Ensemble Verification I

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ECMWF

Training Course 2024

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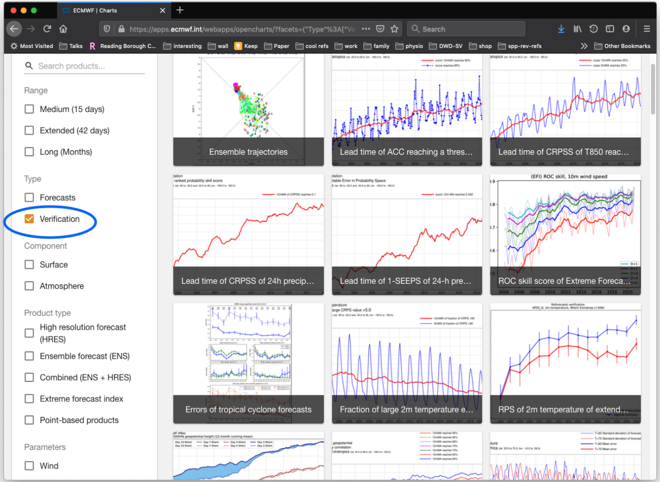
1. introduction
2. reliability (statistical consistency)
3. dichotomous predictands (yes/no)
   - contingency tables
   - Brier score
   - relative operating characteristic (ROC)
   - logarithmic score
4. sensible probabilities: p=0 and p=1?

Examples

`https://charts.ecmwf.int`



`https://www.ecmwf.int/en/forecasts/quality-our-forecasts`

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Objectives of verification (. . . evaluation and diagnostics)

Assess the quality of a forecast system for

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    - tool to monitor the system

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  - Whether a forecast is useful or valuable for a specific user depends on error characteristics but also what other information the user has (eg. climatology) and the particular decision that (s)he needs to make.

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  - An accurate forecast can be of little value (blue desert sky)
  - An inaccurate forecast can be of high value (an intense storm that is predicted but with position error)
  - The actual forecast value may differ from the potential forecast value (uncalibrated raw forecasts, availability of relevant fc information, user's constraints: economic, time limits, lack of training, etc.)

- Forecast verification is the investigation of the properties of the **joint distribution of forecasts and observations** (Murphy & Winkler 1987)

Concepts

Forecast attributes and forecast skill

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- Scalar aspects (attributes) of the forecast quality include:
  - accuracy (e.g. mean absolute error, mean squared error, threat score)
  - bias
  - reliability
  - resolution
  - discrimination
  - sharpness (property of forecast only, e.g. ensemble spread)

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  - resolution
  - discrimination
  - sharpness (property of forecast only, e.g. ensemble spread)
- Forecast skill: relative accuracy of one forecast system with respect to a reference forecast (e.g. climatology)
- More generally: observations $\rightarrow$ estimates of the true state (e.g. also analyses)

Concepts (II)

Examples of scores for single forecasts

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Examples of scores for single forecasts

sample of N forecast-observation pairs (x_j, y_j):

- root mean square error $\left(\frac{1}{N}\sum_{j=1}^{N}(x_j - y_j)^2\right)^{1/2}$
- mean absolute error $\frac{1}{N}\sum_{j=1}^{N}|x_j - y_j|$
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- anomaly correlation coefficient

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- anomaly correlation coefficient
- scores for dichotomous events (e.g. rain/no rain)
  - Peirce skill score (= Hansen-Kuipers, true skill statistic)
  - Gilbert skill score (Equitable threat score)
  - frequency bias
- All of these scores can be applied to the ensemble mean.

- The ensemble predicted rain with a probability of 10%.
- It did rain on the day
- Is this a good forecasts?
  - Yes
  - No
  - I don't know

Concepts (III)

Probabilistic forecasts and ensemble forecasts

- The ensemble predicted rain with a probability of 10%.
- It did rain on the day
- Is this a good forecasts?
  - Yes
  - No
  - I don't know

For probabilistic forecast, the prediction (an ensemble or a probability distribution) and the observation (a value) are different objects. The distribution is not known more precisely after the verifying observation becomes available.

Statistical consistency and reliability

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    - "spread" versus "error"
    - rank histogram
    - reliability diagram (for dichotomous (binary) prediction, e.g. rain/no rain or 0/1)

Statistical consistency and reliability

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  - reliability diagram (for dichotomous (binary) prediction, e.g. rain/no rain or 0/1)
- Reliability alone does not imply skill. The climatological distribution is perfectly reliable for a stationary climate.

