Wave-space diagnostics of numerical weather prediction systems

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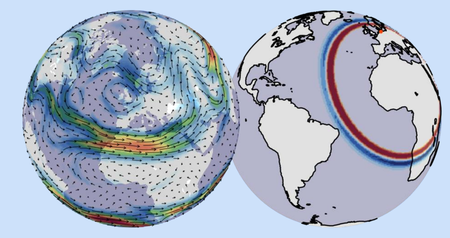


Numerical Weather Prediction (NWP) analyses and forecasts have now achieved a resolution at which the unbalanced circulation across many scales, from the surface to the mesopause, is well-resolved. This advancement enables the quantification of the impact of inertia-gravity (IG) waves, equatorial waves, and ageostrophic circulation within the global atmosphere, as well as the validation of present-day climate simulations produced by coupled climate models.

However, our confidence in weather and climate predictions continues to depend heavily on theoretical frameworks and simplified models that can provide mechanistic descriptions of scale interactions and offer causal explanations for atmospheric behavior. Similarly, fully leveraging recently available global simulations at kilometer-scale resolution requires sophisticated strategies for their evaluating and process diagnostics.

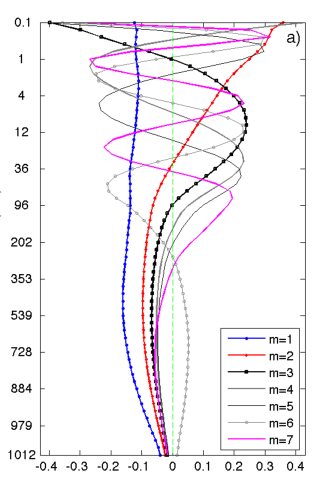
At the linear level, geophysical flows can be decomposed into the eigenmodes of the linearized primitive equations, which, when applied to a spherical domain, separate the flow into Rossby, inertia-gravity, Kelvin, and mixed Rossby-gravity (MRG) modes. Over the past decade, the Linear Normal Mode Decomposition (LNMD) performed by the MODES software [1] has been shown to filter out physically interpretable wave structures in both tropical and mid-latitude regions and enhances our understanding of atmospheric dynamics and predictability, especially in the tropics.

This poster presents snapshots of recent or ongoing research using MODES, including developments towards MODES v2 in 2025.

How does the MODES wave decomposition work? *by Nedjeljka Žagar et al.*

MODES performs the multivariate projection of 3D spatial data (winds and geopotential height) on to eigensolutions of linearized primitive equations on the sphere, a product of the data-specific vertical structure functions and the Hough harmonics [2]. No filtering in time in involved and the representation is complete.

Example of the vertical structure functions.



Examples of the Hough harmonics for k=1

Horizontal projection:

$$\hat{c}_n^k \, Q(j)\, e^{ikx}$$

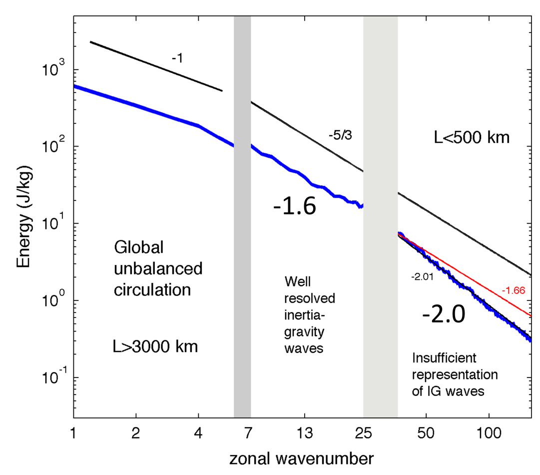
Hough harmonics

Complex expansion coefficient. For spherical wave flows (Rossby, IG, Kelvin and MRG waves), it plays the role of the stream function in quasi-geostrophic models.

k - zonal wavenumber
n - meridional mode index

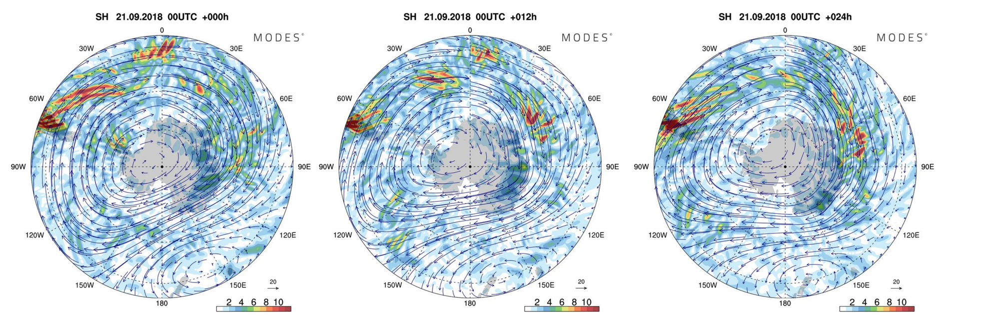
Ten years of the real-time wave decomposition of the operational ECMWF forecasts *by Frank Sielmann et al.*

MODES can quantify the progress in the ECMWF forecast system by analysing the energy spectra of Rossby and IG modes and comparing them with theoretically expected spectral slopes [3].

