



# Research on Error Traceability Technology Based on Scale Analysis

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**10SEP.2024**

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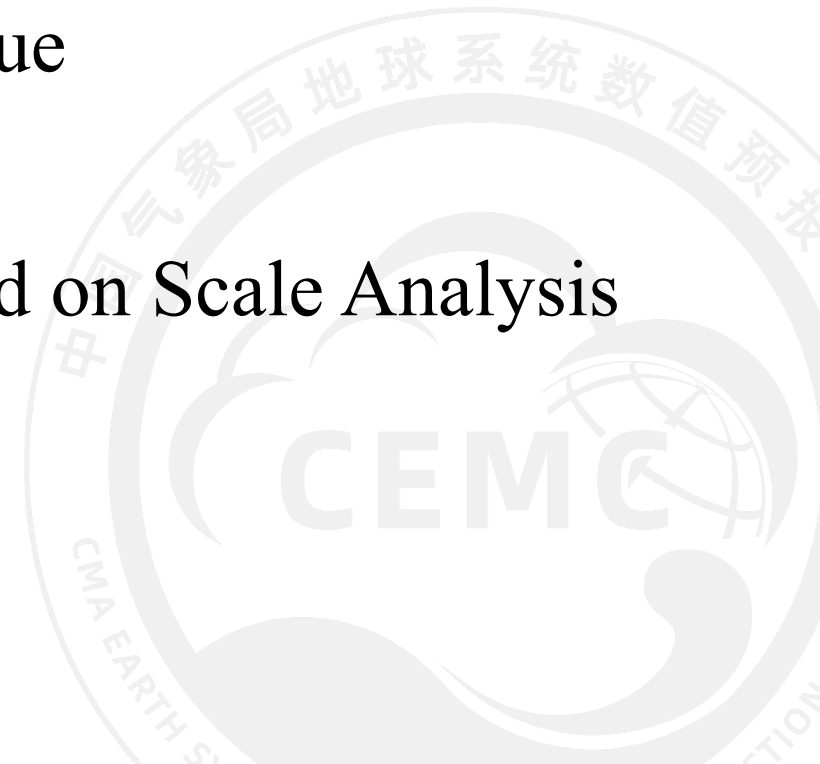
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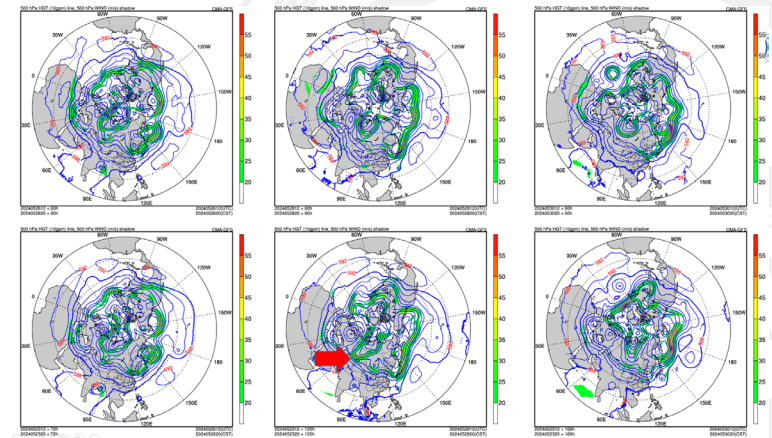
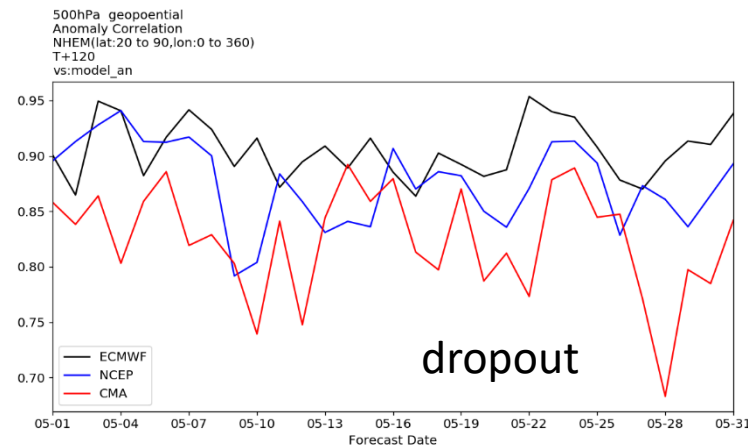
Conclusions



# Motivation

## How to trace error?

- ❑ Analysis the evolution of forecast errors for different time periods directly
- ❑ Consistency analysis of forecast characteristics in different models
- ❑ Traceability analysis based on diagnostic technology
  - ✓ Relaxation method
  - ✓ Scale analysis
  - ✓ Cluster analysis

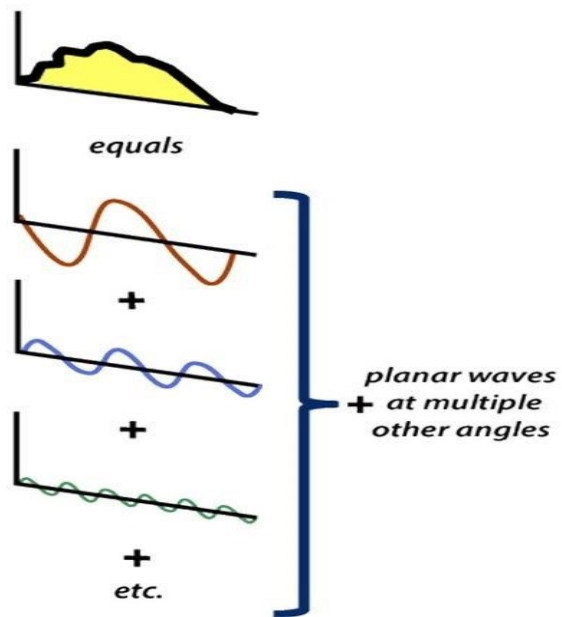


Search for possible reason

# Scale Analysis Method

- Fourier analysis can be used as a filter to achieve spatiotemporal scale separation and obtain statistical information at different scales, which is very important for error tracing analysis.
- Saltzman was one of the earliest to use latitudinal Fourier decomposition to obtain the balance equations of kinetic energy and effective potential energy in the wavenumber domain, and to investigate the role of specific scale eddies in energy conversion.
- Saltzman and Teweles used the latitudinal Fourier transform to obtain the distribution characteristics of energy spectral balance with wave number.

1D Fourier Projection



$$f(x), 0 \leq x \leq N - 1$$

$$\text{Fourier transform: } F(u) = \sum_{x=0}^{N-1} f(x) e^{-j\frac{2\pi}{N}ux}$$

$$\text{Inverse Fourier transform: } f(x) = \frac{1}{N} \sum_{u=0}^{N-1} F(u) e^{j\frac{2\pi}{N}ux}$$

$$\text{Amplitude spectrum: } |F(u)| = [R^2(u) + I^2(u)]^{\frac{1}{2}}$$

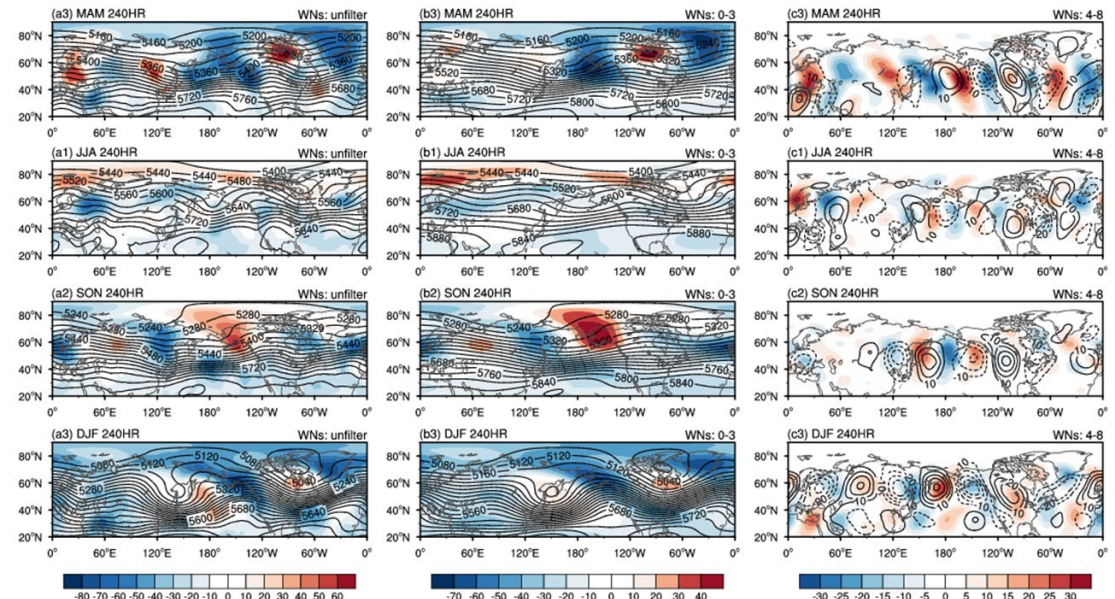
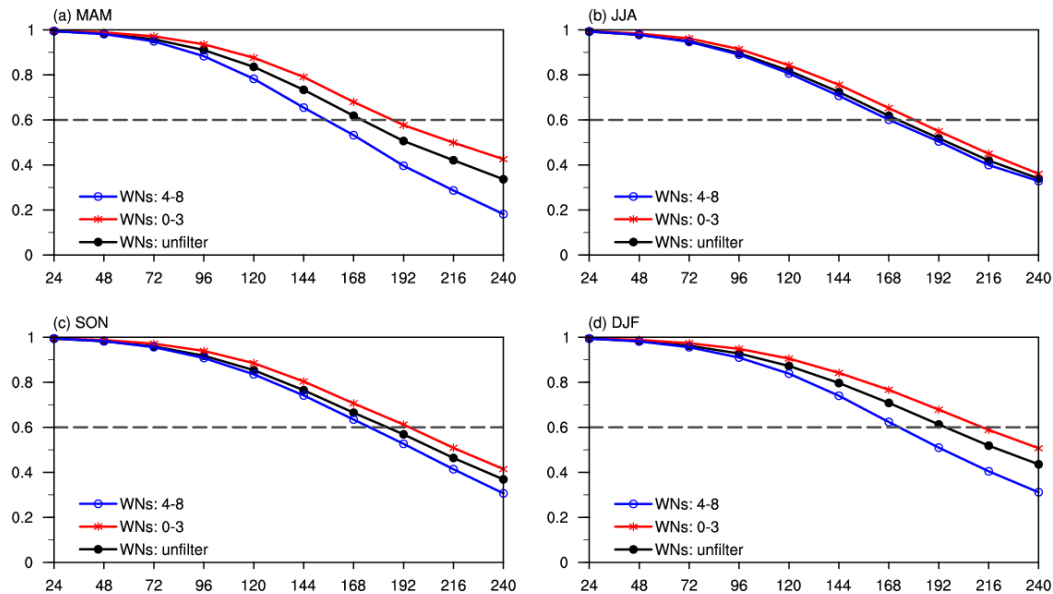
$$\text{phase spectrum: } \varphi(u) = \tan^{-1} \left[ \frac{I(u)}{R(u)} \right]$$

$$\text{Power spectrum: } E(u) = R^2(u) + I^2(u)$$

# Multiscale feature analysis of forecast errors of 500 hPa geopotential height for the CMA-GFS model

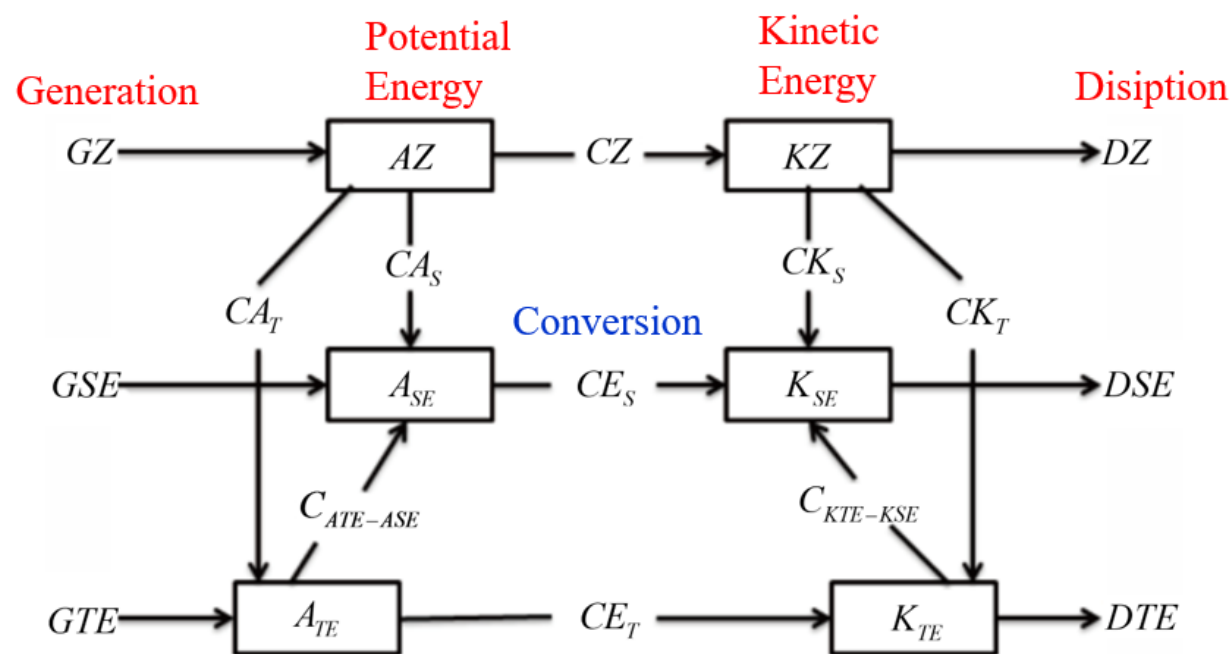
During: 2021.03-2022.02

- ✓ There are no significant seasonal or scale differences in the ACC evolution of 500 hPa geopotential height and its multiscale components, and they all decrease with the extension of forecast lead time and the effective forecast *skills are the highest* in winter and the planetary-scale components.
- ✓ The forecast error distribution of 500 hPa geopotential height and its multiscale components show significant seasonal differences in the location of the error extremum centers. The forecast errors for the original field and planetary-scale components mainly reflect the inadequate prediction of the intensity of large-scale trough and ridge systems at middle and high latitudes.



