Planetary Boundary Layer 2

Surface layer and empirical functions

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Contents

- What do we need from a turbulence parameterization?
- Local eddy diffusion (K-profile)
- Surface layer similarity theory
- Roughness length
- Empirical stability function “cookbook” and history

Set of equations to solve in the model

Momentum

$$\frac{\partial \bar{u}}{\partial t} = -\frac{1}{\rho}\left[\nabla \cdot (\rho \overline{u\boldsymbol{u}}) + \frac{\partial \rho \overline{u'w'}}{dz}\right] + fv - \frac{1}{\rho}\frac{\partial p}{\partial x}$$

$$\frac{\partial \bar{v}}{\partial t} = -\frac{1}{\rho}\left[\nabla \cdot (\rho \overline{v\boldsymbol{u}}) + \frac{\partial \rho \overline{v'w'}}{dz}\right] + fu - \frac{1}{\rho}\frac{\partial p}{\partial y}$$

$$\frac{\partial \bar{p}}{\partial z} = -\rho g$$

Thermodynamics

$$\frac{\partial \bar{\theta}}{\partial t} = -\frac{1}{\rho}\left[\nabla \cdot (\rho \bar{\theta}\overline{\boldsymbol{u}}) + \frac{\partial \rho \overline{\theta' w'}}{dz}\right] + S_\theta$$

Moisture

$$\frac{\partial \bar{q}}{\partial t} = -\frac{1}{\rho}\left[\nabla \cdot (\rho \overline{q\boldsymbol{u}}) + \frac{\partial \rho \overline{q'w'}}{dz}\right] + S_q$$

Large scale terms – resolved by model

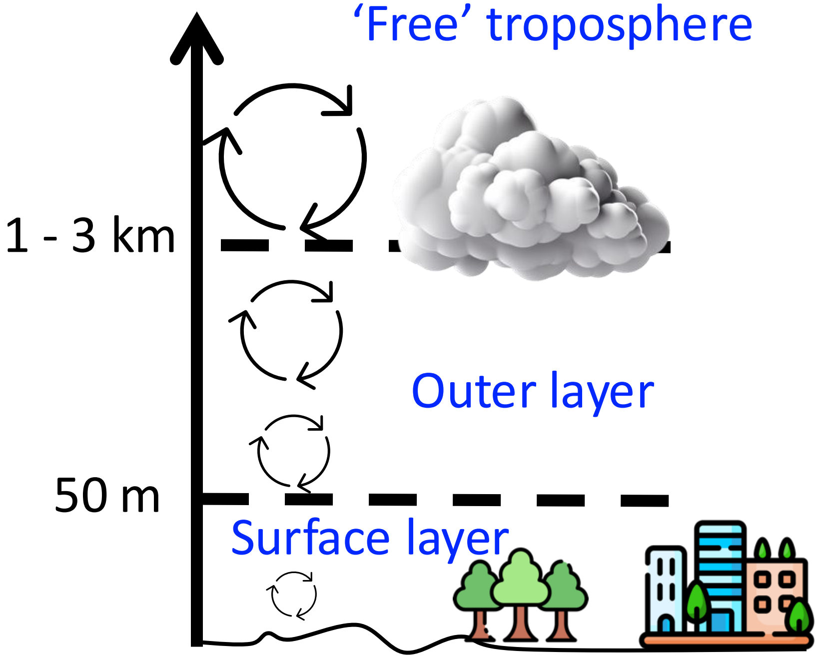
Small scale turbulent fluxes – must be parametrized

Sources and sinks (e.g. heating and cooling from radiation)

What do we need from a turbulence parametrization scheme?

- Provide turbulent exchange of heat, momentum, moisture (and tracers) between the surface and the upper atmosphere

