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TOWARDS ENERGETICALLY CONSISTENT COUPLING

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Universität Hamburg
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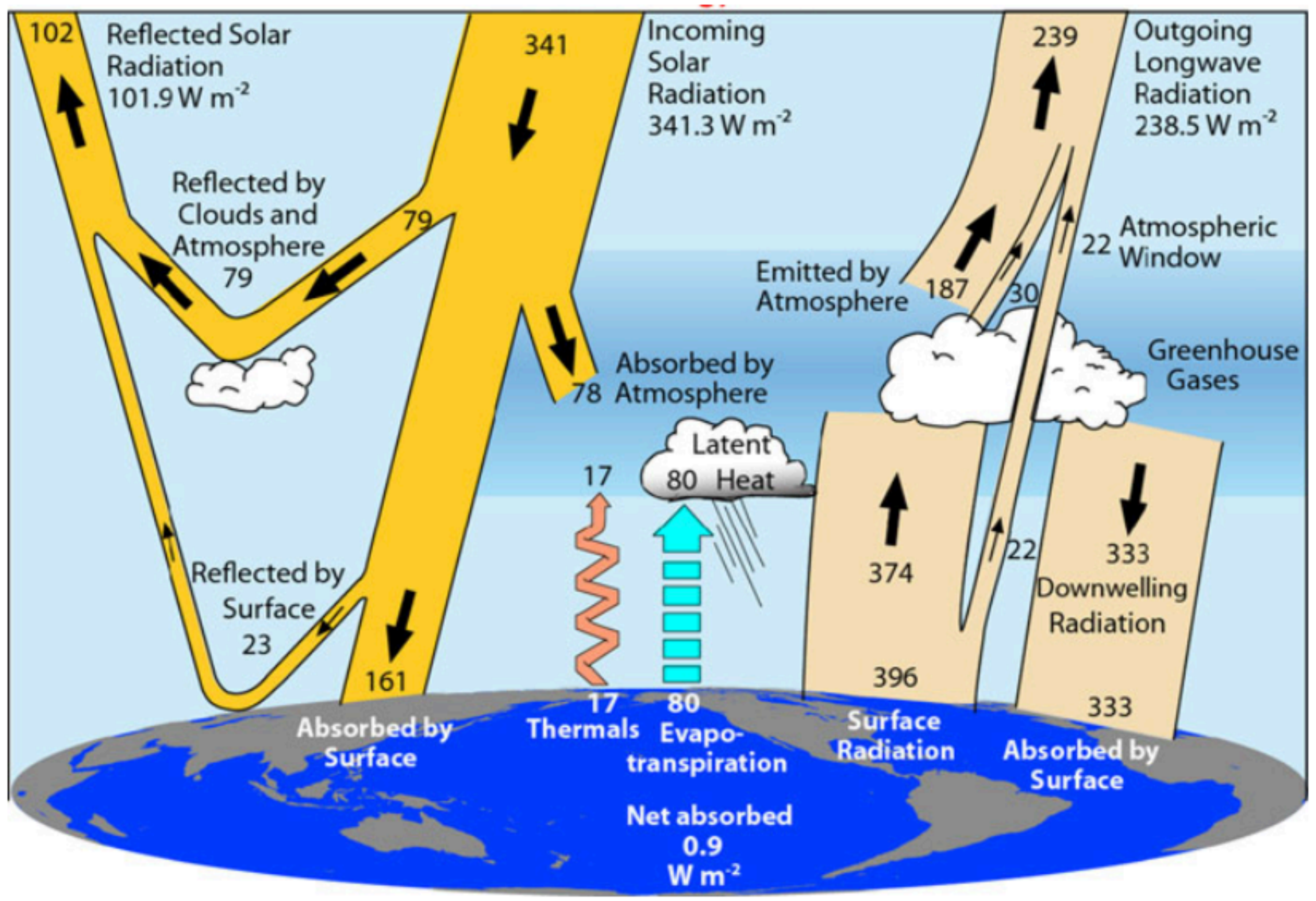
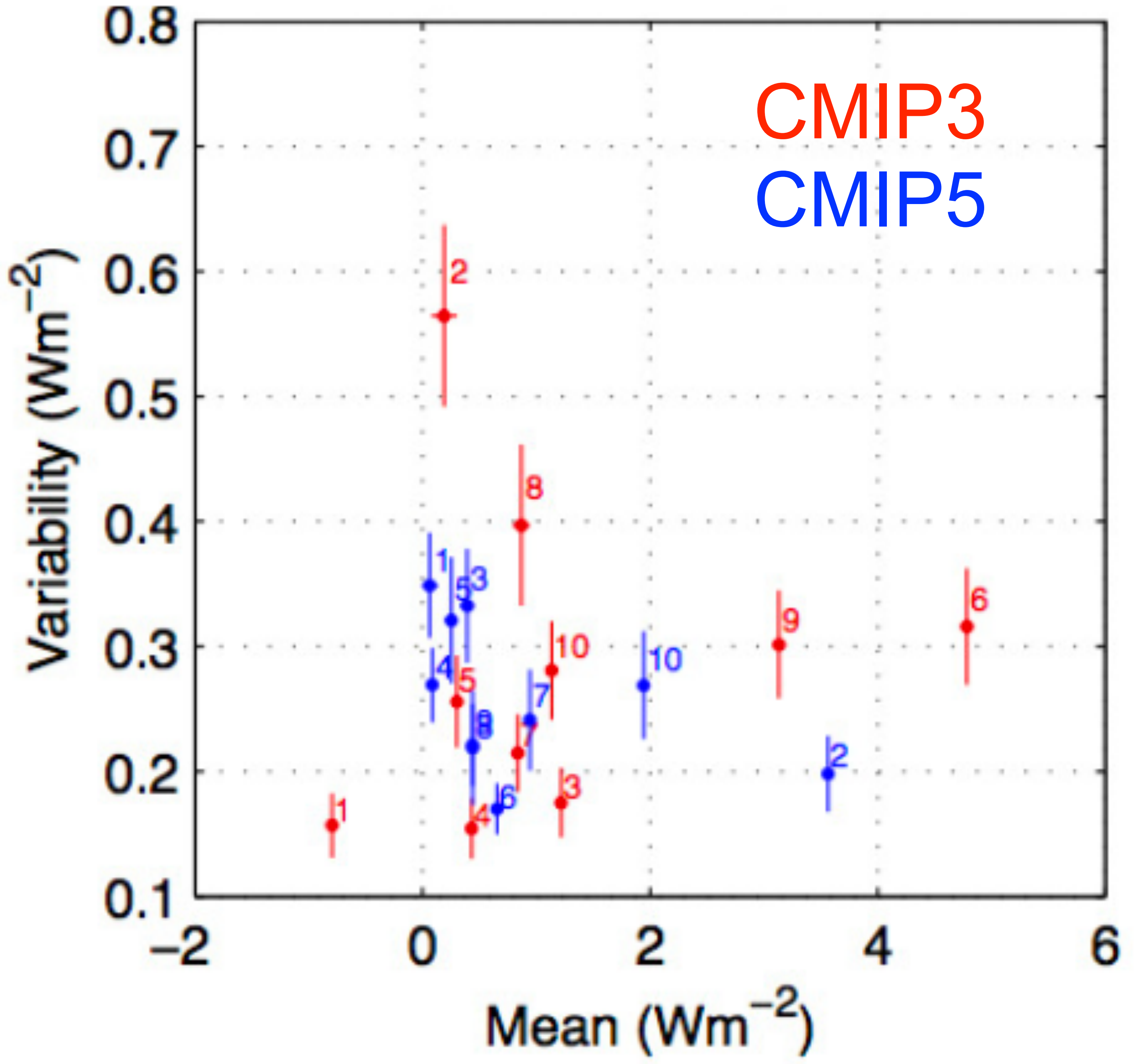
Motivation

