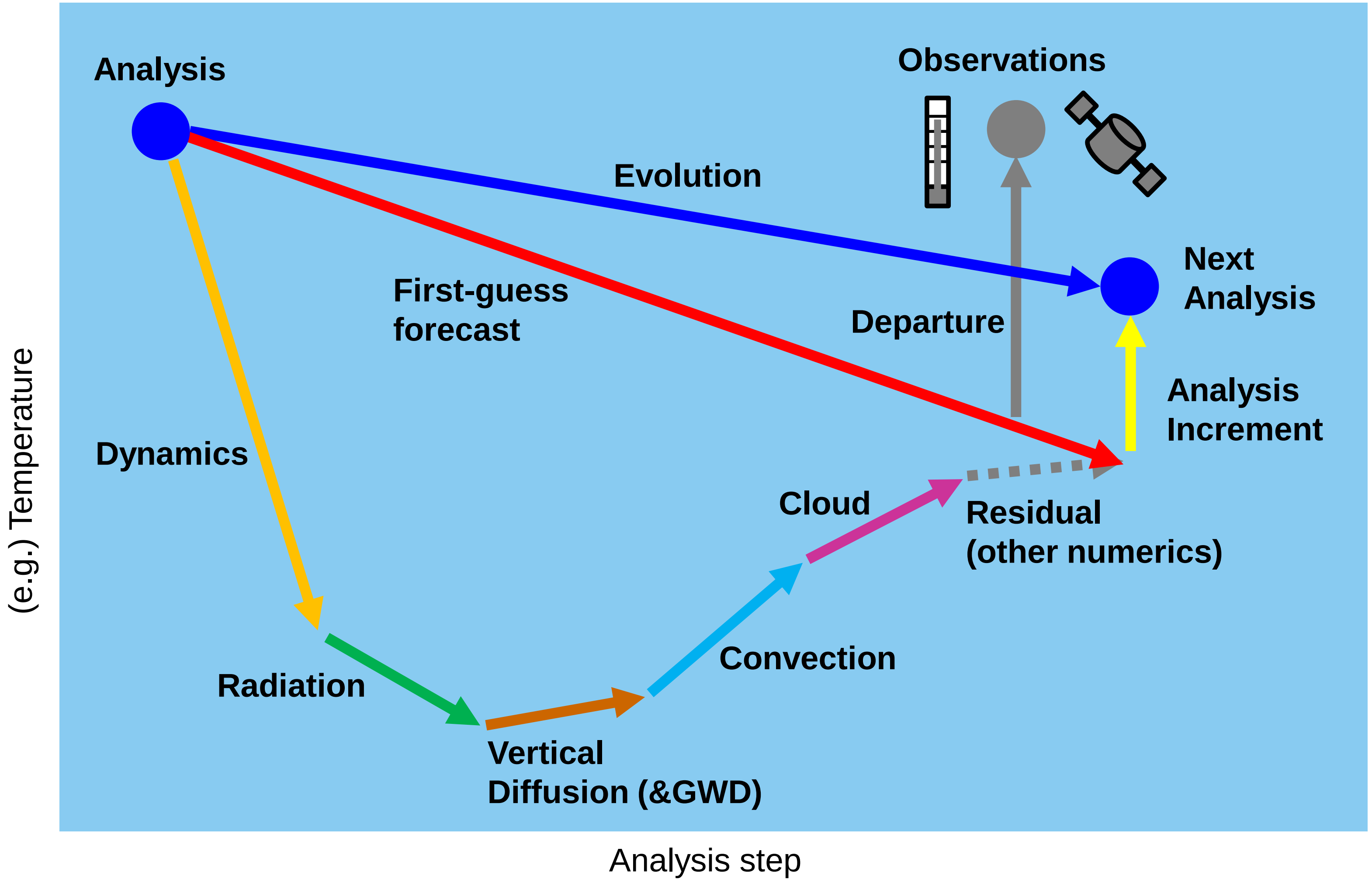
The Initial Tendency approach to diagnosing model error

Analysis increment corrects first-guess error, and draws next analysis closer to observations.

First-guess = sum of all processes

Relationship between increment and individual process tendencies can help identify key errors.