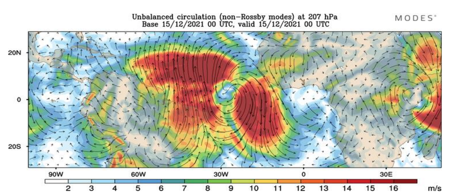


Globally-averaged spectrum of linear inertia-gravity waves in operational ECMWF analyses in boreal winter 2015/2016.

MODES filters gravity waves and equatorial waves. The MODES webpage http://modes.cen.uni-hamburg.de has been continuously updated with products such as more IG modes, temperature perturbations and vertical velocity. In February 2024, the decomposition was switched from the sigma to pressure coordinate system. MODES inspires new ideas, e.g. the excitation mechanism and scale selection of the MRG waves in the upper troposphere and upper stratosphere [4].



Gravity waves emerging from the Andes "hot spot" advected by the westerlies in the operational forecast.



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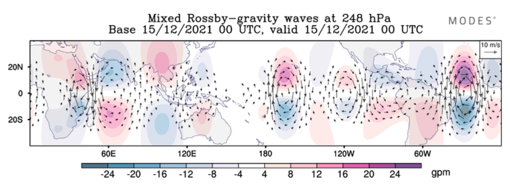
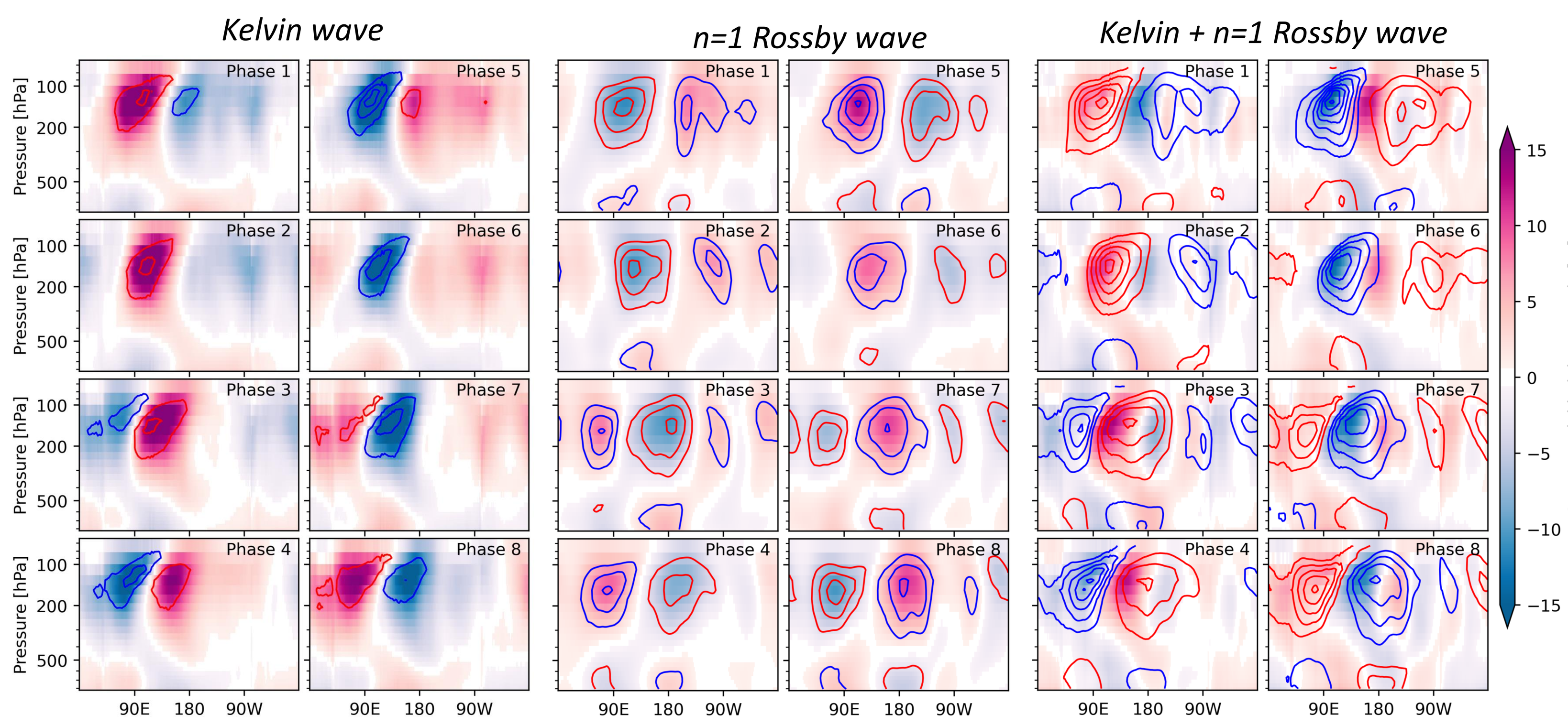