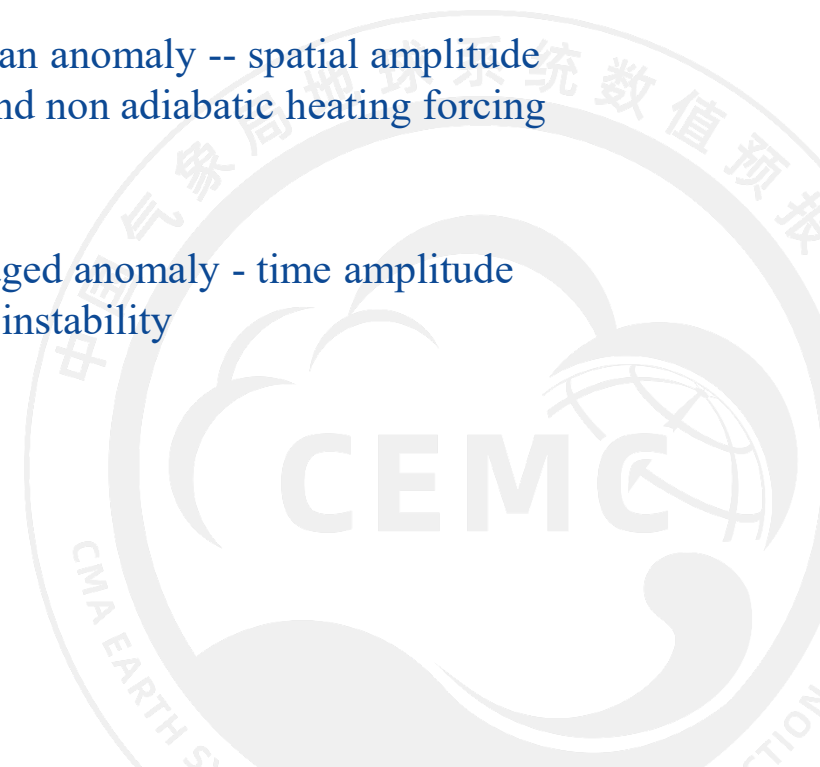
From: Siyuan Sun, Li Li, Bin Zhao\*, Yiyi Ma, Jianglin Hu. 2023. Multiscale feature analysis of forecast errors of 500 hPa geopotential height for the CMA-GFS model. Atmospheric Science Letters, 24(10): e1174.

# Mixed Space-Time Domain Energy Cycle Framework



**Mixed Space-Time Domain energy cycle**

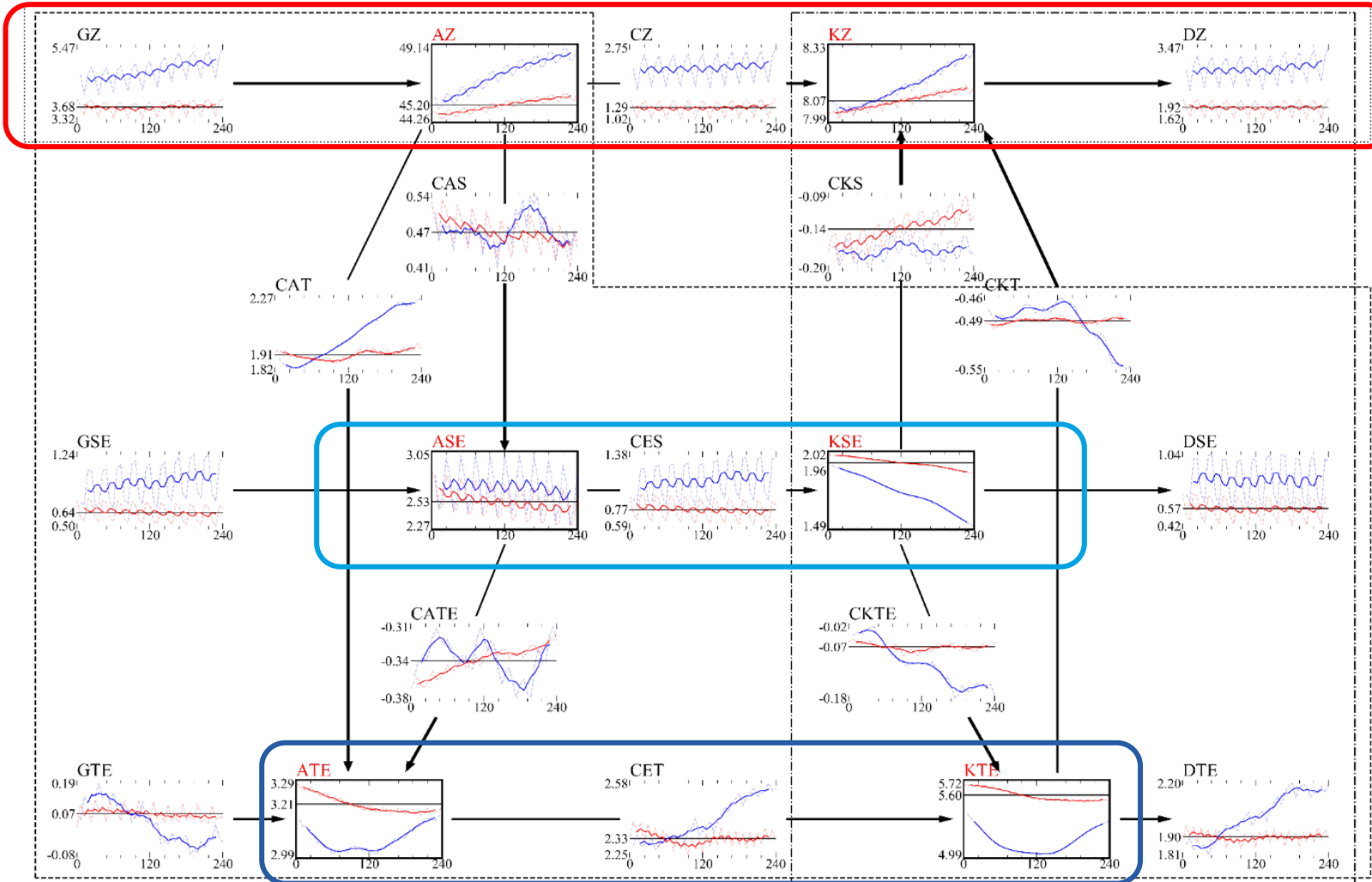
From: Ulbrich and Speth 1991



# Energy Cycle Method based on Scale Analysis

—CMA-GFS

—ERA5



Stronger circulation

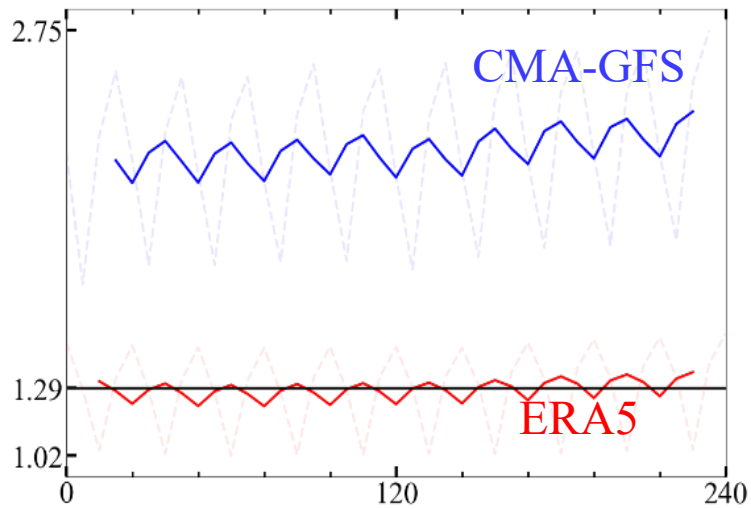
Diurnal variation of temperature  
Stronger ASE and Weaker KSE

Weaker transient energy

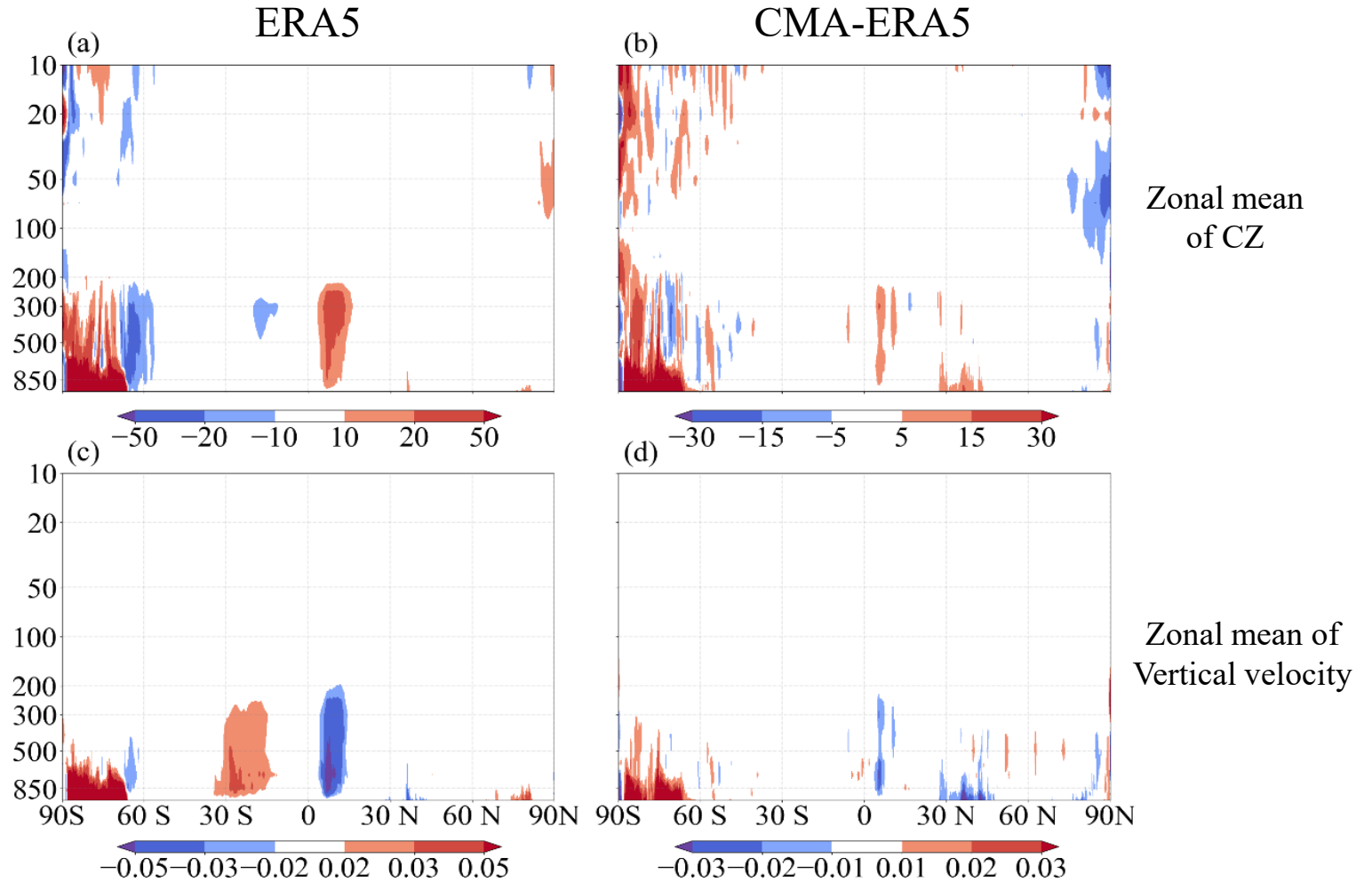
Variation of energy cycle for ERA5 and CMA-GFS with lead time in Jul2022

# CZ: conversion from AZ to KZ

$$CZ = -([\bar{\omega}] - \{\bar{\omega}\})([\bar{T}_v] - \{\bar{T}_v\}) \frac{R}{g \cdot p}$$

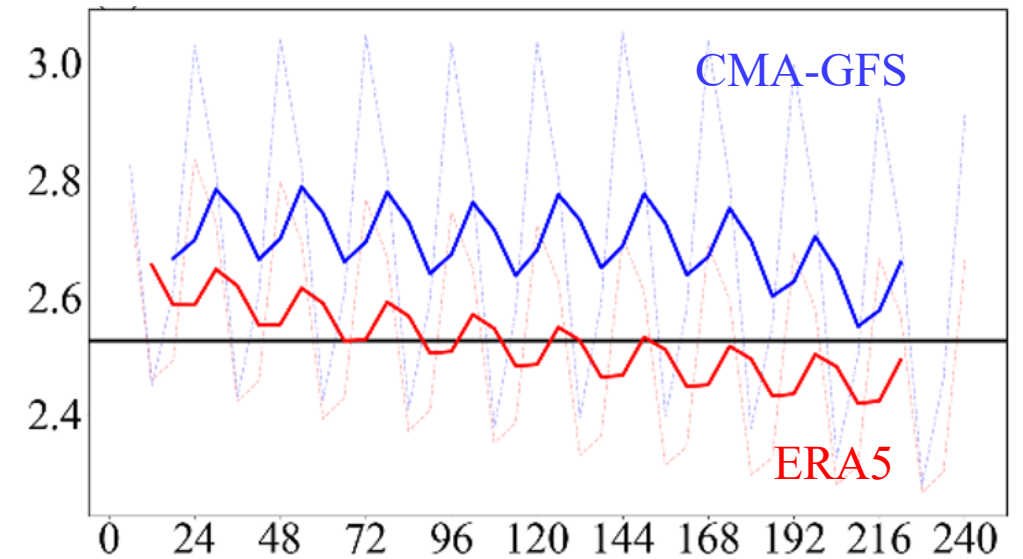
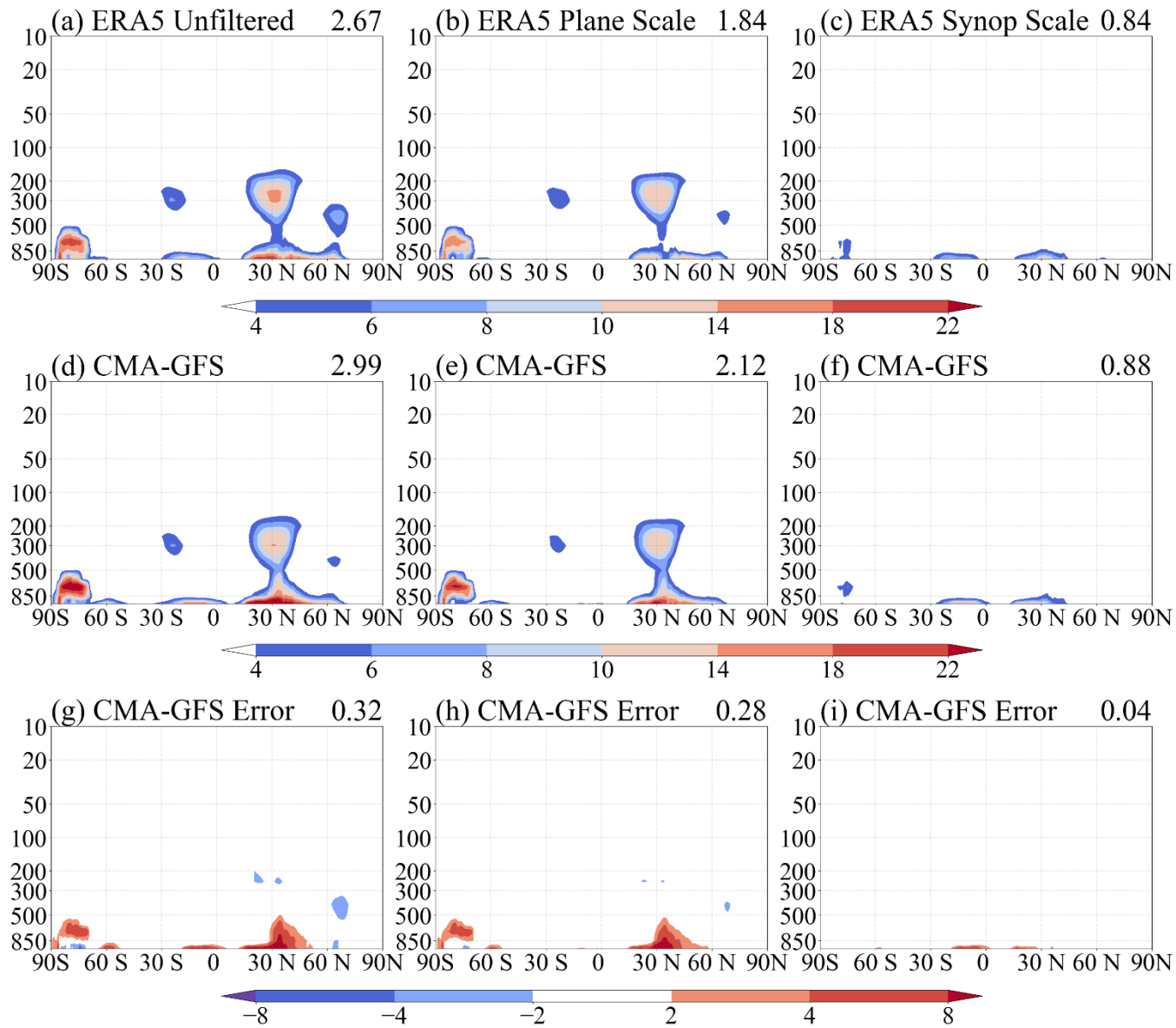