- Account for differences in stability and surface properties

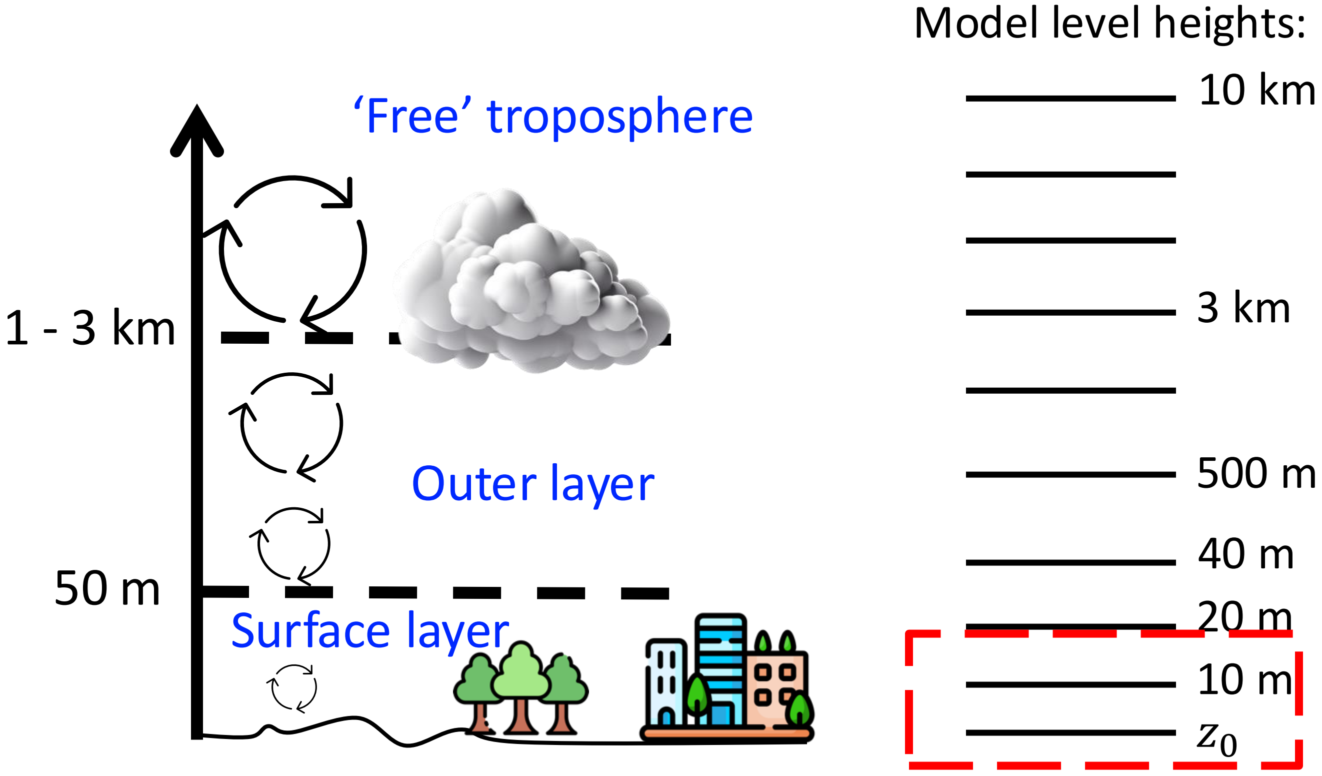


What do we need from a turbulence parametrization scheme?

- Provide turbulent exchange of heat, momentum, moisture (and tracers) between the surface and the upper atmosphere

- Account for differences in stability and surface properties

- Provide profiles of winds and temperatures at the surface, where the model does not resolve in the vertical