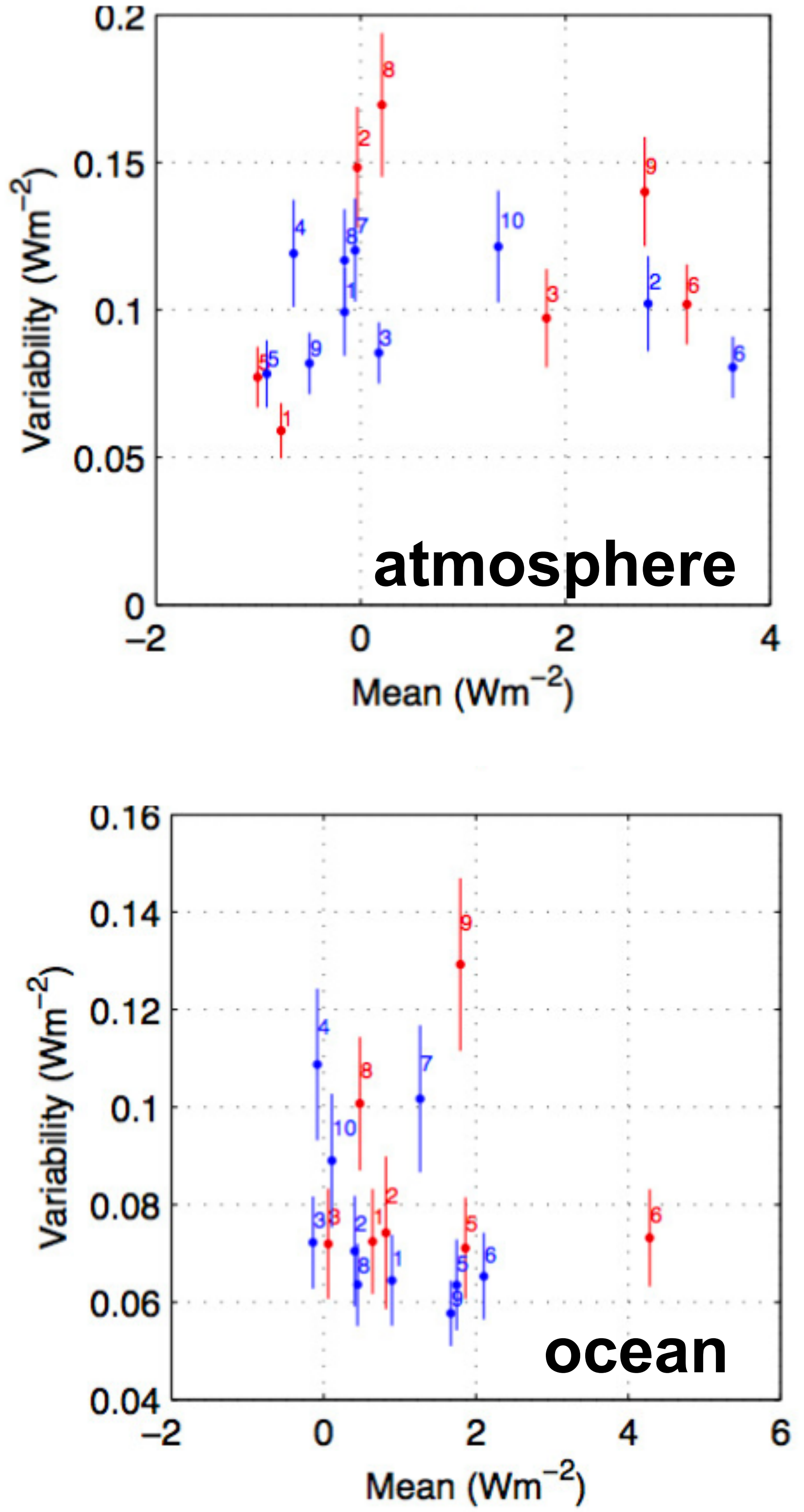
Energy budgets of Earth System (Models) equals the energy flux on TOA:



Trenberth&Fasullo (2012):
small imbalance (1-2 Wm^-2)
due to greenhouse gas forcing

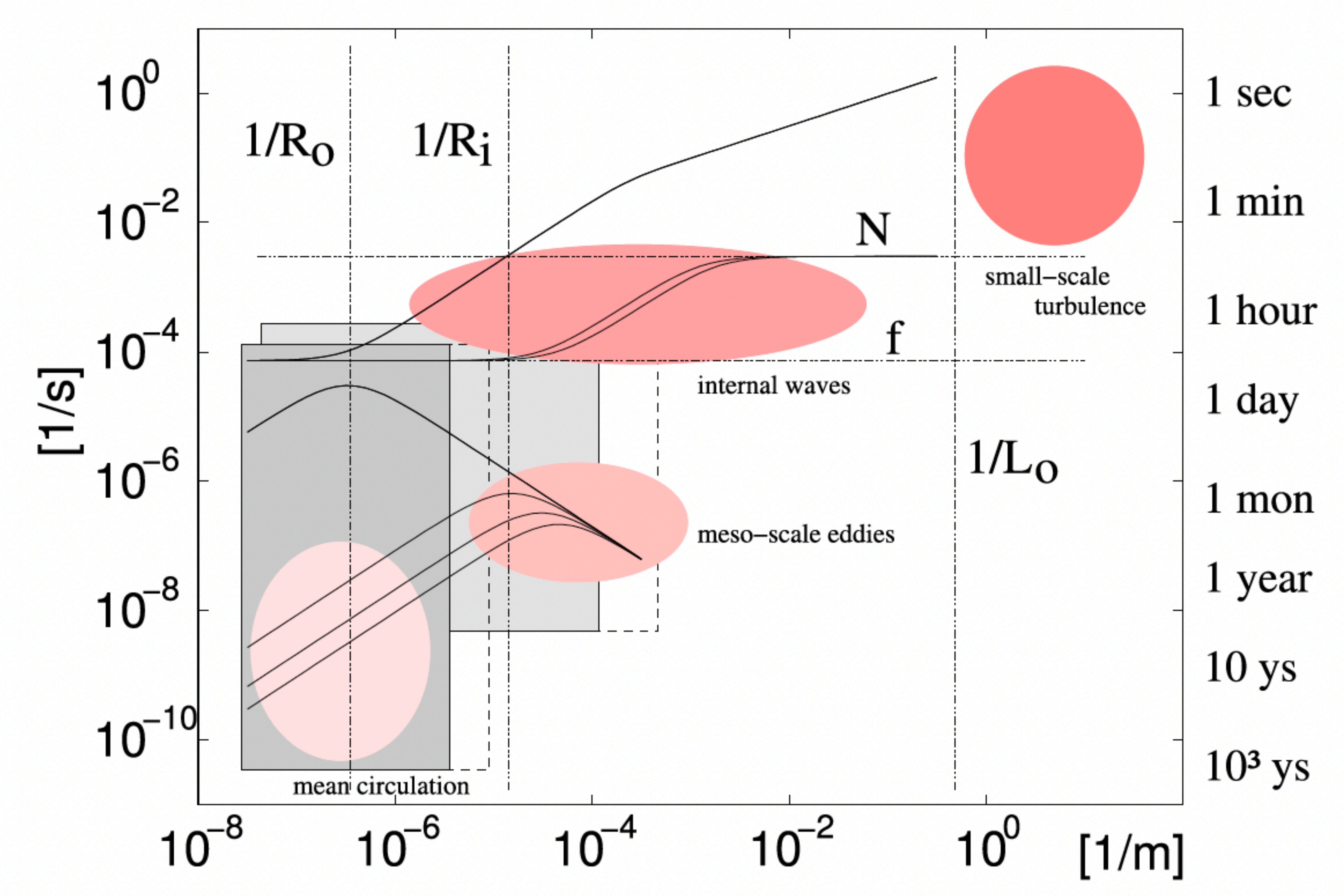


Lucarini et al (2014):
energy bias in steady state
CMIP runs



Motivation

Collaborative research center TRR181:
- Improve climate predictions by removing spurious energy sources and sinks and provide energetically consistent parameterisations



Eden et al. (2014)

next step: coupling to surface wave model

- identify energy transfers
- provide a simple, but energetically consistent coupling framework

Phase-averaged Boussinesq equations

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{u} + \mathbf{f} \times \mathbf{u}^L = b\mathbf{z} - \nabla p^* + \mathbf{u}^S \times (\nabla \times \mathbf{u}) + \mathbf{D}_u$$

$\mathbf{u}^L = \mathbf{u} + \mathbf{u}^S$, with Stokes drift $\mathbf{u}^S$ from irrotational waves.

