

# Horizontal and vertical resolution, hydrostatic and non-hydrostatic dynamics

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Numerical Methods Training Course, November 2024

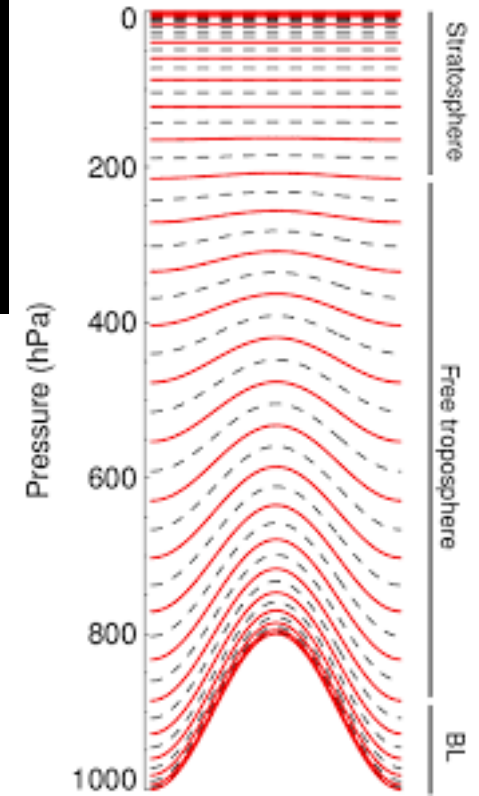


# Outline

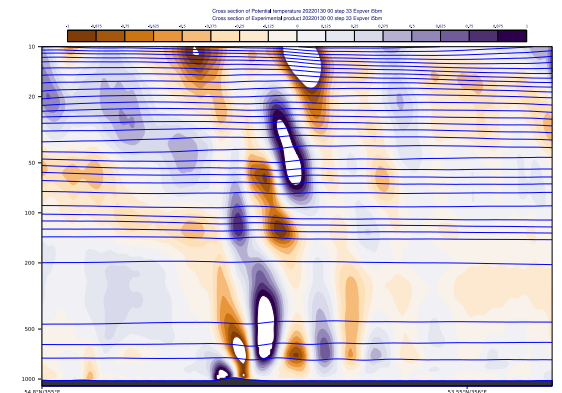
## 1) Horizontal resolution at ECMWF



## 2) Vertical resolution at ECMWF

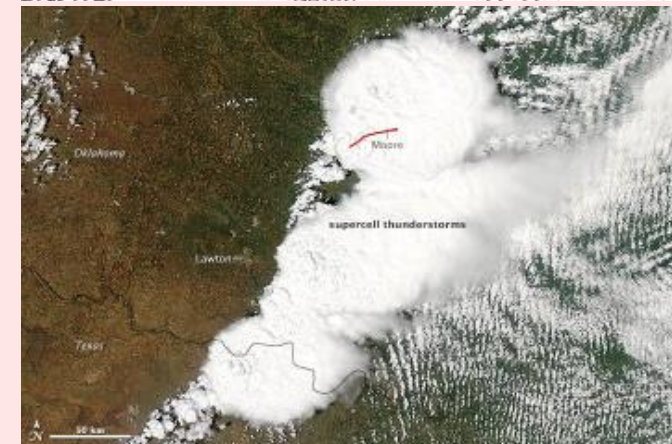
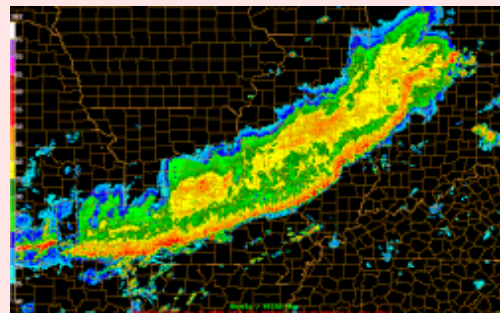
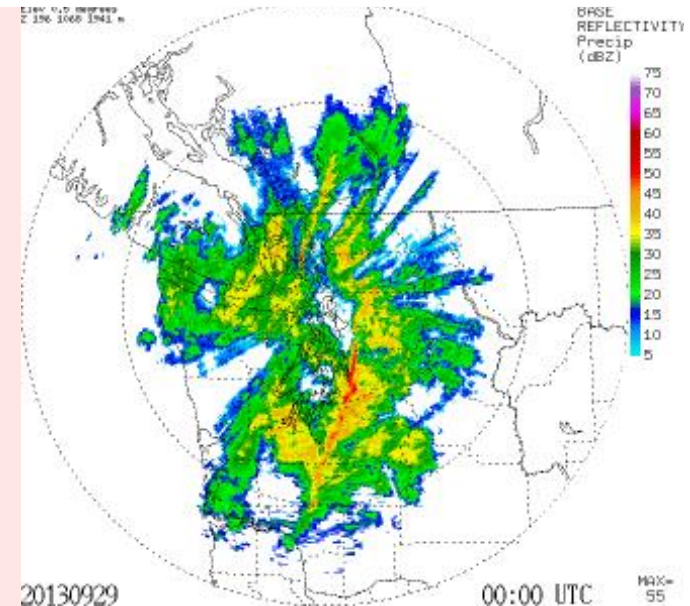


## 3) Hydrostatic vs. non-hydrostatic dynamics

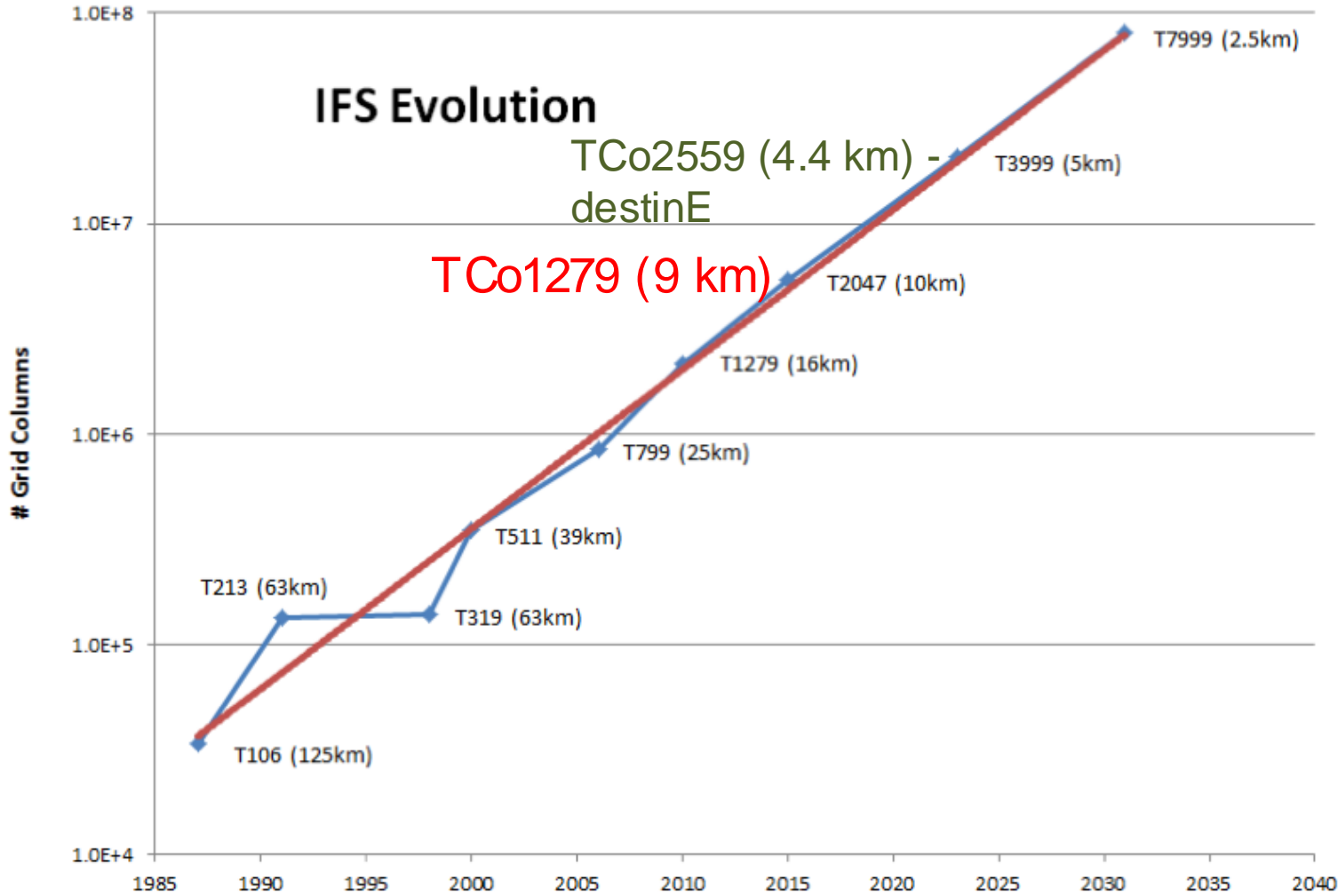


# Why strive for higher and higher resolution?

- To better resolve high-impact weather processes, such as mountain waves, tropical cyclones, squall lines, tornadoes etc.



# History and future of horizontal resolution at ECMWF



- 'T' is the truncation in the spherical harmonic expansion. The higher the 'T', the higher the horizontal resolution
- Since 2016 ECMWF has used cubic octahedral grid (TCo grid) for grid-point calculation.