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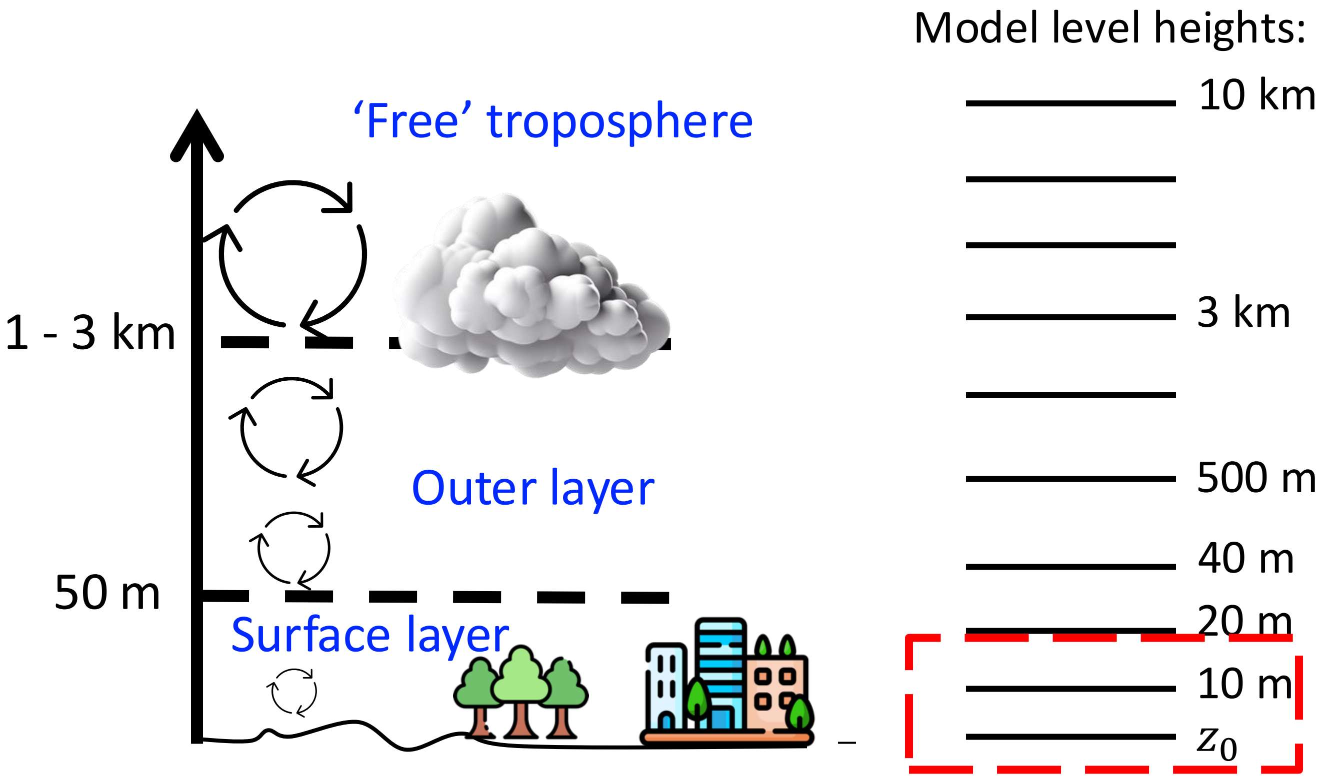
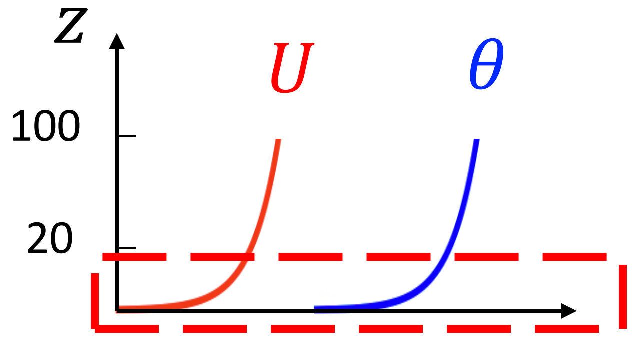


- Model does not resolve surface layer
- There are strong gradients and is where people live
- Requires diagnosis of profiles below 10m



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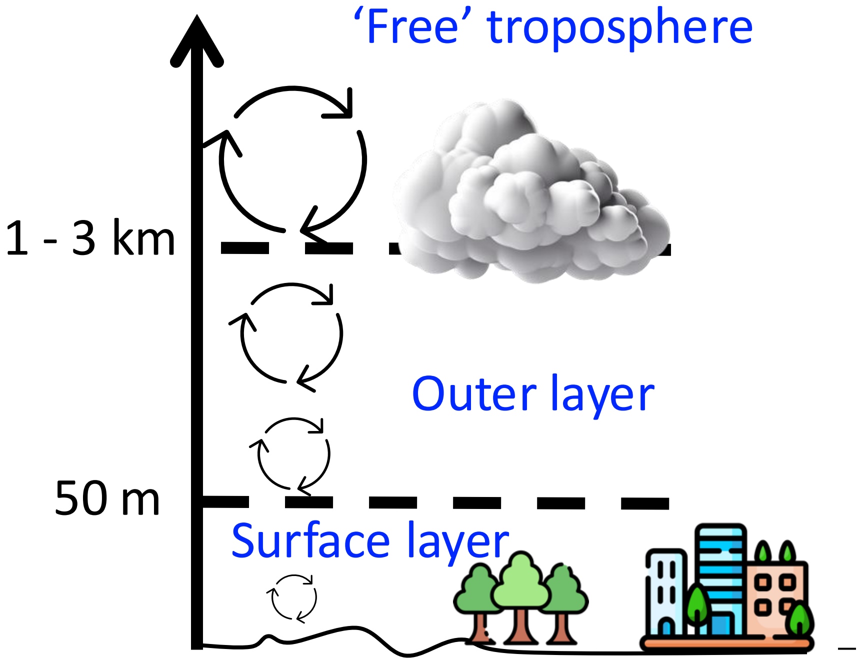
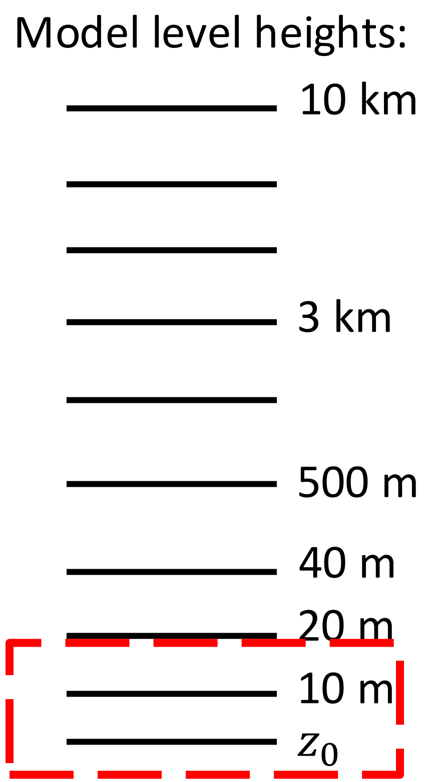
- Provide turbulent exchange of heat, momentum, moisture (and tracers) between the surface and the upper atmosphere
- Account for differences in stability and surface properties
- Provide profiles of winds and temperatures at the surface, where the model does not resolve in the vertical
- Provide turbulent mixing throughout the entire atmosphere – the mixed layer, the cloud layer and the stratosphere