$$p^* = p + \frac{1}{2}|\mathbf{u}^L|^2 - \frac{1}{2}|\mathbf{u}|^2$$

$$\partial_t \mathbf{u}^L + (\mathbf{u}^L \cdot \nabla)\mathbf{u}^L + \mathbf{f} \times \mathbf{u}^L = b\mathbf{z} - \nabla p - \mathbf{u}^L \times (\nabla \times \mathbf{u}^S) + \mathbf{D}_u + \partial_t \mathbf{u}^S$$

e.g. Craik & Leibovich (1976), Leibovich (1980), Holm (1996)

$$MKE_L = \frac{1}{2}\overline{\mathbf{u}^L} \cdot \overline{\mathbf{u}^L} \qquad MKE_E = \frac{1}{2}\overline{\mathbf{u}} \cdot \overline{\mathbf{u}}$$

Mean kinetic energy

$$MKE_L = \frac{1}{2}(\overline{\mathbf{u}} + \overline{\mathbf{u}^S}) \cdot (\overline{\mathbf{u}} + \overline{\mathbf{u}^S}) = MKE_E + MKE_S + MKE_{ES}$$

$$MKE_L = \frac{1}{2}\overline{\mathbf{u}^L} \cdot \overline{\mathbf{u}^L}$$

$$MKE_E = \frac{1}{2}\overline{\mathbf{u}} \cdot \overline{\mathbf{u}}$$

$$MKE_S = \frac{1}{2}\overline{\mathbf{u}^S} \cdot \overline{\mathbf{u}^S}$$

$$MKE_{ES} = \overline{\mathbf{u}} \cdot \overline{\mathbf{u}^S}$$

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Eulerian energy budget

$$\frac{\partial}{\partial t}MKE_E + Tr = \overline{u_i'u_j'}\frac{\partial \overline{u}_i}{\partial x_j} + \delta_{i,3}\overline{b}\overline{u}_i - \mu\frac{\partial \overline{u}_i}{\partial x_j}\frac{\partial \overline{u_i^L}}{\partial x_j} + \epsilon_{ijk}\overline{u}_i f_j \overline{u_k^S}$$

$\epsilon_{ijk}\overline{u}_i f_j \overline{u_k^S}$, work done by Coriolis-Stokes (Hasselmann) force is an energy source

,e.g. Liu et al. (2009) , Sayol et al. (2016), Suzuki&Fox-Kemper (2016), Zhang et al. (2019)

$$\frac{\partial}{\partial t}TKE_E + Tr = -\overline{u_i'u_j'}\frac{\partial \overline{u}_i}{\partial x_j} - \overline{u_i'u_j'}\frac{\partial \overline{u_i^S}}{\partial x_j} + \delta_{i,3}\overline{b'u_i'} - \mu\overline{\frac{\partial u_i'}{\partial x_j}\frac{\partial u_i'}{\partial x_j}}$$

$$MKE_E = \frac{1}{2}\overline{\mathbf{u}} \cdot \overline{\mathbf{u}} \qquad TKE_E = \frac{1}{2}\overline{\mathbf{u}' \cdot \mathbf{u}'}$$