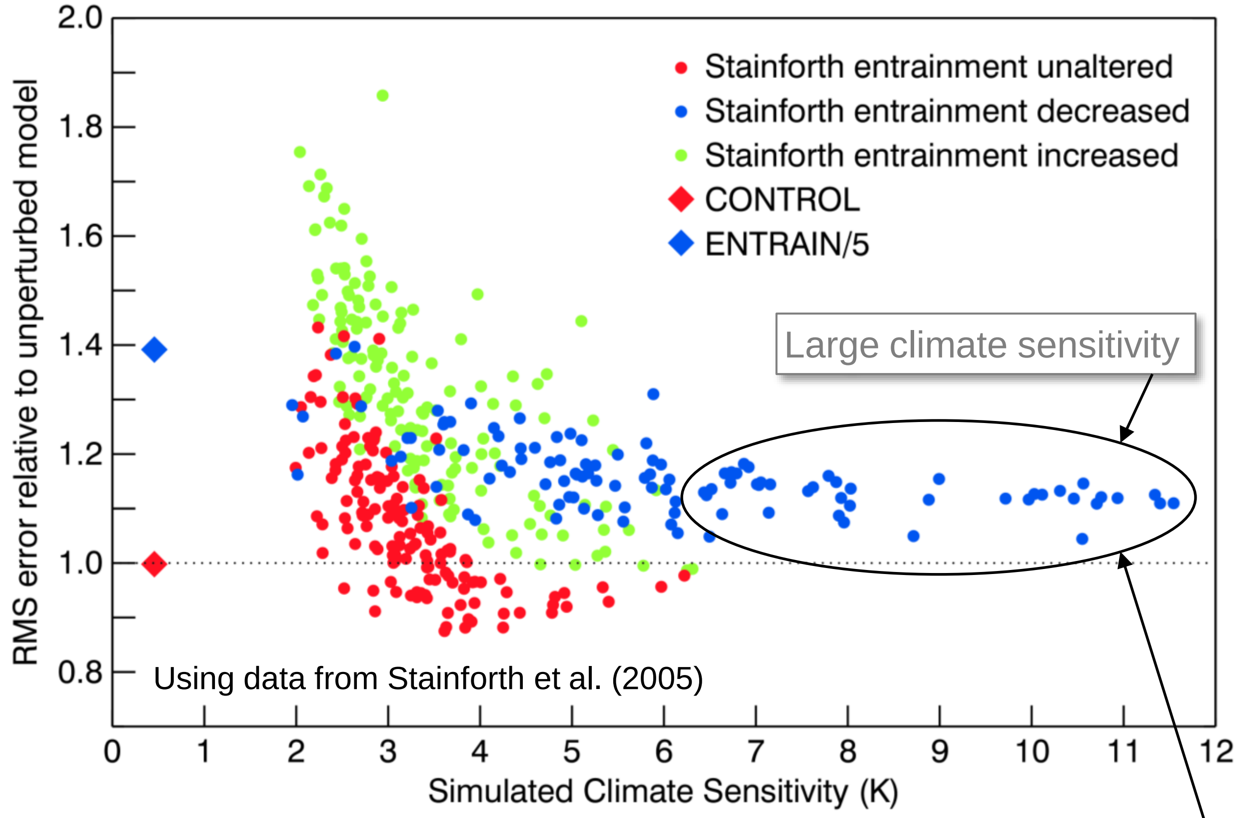
Schematic of the data assimilation process – a diagnostic perspective