Reliability of the ensemble spread

- Consider ensemble variance ("spread") for an M-member ensemble

$$\frac{1}{M}\sum_{j=1}^{M}(x_j - \overline{x})^2$$

and the squared error of the ensemble mean

$$(\overline{x} - y)^2$$

- Average the two quantities for many locations and/or start times.
- The averaged quantities have to match for a reliable ensemble (within sampling uncertainty).

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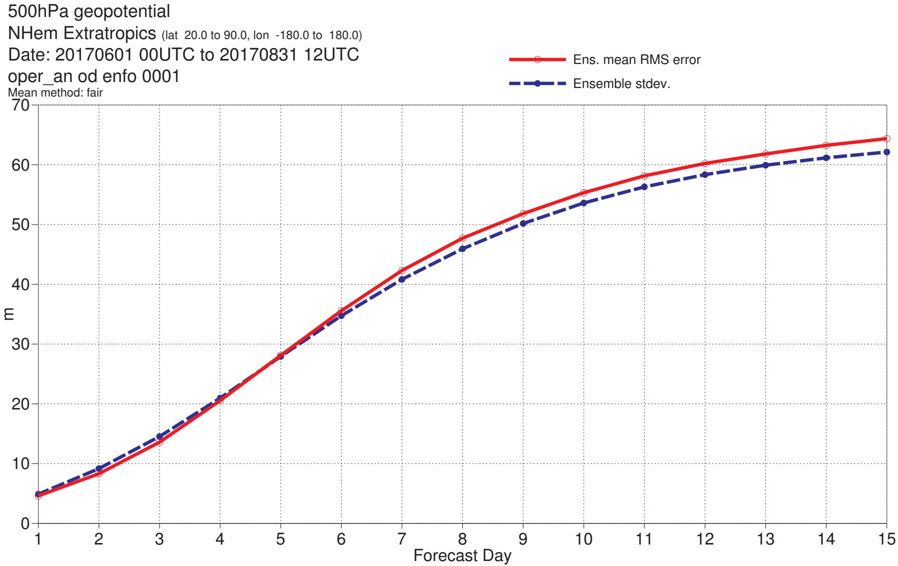
- Average the two quantities for many locations and/or start times.
- The averaged quantities have to match for a reliable ensemble (within sampling uncertainty).
- Finite ensemble size can be corrected for in the estimation of the error of the ensemble mean and the ensemble variance.
- **Cave:** Even in a perfect ensemble, the correlation of ensemble spread and rms error is not 1.