+ IG and Kelvin modes

Non-Rossby modes (linearly unbalanced flow) *MRG wave part of the unbalanced flow*

Filtering large-scale equatorial waves through the eight phases of the Madden-Julian Oscillation in ERA5 *by Katharina M. Holube et al.*

The zonal wind (contours, every ±1.5 m/s, and no zero contour) and geopotential height perturbations (shaded, see colorbar) along the equator of the Kelvin wave and n=1 Rossby wave during the eight phases of the MJO in boreal winter. The phases are defined by an OLR-based index, and the perturbations are averaged between 5⁰N and 5⁰S. The sum illustrates that the height perturbations of the two waves cancel out, whereas their winds sum up.



Wavenumber filtering by MODES versus frequency-wavenumber filtering using the Fourier expansion in longitude and time *by Frank Lunkeit et al.*

Wavenumber decomposition (WD) vs. wavenumber - frequency decomposition (WFD): a primer

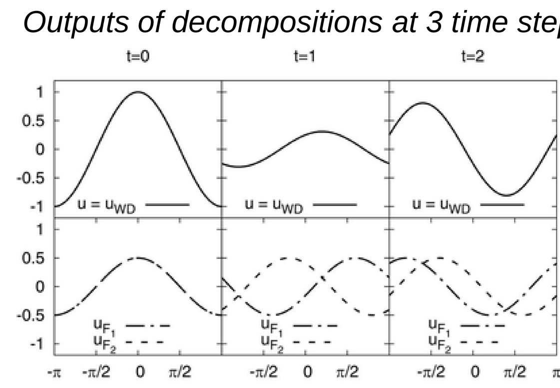
$$u(\lambda,t) = A\cos(\omega_a t)\cos(k\lambda - \omega_p t).$$

A wave with k=1 and freqeuncy ω_p has the amplitude which is also harmonic with frequency ω_a. The wave can be written as a sum of two waves:

$$u_{F_1}(\lambda,t) = \frac{A}{2}\cos(k\lambda - \omega_{F_1} t) \quad + \quad u_{F_2}(\lambda,t) = \frac{A}{2}\cos(k\lambda - \omega_{F_2} t)$$

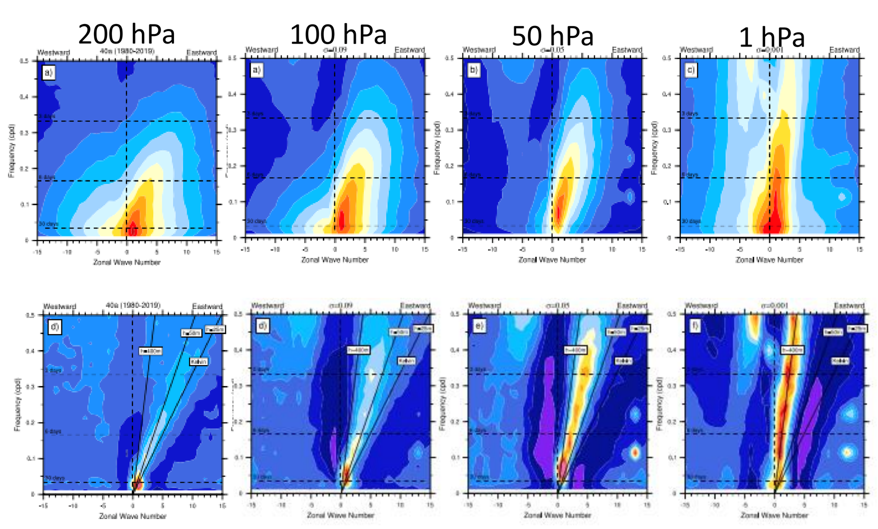
where $\omega_{F_1} = \omega_p + \omega_a$ $\quad \omega_{F_2} = \omega_p - \omega_a$

Outputs of decompositions at 3 time steps



The Fourier series decomposition in time wrongly suggests that the signal is a superposition of two waves of the same amplitude but propagating at different phase speeds.