CZ error is caused by the strong direct thermal circulation



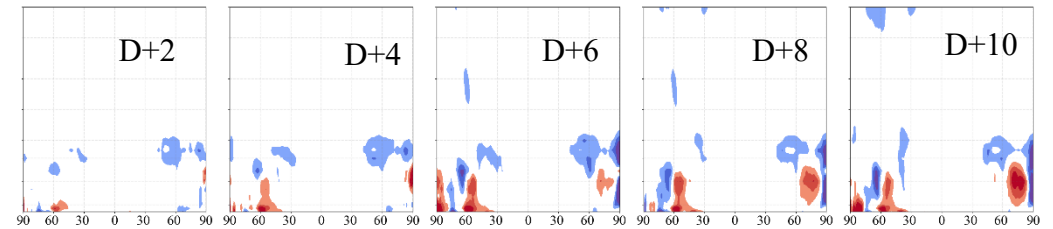
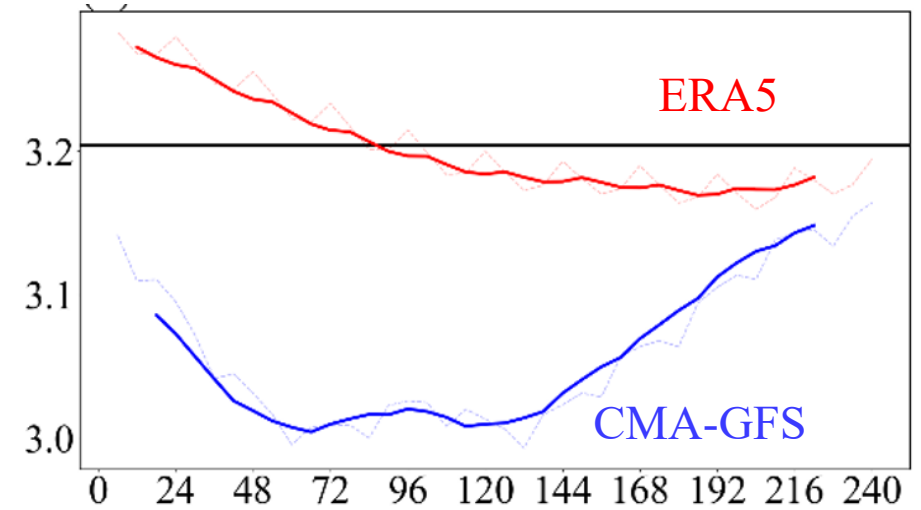
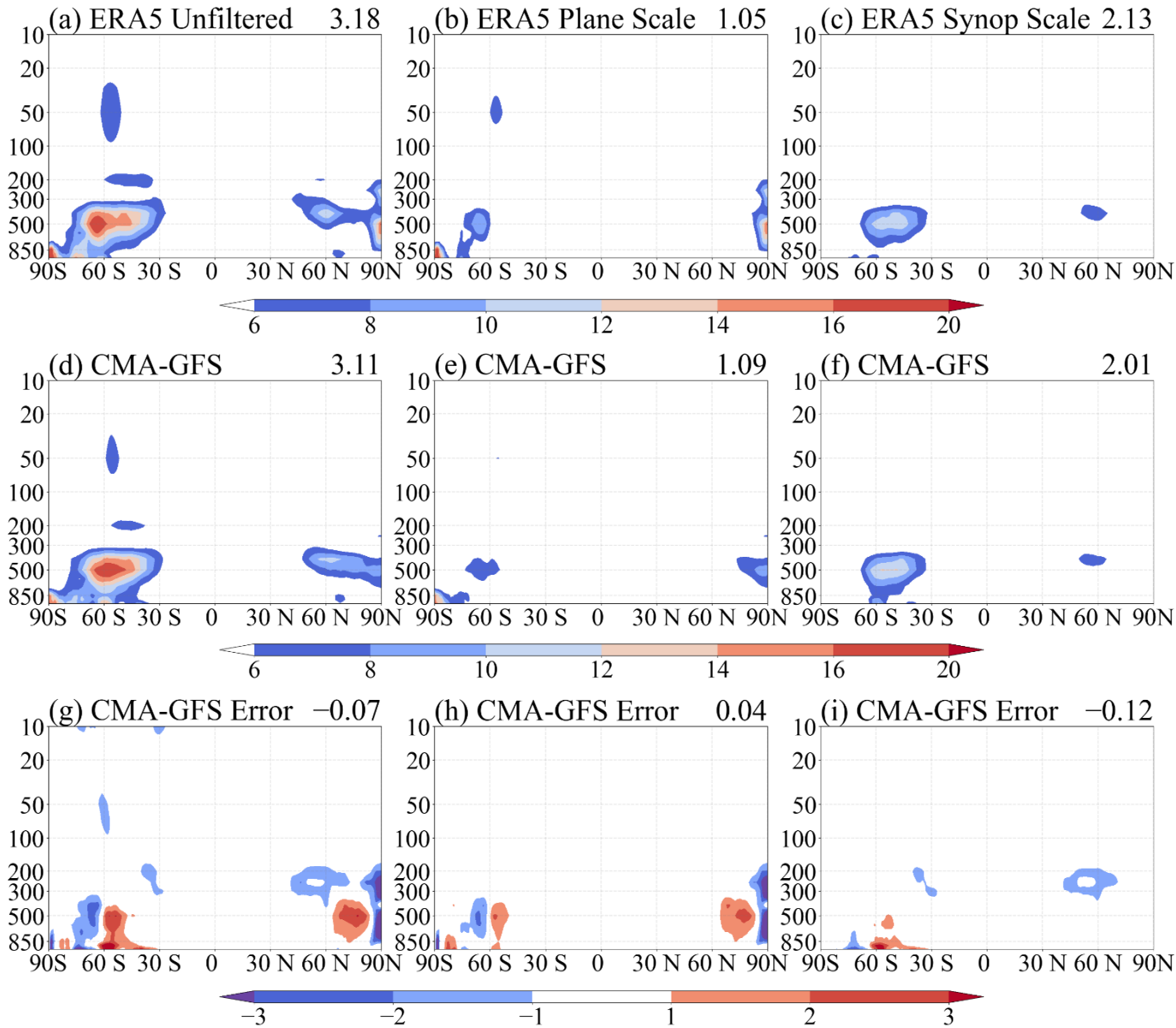


# ASE: Stationary eddy available potential energy



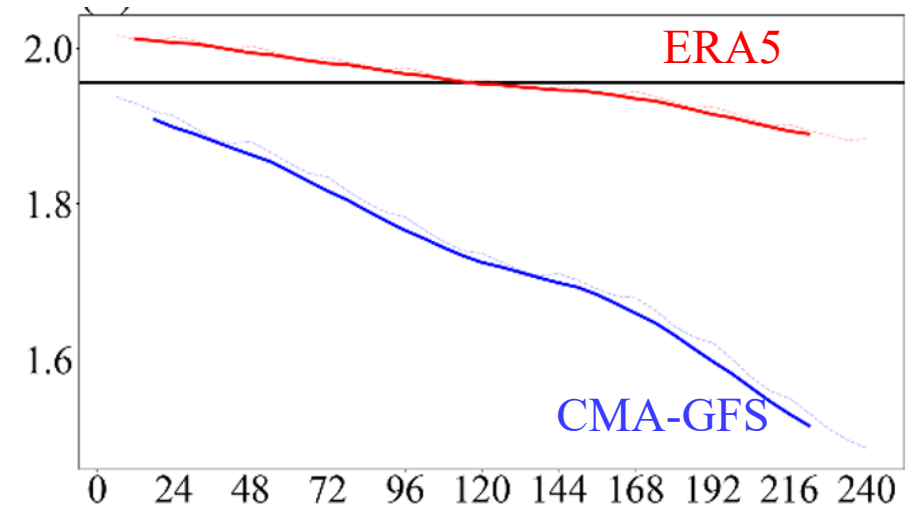
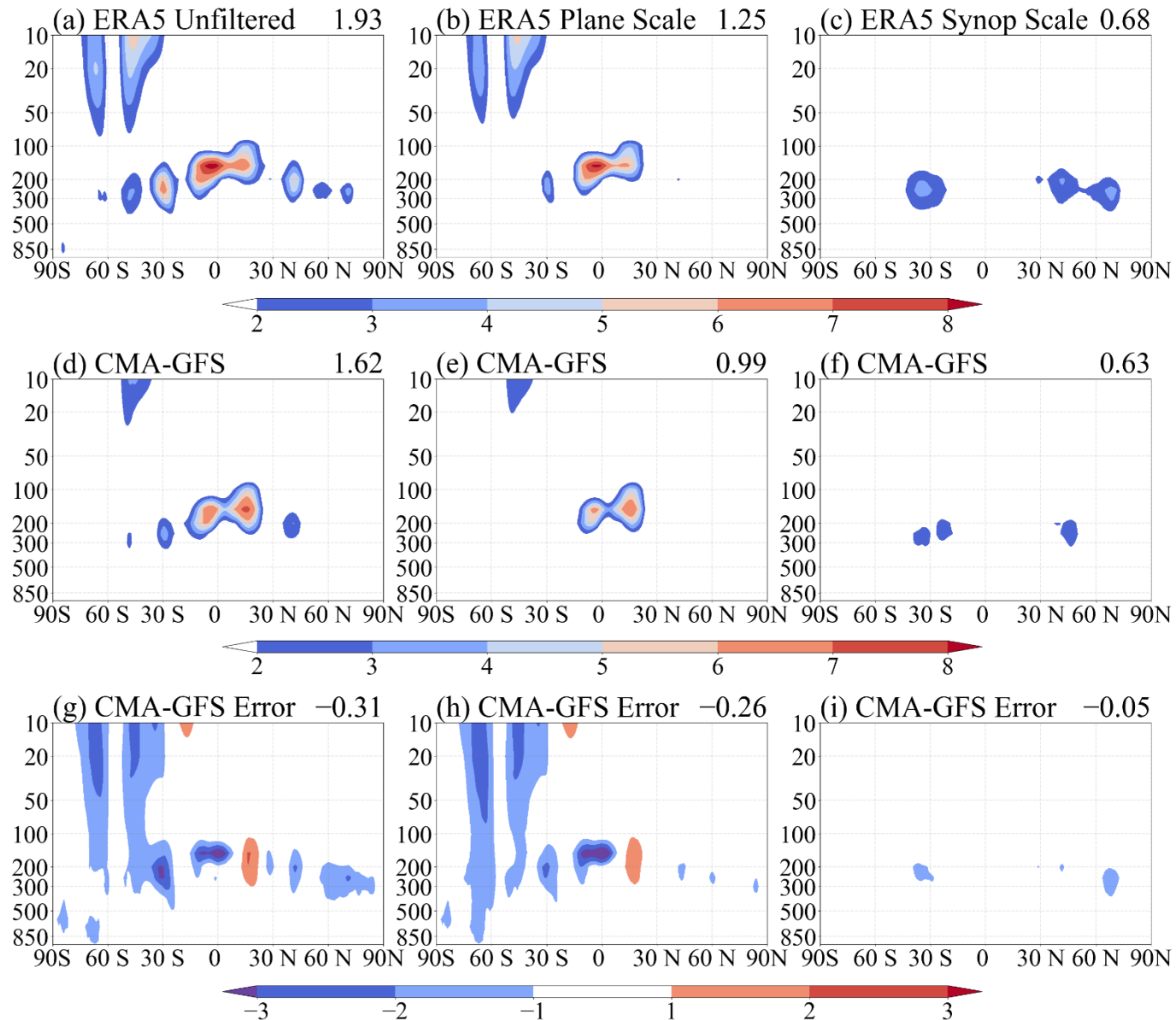
- Planetary scale accounts for 70% and contributes over 90% of the error, while ASE is mainly dominated by planetary scale
- Planetary scale errors are related to terrain and non adiabatic heating;
- The synoptic scale error is caused by non adiabatic heating