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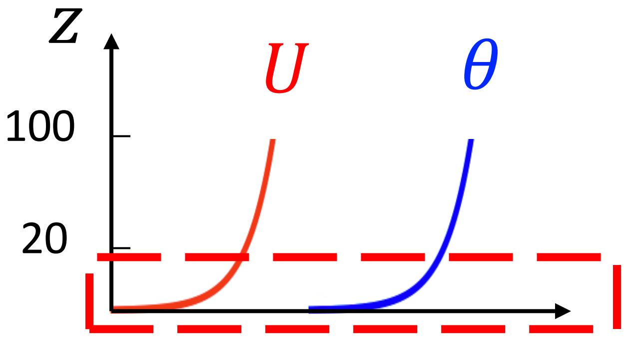

‘Local’ turbulence closure: eddy diffusion (K-profile)

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Any quantity ϕ:

$$\frac{\partial \overline{\phi}}{\partial t} = -\frac{1}{\rho}\left[\frac{\partial \rho \overline{\phi' w'}}{dz}\right]$$

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$$\frac{\partial\overline{\phi}}{\partial t} = -\frac{1}{\rho}\left[\frac{\partial\rho\overline{\phi' w'}}{dz}\right] \sim -\frac{1}{\rho}\frac{\partial}{dz}\left(-\rho K_{\phi}\frac{\partial\overline{\phi}}{\partial z}\right)$$

The magnitude of K_{ϕ} determines the ‘stirring’ of these conserved quantities by turbulent eddies

'Local' turbulence closure: eddy diffusion (K-profile)

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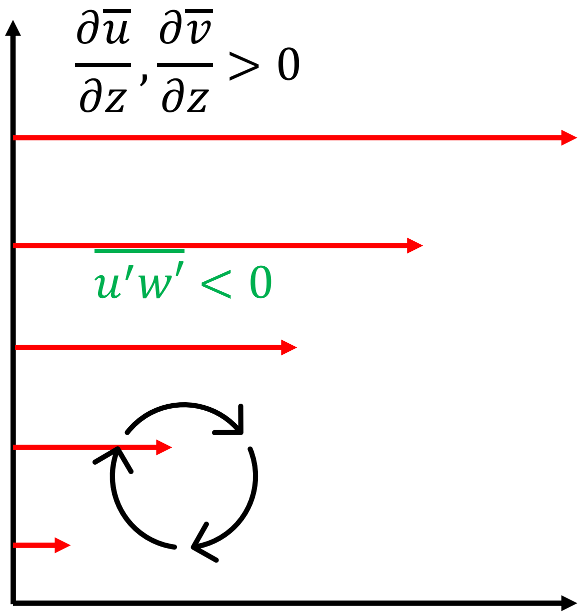