Wavenumber-frequency filtering of 40-year Kelvin wave in ERA5

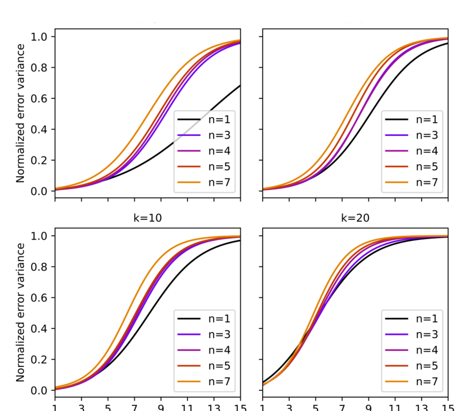


The WFD of tropospheric Kelvin wave assigns excessive variance to periods about 45 days. The results are largely defined by the wave phases [5]. The 1D Fourier time series representation can lead to frequencies and equivalent depths considerably different from the solutions derived from linear wave theory for the same wavenumbers and wave types. The differences can be expected to be even greater for other equatorial waves that are more dispersive than the Kelvin wave.

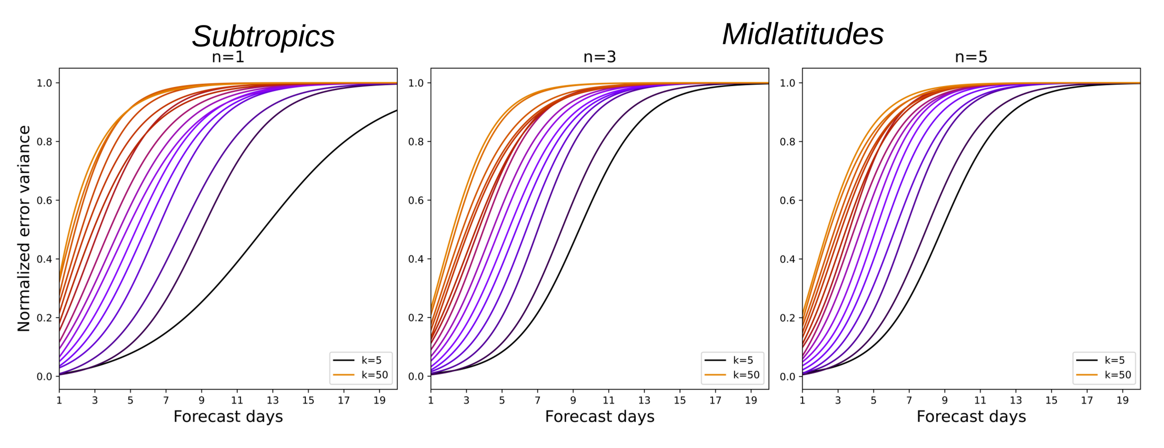
Scale-dependent growth of errors in operational L137 forecasts *by Ole Merkes et al.*

The forecast errors growth depends on the scale. Here, we follow [6,7] to evaluate the growth of errors in deterministic L137 forecasts by comparing them with the analyses. The error variance curves are fit by the *tanh* parametric model [6]. The analyical model provides the saturation error as well as initial error estimates. The results are averaged over about 4 years of data since 2019.

The error variance in every zonal wavenumber is averaged over tropospheric Rossby modes for different meridional modes. The n=1 Rossby has the strong signal in the subtropics. As n increases, the Rossby mode signals become stronger further poleward.



Subtropics *Midlatitudes*



For fixed zonal wavenumber k, the synoptic-scale errors (k=5,7) grow faster for larger n associated with baroclinic instability in middle latitudes. As the scale decreases (k=20), the error growth during the first week of the forecast becomes largely independent of n.

Ongoing work considers forecast errors for other waves, especially equatorial waves, as a function of latitude and pressure level using data since February 2024, when the pressure system became operational.

Recent and fort coming code developments towards MODES v2 *by Sergiy Vasylkevych et al.*

- modal decomposition in the pressure system (in addition to the terrain-following sigma coordinate)
- decompositon of the horizontal wind divergence [8]
- decompositon of the vertical velocity [9]
- decomposition of the vertical momentum fluxes
- filtering of wave temperature perturbations
- accuracy Improvements of the vertical projection
- capability of running MODES in Python
- extension from the LNMD to the non-linear NND (NNMD)
- application of the NNMD to ERA5: updated quantification of balanced/unbalanced flow across scales
- making MODES v2 an open-access software
- organisation of the MODES training workshop (tell us if interested)

References

[1] https://doi.org/10.5194/gmd-8-1169-2015

[2] https://doi.org/10.1007/978-3-030-60963-4

[3] https://doi.org/10.1175/JAS-D-16-0341.1

[4] https://doi.org/10.1002/qj.4742

[5] https://doi.org/10.1175/JCLI-D-21-0342.1

[6] https://doi.org/10.1080/16000870.2017.1287492

[7] https://doi.org/10.1175/JAS-D-19-0166.1

[8] https://doi.org/10.1029/2023JD040427.

[9] https://doi.org/10.1175/JAS-D-23-0090.1

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