# ATE: Transient eddy available potential energy



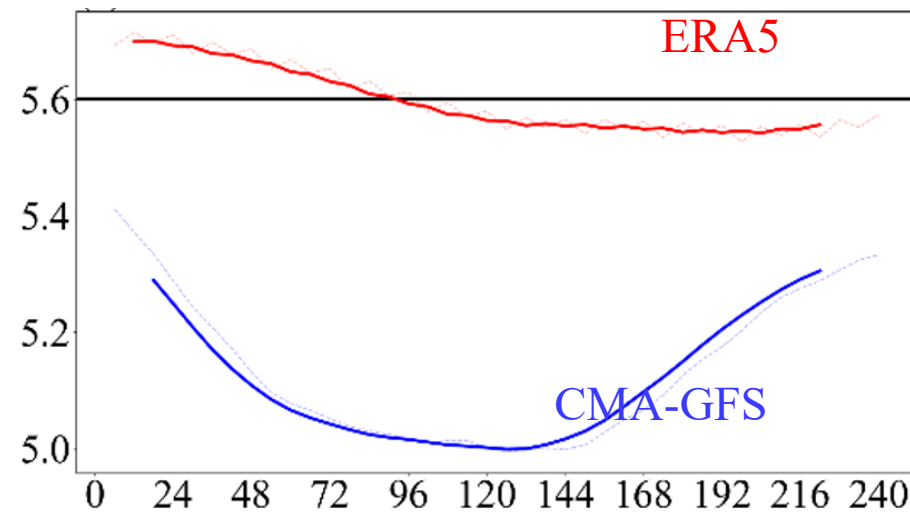
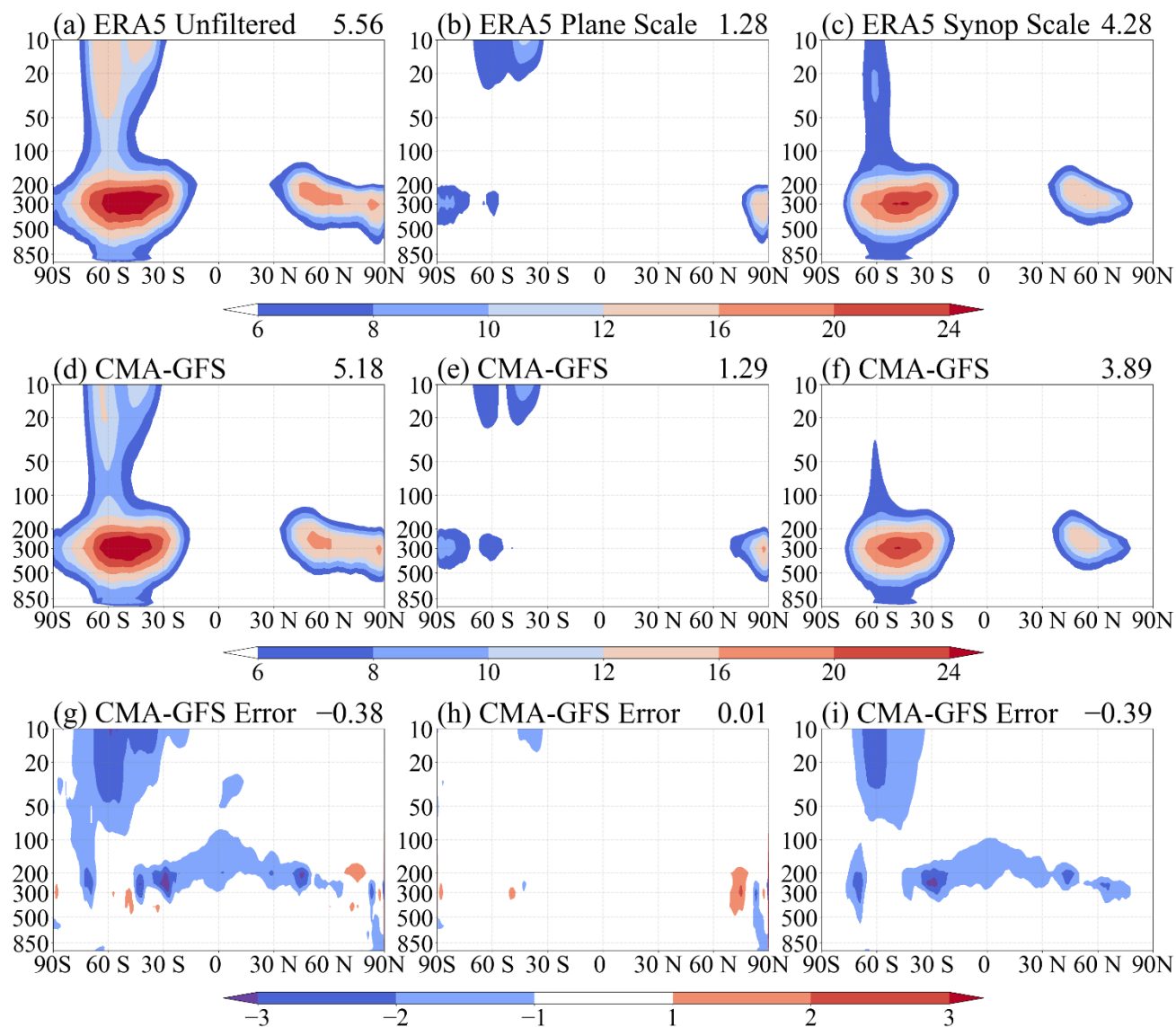
- The error is mainly related to the thermodynamic conditions of the circulation;
- The growth of ATE in the later stage is due to the increase of meridional temperature gradient.

# KSE: Stationary eddy kinetic energy



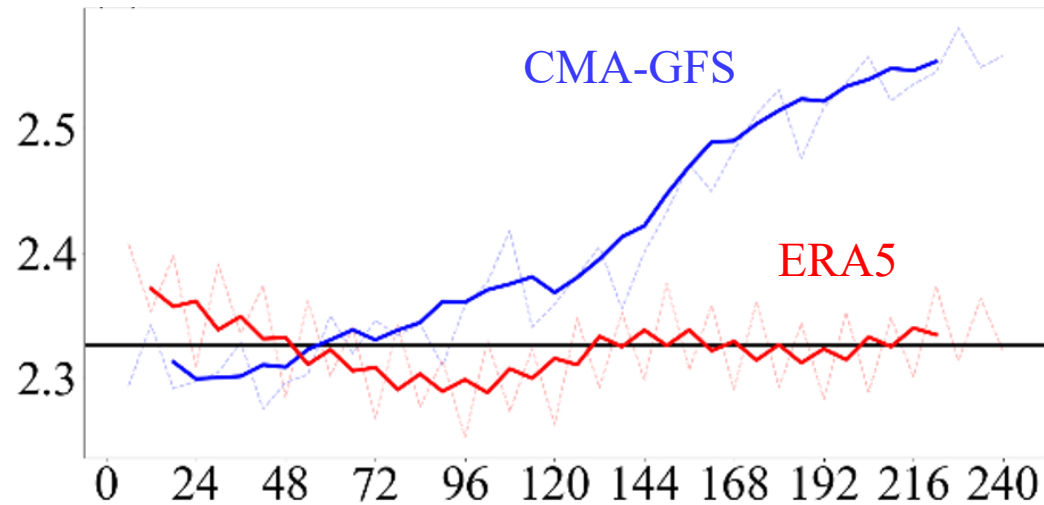
- The KSE extremum is located at the top of the tropical troposphere in the Southern Hemisphere, while other high value areas coincide with the position of the tropospheric jet stream.
- Extending downwards from the top of the stratosphere at high latitudes in the southern hemisphere, it is distributed on both sides of the polar night jet stream
- Planetary scale accounts for 86% of the total error

# KTE: Transient eddy kinetic energy



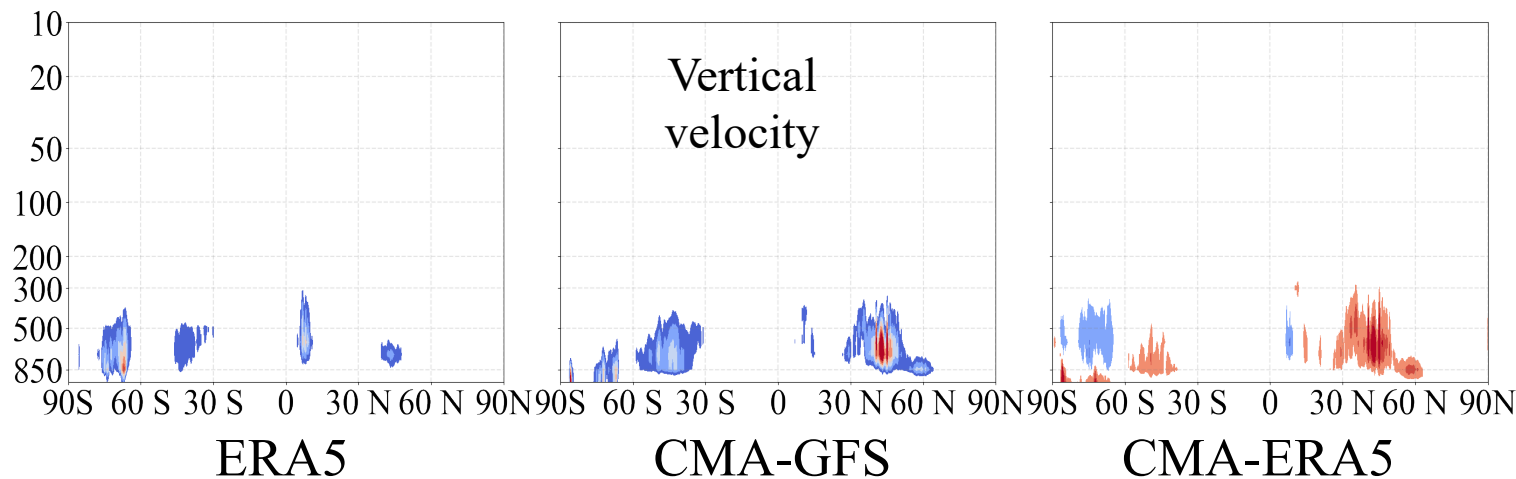
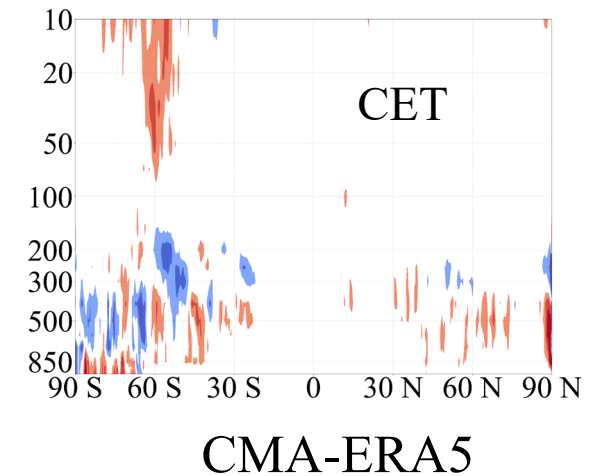
- The maximum value occurs in the upper troposphere of high latitudes in the Southern Hemisphere, which is associated with the subtropical jet stream
- The planetary scale components are mainly located in the upper stratosphere of the southern hemisphere and the polar regions of both hemispheres, which may be due to the dense grid pattern in the high latitude latitudinal grid, resulting in small spatial scales being identified as larger spatial scales in the spectral space
- The negative error is entirely caused by the synoptic scale

# CET: conversion from ATE to KTE



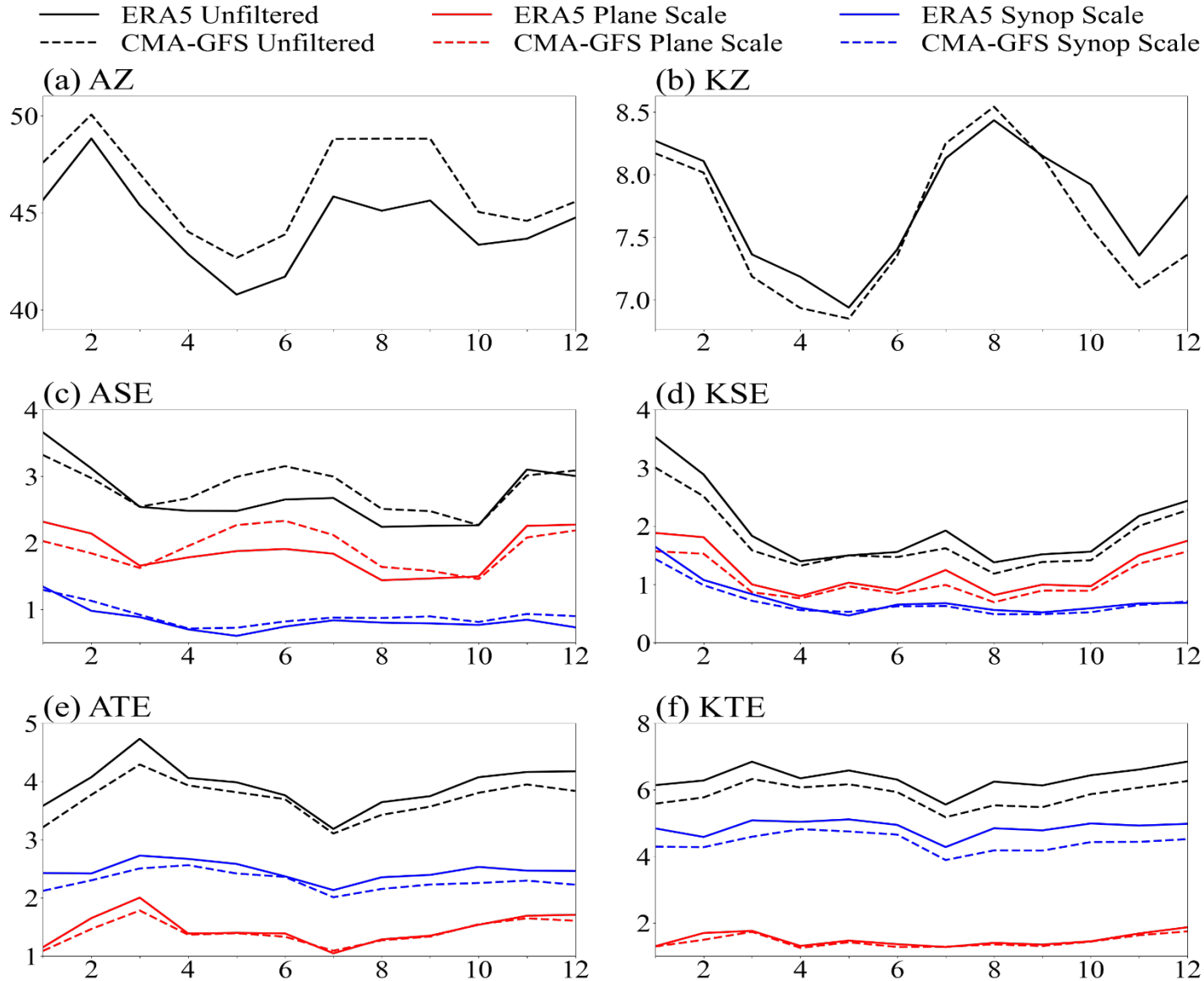
Why it is so large in CMA-GFS?

