Predictability and Ensemble Forecasting with Lorenz-96 systems

Martin Leutbecher

Training Course 2024

- Part 1: The L96 model, chaos, error growth, ensemble forecast experiments (with a perfect model)
- Part 2: How to experiment with L96 using scilab, proposed activities
- Part 3: An imperfect forecast model and the representation of model uncertainties
- Part 4: Proposed activities with an imperfect model

Predictability: 1995 Annual Seminar at ECMWF

PREDICTABILITY—A PROBLEM PARTLY SOLVED

Edward N. Lorenz

Massachusetts Institute of Technology

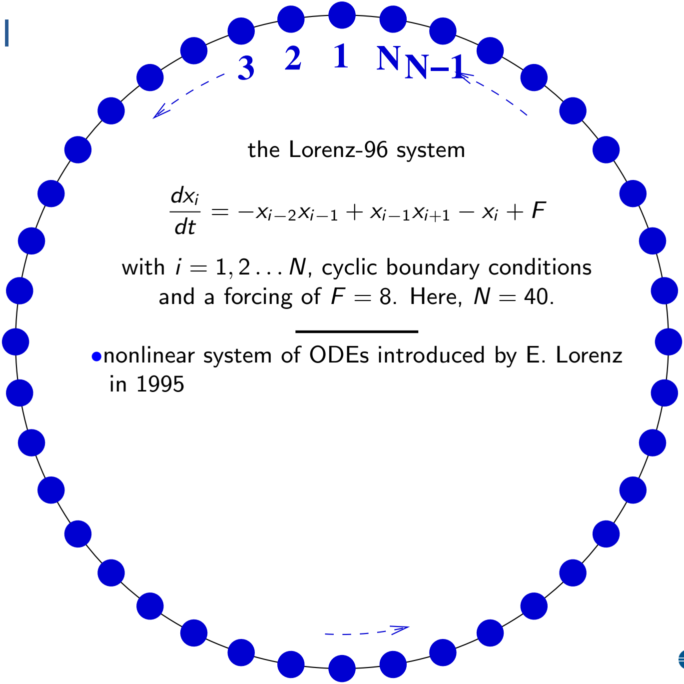
Cambridge, Massachusetts, USA

Summary: The difference between the state that a system is assumed or predicted to possess, and the state that it actually possesses or will possess, constitutes the *error* in specifying or forecasting the state. We identify the rate at which an error will typically grow or decay, as the range of prediction increases, as the key factor in determining the extent to which a system is predictable. The long-term average factor by which an infinitesimal error will amplify or diminish, per unit time, is the leading Lyapunov number; its logarithm, denoted by λ_1, is the leading Lyapunov exponent. Instantaneous growth rates can differ appreciably from the average.

With the aid of some simple models, we describe situations where errors behave as would be expected from a knowledge of λ_1, and other situations, particularly in the earliest and latest stages of growth, where their behavior is systematically different. Slow growth in the latest stages may be especially relevant to the long-range predictability of the atmosphere. We identify the predictability of long-term climate variations, other than those that are externally forced, as a problem not yet solved.

The Lorenz-96 systems appear first in these proceeding.
They are an ideal toy for the training course.

System I



the Lorenz-96 system

$$\frac{dx_i}{dt} = -x_{i-2}x_{i-1} + x_{i-1}x_{i+1} - x_i + F$$

with $i = 1, 2 \ldots N$, cyclic boundary conditions and a forcing of $F = 8$. Here, $N = 40$.