## Integrated Forecast System (IFS) dynamical core

- Operational IFS is a **semi-Lagrangian, semi-implicit, hydrostatic** dynamical core. **Spectral** in the horizontal, **vertical finite elements** with hybrid eta-coordinate in the vertical. Unresolved processes represented by the IFS 'physics' package.

$$\frac{D\mathbf{V}}{Dt} + f\mathbf{k} \times \mathbf{V} + \nabla_h \Phi + R_d T_V \nabla_h \ln p = P_V + K_V,$$

$$\frac{DT}{Dt} - \frac{\kappa T_V \omega}{[1 + (\delta - 1)q]p} = P_T + K_T,$$

$$\frac{\partial}{\partial t} \left( \frac{\partial p}{\partial \eta} \right) + \nabla_h \cdot \left( \mathbf{V} \frac{\partial p}{\partial \eta} \right) + \frac{\partial}{\partial \eta} \left( \dot{\eta} \frac{\partial p}{\partial \eta} \right) = 0,$$

$$\frac{Dq}{Dt} = P_q,$$

$$\frac{\partial \Phi}{\partial \eta} = -R_d T_V \frac{\partial \ln p}{\partial \eta},$$

$$T_V = T \left[ 1 + \left( \frac{R}{R_d} - 1 \right) q \right]$$

# Spectral vs. grid-point representation in the IFS dynamical core

- **Derivatives, dissipation and semi-implicit solver** calculated in **spectral** space. Only VOR, DIV,  $T_v$  and  $p_s$  have spectral representations.
- **Nonlinear terms and semi-Lagrangian advection** calculated in **grid-point** space. Physical parametrizations applied in grid-point space → Need a grid to convert from spectral space.
- For every spectral truncation there is a **physical space grid**.

$$\frac{DV}{Dt} + f\mathbf{k} \times \mathbf{V} + \nabla_h \Phi + R_d T_V \nabla_h \ln p = P_V + K_V,$$

$$\frac{DT}{Dt} - \frac{\kappa T_V \omega}{[1 + (\delta - 1)q]p} = P_T + K_T,$$

$$\frac{\partial}{\partial t} \left( \frac{\partial p}{\partial \eta} \right) + \nabla_h \cdot \left( \mathbf{V} \frac{\partial p}{\partial \eta} \right) + \frac{\partial}{\partial \eta} \left( \dot{\eta} \frac{\partial p}{\partial \eta} \right) = 0,$$

$$\frac{Dq}{Dt} = P_q,$$

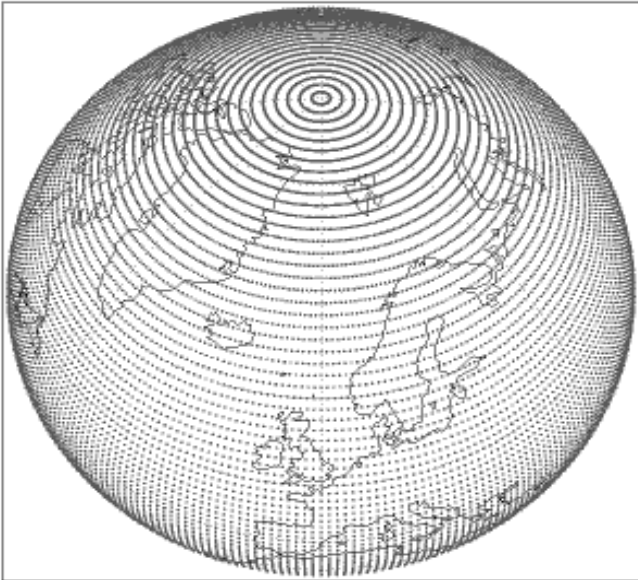
$$\frac{\partial \Phi}{\partial \eta} = -R_d T_V \frac{\partial \ln p}{\partial \eta},$$

$$T_V = T \left[ 1 + \left( \frac{R}{R_d} - 1 \right) q \right]$$

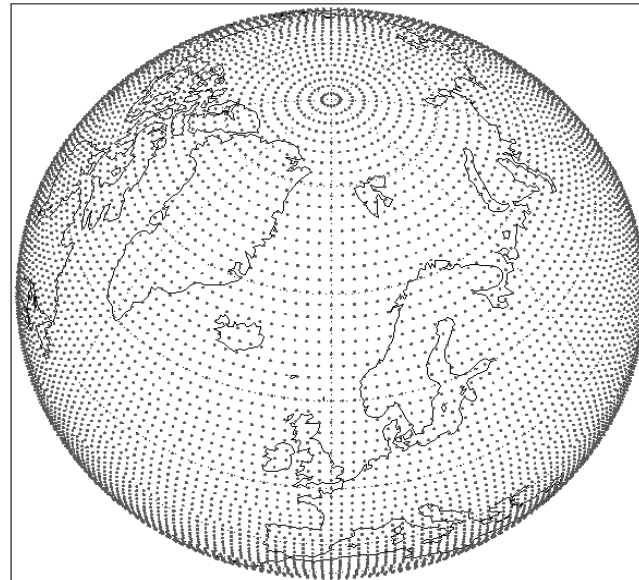
## IFS Gaussian grids

- **Regular (full) grid:** Same number of points around each latitude circle (i.e., crowded near the poles). No aliasing of quadratic terms (i.e. product of 2 variables).
- **Reduced:** Number of points per latitude circle decreases towards the pole, such as to maintain quasi-regular grid spacing  $dx \approx dy$ .
  - **Cubic:** No aliasing of cubic terms (i.e., product of 3 variables)

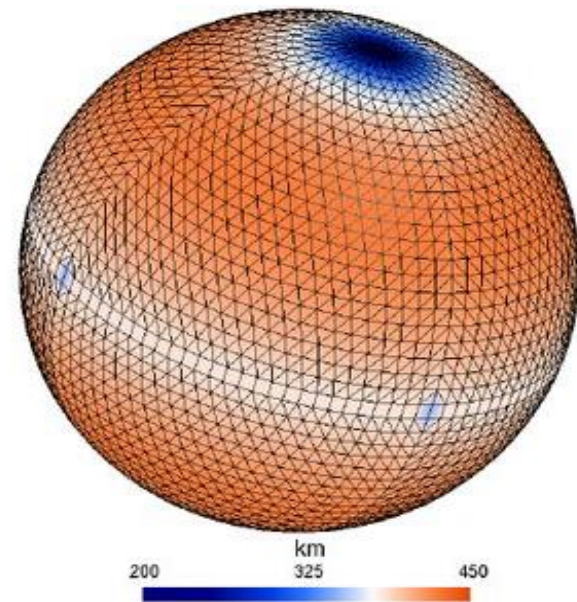
Full: Regular quadratic grid



Reduced: linear grid



Reduced: Cubic octahedral grid



# IFS grid to spectral truncation (NSMAX) pairing

## Pairing grid/truncation

linear: the smallest wavelength  $\lambda_{min} = (2 * \pi * RA) / NSMAX$  is sampled on the grid, along the equator, by 2 points  
 $\Rightarrow NDLON_{lin} \simeq 2 * NSMAX$

quadratic: by 3 points  $\Rightarrow NDLON_{quad} \simeq 3 * NSMAX$

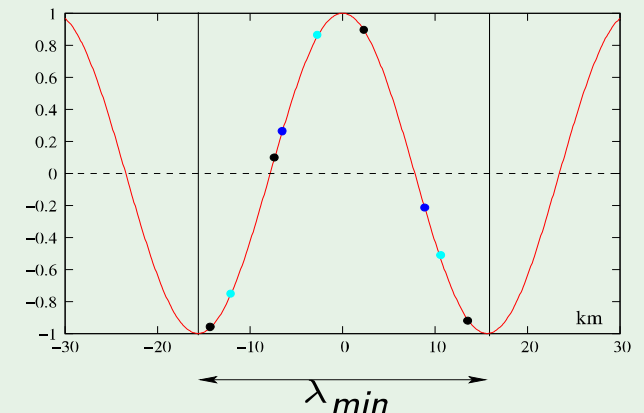
cubic: by 4 points  $\Rightarrow NDLON_{cub} \simeq 4 * NSMAX$

$$NDLON_L = 2560$$

$$T1279 \Rightarrow NDLON_Q = 3840$$

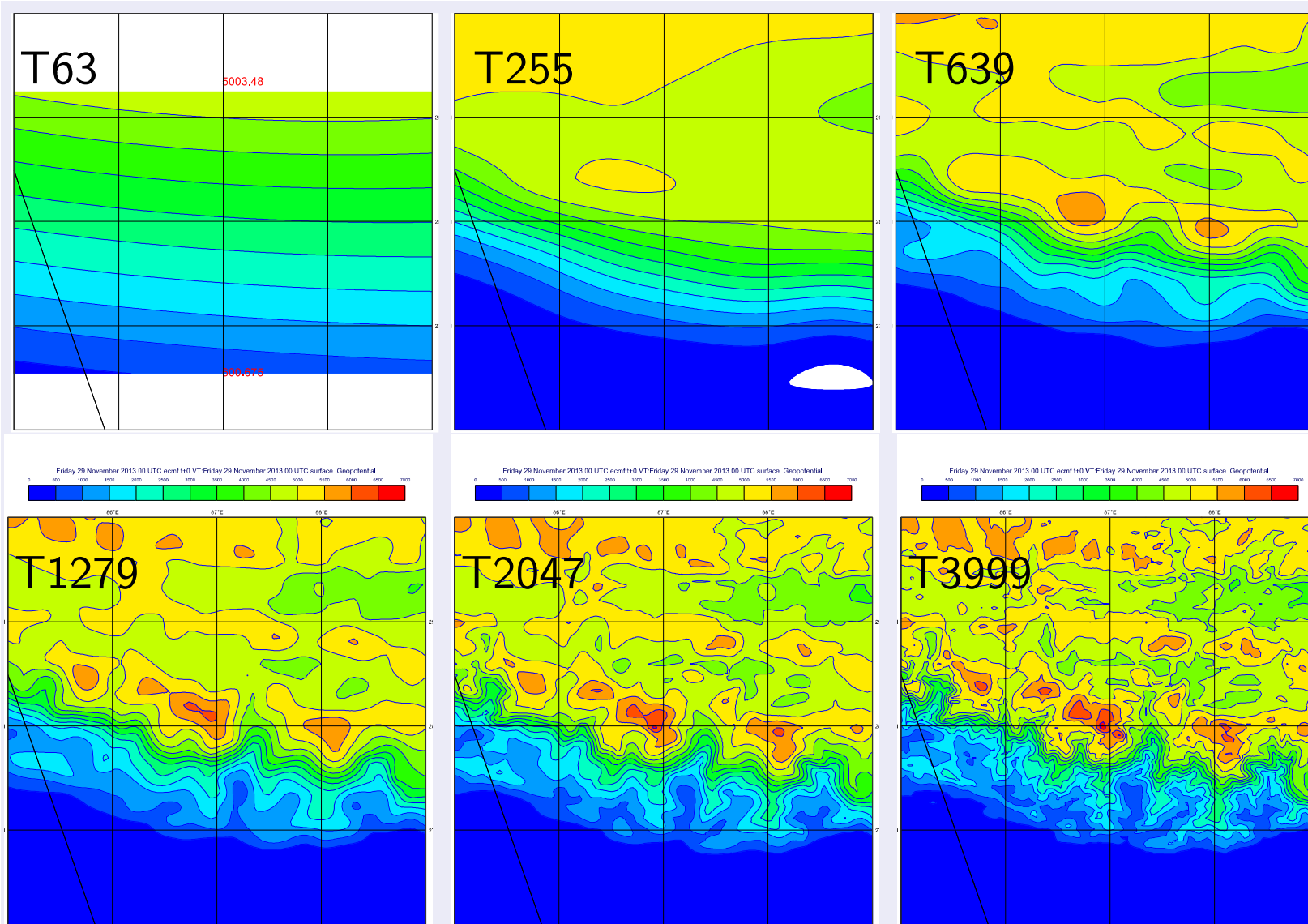
$$NDLON_C = 5120$$

$$NDLON = 2560 \Rightarrow TL1279 \text{ or } TC639$$





# Example of increasing horizontal resolution



Surface geopotential height (i.e., resolved orography) around Mount Everest.

