

# Machine Learning for Model Error Estimation and Prediction

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## Data-driven model error correction



- The physical model will always be affected by systematic, predictable errors due to a variety of causes (unresolved, unrepresented physics, discretisation errors, wrong model forcings, etc.)
- The NN is used as a <u>data-driven (statistical) online correction</u> to the time tendencies forecasted by the physical model:

$$\frac{d\mathbf{x}}{dt} = \Phi_{adv}(\mathbf{x}(t)) + \Phi_{phys}(\mathbf{x}(t)) + \Phi_{ML}(\mathbf{x}(t))$$

• Is model error a big deal?

#### Model Error: Atmosphere

Time series of the difference between observations and temperature first-guess trajectory from the ECMWF operational analysis cycle for radiosonde (top) and Radio Occultations (bottom)





Laloyaux et al., 2020

#### Model Error: Atmosphere

- Traditionally, DA algorithms have used a perfect model assumption and aimed at reducing random initial conditions errors: this approach brings steady progress up to ~2 weeks, but not beyond
- Can Machine Learning help break the 2-week Lorenz predictability barrier?



#### From Frederic Vitart and Thomas Haiden

#### Model Error: Ocean

#### NOAA drifter climatology

#### ORAS5 1985-2012.

40°N

38°N

36°N

34°N

32°N 4

78°W





#### -0.95 -0.75 -0.55 -0.35 -0.15 0.05 0.25 0.45 0.65 0.85





#### -0.95-0.75-0.55-0.35-0.15 0.05 0.25 0.45 0.65 0.85



74°W

72°W

76°W

vozocrte 1979-2014

From Hao Zuo



70°W



- The first challenge is the estimation of model error
- Using a statistical model like a NN requires choosing a ground truth to build the training dataset
- We have different options here...



- One option is to train the NN using a more complex, more accurate, higher resolution version of the model as a proxy for the truth
- The goal is to train the NN to learn the effect of the parameterised and unresolved dynamical and physical processes onto the resolved dynamics (e.g., Brenowitz and Bretherton, 2018, 2019; Rasp et al., 2018)



- In Ocean Modelling one of the main sources of errors come from insufficient resolution -> need to
  account for unresolved physical processes
- This problem has been tackled with ML tools (CNN, DNN, RVM) to build offline data-driven parameterizations of the unresolved dynamics for a fully observed system (Bolton and Zanna, 2019, Zanna and Bolton, 2020, Kutz, 2017, Ling et al., 2016)



- Another option is to train the NN using <u>observations</u> of the system as a proxy for the truth
- The training dataset is built using observations minus (short range) forecasts (O-B departures)
- The NN is trained to learn the O-B departures from a set of available predictors (state, location, time of day, season, etc.)

ECMWF Observation Monitoring



- The idea of using observations as the truth is attractive because it directly introduces independent information on the model errors we want to correct
- It also comes with its own caveats: observation errors, observation coverage, how to extrapolate the corrections in space, time and to other variables in a physically consistent manner?

O-B departures on 01-05-2019 averaged between 10S and 20N (left) and their prediction with a Convolutional Neural Network (right) NN validation NN output 2.0 20 2.0 20 25 25 1.5 1.5 30 30 1.0 1.0 35 35 0.5 0.5 Iaval labom Model level 45 0.0 0.0 -0.5 -0.5 50 50 -1.0-1.055 55 -1.5-1.560 60 65 -2.0 -2.0 65 150 -150 -100 -50 50 100 150 -150 -100 -50 50 100 0 n Longitude Longitude

**C**ECMWF

From P. Laloyaux, 2021

- Option #3 is to train the NN using <u>analysed</u> states of the system as a proxy for the truth
- The training dataset is built using analysis minus (short range) forecasts (<u>A-B increments</u>)
- The NN is trained to learn the A-B increments from a set of available predictors (state, location, time of day, season, local solar zenith angle, SST, etc)
- Advantages:
  - 1. The model error estimates are directly available in model space and globally;
  - 2. Analyses are more accurate than any individual observation type
- Disadvantages:
  - 1. The analyses will be affected to some extent by the model error we want to estimate
  - 2. The (A-B) increments will also be affected by "errors of the day" (i.e., initial condition errors)

# Model error estimation: weak constraint 4D-Var

We already do model error estimation inside the 4D-Var analysis cycle: it is called <u>weak constraint 4D-Var</u>!