- nonlinear system of ODEs introduced by E. Lorenz in 1995

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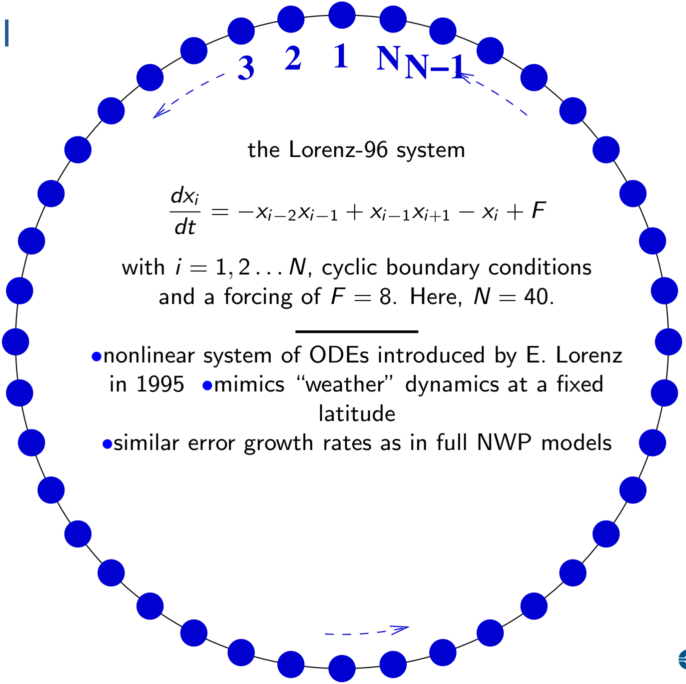

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- mimics "weather" dynamics at a fixed latitude
- similar error growth rates as in full NWP models

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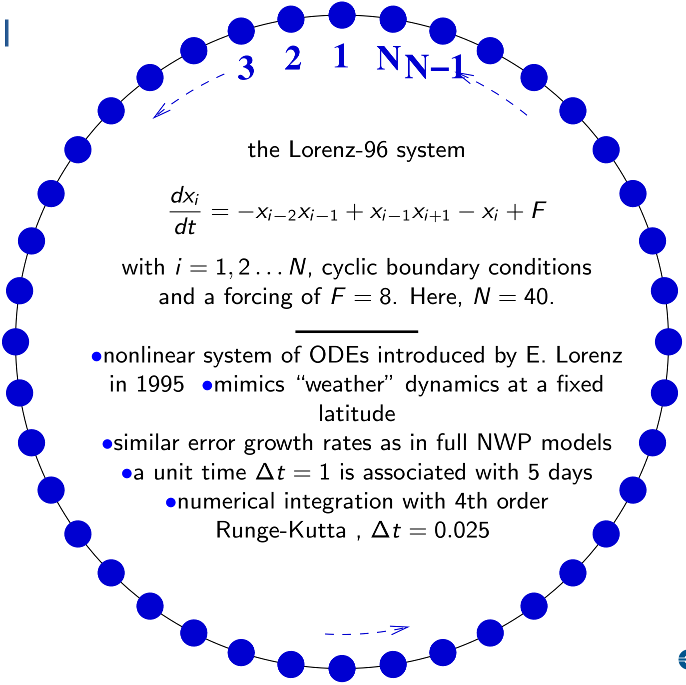

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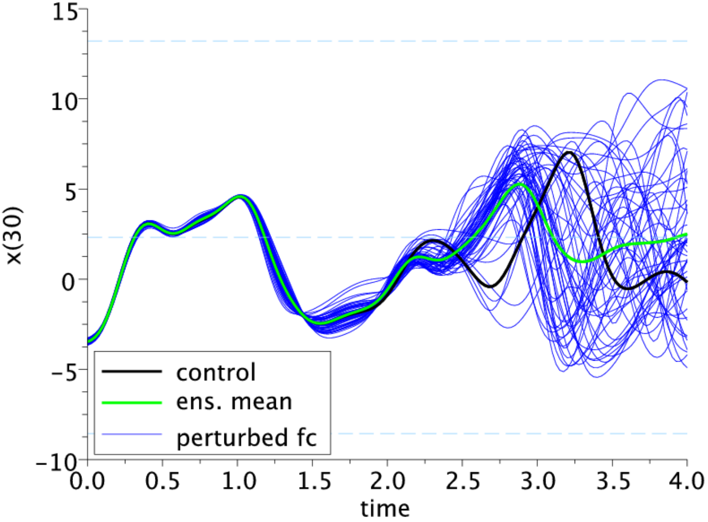
- nonlinear system of ODEs introduced by E. Lorenz in 1995
- mimics "weather" dynamics at a fixed latitude
- similar error growth rates as in full NWP models
- a unit time $\Delta t = 1$ is associated with 5 days
- numerical integration with 4th order Runge-Kutta , $\Delta t = 0.025$