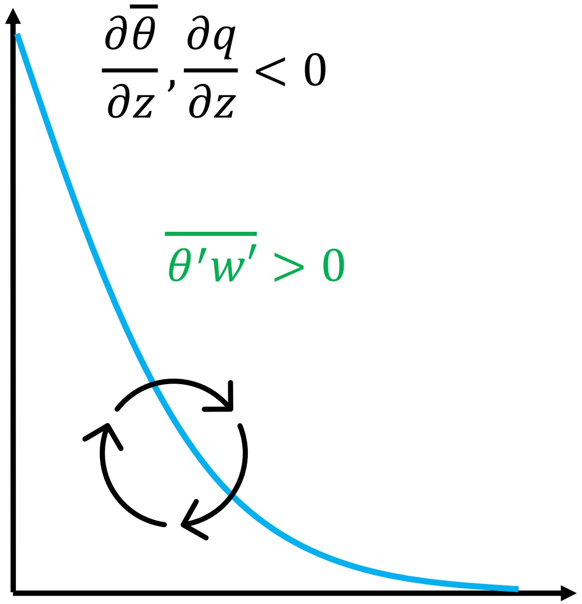

Wind / temperature gradient with turbulent eddies will generate mixing

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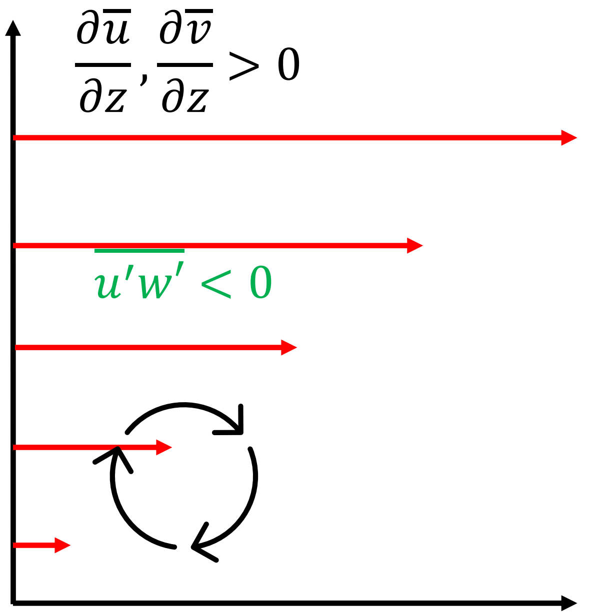