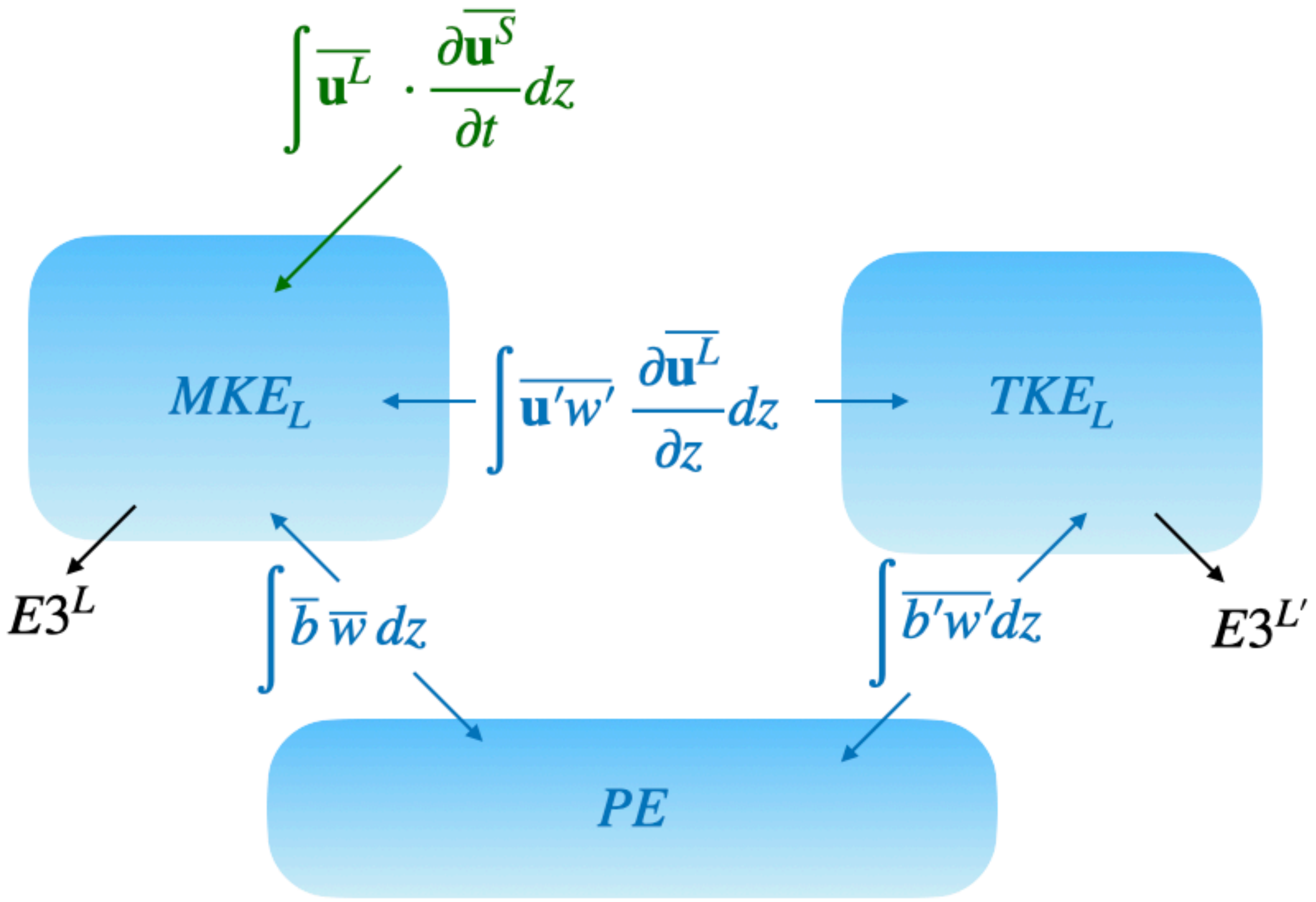
Lagrangian energy budgets

$$\frac{\partial}{\partial t}MKE_L + Tr = \overline{u_i'u_j'}\frac{\partial \overline{u_i^L}}{\partial x_j} + \delta_{i,3}\overline{b}\,\overline{u_i^L} - \mu\frac{\partial \overline{u_i^L}}{\partial x_j}\frac{\partial \overline{u_i^L}}{\partial x_j} + \overline{u_i^L}\frac{\partial \overline{u_i^S}}{\partial t}$$

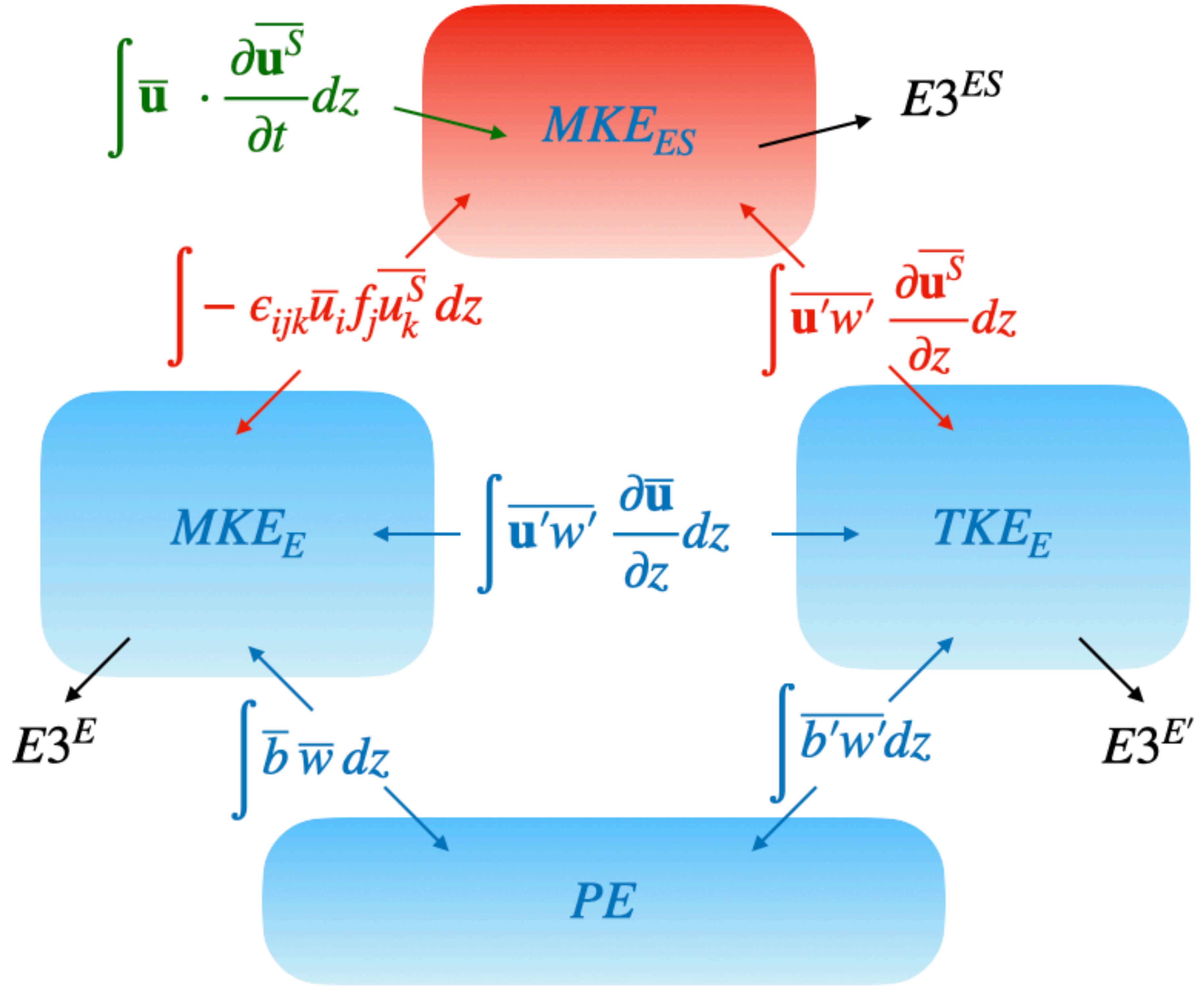
$$\frac{\partial}{\partial t}TKE_L + Tr = -\overline{u_i'u_j'}\frac{\partial \overline{u_i^L}}{\partial x_j} + \delta_{i,3}\overline{b'u_i'} - \mu\overline{\frac{\partial u_i'}{\partial x_j}\frac{\partial u_i'}{\partial x_j}}$$

$$MKE_L = \frac{1}{2}\overline{\mathbf{u}^L}\cdot\overline{\mathbf{u}^L} \qquad TKE_E = \frac{1}{2}\overline{\mathbf{u}^{L'}\cdot\mathbf{u}^{L'}}$$