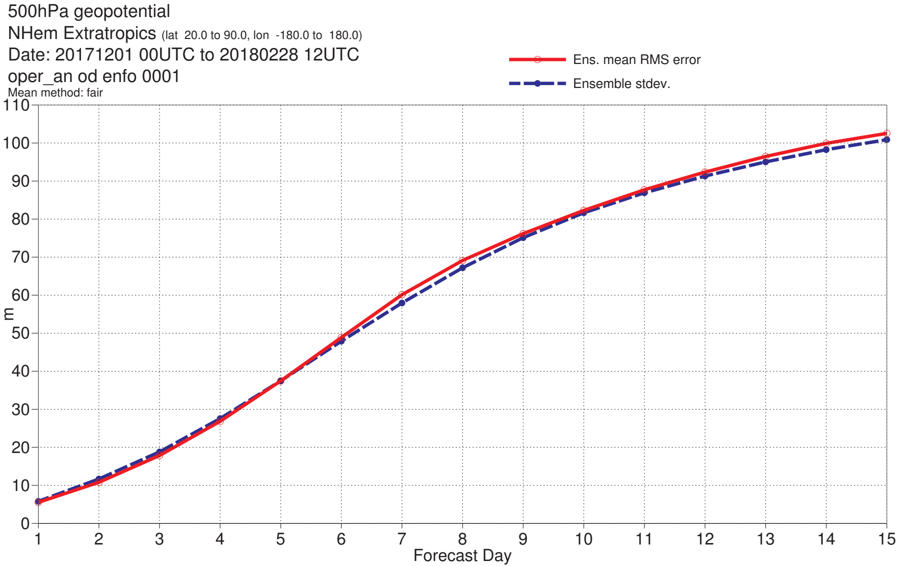
Examples of spread and error

ECMWF EPS — 500 hPa geopotential, JJA 2017



Examples of spread and error

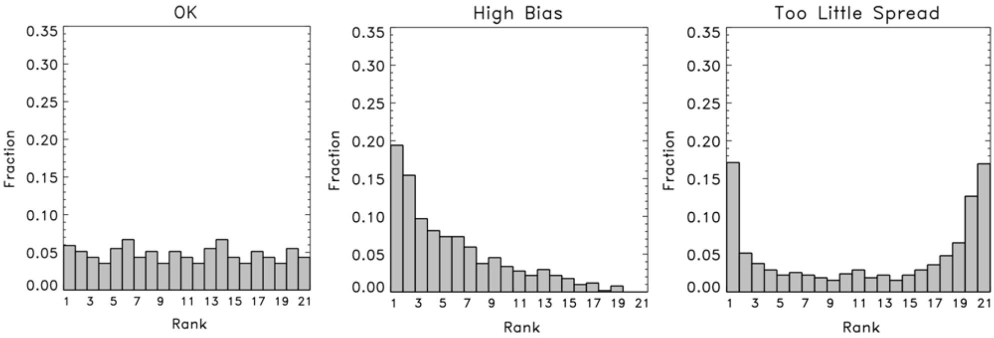
ECMWF EPS — mean sea level pressure, DJF 2018



Rank Histogram

- Are the ensemble members statistically indistinguishable from the verification data?
- Determine where **observation** lies with respect to the ensemble members:

Rank Histogram



OBS is indistinguishable from any other ensemble member

OBS is too often below the ensemble members (biased forecast)

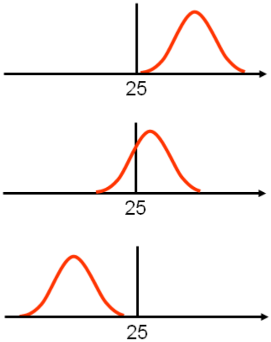
OBS is too often outside the ensemble spread

A uniform rank histogram is a necessary but not sufficient criterion for determining that the ensemble is reliable (see also: T. Hamill, 2001, MWR)

Dichotomous predictands

Joint distribution of forecasts and obs

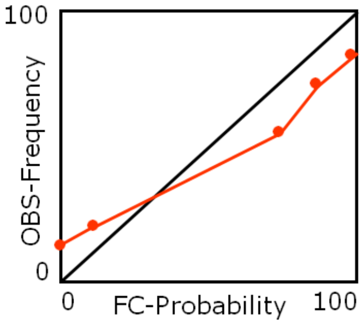
- Consider the probabilistic prediction of the event that the temperature exceeds 25° C.
- Hypothetical verification sample of 30 start dates and 2200 grid points = 66000 forecasts.
- How often was the event ($T > 25°$ C) predicted with probability p?



FC Prob.	# FC	OBS-Frequency (perfect model)	OBS-Frequency (imperfect model)
100%	8000	8000 (100%)	7200 (90%)
90%	5000	4500 (90%)	4000 (80%)
80%	4500	3600 (80%)	3000 (66%)
....	....	....	....
....	....	....	....
....	....	....	....
10%	5500	550 (10%)	800 (15%)
0%	7000	0 (0%)	700 (10%)

Dichotomous predictands

Reliability diagram



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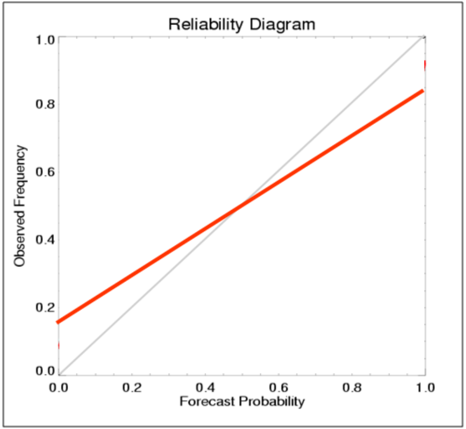
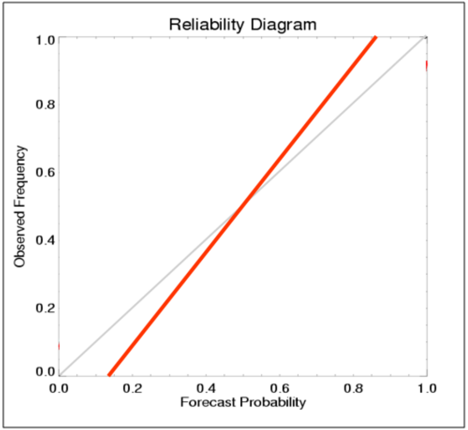
Over- and under-confidence