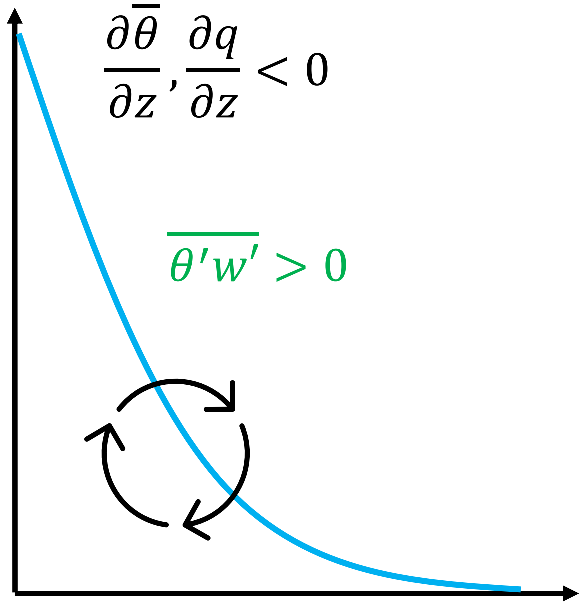

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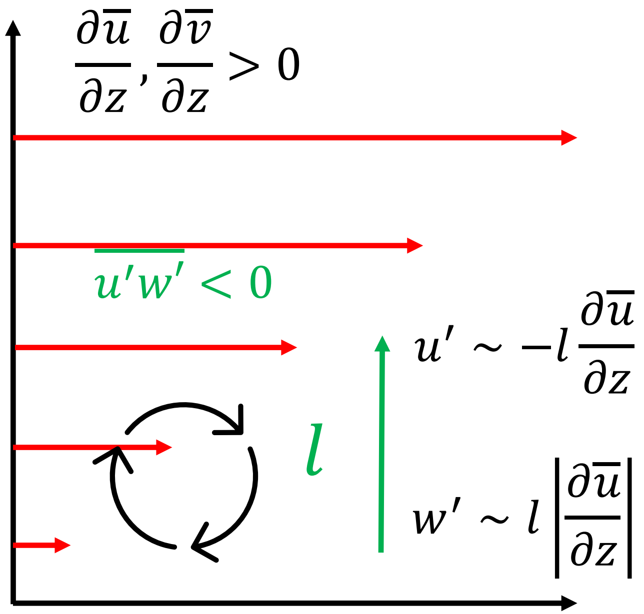
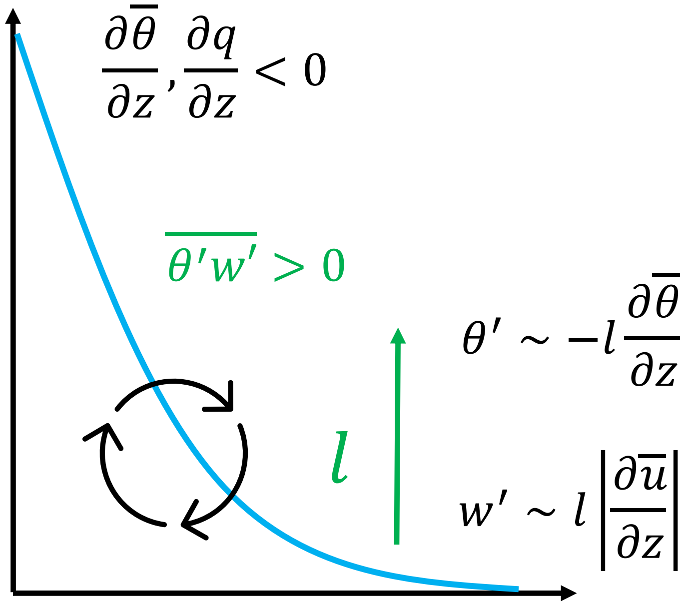
Mixing occurs over a certain lengthscale l, related to size of eddies





Wind / temperature gradient with turbulent eddies will generate mixing

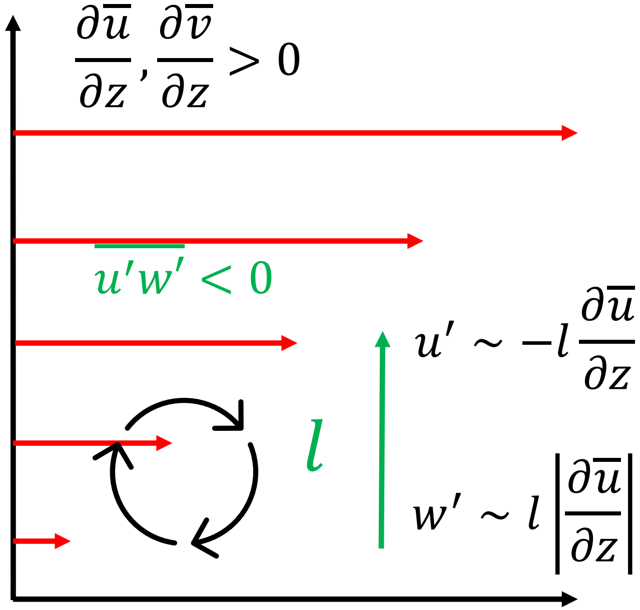
‘Local’ turbulence closure: eddy diffusion (K-profile)

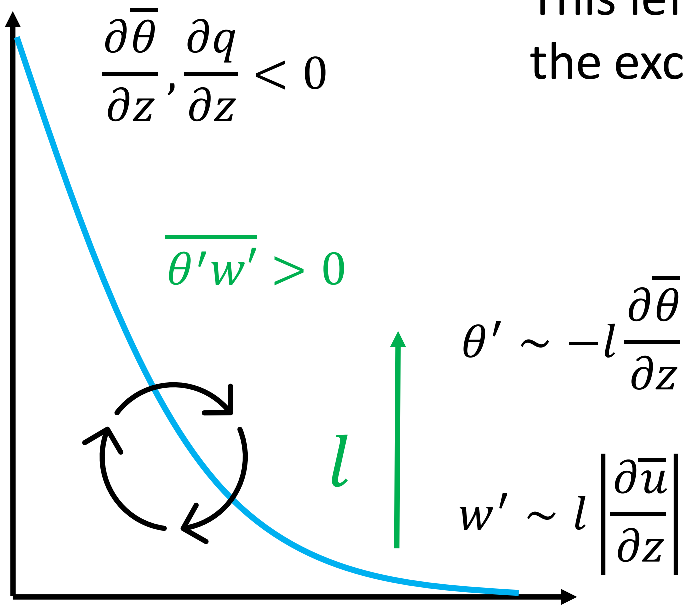
Any quantity ϕ:

$$\overline{\phi' w'} = -K_\phi \frac{\partial \overline{\phi}}{\partial z} = -l^2 \left| \frac{\partial \overline{u}}{\partial z} \right| \frac{\partial \overline{\phi}}{\partial z}$$