$$\text{CET} = - \left[ \overline{\omega' T'_v} \right] \frac{R}{g \cdot p_v}$$



- The strong transient ascending motion in mid to high latitudes leads to a stronger CET, which continues to increase with the forecast time
- CMA-GFS has a stronger forecast for convective systems in mid latitudes, which is the main reason for the stronger CET
- The upward movement in tropical regions of the northern hemisphere is weaker and the extreme values are more dispersed, while the negative deviation in high latitudes of the southern hemisphere may be related to terrain

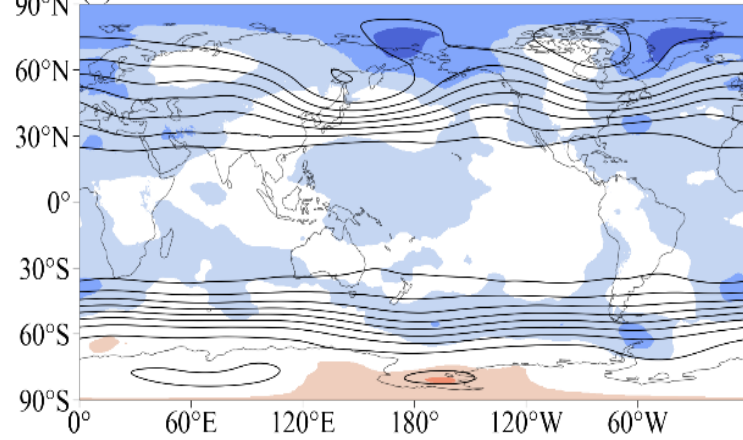
# Multi scale seasonal variation



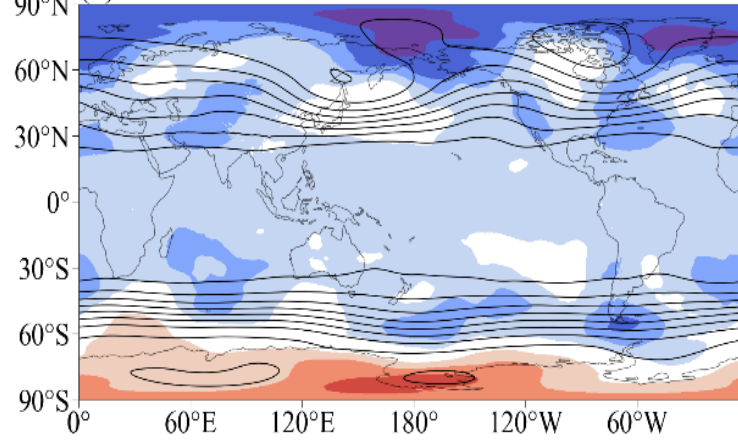
- The planetary scale dominates the stationary vorticity energy (ASE and KSE), while the synoptic scale dominates the transient vorticity energy (ATE and KTE).
- The monthly variation of eddy energy is mainly reflected in the fact that stationary eddies are higher in winter and summer, and lower in other seasons.
- The peak of transient energy occurs in the spring and the minimum in the summer.

# Systematic error in 500hPa geopotential height forecasting

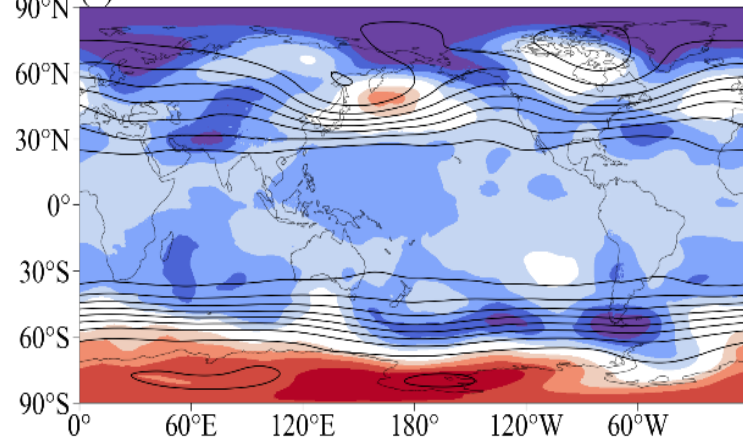
(a) DJF 3d Unfiltered



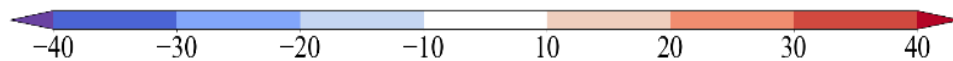
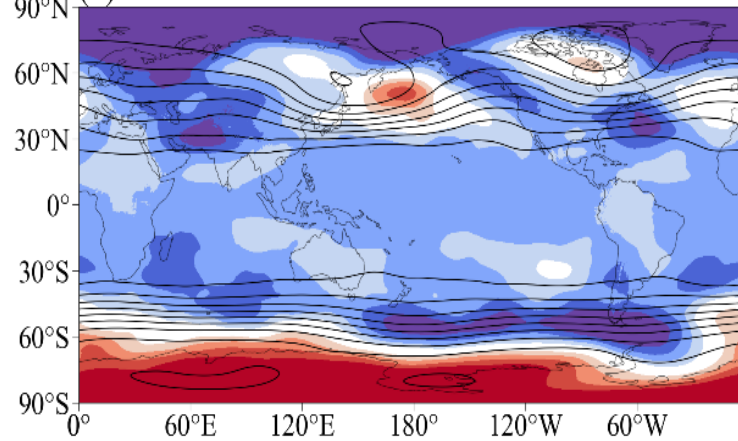
(b) DJF 5d Unfiltered



(c) DJF 8d Unfiltered

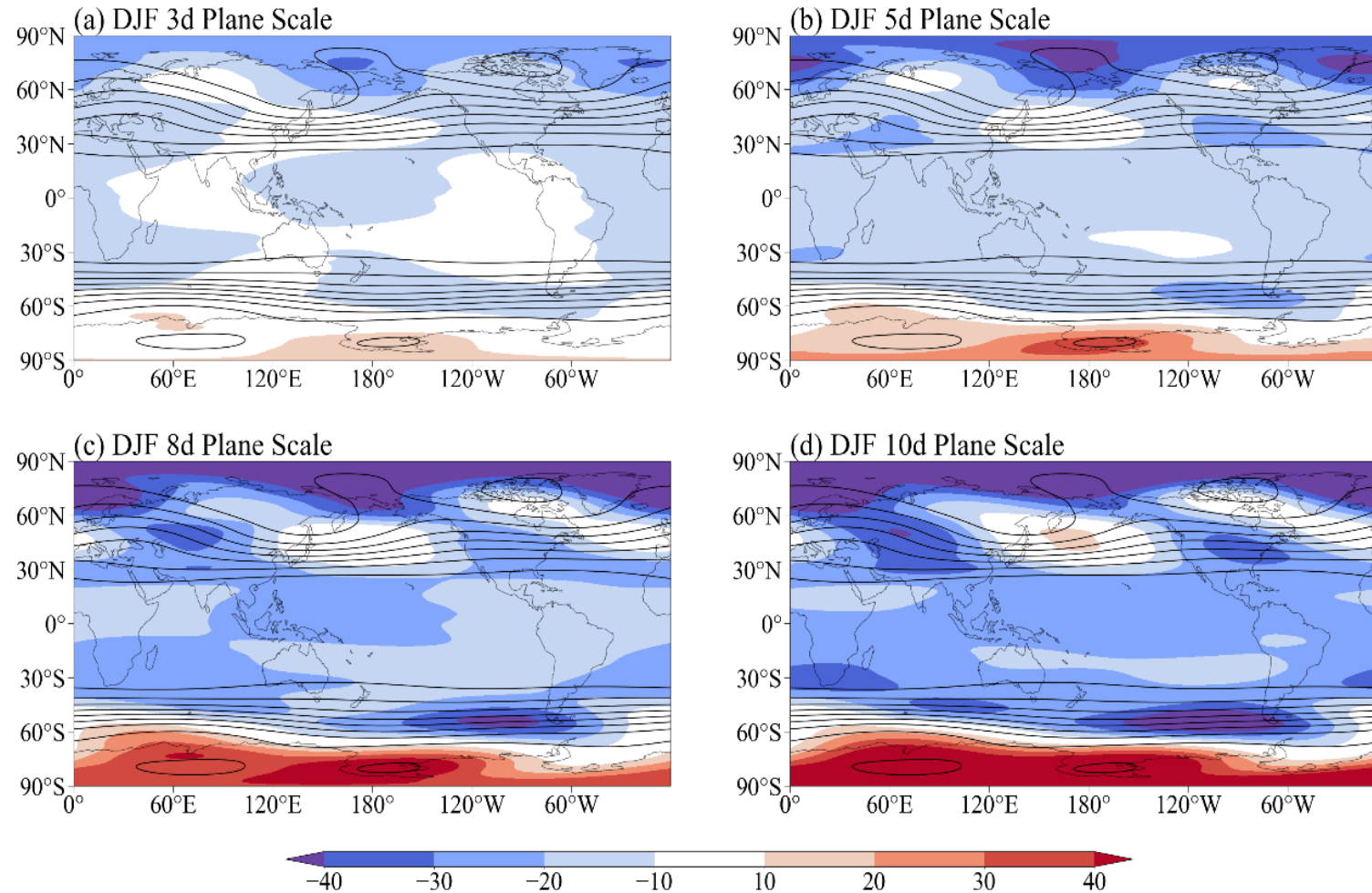


(d) DJF 10d Unfiltered



- The overall deviation is related to the errors in the northern hemisphere and the troughs and ridges;
- The southern hemisphere shows a banded distribution;
- The error in mid to high latitudes increases rapidly.

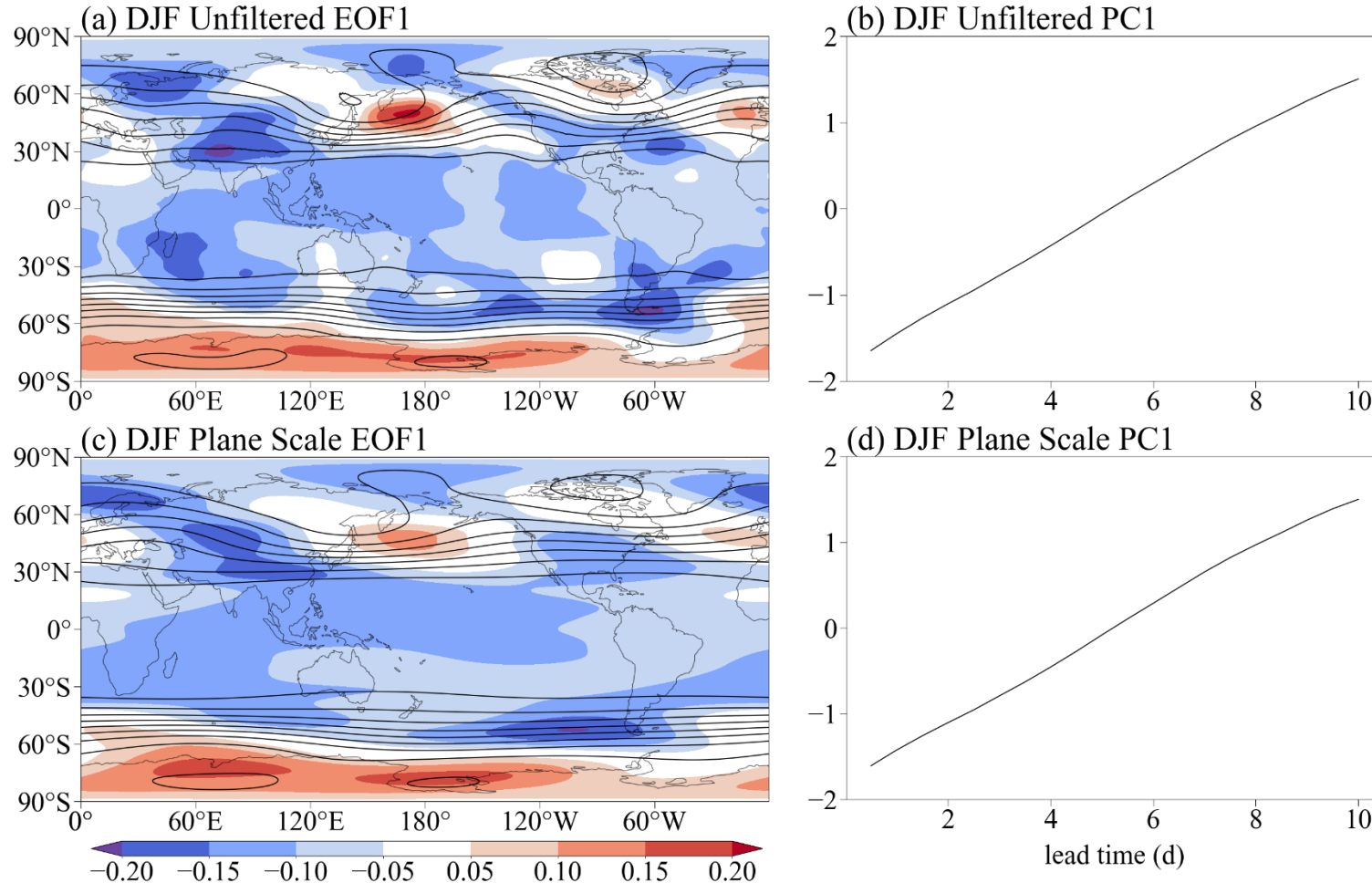
# Systematic error in winter 500hPa geopotential height forecasting



Similar to the original field error, it mainly indicates the strength and position error of the trough and ridge in the northern hemisphere