As the horizontal resolution is increased, more realistic details are represented by the model.

## Different ways to increase horizontal resolution at ECMWF

- Increase the spectral truncation (i.e., the amount of waves retained in the spherical harmonic expansion) but keep the same grid: What we did in 1999.
- Increase the spectral truncation as well as the grid-point resolution: What we did from 1999 to 2016.
- Keep the same spectral truncation, but resolve better in grid-point space by increasing the grid-point resolution: This is what we did in 2016.

### Latest resolution upgrade

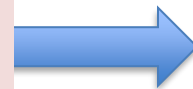
**FROM:**

**4DV:** TCo1279/TL255-319-399

**HRES:** TCo1279

**EDA:** TCo639

**ENS:** TCo639/TCo319 (d1-10/d11-30)



**TO:**

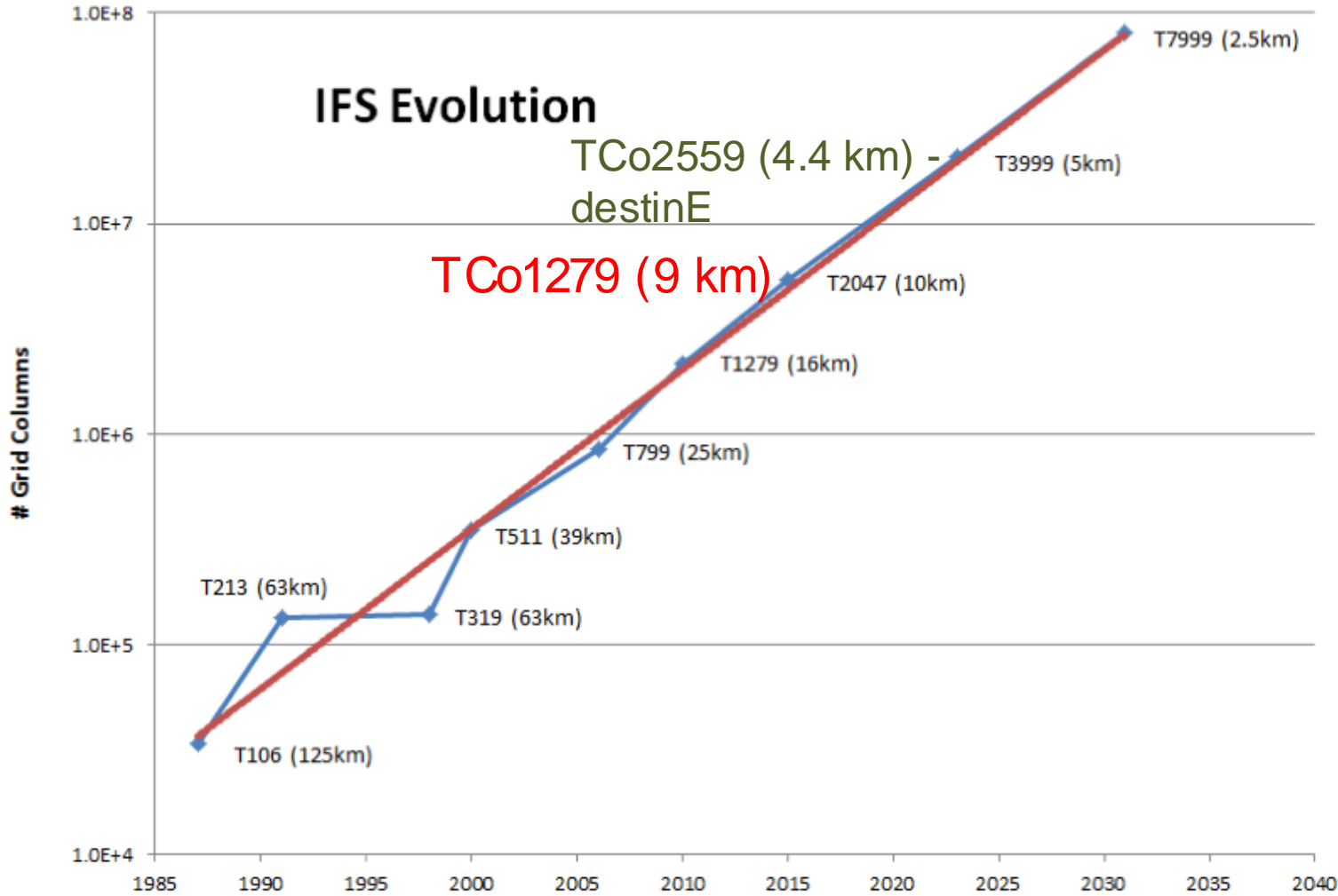
**4DV:** TCo1279/TL255-319-399-511

**DestinE:** TCo2559

**EDA:** TL399

**ENS:** TCo1279/TCo319 (d1-15/d1-45)

# History and future of horizontal resolution at ECMWF

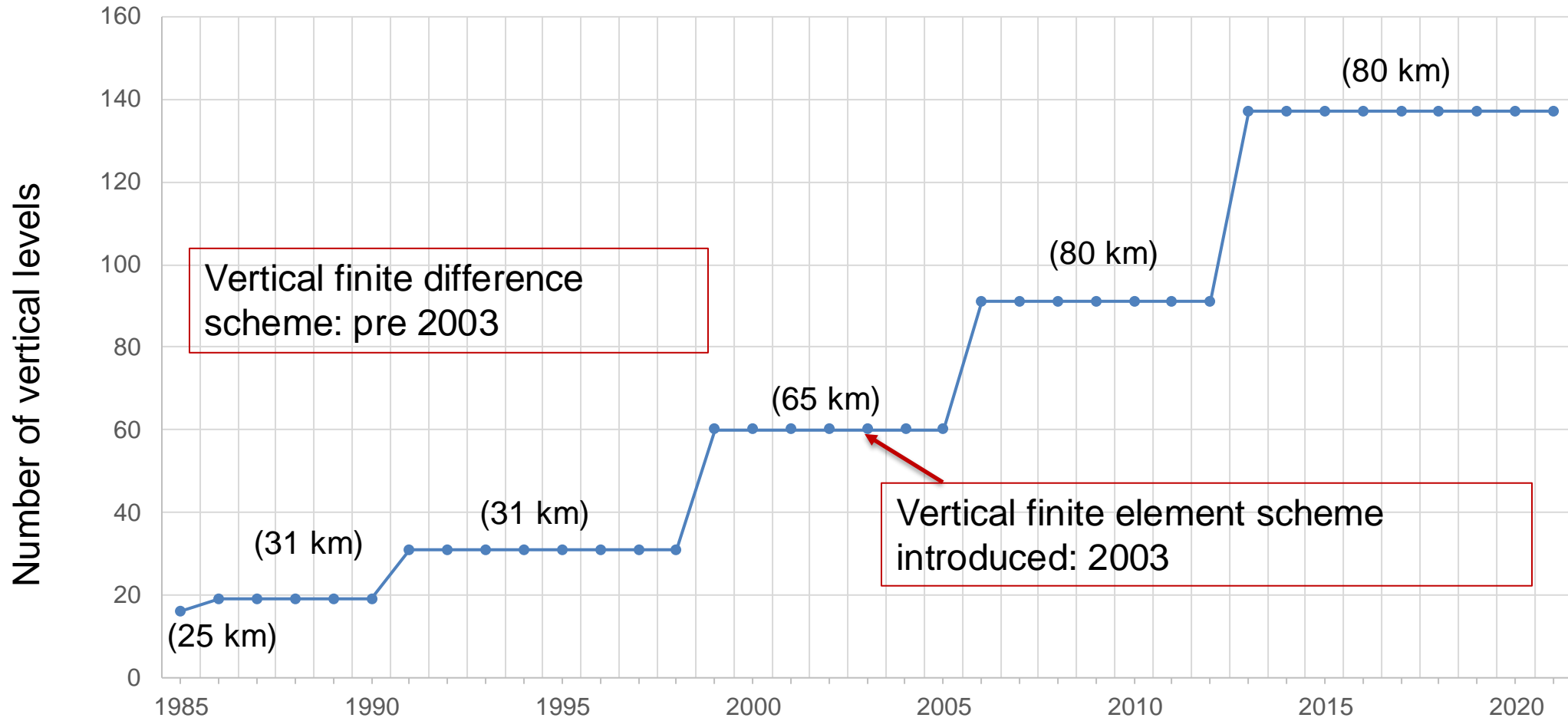


- 'T' is the truncation in the spherical harmonic expansion. The higher the 'T', the higher the horizontal resolution
- Since 2016 ECMWF has used cubic octahedral grid (TCo grid) for grid-point calculation.

## Part I: Recap

- High horizontal resolution needed to resolve high impact weather events better.
- Horizontal resolution at ECMWF can be increased by:
  1. Increasing the truncation wavenumber in the spherical harmonic expansion but keeping the grid resolution unchanged.
  2. Increasing the truncation wavenumber in the spherical harmonic expansion AND grid resolution.
  3. Increasing the grid resolution, but keeping the truncation wavenumber in the spherical harmonic expansion unchanged.
- Currently, cubic-octahedral grid is used at ECMWF. This grid prevents aliasing of products of 3 variables and represents the shortest resolved wavelength by 4 points along the equator.