ECMWF

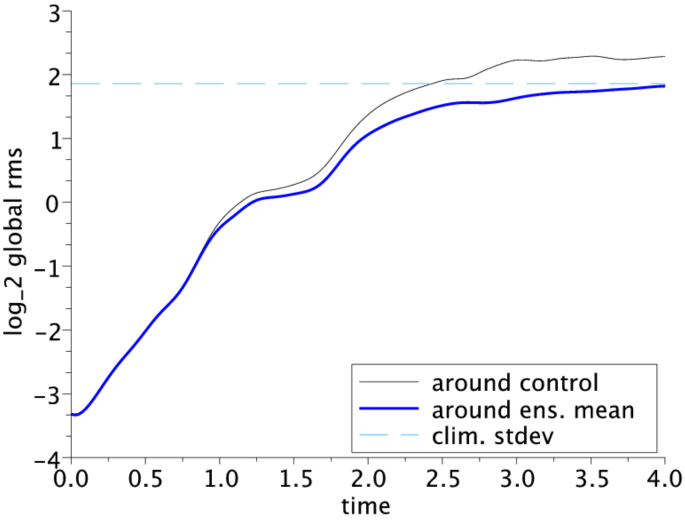
System I is chaotic

Sensitive dependence on initial conditions

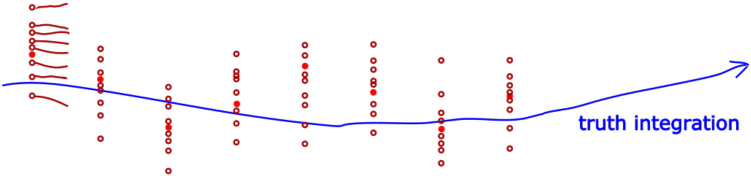


System I is chaotic

Perturbation growth



Ensemble forecasting — a schematic

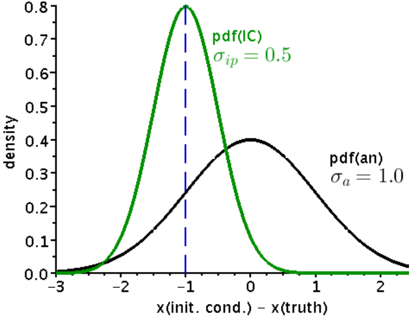


Initial conditions and their uncertainty

- Initial states ("analyses") have been prepared. Values of the standard deviation of the analysis uncertainty of

$\sigma_a = 0.01, 0.02, 0.05, 0.1, 0.2, 0.5$ and 1.0

can be selected.



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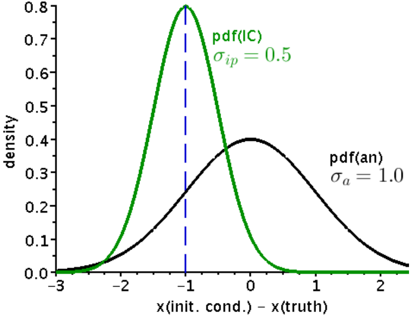

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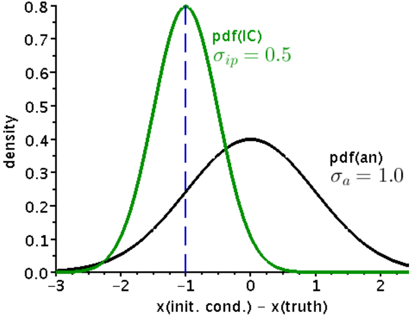

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- Ensemble forecasts are started every 0.4 time units (2 days).

Verification statistics

- all metrics will be explained in the verification lectures
- in the practical activities you can explore how everything is linked
  - representation of initial uncertainties
  - accuracy of initial conditions
  - error growth and ensemble perturbation growth
  - comparison of different sets of ensemble forecasts using scores

References

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