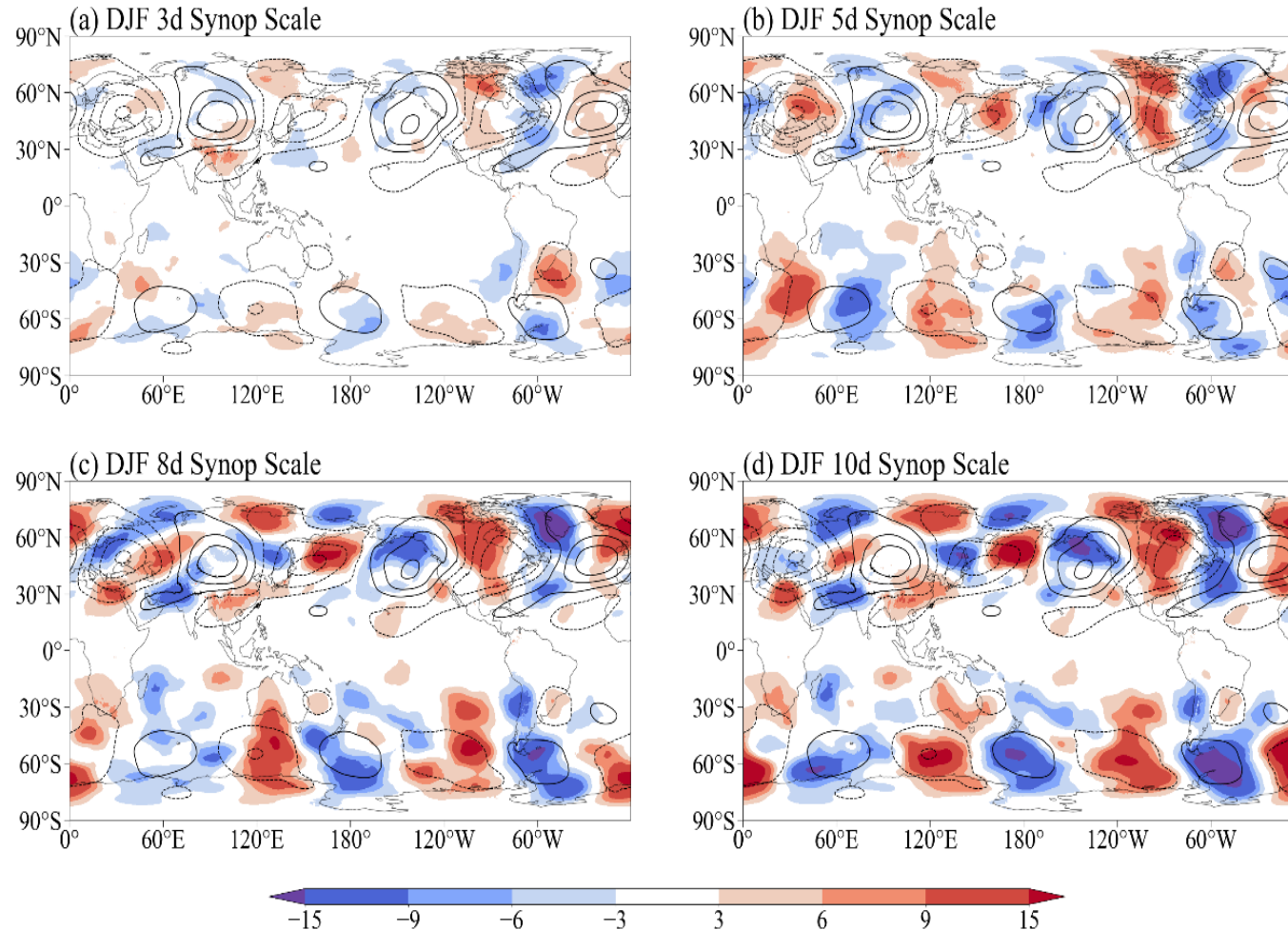


# EOF characteristic mode



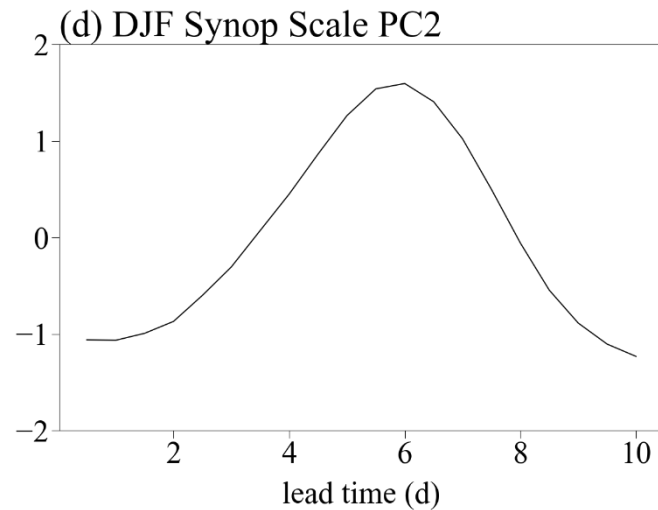
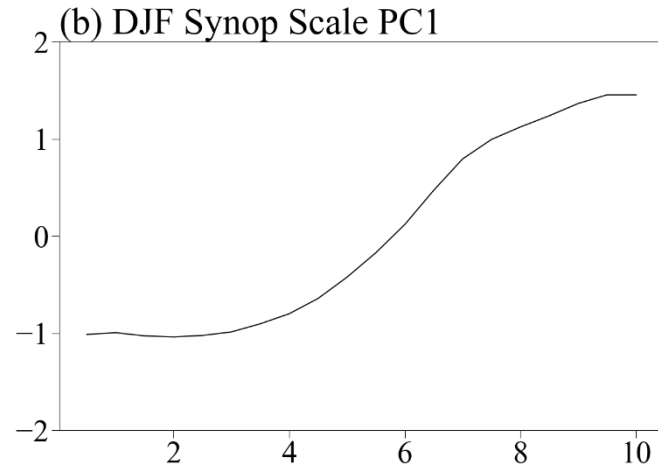
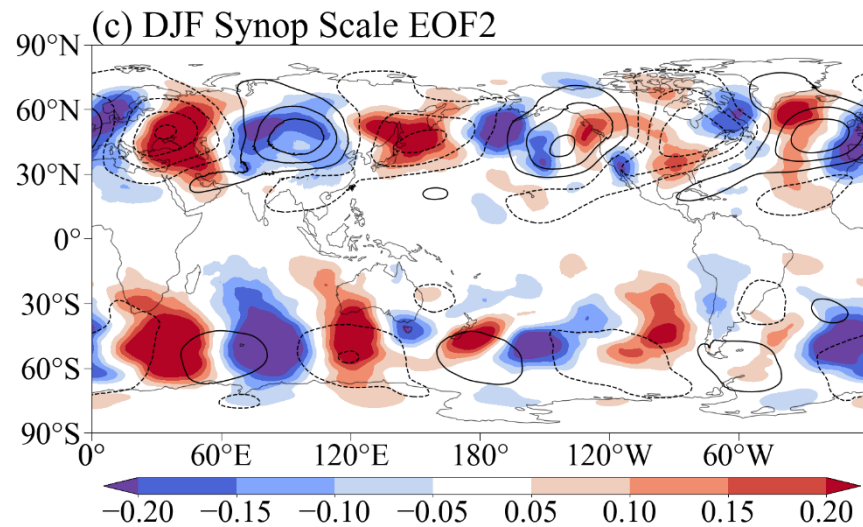
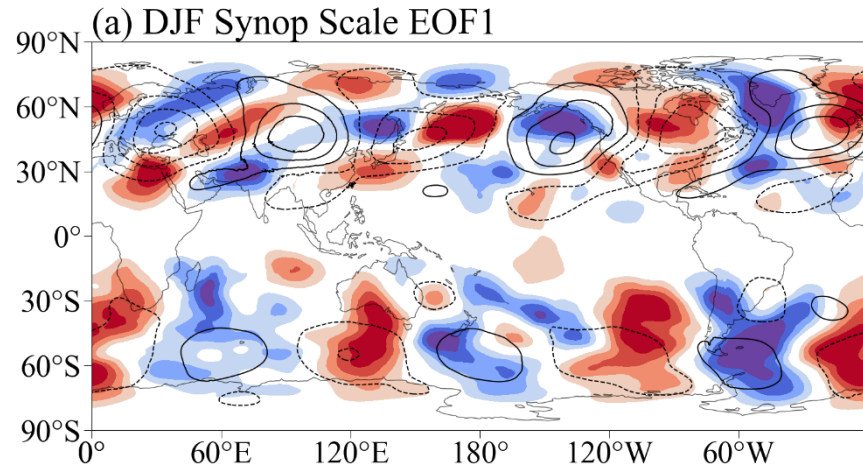
The errors of the unfiltered field and planetary scale components mainly increase linearly according to the first mode

# Systematic error in winter 500hPa geopotential height forecasting



- Errors in positive and negative wave train patterns;
- The central position and intensity vary with the forecast time

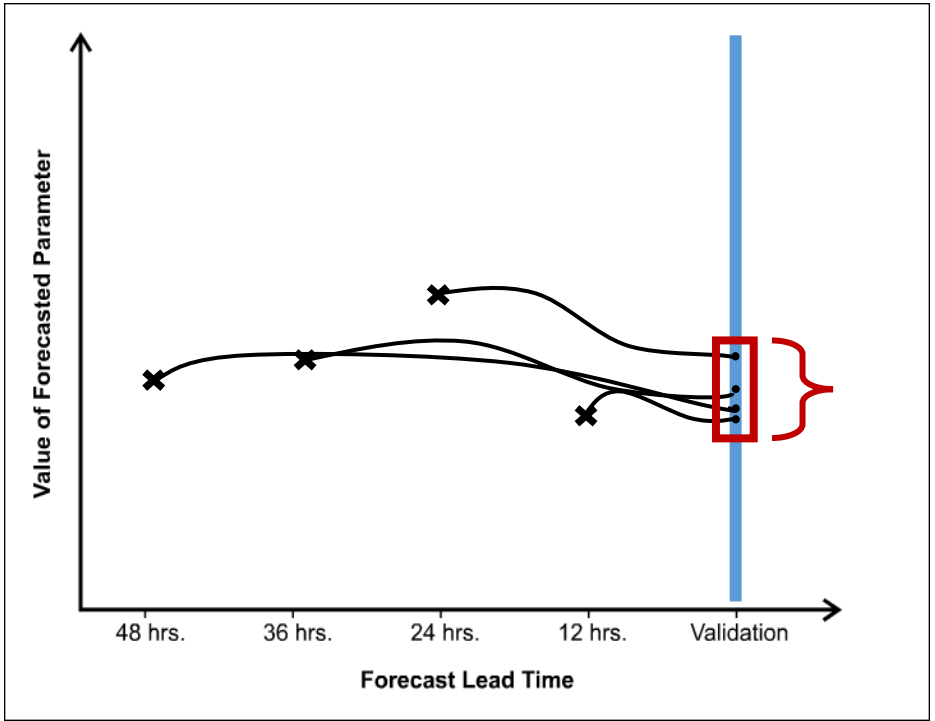
# EOF characteristic mode



- The leading mode mainly shows a linear growth trend;
- The second mode presents periodic changes

- ❑ The area with large error values is mainly related to the position of the trough and ridge. The center of the large error values of the original field and planetary scale components does not change significantly with the forecast time, while the center of the synoptic scale and finer scale components changes with the forecast time;
- ❑ With the forecast time increasing, the error values of the original field and planetary scale components continue to increase, but the distribution of large value centers does not show significant changes, which can be characterized by linear growth in the leading mode; The center and structure of the large error components of synoptic and finer scales will change, and the periodic changes represented by the second mode need to be considered.

# Forecast consistency issue



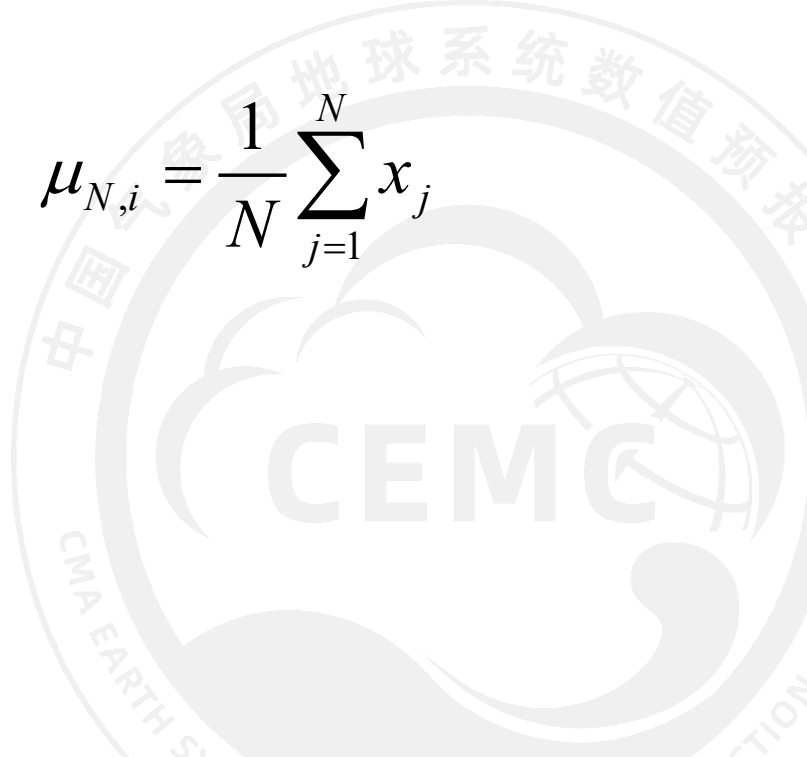
The ratio of standard deviation to mean error