Mixing occurs over a certain lengthscale l, related to size of eddies

This lengthscale can be used to determine the exchange coefficients (K_ϕ)





Wind / temperature gradient with turbulent eddies will generate mixing

'Local' turbulence closure: eddy diffusion (K-profile)

Momentum

$$\overline{u'w'} = -K_M \frac{\partial \overline{u}}{\partial z} = -l^2 \left| \frac{\partial \overline{u}}{\partial z} \right| \frac{\partial \overline{u}}{\partial z}$$

Thermodynamics

$$\overline{\theta' w'} \sim -K_H \frac{\partial \overline{\theta}}{\partial z} = -l^2 \left| \frac{\partial \overline{u}}{\partial z} \right| \frac{\partial \overline{\theta}}{\partial z}$$

K_M, K_H and K_q are the exchange coefficients of momentum, heat and moisture

Generally assumed that diffusion of heat == diffusion of moisture

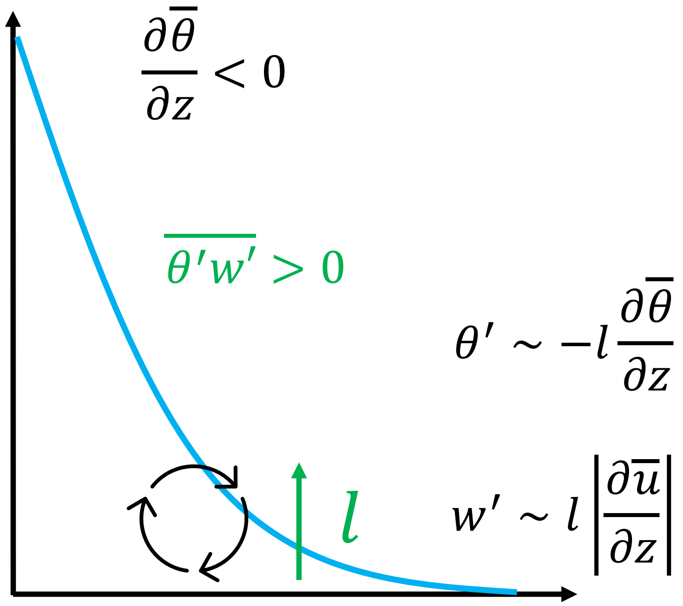
$$K_H = K_q$$

‘Local’ turbulence closure at the surface

What is l at the surface?

Any quantity ϕ:

$$\overline{\phi' w'} = -K_\phi \frac{\partial \overline{\phi}}{\partial z} = -l^2 \left|\frac{\partial \overline{u}}{\partial z}\right| \frac{\partial \overline{\phi}}{\partial z}$$



Size of eddies are constrained by the surface itself:

$$l \sim \kappa z,$$

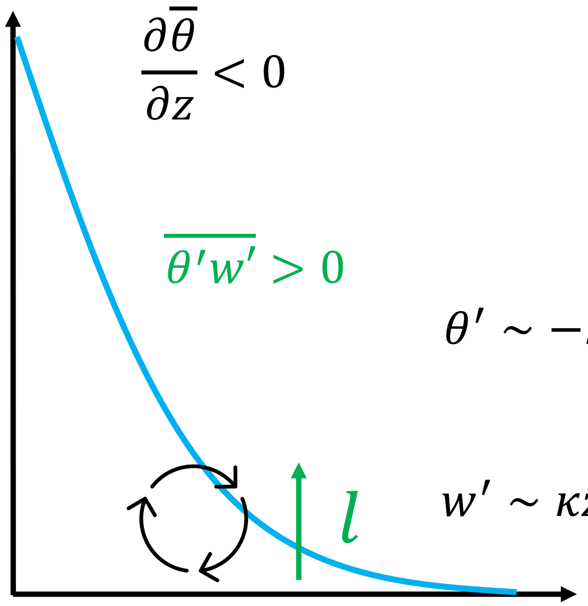
$\kappa =$von-Karman constant

– determined from observations

What is l at the surface?