# History of vertical resolution at ECMWF (model top in parentheses)



# Vertical Finite Differences vs. Vertical Finite Element

- Pre 2003, IFS used the Simmons & Burridge (1981) vertical FD discretization.

- **Idea:** All the vertical derivatives calculated as **centered finite differences**.

$$\partial_x f \approx \frac{f(x + dx) - f(x - dx)}{2dx}$$

### MIDPOINT RULE

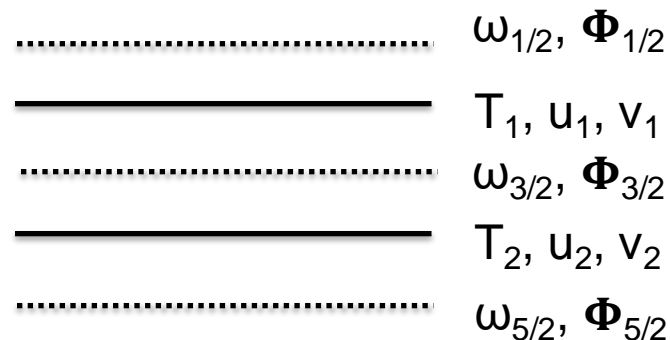
$$\int_a^b f(x) dx \approx M_n = \Delta x [f(\bar{x}_1) + f(\bar{x}_2) + \dots + f(\bar{x}_n)]$$

where  $\Delta x = \frac{b - a}{n}$

and  $\bar{x}_i = \frac{1}{2}(x_{i-1} + x_i) = \text{midpoint of } [x_{i-1}, x_i]$

All the vertical integrals performed by the **mid-point rule**.

- Order of accuracy: 2
- Staggering of variables: Yes, **Lorenz staggering** → introduces a computational  $2\Delta z$  mode. A solution to the  $2\Delta z$  mode is to use Charney-Phillips staggering (e.g. the UKMO)

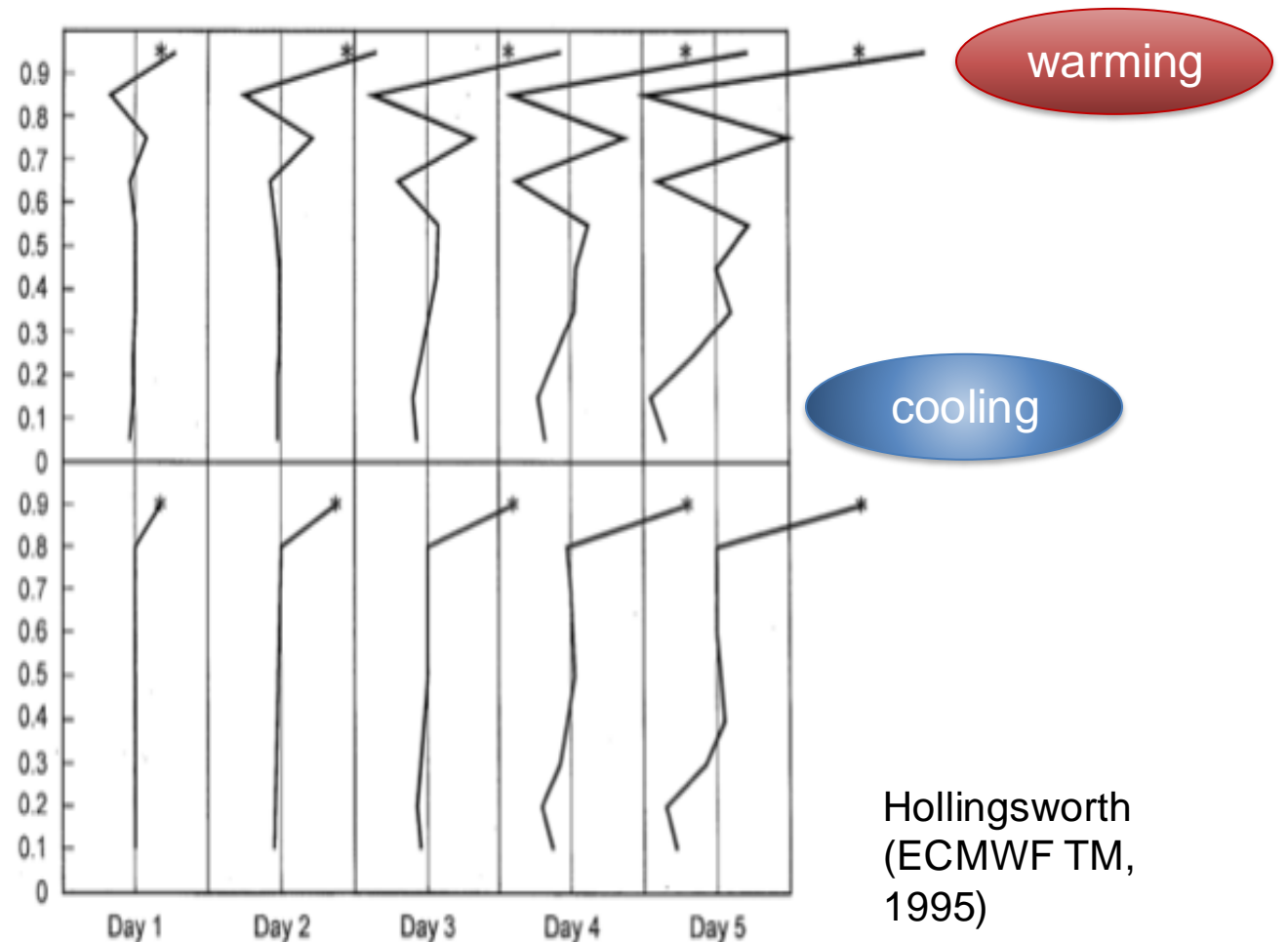


## Impact of the $2\Delta z$ computational mode

Lorenz staggering permits  $2\Delta z$  computational mode **in temperature**. If excited, can generate spurious cooling/heating.

**Lorenz staggering** produces spurious heating/cooling, due to  $2\Delta z$  noise

**Charney-Phillips staggering**, which eliminates  $2\Delta z$  noise, produces correct response



# Vertical Finite Differences vs. Vertical Finite Element

- Post 2003, IFS uses Untch & Hortal (2004) vertical FE discretization

- **Idea:** Equations discretized without vertical derivatives.

Vertical integrals

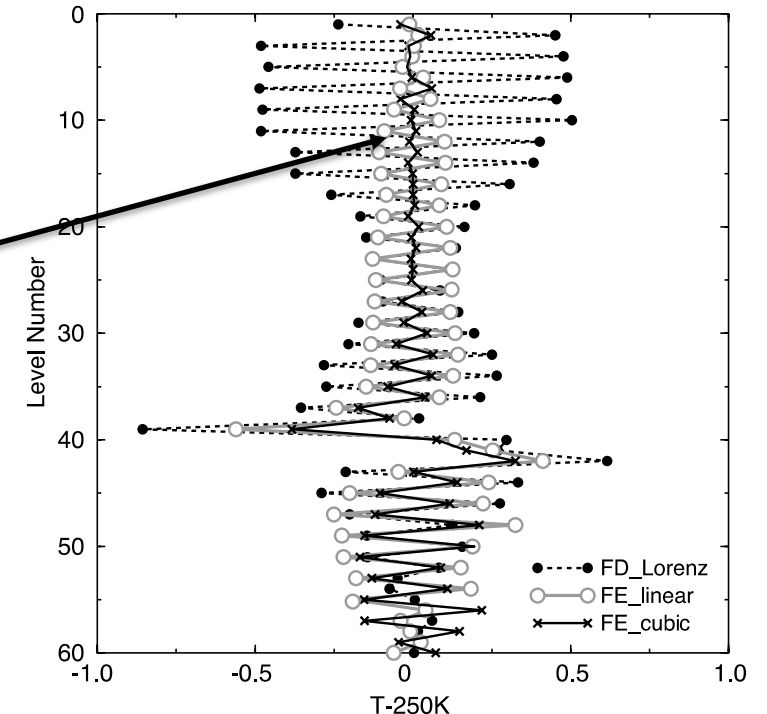
$$F(\eta) = \int_0^\eta f(x) dx$$

approximated as

$$\sum_{i=K_1}^{K_2} C_i d_i(\eta) = \sum_{i=M_1}^{M_2} c_i \int_0^\eta e_i(x) dx,$$

$C_i$ : coeff. of expansion of  $F(\eta)$   
 $d_i(\eta)$ : basis functions  
 $c_i$ : coeff. of the expansion of  $f(\eta)$  as a linear combination of basis functions  $e_i(\eta)$ , which are **cubic splines** in IFS

- Order of accuracy: 8
- Staggering of variables:  
No. Smooths the  $2\Delta z$  computational mode.