$$SPRDI_{N,i} = \frac{\sigma_{N,i}}{\mu_{N,i}}$$

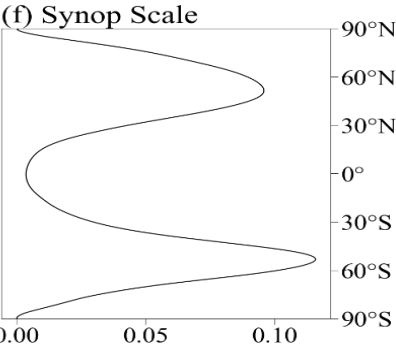
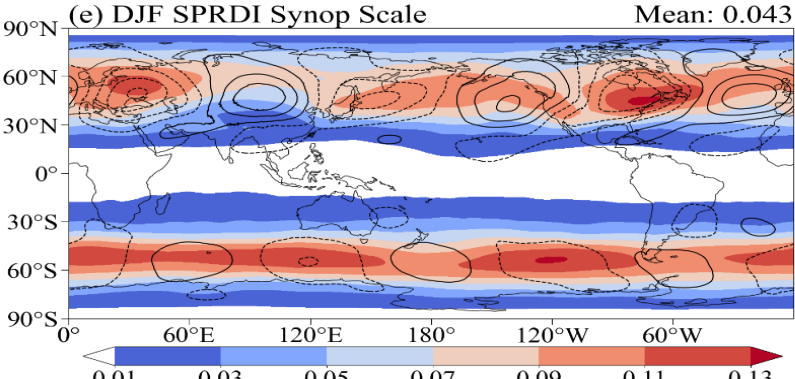
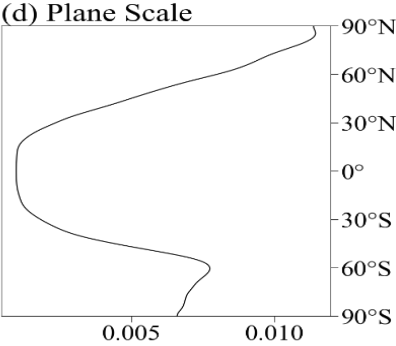
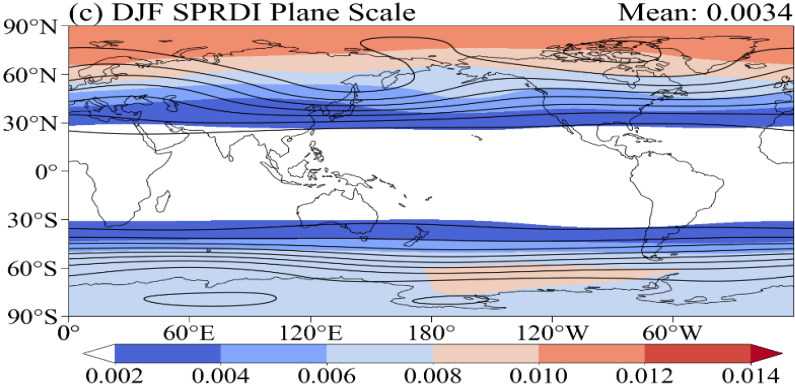
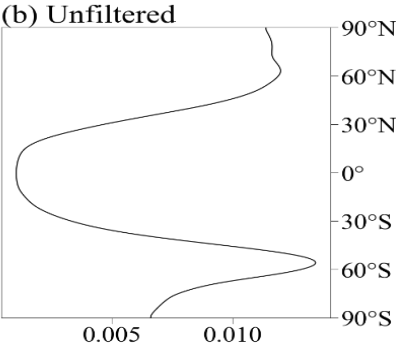
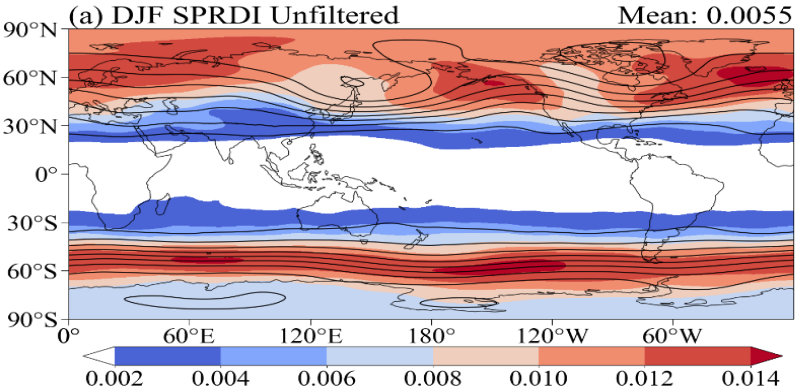
$$\sigma_{N,i} = \sqrt{\frac{1}{N} \sum_{j=1}^N (x_j - \mu_{N,i})^2}$$

$$\mu_{N,i} = \frac{1}{N} \sum_{j=1}^N x_j$$

- ❑ The smaller the index, the better the consistency;
- ❑ Non dimensional quantity, which can compare the inconsistency of different forecast elements and scales;
- ❑ We can obtain spatial distribution characteristics



# Inconsistencies in winter 500hPa geopotential height forecasting



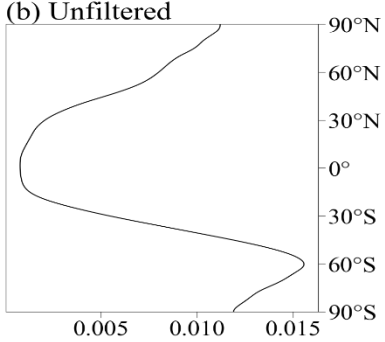
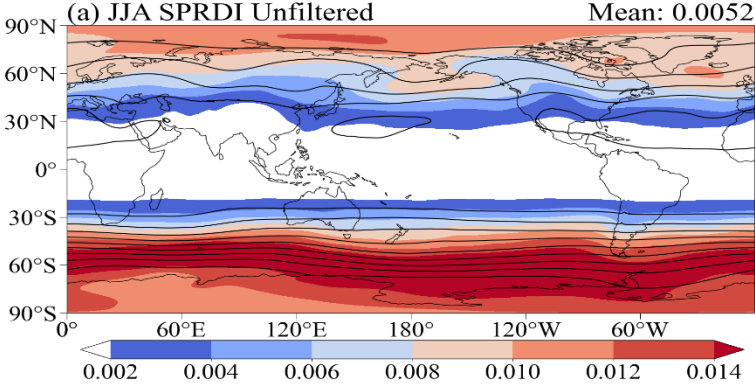
**Spatial distribution:**

There is significant inconsistency between the low value center of the synoptic scale, the planetary scale ridge, and the back and front of the unfiltered trough and ridge, respectively

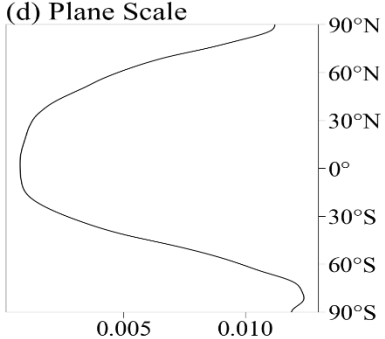
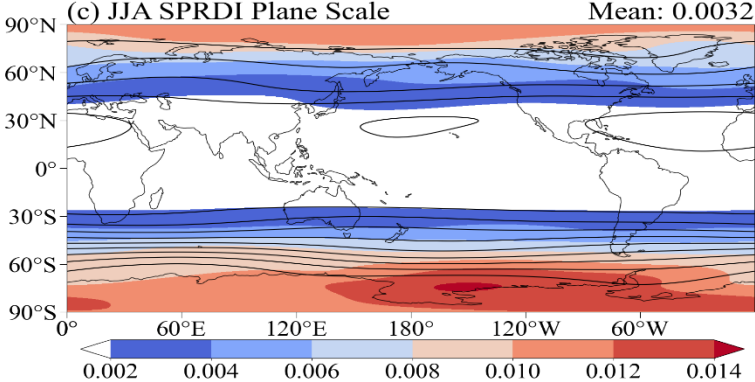
**Mean:**

The greatest inconsistency in synoptic scale

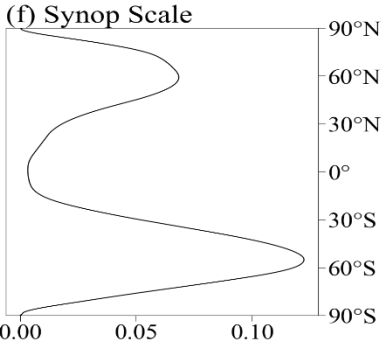
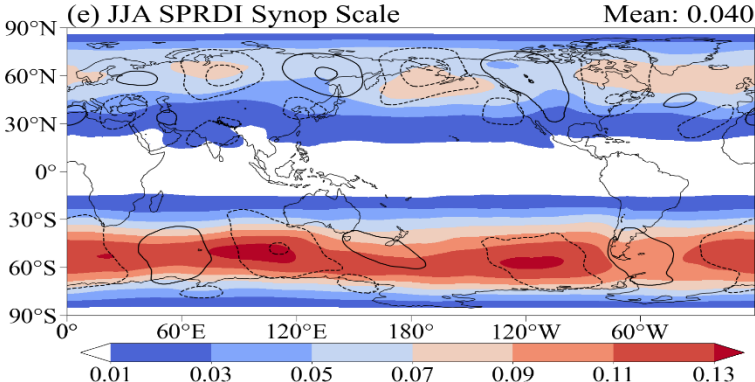
# Inconsistencies in summer 500hPa geopotential height forecasting



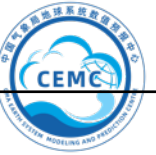
Spatial distribution:  
Banded distribution  
Mean:



The greatest inconsistency in synoptic scale



# Spatial verification for heavy rainfall based on scale analysis



**Method:** Harr filtering, wavelet

**Purpose:** To quantitatively evaluate the contribution of precipitation errors at different scales (orthogonal components)

**Advantages:** Method shaping, wide application, and easy access to refined information

Harr wavelet technique:

Harr wavelet: 1D→2D

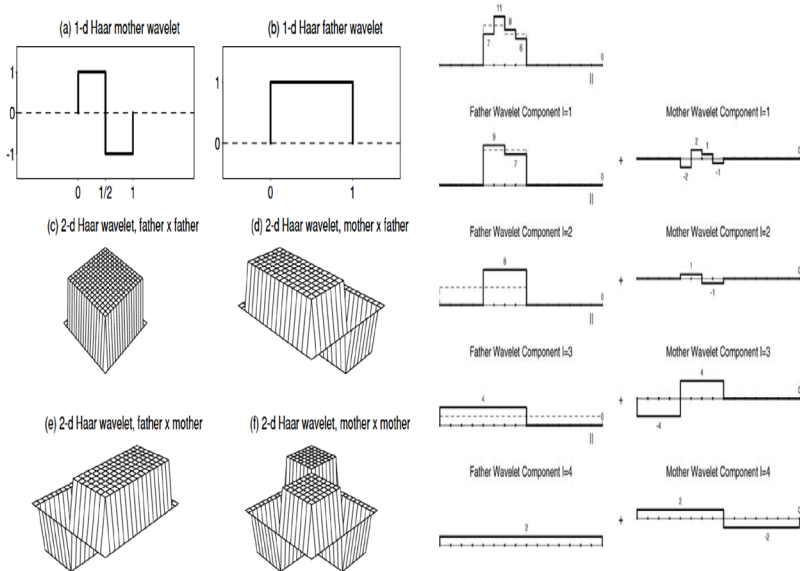
$$W = \sum_{l=1}^L W_{m,l} + W_{f,L+1}$$

$2^L \times 2^L$  GRID:

$$Z_u = \sum_{l=1}^{L+1} Z_{u,l}$$

Scales  $l : 1, 2, 3, \dots, L, L+1$

Resolution:  $2^{l-1} : 1, 2, 4, \dots, 2^{L-1}, 2^L$



## RMSE

Orthogonality of wavelet components :

$$MSE = \sum_{l=1}^L \sum_{l'=1}^L \overline{Z_l Z_{l'}} = \sum_{l=1}^L \overline{Z_l^2}$$



Additivity of scale components

$$MSE_u = \sum_{l=1}^{L+1} MSE_{u,l}$$



Mean square error additivity :

$$MSE\%_{u,l} = \left( \frac{MSE_{u,l}}{MSE_u} \right) \times 100$$

## Energy

$$En(X) = \frac{1}{n} \sum_{i=1}^n x_i^2$$

$$En\%_{u,l} = \left( \frac{En_{u,l}}{En_u} \right) \times 100$$

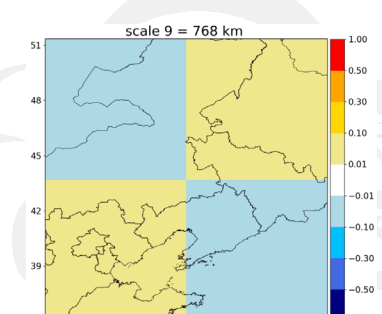
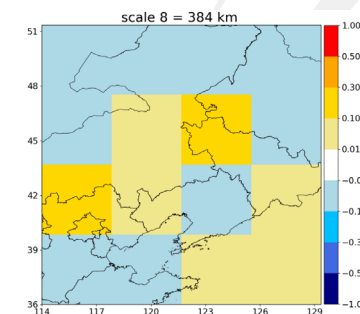
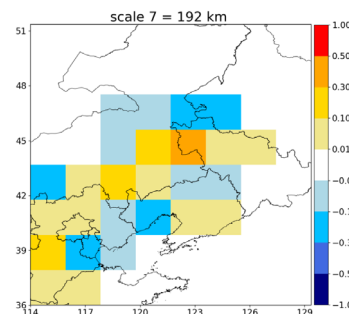
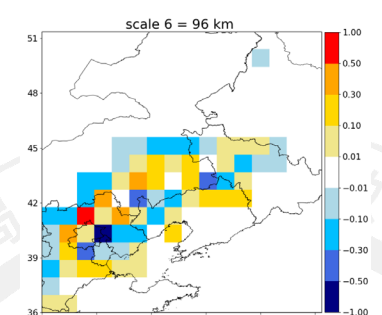
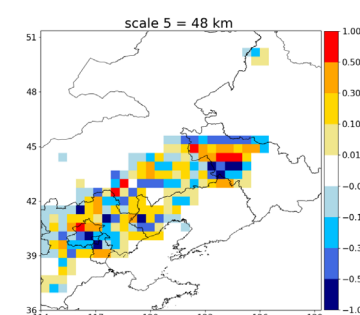
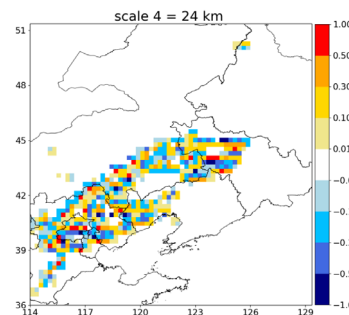
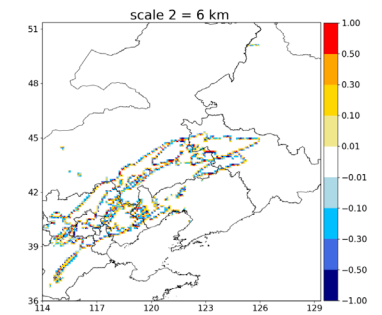
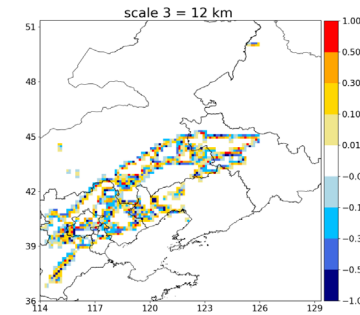
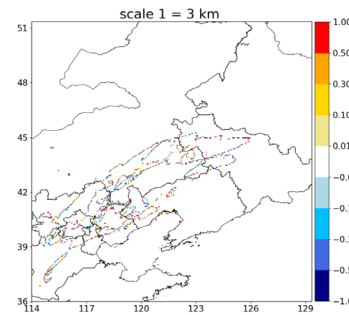
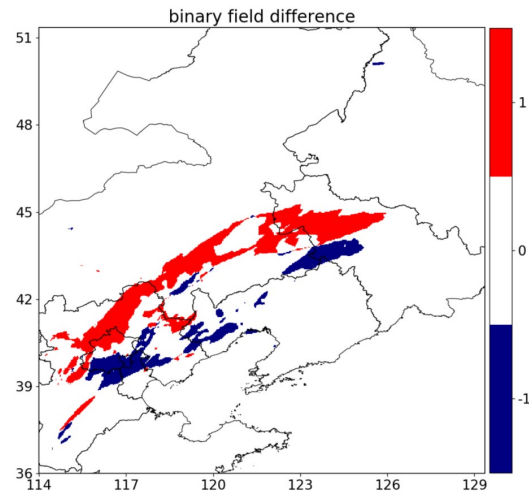
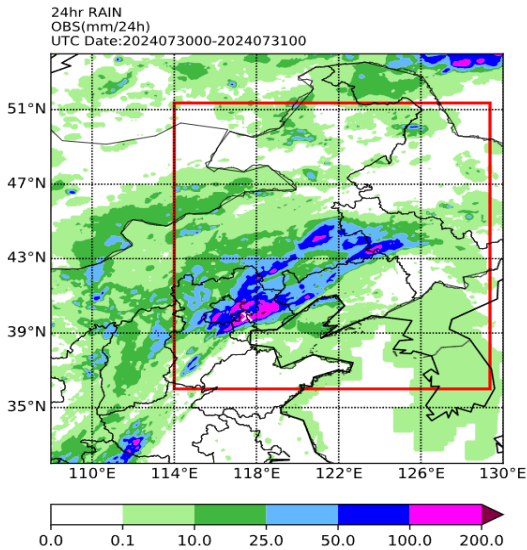
$$En \text{ rel diff} = \frac{En(F) - En(O)}{En(F) + En(O)}$$