$$J_{wc4DVar}(\boldsymbol{x}_{0},\boldsymbol{\eta}) = J_{B} + J_{O} + J_{Q} = \frac{1}{2} (x_{0} - x_{0}^{b})^{T} B^{-1} (x_{0} - x_{0}^{b})$$
$$+ \frac{1}{2} \sum_{i=0}^{N} (H_{i}(x_{i}) - y_{i})^{T} R_{i}^{-1} (H_{i}(x_{i}) - y_{i})$$
$$+ \frac{1}{2} \sum_{i=1}^{N} (x_{i} - M_{i}(x_{i-1},\boldsymbol{\eta}))^{T} Q_{i}^{-1} (x_{i} - M_{i}(x_{i-1},\boldsymbol{\eta}))$$

 $x_0 = state \ estimate$  $\eta = model \ error \ estimate$ 

Loss functions common in ML/DL can be obtained as particularisations of wc-4DVar (Brajard et al., 2020; Bocquet et al., 2020; Farchi et al., 2020):

- 1. ML/DL models are not used for state estimation  $\rightarrow J_B = 0$
- 2. ML/DL loss functions typically assume full, noiseless observations  $(H_i = I, R_i \rightarrow 0) \rightarrow J_0 = 0$
- 3. ML/DL models can optionally have a regularization term function of the NN model parameters  $L(\eta) = L(W, b)$ , e.g. Tikhonov regularisation, drop-out, etc.

## Model error estimation: weak constraint 4D-Var

- wc-4DVar progressively learns a model error tendency correction and applies it to subsequent background forecasts in the DA cycle
- wc-4DVar is an <u>online machine</u> <u>learning algorithm</u> for model error estimation and correction!



#### Model error estimation: weak constraint 4D-Var

- What can Machine Learning bring to table?
- wc-4DVar produces an <u>estimate</u> of model error valid over the length of the assimilation window:  $J_{wc4DVar}(\mathbf{x}_0, \boldsymbol{\eta})$
- The NN will produce a model of model error, which can be applied and used at any point in time:

 $M_w(predictors) = M_w(x, time, lat, lon, SST, ...)$ 



# Hybrid ML-DA

- Option #3 is to train the NN using <u>analysed</u> states of the system as a proxy for the truth
- In this approach we have a number of possibilities on how to fuse the Data Assimilation system with the ML estimate of model error:
  - 1. Offline training of NN and use it in a data assimilation cycle (e.g, Bonavita and Laloyaux, 2020, Watson, 2019);
  - 2. Online training of NN in a data assimilation cycle with a coordinate descent approach (e.g. Brajard et al., 2020, Bocquet et al., 2020),
  - 3. Full online training of NN inside data assimilation system (e.g., Farchi et al., 2021b)



In all generality we would like to minimise the following cost function of the state (x<sub>0:N</sub>) and the NN parameters (p) (Farchi et al., 2021):

$$\mathcal{J}(\mathbf{p}, \mathbf{x}_{0:N_{t}}) \triangleq \frac{1}{2} \|\mathbf{x}_{0} - \mathbf{x}_{0}^{\mathsf{b}}\|_{\mathbf{B}^{-1}}^{2} + \frac{1}{2} \sum_{k=0}^{N_{t}} \|\mathbf{y}_{k} - \mathcal{H}_{k}(\mathbf{x}_{k})\|_{\mathbf{R}_{k}^{-1}}^{2} + \frac{1}{2} \sum_{k=0}^{N_{t}-1} \|\mathbf{x}_{k+1} - \mathcal{M}_{k}(\mathbf{p}, \mathbf{x}_{k})\|_{\mathbf{Q}_{k}^{-1}}^{2} + \mathcal{L}(\mathbf{p}),$$
(4)

- Here the background error (x<sub>0</sub>- x<sub>0</sub><sup>b</sup>)~N(0,B), the observations are sparse and are related to the system state by y<sub>k</sub>=H<sub>k</sub>(x<sub>k</sub>)+v<sub>k</sub>, (v<sub>k</sub>~N(0,R))
- Model errors  $(x_{k+1}-M(p,x_k))$  follow a Gaussian distribution with covariance  $Q_k$ , and are assumed uncorrelated with other error sources

- How to optimise this cost function of  $(p, x_{0:N})$ ?
- One way is to alternate DA steps (to estimate x0:N) and ML steps (to estimate the model error parameters p) in a coordinate-descent framework:



• This coordinate-descent idea was used with good results, e.g. in Brajard et al., 2021 to reconstruct the state <u>and</u> the dynamics of Lorenz-96 model using convolutional neural networks