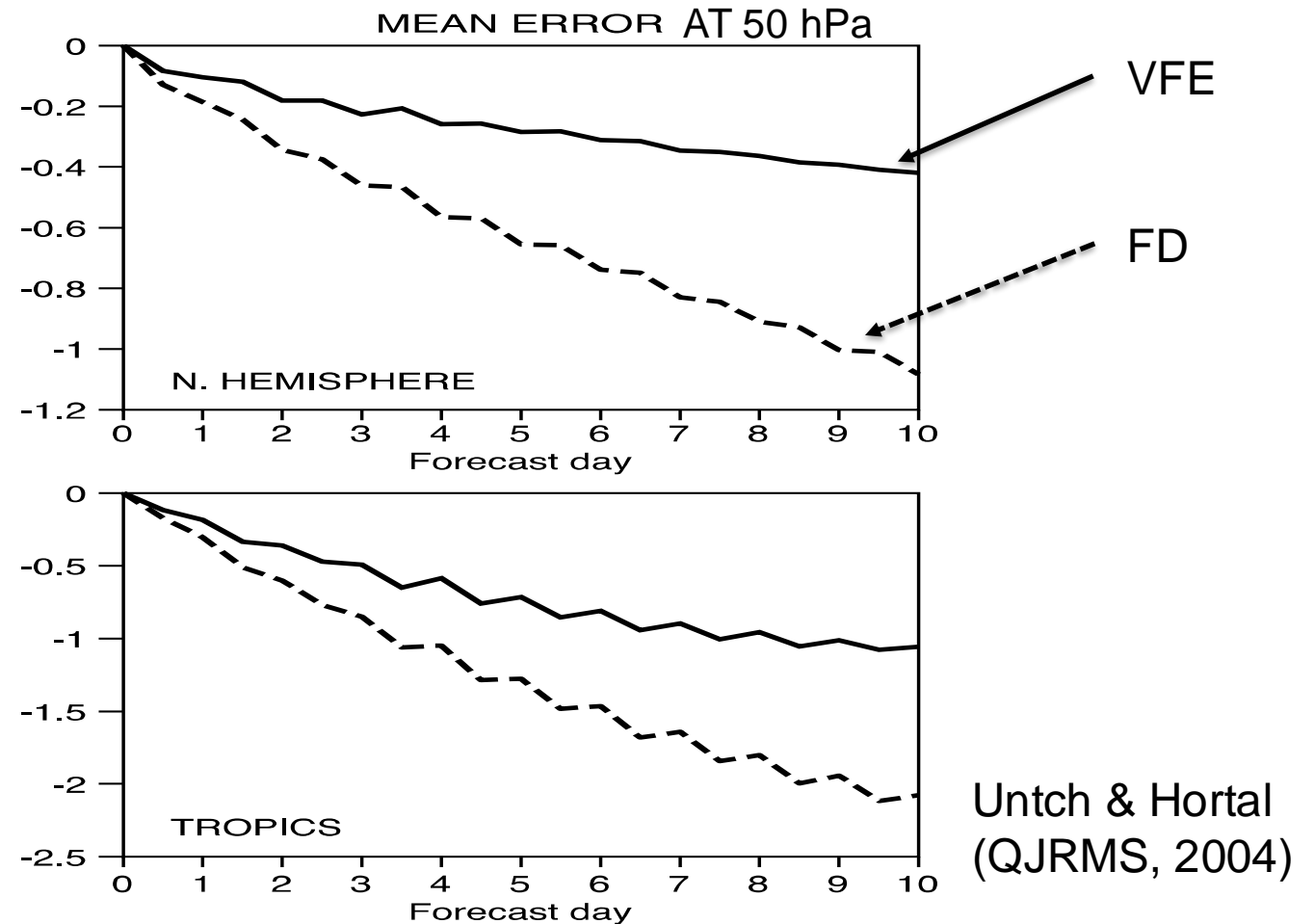




## Impact of eliminating $2\Delta z$ computational mode

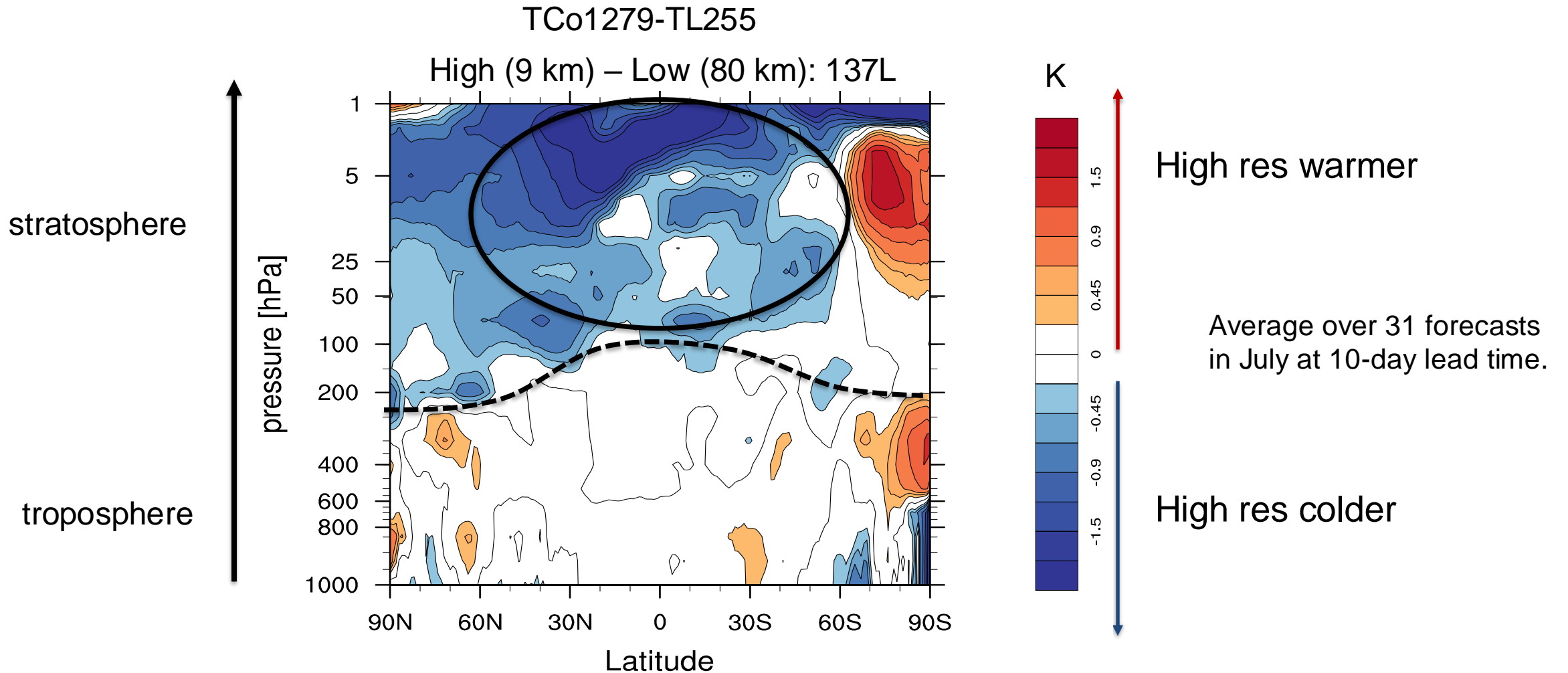
- Vertical finite element vertical discretization **damps** the  $2\Delta z$  noise, but **does not fully eliminate** it → Aliasing of vertically unresolved waves onto the  $2\Delta z$  mode can still generate spurious thermal response.

- Damping the computational mode via VFE reduced spurious global-mean cooling in the lower to mid stratosphere in IFS in comparison to FD with Lorenz staggering.



# Stratosphere at ECMWF: The $2\Delta z$ problem returns at high horizontal resolution

- **Question:** Does the cooling at higher horizontal resolution arise due to inadequate horizontal to vertical resolution aspect ratio? OR: Is the vertical resolution too coarse?



## Theoretical considerations: Horizontal to vertical resolution aspect ratio

- Aspect ratio relevant for balanced **quasi-geostrophic** dynamics (Lindzen & Fox-Rabinovitz, 1989) and **inertia-gravity** wave dispersion relation

$$\frac{\Delta z}{\Delta x} \sim \frac{f}{N} \quad \Delta z \sim 200 \text{ m, for } \Delta x \sim 18 \text{ km}$$

- **Stratified turbulence** develops shear layer of thickness (e.g., Waite & Bartello, 2004)

$$L_b \equiv 2\pi U / N \quad \text{In the stratosphere, } L_b \sim 1 \text{ km, need } 4\text{-}6 \Delta z \text{ to resolve } \rightarrow \Delta z \sim 200 \text{ m}$$

- **Gravity wave propagation:**

Dispersion relation (medium-frequency)

$$|\lambda_z| \sim |c - U| / N \quad \text{If vertical resolution not adequate to resolve } \lambda_z, \text{ discretization errors occur}$$

## Stratosphere in the IFS: Horizontal to vertical resolution aspect ratio

- Perform **10-day forecast** experiments at **high (9 km)** and **low (80 km)** horizontal resolution and gradually increasing **vertical resolution** in the lower to mid-stratosphere.