Reliability diagram

over-confident model



under-confident model



Scores for dichotomous predictions

- Extended contingency tables
- Scores
  - Brier score (reliability and resolution)
  - Logarithmic score (reliability and resolution)
  - Relative Operating Characteristic (discrimination)

Contingency table

single forecast

- Consider an event e (e.g. $T > 25^\circ$ C)
- The joint distribution of forecasts and observations can be condensed in a 2×2 contingency table:

	e observed	
e predicted	Yes	No
Yes	hits a	false alarms b
No	misses c	correct rejections d

- hit rate $H = \frac{a}{a+c}$
- false alarm rate $F = \frac{b}{b+d}$
- $N = a + b + c + d$ sample size

(Extended) contingency table

The joint distribution of forecasts and observations for a M-member ensemble can be summarized in a $(M+1) \times 2$ contingency table $\mathbf{T}$

e pred. by m_e members	e observed Yes	No
M	n_M	$\tilde{n}_M$
$M-1$	n_{M-1}	$\tilde{n}_{M-1}$
$\dots$	$\dots$	$\dots$
j	n_j	$\tilde{n}_j$
$\dots$	$\dots$	$\dots$
1	n_1	$\tilde{n}_1$
0	n_0	$\tilde{n}_0$

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The joint distribution of forecasts and observations for a M-member ensemble can be summarized in a $(M+1) \times 2$ contingency table $\mathbf{T}$

$$\text{sample size } N = \sum_{j=0}^{M} n_j + \sum_{j=0}^{M} \tilde{n}_j$$

Each row corresponds to a probability value, e.g. $p = j/M \quad \longrightarrow$

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(Extended) contingency table

ensemble

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Contingency tables are additive:

$$\mathbf{T}(\text{sample1} \cup \text{sample2}) = \mathbf{T}(\text{sample1}) + \mathbf{T}(\text{sample2})$$

Brier score

definition and decomposition

$$\mathrm{BS} = \frac{1}{N} \sum_{k=1}^{N} (p_k - o_k)^2$$

- p_k is the predicted probability of the k-th forecast and $o_k = 1$ (0) if the event occurred (did not occur)
- The Brier score BS is the **mean squared error** of the probability forecast.

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- The Brier score BS is the **mean squared error** of the probability forecast.
- The BS can be decomposed in three components that measure
  - reliability
  - resolution
  - uncertainty

Brier score components

BS=REL−RES+UNC

stratify sample in terms of the rows j in the contingency table

Reliability: deviation of observed relative frequency from forecasted probability

$$\mathrm{REL} = \frac{1}{N}\sum_{j=0}^{M} \ell_j (\overline{o}_j - p_j)^2$$

N	total number of cases
M	number of probability bins -1
p_j	$= j/M$ probability in bin j
ℓ_j	$= n_j + \tilde{n}_j$ number of cases in bin j
$\overline{o}_j$	$= n_j/\ell_j$ frequency of event occuring when fore-casted with probability p_j

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Resolution: ability of forecast to identify periods in which observed frequencies differ from average

$$\mathrm{RES} = \frac{1}{N}\sum_{j=0}^{M} \ell_j(\overline{o}_j - \overline{o})^2$$

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$\overline{o}$	event frequency in whole sample

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Uncertainty: Variance of obs. (0/1) in sample

$$\mathrm{UNC} = \overline{o}(1 - \overline{o})$$

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Brier Skill Score

- Skill scores are used to compare the performance of forecasts with that of a reference forecast (e.g. climatological distribution)
- They are defined so that the perfect forecast has a skill score of 1 and the reference forecast has the skill score of 0

$$\text{skill score} = \frac{\text{actual fc} - \text{ref}}{\text{perfect fc} - \text{ref}}$$