• However: 1) the initialization of **p** is critical and cold-starting can easily lead to divergence, and 2) the number of DA-ML cycles required to reach convergence can be high, problematic for a realistic application

- Farchi et al., 2021a, have used an offline training approach to construct a hybrid ML-physics model for a quasi-geostrophic system
- Their surrogate model based on Convolutional Neural Networks was shown to be able to significantly improve on the original perturbed model both in forecast mode and in the assimilation cycle



Forecast skill of the original model (dotted line) and the hybrid surrogate model (continuous line) as a function of the lead time in days

From Farchi et al., 2021a

- In Farchi et al., 2021b, two crucial aspects of the training process have been studied using the two-scale Lorenz model
- One aspect was whether it is more efficient to correct the integrated-in-time model error (i.e., the model resolvent), or the model error tendencies
- Results show that both techniques perform well in forecast mode, but tendency correction is preferable when the hybrid model is used in a DA cycle



Analysis RMSE for the physical model (in blue), the true model (in black), and the trained surrogate models: RC-CNN-a (in green), TC-CNN-b (in red), and TC-CNN-c (in cyan). The surrogate models are trained either with the analysis (left panel) or with the truth (right panel).

From Farchi et al., 2021b

- The other aspect that was investigated was the effectiveness of offline vs online (i.e., inside 4D-Var) model error learning
- Online learning appears to give best results in this experimental setup



Time series of sRMSE (top panel) and tMSE (bottom panel) for the online experiment with TC-CNN-b (in blue). For comparison, the horizontal lines show the scores for the physical model (in cyan), the true model (in black), TC-CNN-b trained offline with the analysis (in green) and trained offline with the truth (in red).

From Farchi et al., 2021b

- How can we adapt these ideas to operational NWP and Climate prediction? There are some issues:
  - 1. Typically there is not enough time to iterate to convergence the DA and ML steps in operational NWP;
  - 2. We have a much more complex model, but a very good one!
  - 3. Most importantly, the size of the model error space is orders of magnitude bigger than in low-order models

- What kind of prior knowledge do we have on the atmospheric model error generating distribution?
- Prior assumptions:
  - 1. We can consider the atmospheric flow to be subject to homogeneous dynamics and heterogeneous forcings;
  - 2. Physical parameterisations of unresolved motions and radiation plus surface forcings are the dominant sources of model error
  - 3. Physical parameterisations are computed and applied over model columns.
- This led us to define a set of predictors made up of the concatenation of <u>climatological predictors</u> (<u>time of day, month, lat, lon</u>) and the <u>vertical columns</u> (137 levels) of the <u>model first guess</u> prognostic variables of the model (t, lnsp, vo, div, q).
- This choice amounts to splitting the full 3d regression problem into a 1d x 2d problem and is conceptually similar to having a separable representation of a 3D covariance matrix



- Dense Neural Network with Relu activations
- <u>Three layers</u> with nonlinear activations give best results: problem with only <u>moderate</u> <u>nonlinearities</u>
- Dropout layers used to control overfitting, input/outputs prenormalised for training, Adam minimiser
- Number of trainable parameters ~6\*10<sup>4</sup>, size of training dataset ~10<sup>6</sup>

Bonavita & Laloyaux, 2020



E 2022

T\_LNSP - Full Regressors



- Training/Testing curves are shown in terms of explained variance (R<sup>2</sup>)
- Saturation of explained variance is used as stopping criterion during training
- Mass (T, Insp) errors can be better predicted (~14-15% explained variance) than wind (~4-5% explained variance) and humidity (~0%) errors.
- State-dependent predictors (first guess values) have more predictive power than climatological predictors
- The NN model provides a state-dependent correction beyond climatological bias correction.

#### Bonavita & Laloyaux, 2020

#### Model Error Estimation and Correction in the IFS: Random errors



Instrument(s): AIREP PILOT PROF TEMP - U V

Area(s): Europe Japan N.Hemis S.Hemis Tropics

Instrument(s): TEMP – T Area(s): N.Hemis S.Hemis Tropics From 00Z 16–Jul–2019 to 12Z 22–Aug–2019



Bonavita & Laloyaux, 2020

#### Model Error Estimation and Correction in the IFS: Forecast Skill

- ANN in combination with Weak Constraint 4DVar improves the fit of observations to the model, both in the mean and in the random component.
- What can the ANN bring to forecast skill?