$$En\% \text{ rel diff} = \frac{En\%(F) - En\%(O)}{En\%(F) + En\%(O)}$$

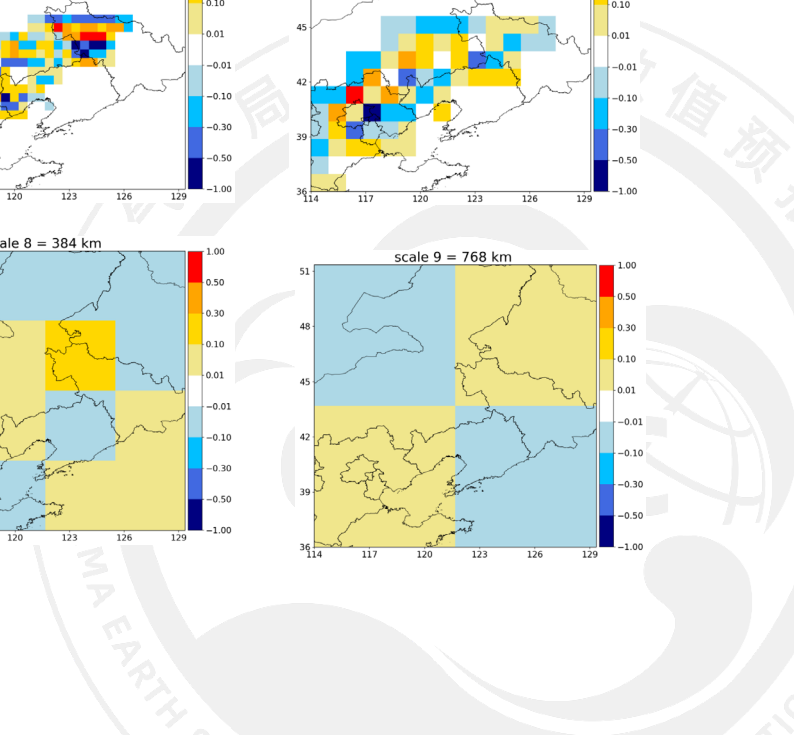


# Case study: North China heavy rainfall in July 2024

nine scales

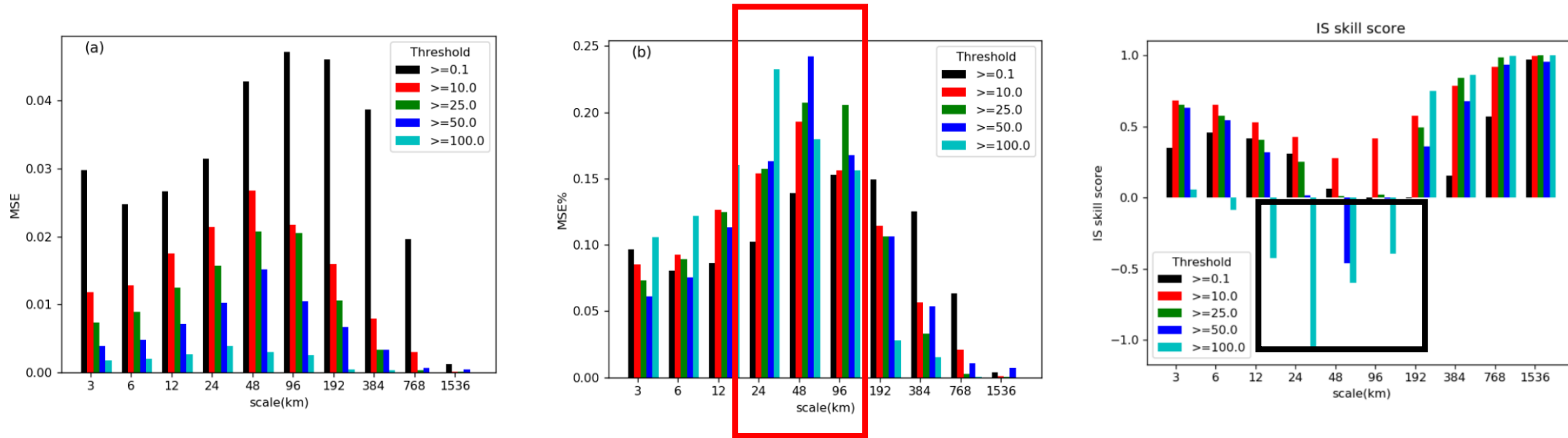


- Resolution:3km CMA-MESO
- Few events and small displacement



# Scale analysis

- The main error scale of the CMA-MESO 3km model's heavy rain is 48km, while heavy rainstorm is 24km, about 25%.
- The moderate rain show highest skill, while 24-96km is also the worst scales
- the weakest scale in precipitation forecast capability.

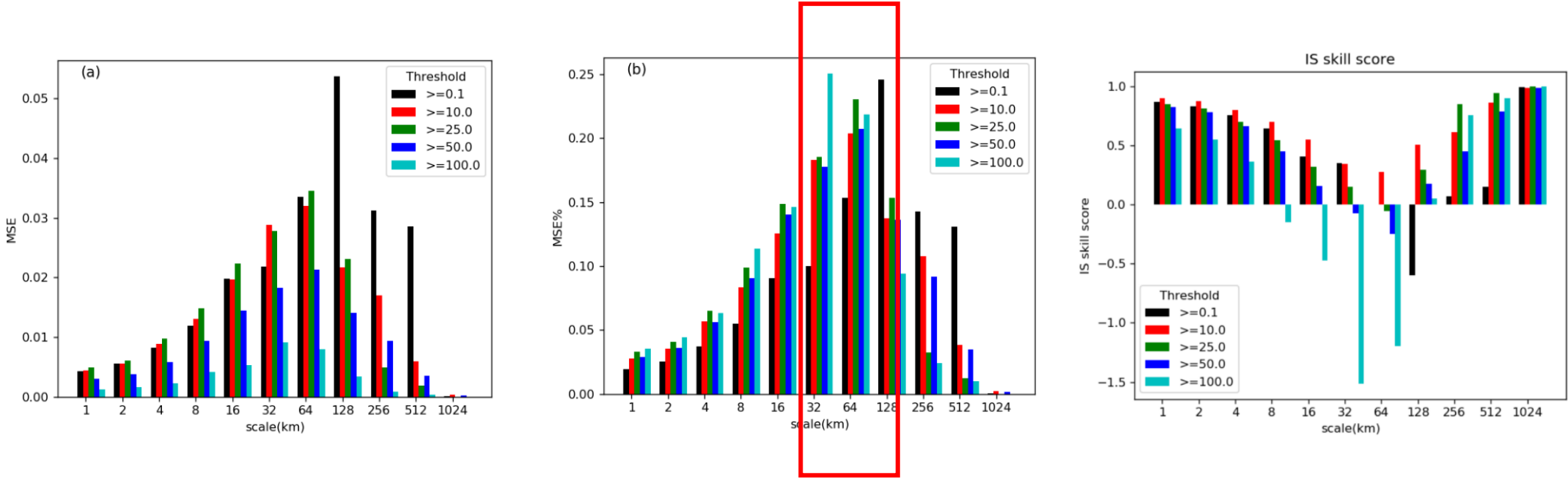


Mean square error (MSE), mean square error percentage (MSE% u, l), and IS score for precipitation

# For higher resolution forecast

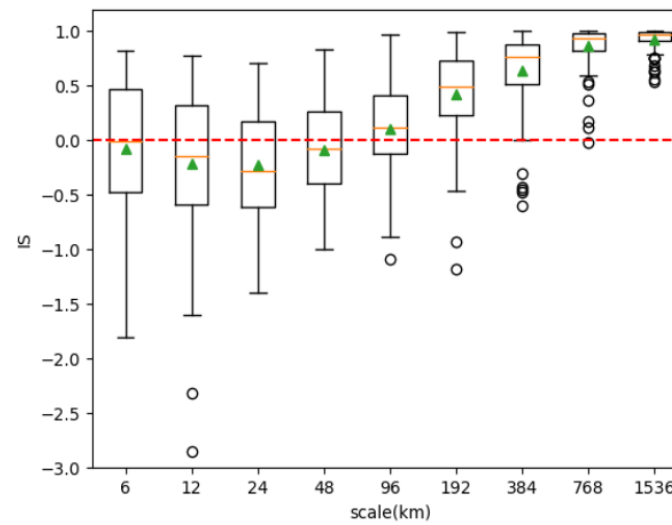
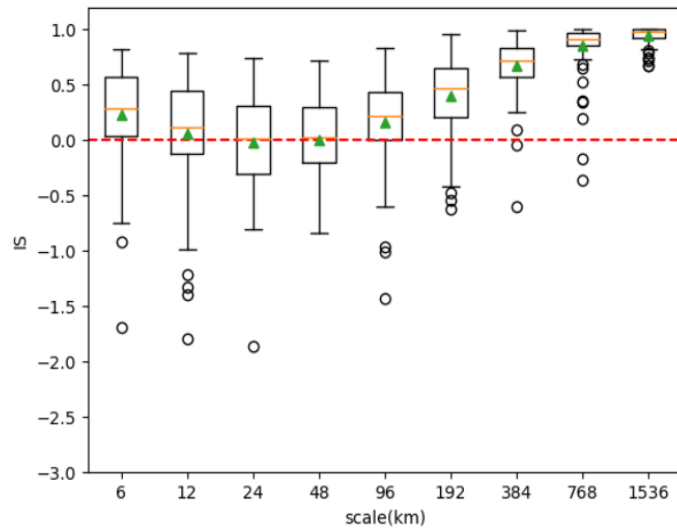
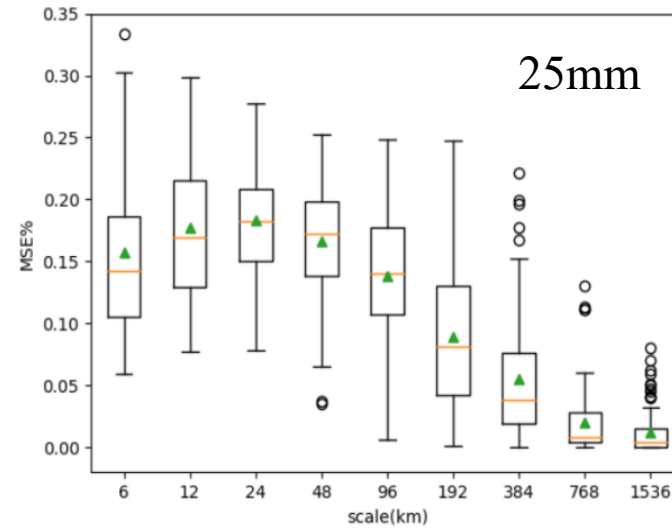
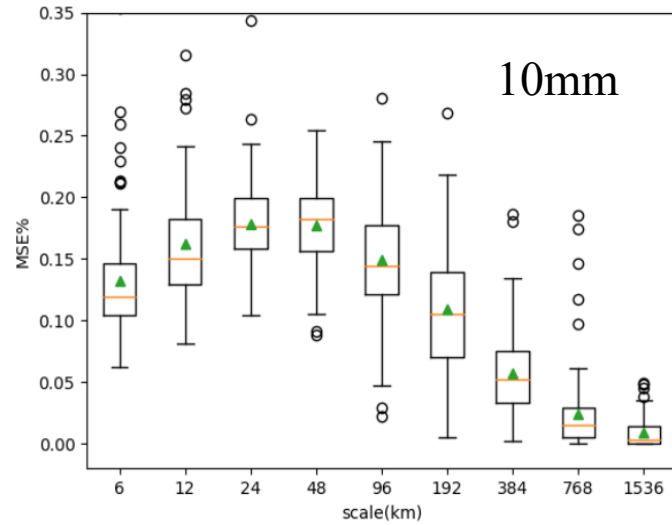
- The main error scale of the 1km precipitation model are 32-64km
- The largest percent for heavy rain is 64km and 32km for heavy rainstorm
- The moderate rain show highest skill.

Basically consistent with that of 3km!!



Mean square error (MSE), mean square error percentage (MSE% u, l), and IS score for precipitation

# JJA analysis



- The evaluation results of the summer precipitation verification in North China in 2023 show that the CMA-MESO model generally performs poorly in predicting the threshold precipitation of 10-25 mm at the 24-48 km scale.
- It reflects the consistency of seasonal assessment

# Summary

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- The inconsistency of synoptic and finer scales is the greatest, while the inconsistency of planetary scale components is the smallest;
- The large inconsistency in forecasts is mainly related to the distribution and intensity of synoptic and finer scale eddies.
- The instability of CMA-GFS forecast results mainly comes from the synoptic scale.
- CMA-MESO shows highest skill in moderate rain forecast. 48km of heavy rain and 24km of heavy rainstorm are the key scale for model adjustment
- The wavelet method shows basically consistent with different resolutions.

Thank you!