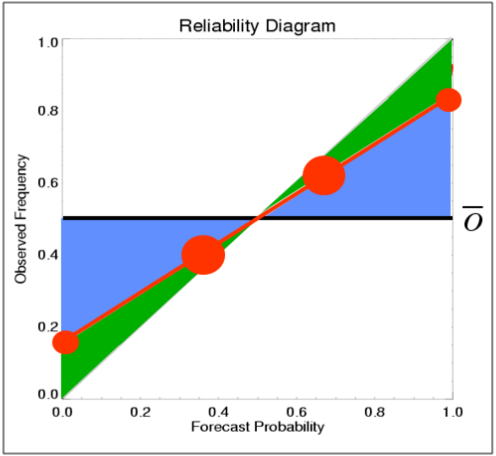
- BS for perfect forecast is 0 $\Rightarrow$

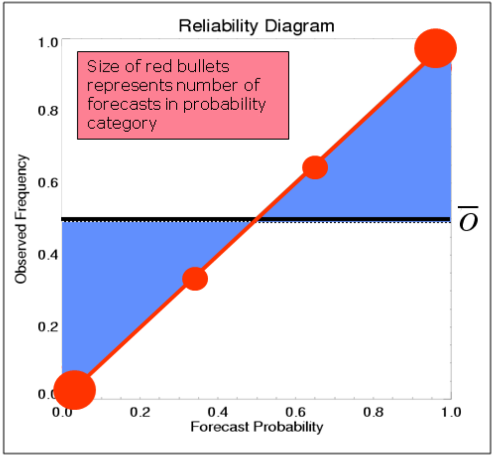
$$\mathrm{BSS} = 1 - \frac{\mathrm{BS}}{\mathrm{BS}_{\mathrm{ref}}}$$

- positive (negative) BSS $\Rightarrow$ forecast is better (worse) than the reference forecast

Reliability score (the smaller, the better)

Resolution score (the bigger, the better)



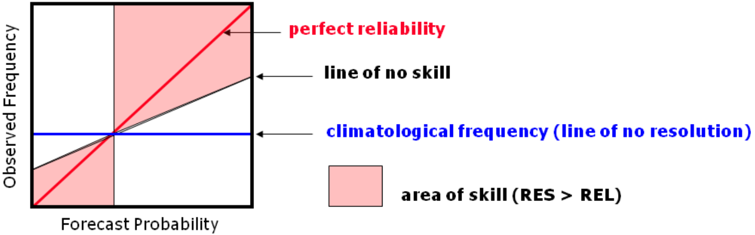


Positive contribution to skill

diagnosed from the attributes diagram

$$BSS = 1 - \frac{BS}{BS_c}$$

$$= 1 - \frac{REL - RES + UNC}{UNC} = \frac{RES - REL}{UNC}$$



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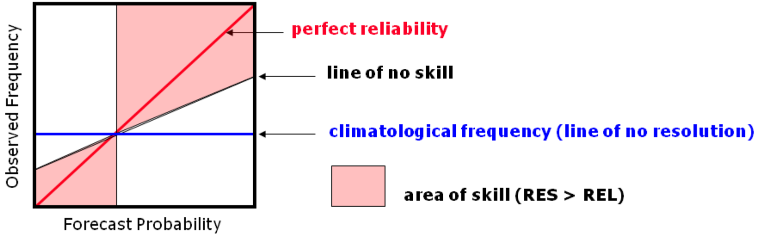

Cave: Using sample climatology as reference can lead to ficticious skill

Discrimination and ROC

- Until now, we asked:
  What is the distribution of observations o if the forecast system predicts an event to occur with probability p?
- To measure the ability of a forecast system to *discriminate* between occurrence and non-occurrence of an event, we have to ask:
  What is the distribution of forecast probabilities when the event occurred and what is the distribution when it did not occur?

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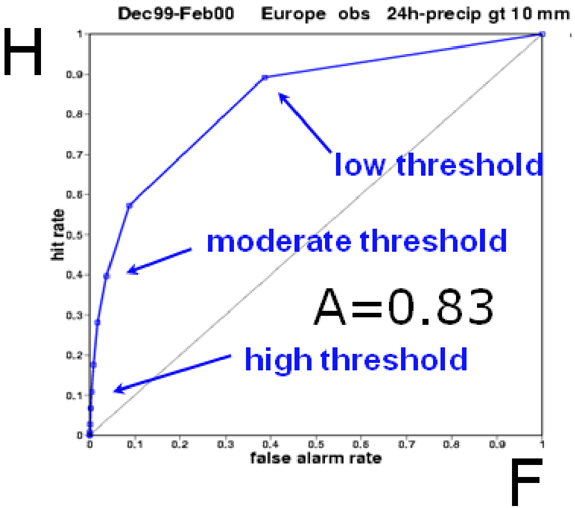
- To measure the ability of a forecast system to *discriminate* between occurrence and non-occurrence of an event, we have to ask:
  What is the distribution of forecast probabilities when the event occurred and what is the distribution when it did not occur?
- For evaluation purposes, let us predict the event when the probability exceeds a threshold p_i.
- For any probability threshold p_i, compute the hit rate $H_i = \frac{a}{a+c}$ and the false alarm rate $F_i = \frac{b}{b+d}$
- The *relative operating characteristic* (ROC, also referred to as receiver operating characteristic) is the diagram that shows H versus F for all probability thresholds.