- The original idea of Bonavita and Laloyaux, 2020 has been further developed
- Specifically, the NN training can now be done inside the IFS 4DVar (NN 4D-Var, Farchi et al., 2023)
- Where are we with this line of development?



Training the NN parameters inside 4D-Var results in further forecast skill improvements for most variables.



Figure: Score card 2022/06/03 to 2022/08/31. 12H assimilation window with NN model error correction trained **online**.



Figure: Z500 NH anomaly correlation. 2022/06/03 to 2022/08/31. 12H assimilation window with NN model error correction trained **online**.

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- Up to now we have been learning model errors optimising NN 4D-Var over the standard 12-hour assimilation window
- The problem with this approach is that some of the errors of the initial conditions (analysis) will get aliased into the model error estimates
- Initial conditions errors will be less of a factor if we do the NN 4D-Var optimisation over a longer assimilation window





Figure: Z500 NH anomaly correlation. 2022/06/03 to 2022/07/28. 24H assimilation window with offline NN model error correction.

Figure: Z500 NH anomaly correlation. 2022/06/03 to 2022/07/28. 24H assimilation window with **online** NN model error correction.



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- Optimising over a 24-hour assimilation window shows clear improvement over 12-hour window (it recovers ~ half the gap in performance between the IFS and GraphCast/AIFS)
- Can we do better with an even longer assimilation window?
- Up to now we have used the original fully connected NN column model in NN 4D-Var. What if we use more expressive 3D NN architectures\* (CNN, Graph networks, Transformers, etc.)?
- To be continued...

\* Note that this is more complex than putting together NN blocks in Tensorflow/Pytorch, as it requires coding the NN machinery inside 4D-Var



### ML forecast models



From Zied Ben Bouallegue (ECMWF)

- Field took off with Keisler, 2022, then in rapid succession FourCastNet (NVIDIA, Pathak et al., 2022), Pangu-weather (Huawei, Bi et al., 2022), GraphCast (Google-DeepMind, Lam et al., 2022), FengWu (Academic, Chen et al., 2023)...
- All trained on ERA5 re-analysis (deterministic + EDA)
- Superior forecast scores, 3-4 order of magnitude cheaper to run (not to train!)
- What's not to like?

#### A look under the hood of MLWP models: Pangu-weather

• Are Pangu-weather (and other DLWP models) realistic atmosphere emulators?



Bonavita, M. (2023) On some limitations of data-driven weather forecasting models. GRL, under review. (arXiv: 10.48550/arXiv.2309.08473)

#### A look under the hood of MLWP models: Pangu-weather

- Are Pangu-Weather (and other DLWP models) realistic atmosphere emulators?
- Pangu-Weather forecasts show sharply decreased levels of variability wrto ERA5 analyses beyond ~wavenumber 50 (~400 km) from the start of the forecast
- Differently from IFS forecasts, which show consistent variability at all forecast ranges, PW forecast variability decreases with forecast range, noticeable jump at t+24h and beyond -> increasing "blurriness" of predictions
- Does it matter?

#### A look under the hood of MLWP models: Pangu-Weather



Typhoon Doksuri (Egay) of the 2023 Pacific typhoon season near its peak intensity while off the coast of Luzon during the afternoon of July 25, 2023. It had 10-min sustained winds of 175 km/h (110 mph) (JMA) and 1-min sustained winds of 230 km/h (145 mph) (JTWC) and an official minimum central pressure of 935 mbar (27.6 inHg) at the time this image was captured.



# Probability (%) of Tropical Cyclone Intensity falling in each category TD[up to 33] TS [34-63] HR1[64-82] HR2 [83-95] HR3 [> 95 kt] 10m Wind Speed (kt) solid=HRES; dot=Ens Mean Eri28 Mean Sea Level Pressure in Tropical Cyclone Centre (hPa) solid=HRES; dot=Ens Me 970-

Fri28

#### A look under the hood of MLWP models: Pangu-Weather

TC Doksuri 26 Jul 2023 12Z Estimated Best Track min mslp: **944** hPa





#### Pangu-Weather dynamical fields

Vertical velocity is not predicted by Pangu-Weather (and others) but can be diagnosed by integrating the continuity equation on forecasted pressure-level fields (Holton and Hakim, 2012):

$$\omega(p) = \omega(p_s) - \int_{p_s}^p \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)_p dp$$

Unsurprisingly, the progressive reduction in the magnitude of the predicted divergence field leads to increasingly weak vertical velocity predictions:



Evolution of stdev of fcst vertical velocity field IFS, ERA5, Pangu

#### Pangu-Weather dynamical fields (3)

#### ERA5 fcst vert. vel. 2023-09-07 00UTC t+120h

#### Vertical velocity 500 hPa 2023-09-07 00UTC t+120 -0.3 -0.27 -0.24 -0.21 -0.18 -0.15 -0.12 -0.09 -0.06 -0.03 0 0.03 0.06 0.09 0.12 0.15 0.18 0.21 0.24 0.27 0.3 120°E 140°E 80°F 100°E 140°W 120°W 100°E 120°E 100°W 80°W 20°E 40°E 60°E 80°E 140°E

# Pangu-Weather fcst vert. vel. 2023-09-07 00UTC t+120h



#### Pangu-Weather dynamical fields (3)

#### Hurricane Lee, 12 September 2023 01UTC

Strongest TC of the 2023 Atlantic Season so far, Category 3 at the time



https://zoom.earth/storms/lee-2023/#map=satellite-hd

#### Pangu-Weather dynamical fields (3)



ERA5 vert. vel. (m/s) + Z500 2023-09-07 00UTC t+120h





#### Pangu-Weather dynamical fields



GraphCast vert. vel. (m/s) + Z500 2023-09-07 00UTC t+120h Pangu-Weather vert. vel. (m/s) + Z500 2023-09-07 00UTC t+120h







#### Take-home Messages

- We are in the Big Data era! There is an unprecedented and rapidly growing amount of geophysical data ready to be used, both online (at ECMWF we use ~80 million obs a day in 4DVar: this is still less than 5% of the total amount of obs that reach the building every day!) and through reanalysis datasets
- This means that we have enough data not only to improve the initial state estimates, but also the models (both forecast model and forward models for observations)
- Machine Learning techniques can play a crucial role in tackling model deficiencies and improving predictive capabilities in both NWP and Climate.
- This field is evolving rapidly, next year's presentation will likely be quite different from this year's! Stay tuned...



#### Outlook

- For the next generation of MLWP models the challenge will be to produce physically consistent forecasts with realistic activity levels <u>and</u> maintain forecast skill
- Can MLWP extend to DA, i.e. from observations to forecasts? (no examples of this yet, contrary to what is sometimes claimed)
- For the traditional DA-NWP community the challenge is to speed up adoption of ML techniques to make traditional DA and NWP processes significantly more effective and efficient: Can we match ML models forecast accuracy <u>and</u> provide physically credible forecasts?
- Too early to say which approach will prevail, but certainly things are moving at speed! 4<sup>th</sup> ECMWF-ESA Workshop on ML for Earth Observation and Prediction, Frascati, Rome, 7-10 May 2024 (https://www.ml4esop.esa.int)



# Thanks for your attention!

Bocquet, M., Brajard, J., Carrassi, A. and Bertino, L., 2019: Data assimilation as a learning tool to infer ordinary differential equation representations of dynamical models. Nonlinear Processes in Geophysics, 26 (3), 143–162. <u>doi:10.5194/npg-26-143-2019</u>.

Bocquet, M., Brajard, J., Carrassi, A. and Bertino, L. (2020). Bayesian inference of chaotic dynamics by merging data assimilation, machine learning and expectation-maximization. Foundations of Data Science, 2 (1), 55–80. doi:10.3934/fods.2020004

Bolton, T., & Zanna, L. (2019). Applications of deep learning to ocean data inference and subgrid parameterization. Journal of Advances in Modeling Earth Systems, 11, 376– 399. <u>https://doi.org/10.1029/2018MS001472</u>

Bonavita, M. and P. Laloyaux, 2020: Machine Learning for Model Error Inference and Correction, JAMES <u>https://doi.org/10.1002/essoar.10503695.1</u>

Bonavita, M., Arcucci, R., Carrassi, A., Dueben, P., Geer, A. J., Le Saux, B., Longépé, N., Mathieu, P., & Raynaud, L. (2021). Machine Learning for Earth System Observation and Prediction, Bulletin of the American Meteorological Society, 102(4), E710-E716, DOI: <u>10.1175/BAMS-D-20-0307.1</u>

Bonavita, M. (2021) Exploring the structure of time-correlated model errors in the ECMWF data assimilation system. Q J R Meteorol Soc, 147(739), 3454–3471. Available from: https://doi.org/10.1002/qj.4137

#### **C**ECMWF

Continued...