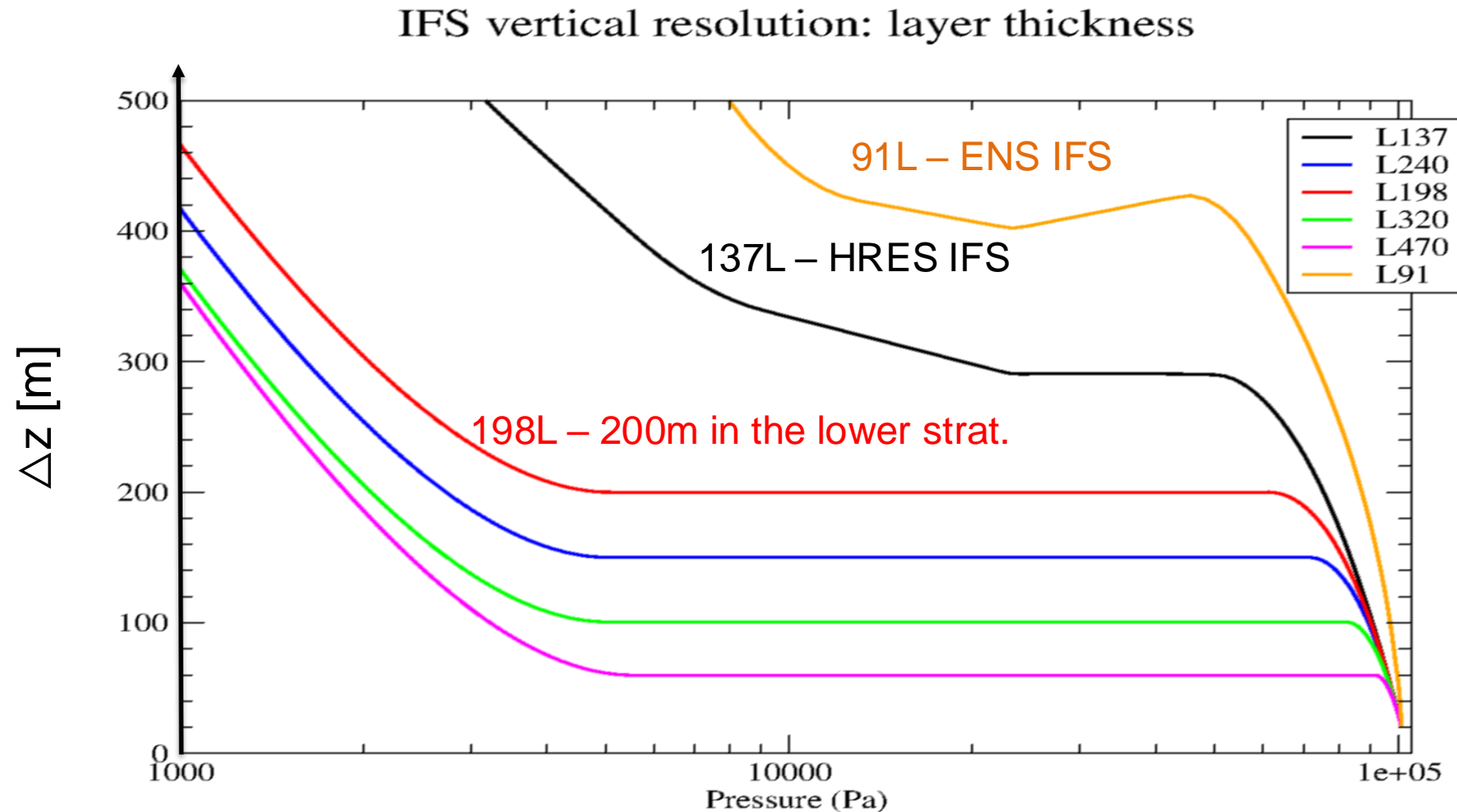
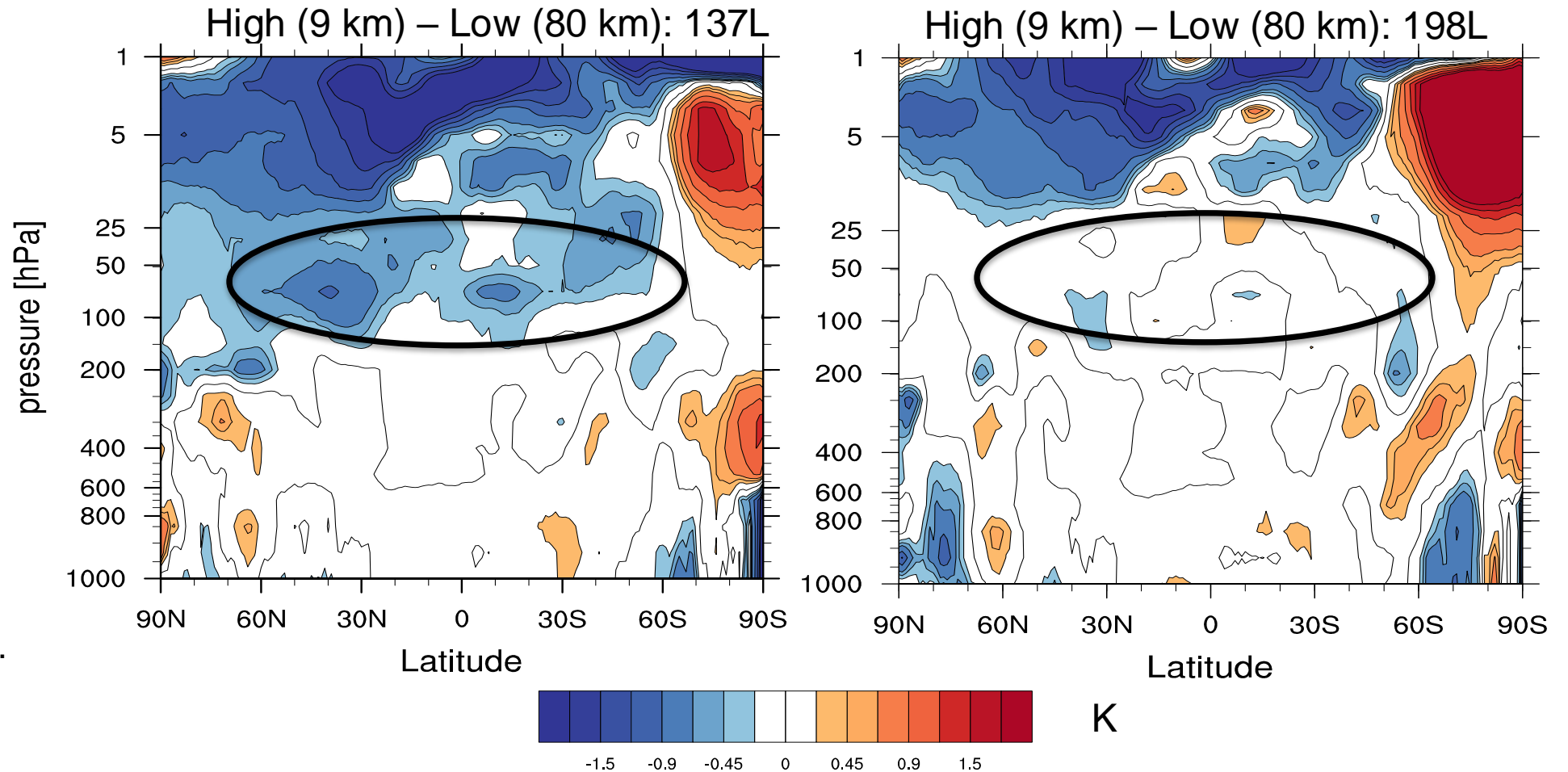


Figure courtesy:  
Tim Stockdale

# Stratosphere in the IFS: Horizontal to vertical resolution aspect ratio

- **Vertical resolution** of **200 m** in the lower to mid stratosphere eliminates global mean cooling there at high horizontal resolution.

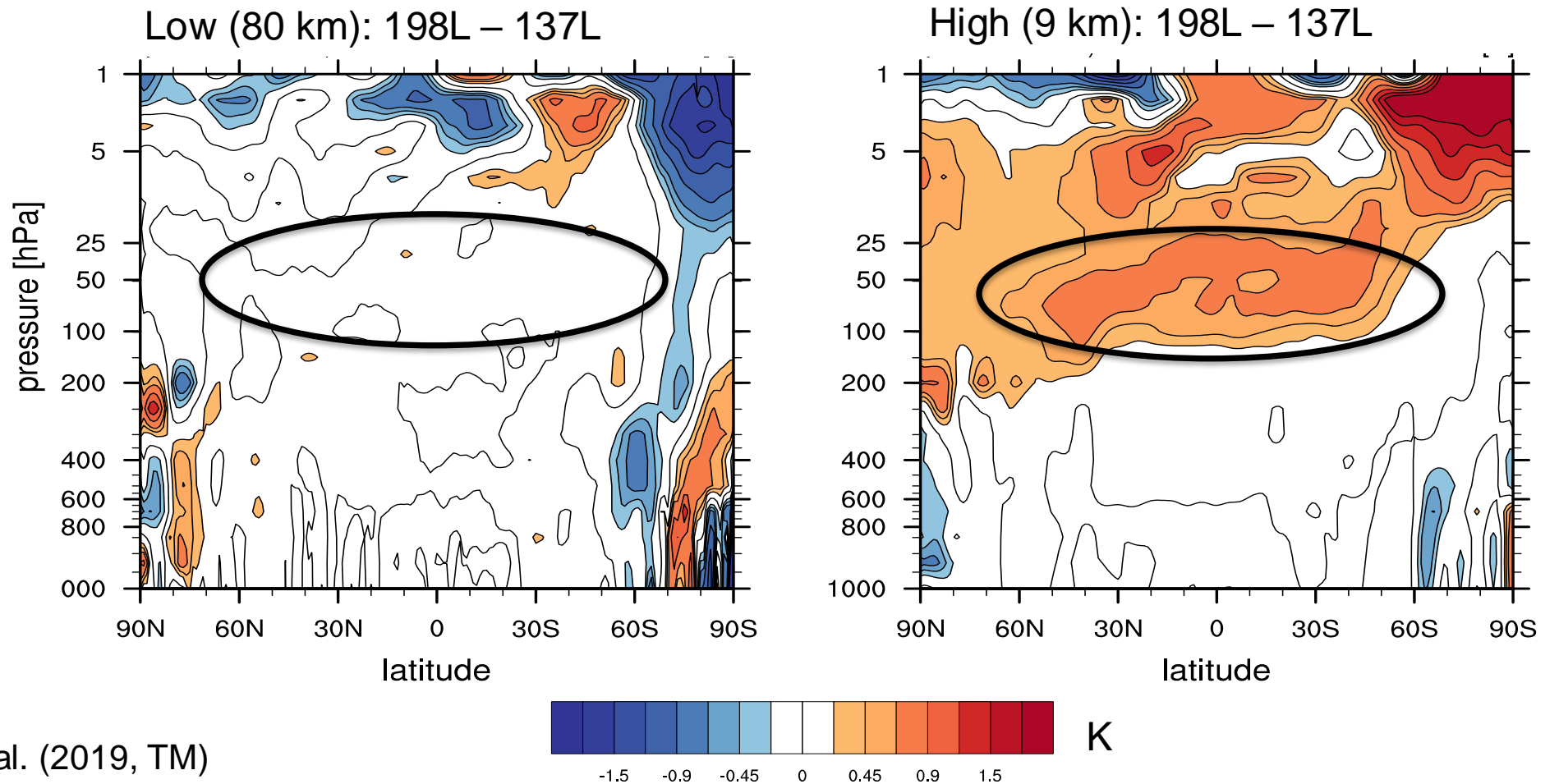
Average over 31 forecasts in July at 10-day lead time.