Relative Operating Characteristic



- random forecast (independent of observed event) on diagonal
- summary measure: area under the ROC $\in [0.5, 1]$

Logarithmic score

- also known as ignorance score (Good 1952, Roulston and Smith 2002)

$$\mathrm{LS} = -\frac{1}{N}\sum_{k=1}^{N}\left[o_k \log p_k + (1-o_k)\log(1-p_k)\right]$$

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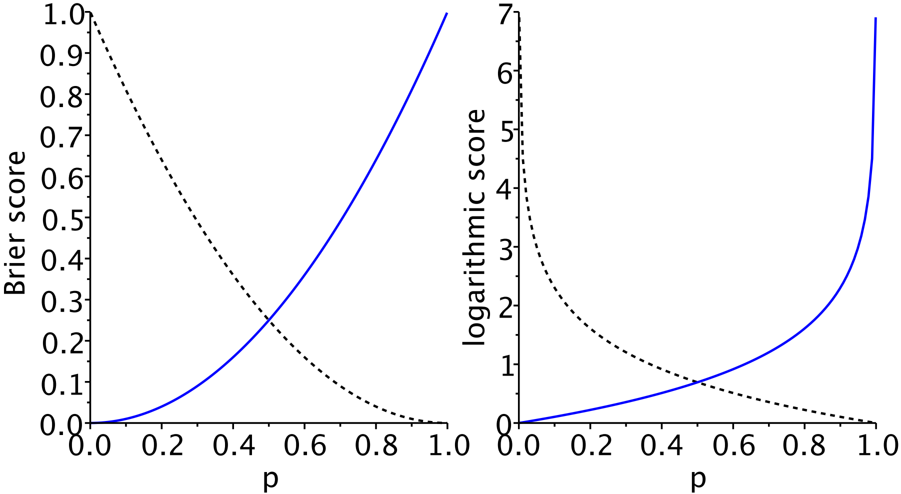
- The score ranges between 0 and ∞. The latter happens if the predicted probability is zero and the event occurs (or if $p = 1$ and the event does not occur).
- The ignorance score is more sensitive to the cases with probability close to 0 and close to 1 than the Brier score.

Brier score versus logarithmic score

event occurs (dotted), event does not occur (solid)

$(p-1)^2$ and p^2 $\qquad$ $-\log(p)$ and $-\log(1-p)$



Sensible probabilities

- Never forecast $p = 0$ or $p = 1$ unless you are really certain!
- If the true probability is not equal to zero (or one), there will still be cases when no member (or all members) predict(s) the event.
  Sampling uncertainty!

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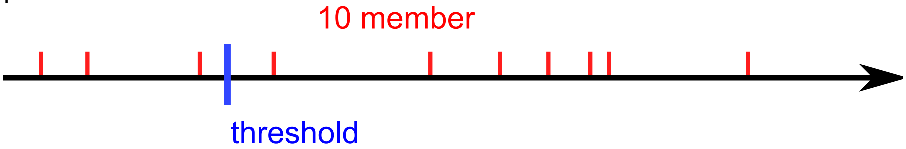
  Sampling uncertainty!
- Wilks proposed to estimate cumulative probabilities using Tukey's plotting positions



- When n members of an M-member ensemble have a value less than the threshold θ, the probability to not exceed θ is set to

$$p^{(T)}(n) = \frac{n + 2/3}{M + 4/3}$$

- Consider for example $M = 10$:

n	0	1	2	3	4	5	6	7	8	9	10
p	0.06	0.15	0.24	0.32	0.41	0.50	0.59	0.68	0.76	0.85	0.94