Bonavita, M. (2023). On some limitations of data-driven weather forecasting models. GRL, under review. https://doi.org/10.48550/arXiv.2309.08473

Brajard, J., Carrassi, A., Bocquet, M. and Bertino, L. (2020). Combining data assimilation and machine learning to emulate a dynamical model from sparse and noisy observations: A case study with the lorenz 96 model. Journal of Computational Science, 44, 101171. <u>doi:10.1016/j.jocs.2020.101171</u>.

Brenowitz, N. D., & Bretherton, C. S. (2018). Prognostic validation of a neural network unified physics parameterization. Geophysical Research Letters, 17, 6289–6298. <u>https://doi.org/10.1029/2018GL078510</u>

Brenowitz, N. D., and Bretherton, C. S. (2019), Spatially Extended Tests of a Neural Network Parametrization Trained by Coarse-Graining, J. Adv. Model. Earth Syst., 11, 2728–2744. <u>https://doi.org/10.1029/2019MS001711</u>.

Chantry, M., S. Hatfield, P. Dueben, I. Polichtchouk, and T. Palmer. Machine learning emulation of gravity wave drag in numerical weather forecasting. Journal of Advances in Modeling Earth Systems, 13(7):e2021MS002477, 2021. doi: https://doi.org/10.1029/2021MS002477

Farchi, A., Laloyaux, P., Bonavita, M. & Bocquet, M.(2021a) Using machine learning to correct model error in data assimilation and forecast applications. Q J R Meteorol Soc, 147(739), 3067–3084. Available from: <u>https://doi.org/10.1002/qj.4116</u>

Farchi, A., M. Bocquet, P. Laloyaux, M. Bonavita and Q. Malartic, (2021b)A comparison of combined data assimilation and machine learning methods for offline and online model error correction, Journal of Computational Science, Volume 55, 2021,101468, ISSN 1877-7503, <u>https://doi.org/10.1016/j.jocs.2021.101468</u>.

#### **C**ECMWF

Keisler, R., (2021) Forecasting Global Weather with Graph Neural Networks, https://arxiv.org/pdf/2202.07575.pdf

Kutz, J. N. (2017). Deep learning in fluid dynamics. Journal of Fluid Mechanics, 814, 1–4.

Laloyaux, P, Bonavita, M, Dahoui, M, et al. 2020: Towards an unbiased stratospheric analysis. Q J R Meteorol Soc. 146: 2392–2409. <u>https://doi.org/10.1002/qj.3798</u>

Ling, J., Kurzawski, A., & Templeton, J. (2016). Reynolds averaged turbulence modelling using deep neural networks with embedded invariance. Journal of Fluid Mechanics, 807, 155–166.

Navon, I.M., 1998: Practical and theoretical aspects of adjoint parameter estimation and identifiability in meteorology and oceanography, Dynamics of Atmospheres and Oceans, Volume 27, Issues 1–4, 55-79

Rasp, S., Pritchard, M. S., & Gentine, P. (2018). Deep learning to represent subgrid processes in climate models. Proceedings of the National Academy of Sciences of the United States of America, 115(39), 9684–9689. https://doi.org/10.1073/pnas.1810286115

Ruiz, J.J., Pulido, M. and T. Miyoshi, 2013: Estimating Model Parameters with Ensemble-Based Data Assimilation: A Review, Journal of the Meteorological Society of Japan. Ser. II, 91, 2, 79-99. <u>https://doi.org/10.2151/jmsj.2013-201</u>



#### **Additional Material**



#### A look under the hood of MLWP models: Pangu-Weather

Unrealistic forecast energy spectra imply dynamically inconsistent forecast fields

#### Pangu-Weather dynamical fields (1)

Geostrophic wind  $(\mathbf{V}_g = \frac{1}{f} \hat{\mathbf{k}} \times \nabla_p \Phi)$  vs ageostrophic wind  $\mathbf{V}_{ag} \equiv \mathbf{V} - \mathbf{V}_g$ 



#### Pangu-Weather dynamical fields (1)



 $|\mathbf{V}_{ag}|$ 

#### Pangu-Weather dynamical fields (2)

Vorticity and divergence decomposition of the circulation

 $\boldsymbol{u} = \boldsymbol{u}_d + \boldsymbol{u}_v = -\nabla \chi + \mathbf{k} \times \nabla \psi$  $\nabla^2 \chi = \boldsymbol{\delta}, \ \nabla^2 \psi = \boldsymbol{\zeta}$ 