Polichtchouk et al.  
(2019, TM)

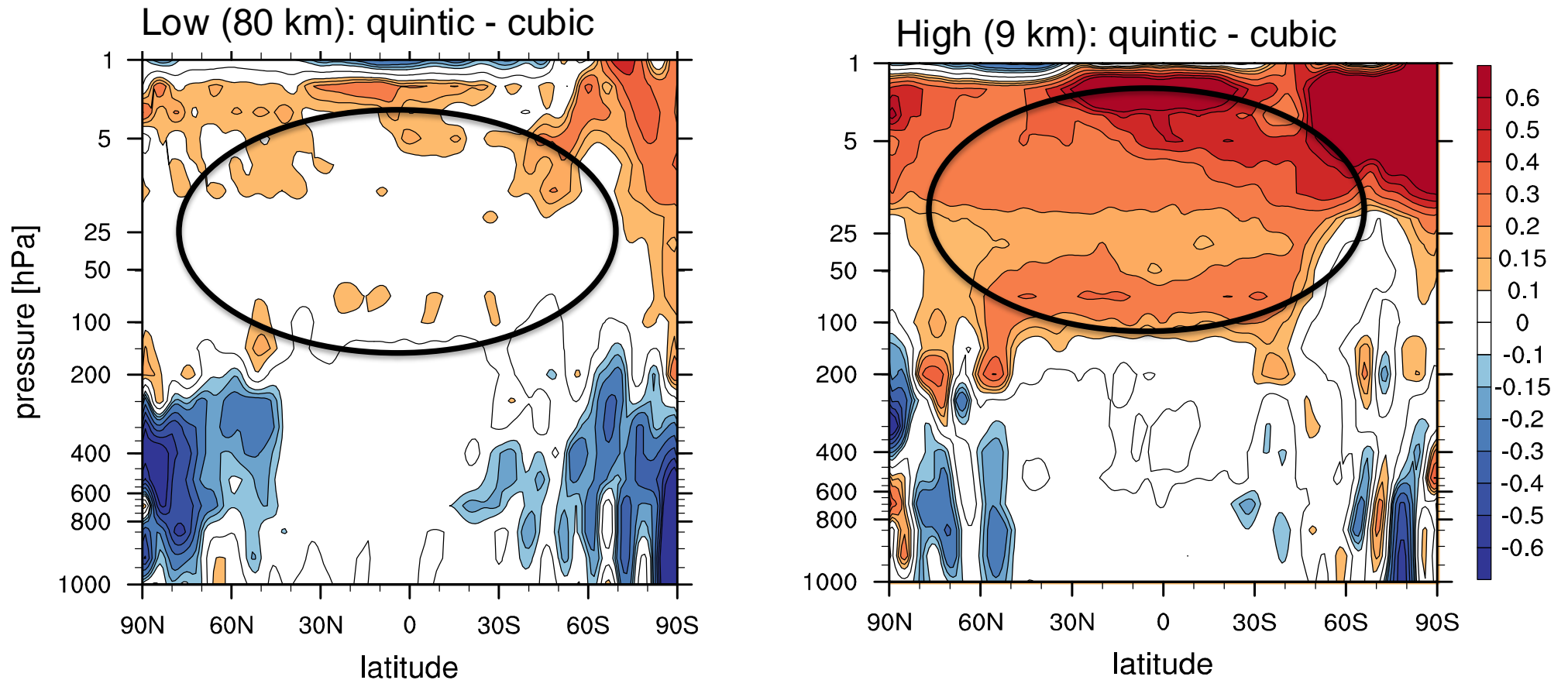
# Stratosphere in the IFS: Horizontal to vertical resolution aspect ratio

- Increase in the **vertical resolution** leads to warming in the stratosphere at **high horizontal resolution**. **No impact at low horizontal resolution.**



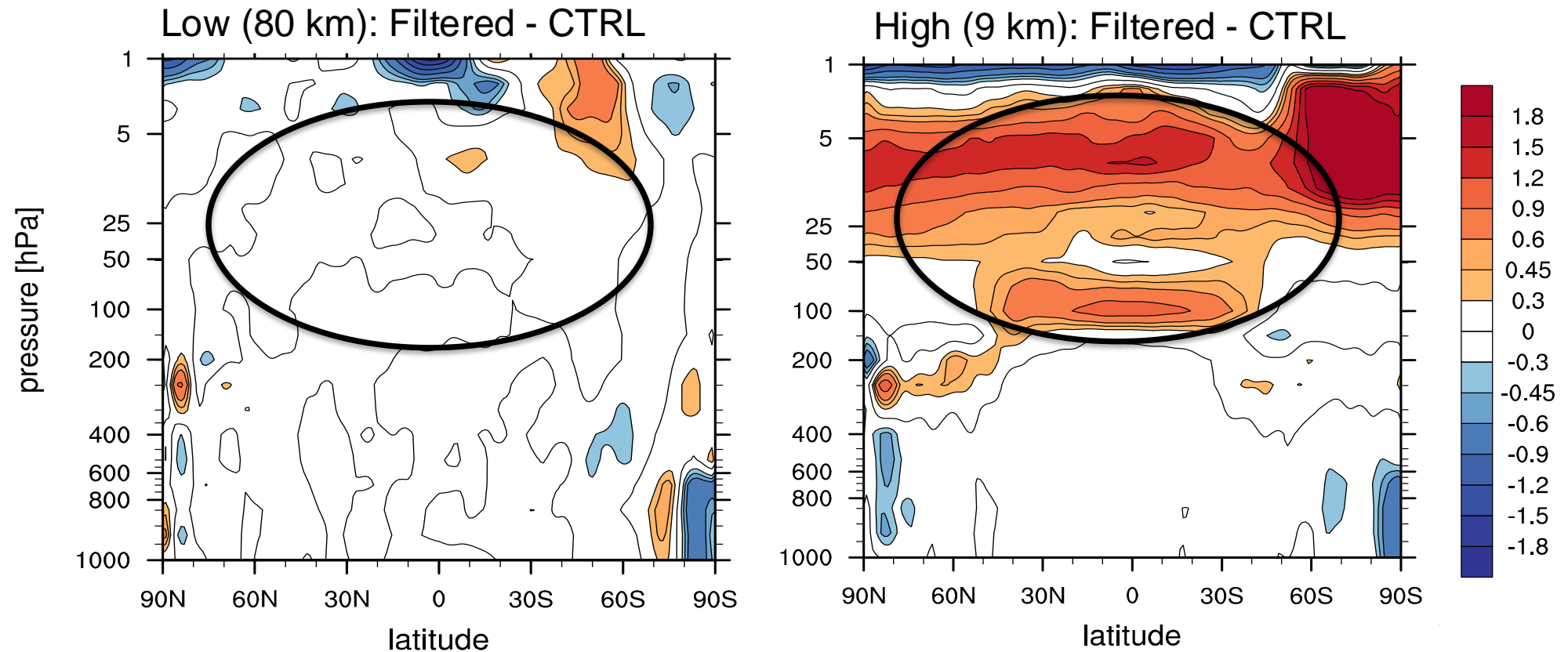
## Higher order vertical Semi-Lagrangian interpolation

- Increasing vertical resolution is expensive → seek cheaper solutions to solve the problem.
- **Question:** Does improving the accuracy of the vertical semi-Lagrangian interpolation help?
- Going from 3<sup>rd</sup> to 5<sup>th</sup> order vertical interpolation helps → Stratosphere warms with higher order interpolation at high horizontal resolution.



## Filtering $2\Delta z$ noise during semi-Lagrangian interpolation

- **Question:** Does **filtering  $2\Delta z$ -noise** in temperature via semi-Lagrangian vertical filter help horizontal resolution sensitivity?
- Filtering warms high horizontal resolution forecasts, **no impact on low horizontal resolution.**





## Part 2: Recap

- Current vertical resolution at ECMWF is **137 levels**. Vertical discretization by **vertical finite element scheme (VFE)**.
- VFE scheme suppresses  $2\Delta z$  noise inherently present in the vertical finite difference scheme using Lorenz staggering.  $2\Delta z$  noise detrimental for temperature forecasts, especially in the stratosphere leading to unphysical global-mean cooling.
- At high horizontal resolution,  $2\Delta z$  noise returns in the VFE scheme. Due to inconsistent horizontal to vertical resolution aspect ratio for gravity waves  $\rightarrow$  need to increase the vertical resolution together with the horizontal resolution.
- Increasing the **vertical resolution** to **200 m** eliminates the global mean cooling at **higher horizontal resolution**.
- **Filtering out  $2\Delta z$ -noise** or increasing the **order of vertical SL interpolation** also alleviate the global mean cooling at **high horizontal resolution**.

## Hydrostatic vs non-hydrostatic model

- **Question:** As we increase the horizontal and vertical resolutions, do we need to relax the hydrostatic approximation?
- Hydrostatic approximation:  $\frac{Dw}{Dt} \ll \left[ -\frac{1}{\rho} \frac{\delta p}{\delta z} - g \right] \Rightarrow w$  diagnostic  
→ Adjustment to the hydrostatic equilibrium faster than a time step.
- Validity of the hydrostatic approximation:  $\left| \frac{Dw/Dt}{\frac{1}{\rho} \delta p / \delta z} \right| \ll 1$

$$\frac{Dw}{Dt} \sim \frac{UW}{L} = \frac{U^2 H}{L^2}, \quad \frac{1}{\rho} \frac{\delta p}{\delta z} \sim \frac{U^2}{H} \Rightarrow \left| \frac{Dw/Dt}{\frac{1}{\rho} \delta p / \delta z} \right| \sim \frac{U^2 H / L^2}{U^2 / H} \sim (H/L)^2 \ll 1$$

H: vertical length scale (e.g., scale height 10 km)

L : horizontal length scale

$W \sim U H/L$

$T \sim L/U$ : Advective time-scale

$dP \sim \rho U^2$  (scaling from considering horiz. momentum eqn.)

# Hydrostatic vs non-hydrostatic model

- Validity of hydrostatic approximation

$$(H/L)^2 \ll 1$$

→ For  $H=10$  km, hydrostatic approximation valid for  $L \gg 10$  km.

- **Common interpretation:** Hydrostatic approximation valid for  $\Delta x > 10$  km.
- **Recall,** current horizontal resolution at ECMWF is TCo1279 or  $\Delta x \approx 9$  km. Do we need a non-hydrostatic (NH) model?

## Hydrostatic vs non-hydrostatic model

- **Question:** What can we capture with NH model that is not there in the H model?
- In a hydrostatic model, the **adjustment to hydrostatic balance** occurs **faster than a timestep size** of the model. **Sub-time step, unresolved processes** restore the hydrostatic balance by distributing mass via e.g.
  - Convergent/divergent ageostrophic wind and vertical velocity acceleration driven by the small-scale NH pressure gradient forces, which resolved state of the atmosphere never sees explicitly.
- In a NH model, the above sub-time step processes are resolved if the temporal and spatial resolution is fine enough to represent them. If not, NH model should give the same results as the H model.