Energy budgets



Lagrangian energy budget



Eulerian energy budget

Laminar example

$$\partial_t \mathbf{u}^L + (\mathbf{u}^L \cdot \nabla)\mathbf{u}^L + \mathbf{f} \times \mathbf{u}^L = b\mathbf{z} - \nabla p - \mathbf{u}^L \times (\nabla \times \mathbf{u}^S) + \mathbf{D}_u + \partial_t \mathbf{u}^S$$

consider a linear and non-viscous ocean away from lateral boundaries, with $\mathbf{u}^S = \begin{pmatrix} u^S(z,t) \\ 0 \\ 0 \end{pmatrix}$

horizontal momentum equations become:

$$\partial_t u^L - f v^L = \partial_t u^S$$

$$\partial_t v^L + f u^L = 0$$

Hasselmann (1970)

and in steady state:

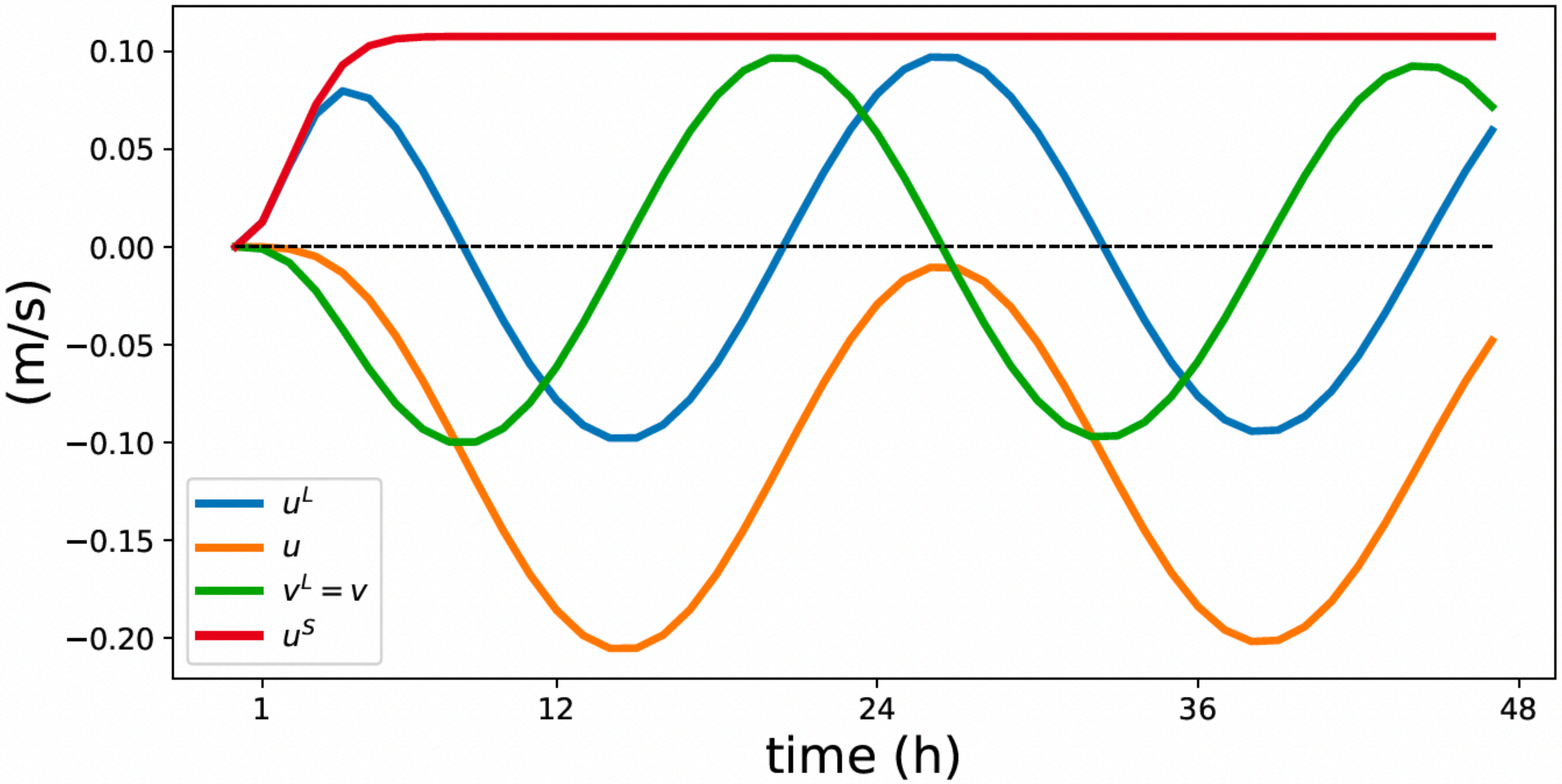
$$u^S = -u$$

$$v = 0$$

Ursell & Deacon (1950), Pollard (1970)

Numerical model: laminar example

$$u^S(z,t) = u_0^S \exp\left(\frac{z}{D_s}\right)\left[1 - \exp\left(\frac{-t^2}{2T_w^2}\right)\right]$$, with growth time scale T_w of 2h