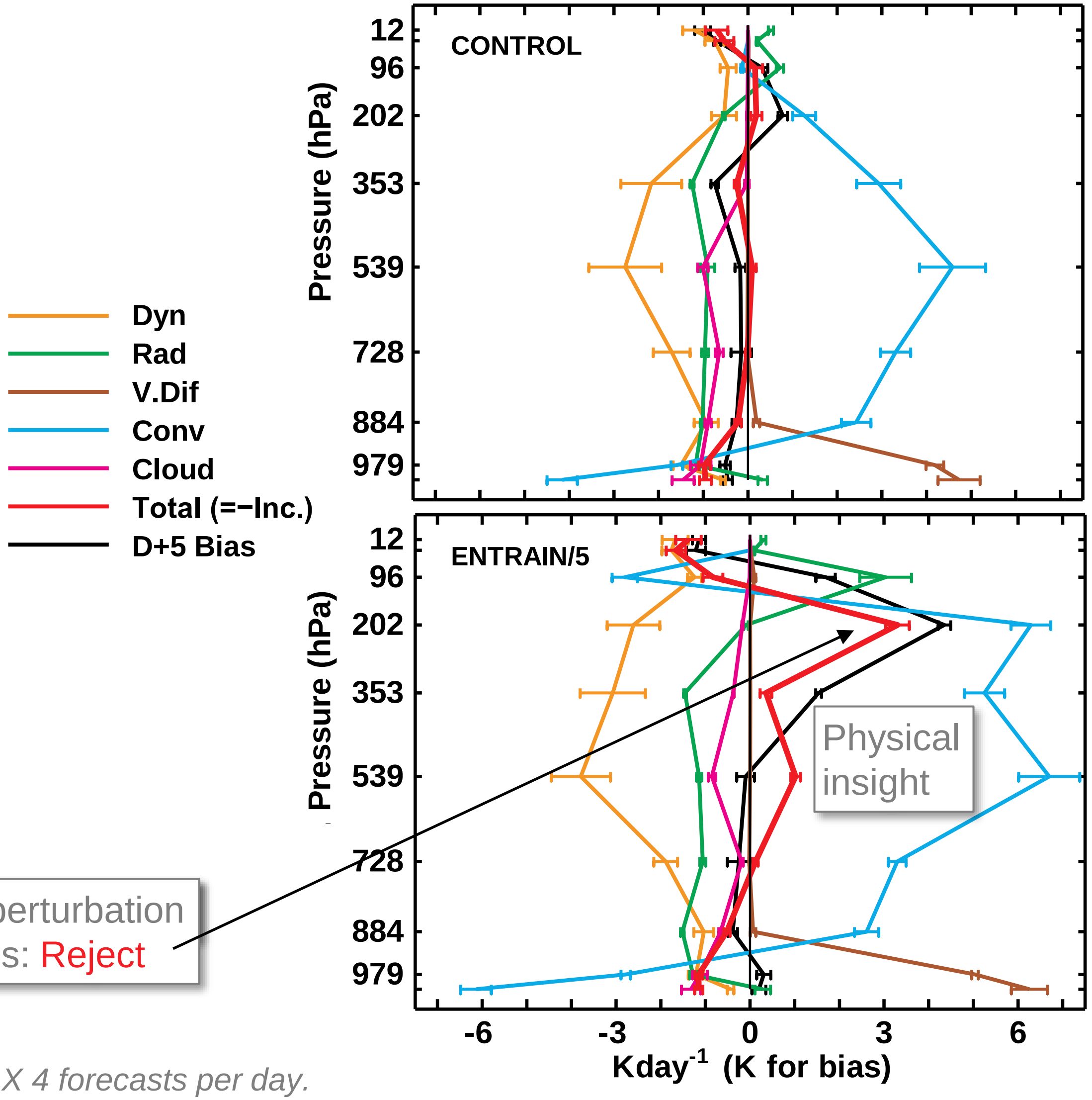
"Initial Tendency" approach discussed by Klinker & Sardeshmukh (1992). Refined by Rodwell & Palmer (2007)

Data assimilation as a means of constraining climate sensitivity

Present-day climate error vs climate sensitivity for perturbed parameter ensemble (HadSM3)



Amazonia mean T initial tendencies (IFS 29r1)



The initial tendency approach was used to investigate the Saharan aerosol result shown earlier

Small present-day climate error: Accept perturbation
Inconsistent with assimilated observations: Reject

Rodwell and Palmer (2007). Amazonia (300-320ºE, 20ºS-0ºN) 6hr tendencies 31d (Jan 2005) X 4 forecasts per day. 70% conf.int. T159, L60,30min. See also Sexton et al. (2019), Klocke and Rodwell (2014)

Summary

- Tropical waves, teleconnections, and the propagation of errors
  - Important for predictability
  - Can complicate the diagnosis of forecast system deficiencies
- Identifying the root-causes of forecast errors and assessing models
  - Diagnosis at short leadtimes (associated with data assimilation) can localise errors (geographically, process-wise, model versus observation) before errors and uncertainties have had time to propagate and interact
  - Don’t need mean error patterns to agree at short-range and long ranges (although sometimes bias patterns do simply grow in magnitude)
- Next lecture: Ensemble aspects, Uncertainty growth, “Classifying and modelling butterflies”