## Non-hydrostatic model of the IFS

- A **non-hydrostatic fully compressible** set of Euler equations has been developed for the limited area version of the IFS dynamical core ALADIN/AROME/HARMONIE (Bubnova et al., 1995), which has been adapted for the global dynamical core (Wedi et al. 2009):
  - spectral semi-implicit, semi-Lagrangian, reduced Gaussian grid, hybrid vertical levels  $p(\eta) = A(\eta) + B(\eta)\Pi_s$  where  $\Pi$  is the hydrostatic part of the true pressure + IFS physics package.
  - 2 more prognostics variables:  $w$  (in practice, the vertical term of the 3D divergence) and the NH pressure departure  $\ln\left(\frac{p-\Pi}{\Pi}\right)$
  - Predictor/corrector scheme: Doubles the cost of dynamics

## Non-hydrostatic vs. hydrostatic model of the IFS: Examples

Convection is a good example to test hydrostatic assumption: Convection is caused by local imbalance between pressure gradient force (buoyancy) and gravity → likely that hydrostatic approximation will distort evolution of explicitly resolved convective systems.

**Example 1:** Falling large cold bubble and rising small warm bubble.

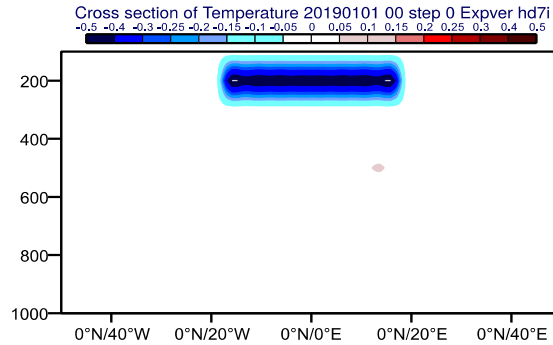
NH model realistically represents descent of cold bubble, while in H model the descent is too fast.

However, despite the inability to appropriately evolve the bubble, H simulation tends to meaningful final state, with stabilized stratification.

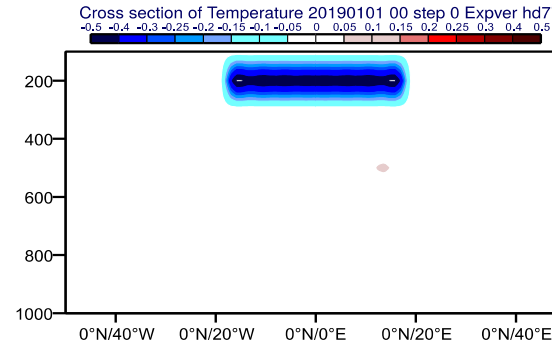
# Non-hydrostatic vs. hydrostatic model of the IFS: Examples

**Example 1:** Falling large cold bubble and rising small warm bubble. Potential temperature evolution.

t=0

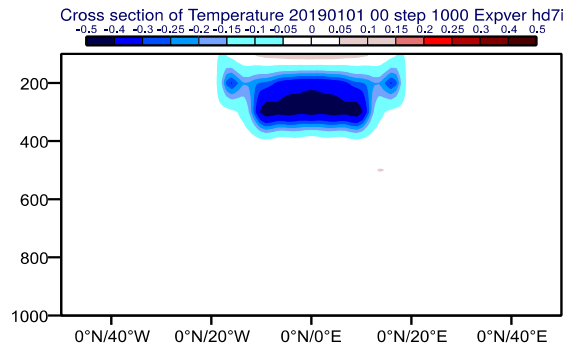


t=0



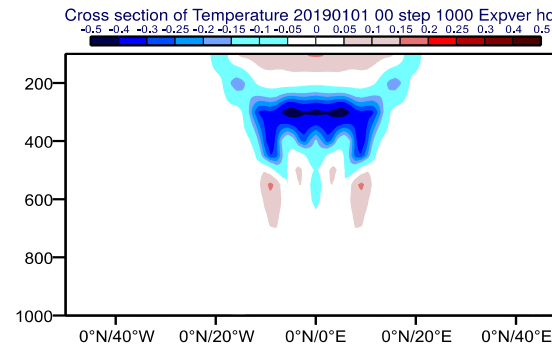
Non-hydrostatic

t=1000s

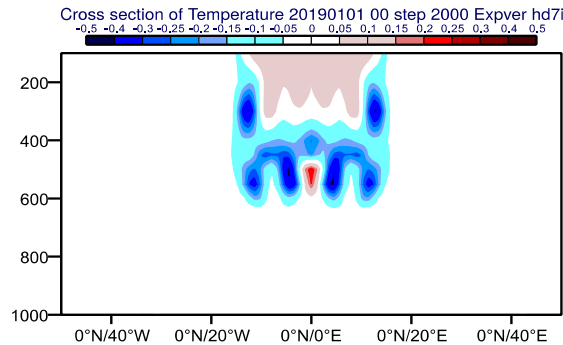


Hydrostatic

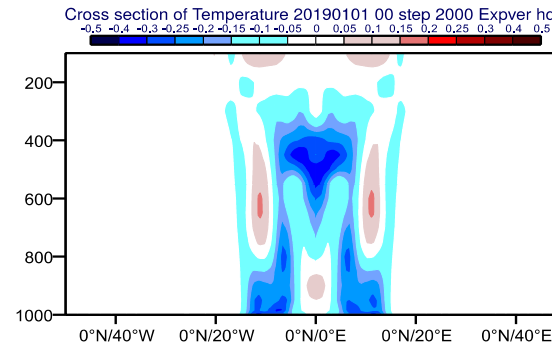
t=1000s



t=2000s



t=2000s



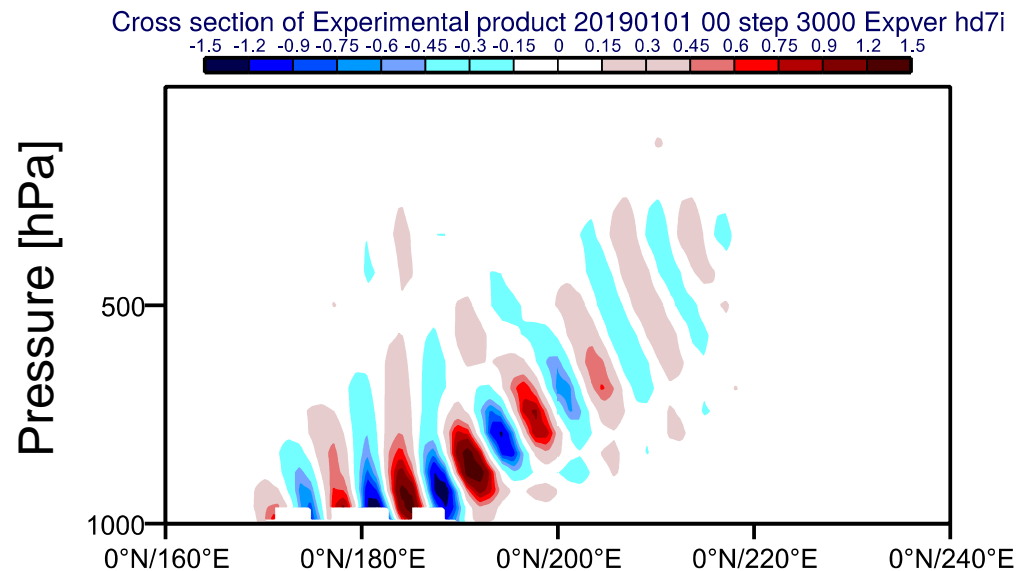
# Non-hydrostatic vs. hydrostatic model of the IFS: Examples

**Example 2:** Flow over idealized orography on a small planet with radius 30 km:

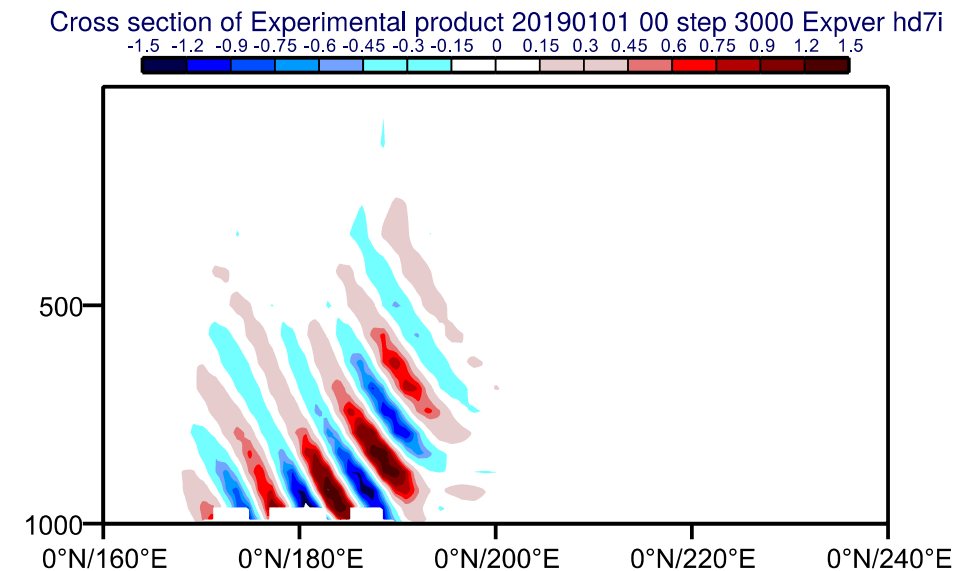
- Dispersion relation for hydrostatic and non-hydrostatic gravity waves is different.
- In practice, non-hydrostatic effects only important at 1km horizontal resolution.

Vertical cross section of vertical velocity of flow over Gaussian mountain with height 250m and width 5km

Non-hydrostatic



Hydrostatic





## Part 3: Recap

- Non-hydrostatic effects believed to become important for horizontal resolution finer than 10 km.
- Non-hydrostatic effects are particularly important for resolved convection as well as small-scale gravity waves.
- There exists a non-hydrostatic equivalent of the IFS dynamical core, but it is expensive due to having two additional prognostic equations AND the predictor-corrector numerics.
- In practice, non-hydrostatic effects emerge in medium-range weather forecasts for horizontal grid-spacings  $< 4.4\text{km}$  and mostly over regions dominated by steep orography only.