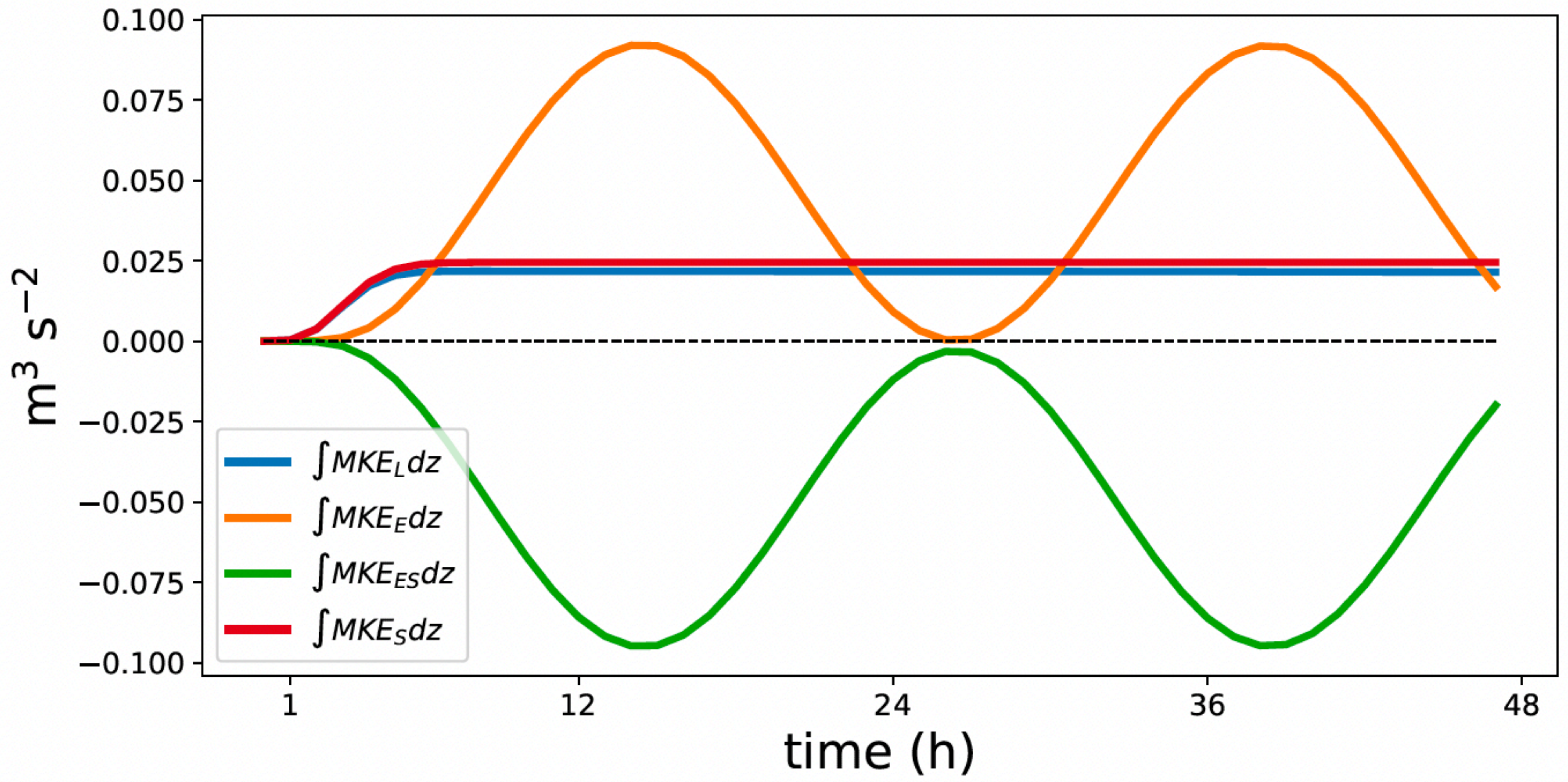


$$\partial_t u^L - fv = +\,\partial_t u^S$$

$$\partial_t v^L + fu + fu^S = 0$$

Laminar energy budgets

- (blue) $\frac{\partial}{\partial t} MKE_L = \overline{\mathbf{u}^L} \cdot \partial_t \overline{\mathbf{u}^S}$
- (orange) $\frac{\partial}{\partial t} MKE_E = -\,\overline{\mathbf{u}} \cdot f\mathbf{z} \times \overline{\mathbf{u}^S}$
- (red) $\frac{\partial}{\partial t} MKE_S = \overline{\mathbf{u}^S} \cdot \partial_t \overline{\mathbf{u}^S}$
- (green) $\frac{\partial}{\partial t} MKE_{ES} = \overline{\mathbf{u}} \cdot f\mathbf{z} \times \overline{\mathbf{u}^S} + \overline{\mathbf{u}} \cdot \partial_t \overline{\mathbf{u}^S}$



Coupling between wave model and large scale ocean model

$$\partial_t \mathbf{u}^L + (\mathbf{u}^L \cdot \nabla)\mathbf{u}^L + \mathbf{f} \times \mathbf{u}^L = b\mathbf{z} - \nabla p - \mathbf{u}^L \times (\nabla \times \mathbf{u}^S) + \nabla \mu_t \nabla (\mathbf{u}^L - \mathbf{u}^S) + \partial_t \mathbf{u}^S$$

assume $\nabla_h u^L \gg \nabla_h u^S$

equations to be used in large-scale ocean models become:

$$\partial_t \mathbf{u}_h^L + (\mathbf{u}^L \cdot \nabla)\mathbf{u}_h^L + \mathbf{f} \times \mathbf{u}_h^L = -\nabla_h p + \nabla \mu_t \nabla (\mathbf{u}_h^L - \mathbf{u}_h^S) + \partial_t \mathbf{u}_h^S + \left[w^L \partial_z \mathbf{u}_h^S \right]$$

$$\partial_z p - b = \left[-u^L \partial_z u^S - v^L \partial_z v^S \right]$$

$$\partial_t b + (\mathbf{u}^L \cdot \nabla) b = \nabla \kappa_t \nabla b$$

$$\nabla \cdot \mathbf{u}^L = 0$$

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Coupling - energy provided by the wave model

$\omega > \omega_c$ *diagnostic range*

$\omega < \omega_c$ *prognostic range*

$$\Phi_{oc} = \rho_w g \int_0^{2\pi} \int_{\omega_c}^{\infty} S_{in} \, d\omega d\theta \; - \; \Phi_{diss}$$

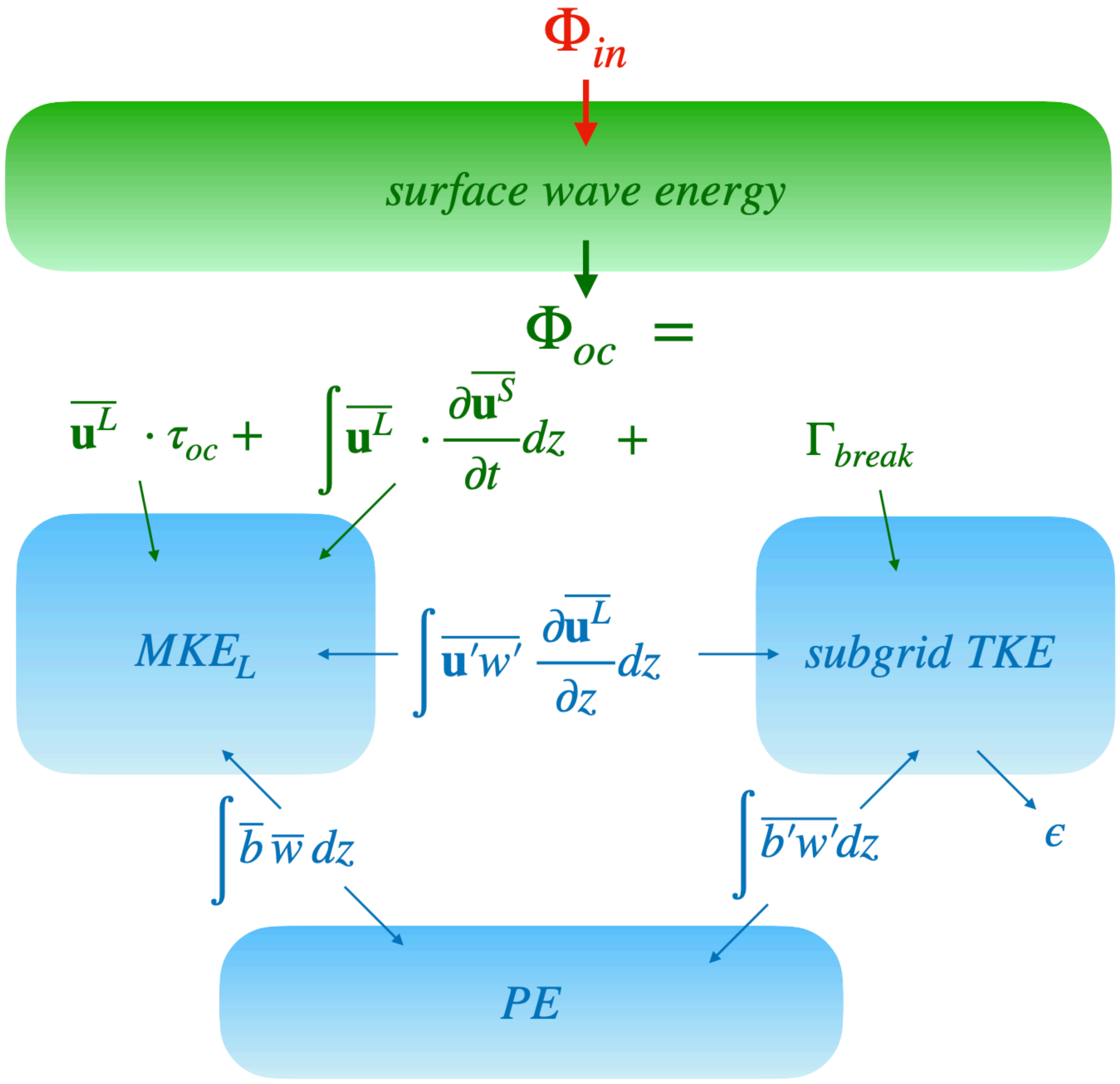
$$\boldsymbol{\tau}_{oc} = \boldsymbol{\tau}_a - \boldsymbol{\tau}_{in} + \boldsymbol{\tau}_{diss}$$

Chalikov & Belevich (1993), Janssen (2012), Breivik et al. (2015)

Φ_{oc} ***is energy gain of ocean water column***

$$\Phi_{oc} = \overline{\mathbf{u}_h^L} \cdot \tau_{oc} + \int \overline{\mathbf{u}_h^L} \cdot \partial_t \overline{\mathbf{u}_h^S} \, dz + \Gamma_{break}$$

Coupling framework



Conclusions

- simple inclusion of sea state impacts in climate models by re-interpreting existing velocity as Lagrangian velocity ($\partial_t \mathbf{u}_h^S$ is only new term in momentum equation)

- energy provided by the wave model is split-up into energy which goes into mean motions and a remainder which goes into turbulence according to

$$\Phi_{oc} = \overline{\mathbf{u}^L} \cdot \tau_{oc} + \int \overline{\mathbf{u}^L} \cdot \partial_t \overline{\mathbf{u}^S} + \Gamma_{break}$$

- energy transfer terms in Lagrangian budget are easier to interpret, for example the Coriolis-Stokes term is absent