Any quantity ϕ:

$$\overline{\phi' w_s'} = -K_\phi \frac{\partial \overline{\phi}}{\partial z} = -\kappa^2 z^2 \left|\frac{\partial \overline{u}}{\partial z}\right| \frac{\partial \overline{\phi}}{\partial z}$$



$$\theta' \sim -\kappa z \frac{\partial \overline{\theta}}{\partial z}$$

$$w' \sim \kappa z \left|\frac{\partial \overline{u}}{\partial z}\right|$$

Size of eddies are constrained by the surface itself:

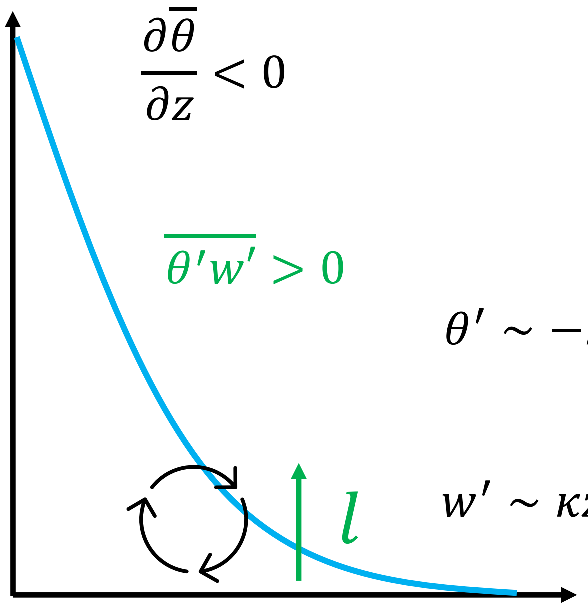
$$l \sim \kappa z,$$

κ =von-Karman constant
– determined from observations

Assume that fluxes are constant with height near the surface

Any quantity ϕ:

$$\left(\overline{\phi' w'}\right)_z = \left(\overline{\phi' w'}\right)_s = -\kappa^2 z^2 \left|\frac{\partial \bar{u}}{\partial z}\right| \frac{\partial \bar{\phi}}{\partial z}$$



$$\theta' \sim -\kappa z \frac{\partial \bar{\theta}}{\partial z}$$

$$w' \sim \kappa z \left|\frac{\partial \bar{u}}{\partial z}\right|$$

Near surface, fluxes are assumed constant with height:

$$\left(\overline{\phi' w'}\right)_s = -\kappa^2 z^2 \left|\frac{\partial \bar{u}}{\partial z}\right| \frac{\partial \bar{\phi}}{\partial z} = u_* \phi_*$$

$$\left(\overline{u' w'}\right)_s = -\kappa^2 z^2 \left|\frac{\partial \bar{u}}{\partial z}\right| \frac{\partial \bar{u}}{\partial z} = u_*^2$$

$$\left(\overline{\theta' w'}\right)_s = -\kappa^2 z^2 \left|\frac{\partial \bar{u}}{\partial z}\right| \frac{\partial \bar{\theta}}{\partial z} = u_* \theta_*$$

Where $u_* = \sqrt{\left(\overline{u' w'}\right)_s} = \kappa z \left|\frac{\partial \bar{u}}{\partial z}\right|$ is the surface frictional velocity

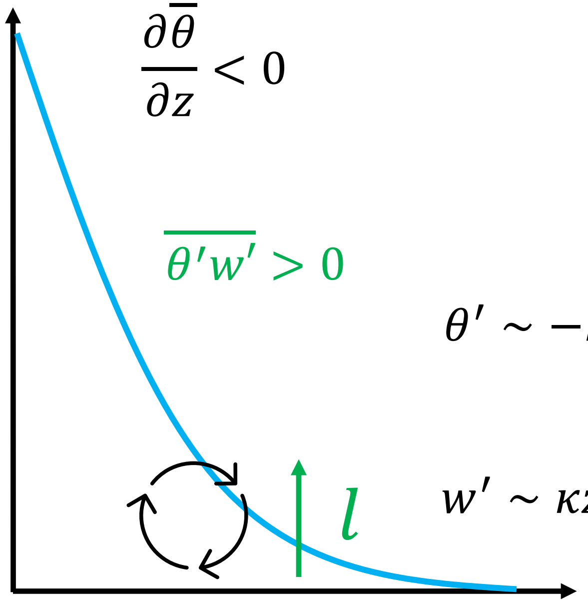
θ_* is the temperature scaling, similarly, q_* is the moisture scaling

This means we can get profiles of $\overline{\phi}$ from flux

Any quantity ϕ:

$$\left(\overline{\phi' w'}\right)_z = -\kappa^2 z^2 \left|\frac{\partial \overline{u}}{\partial z}\right| \frac{\partial \overline{\phi}}{\partial z} = u_* \phi_*$$

Using $u_* = \sqrt{\left(\overline{u' w'}\right)_s} = \kappa z \left|\frac{\partial \overline{u}}{\partial z}\right|$



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This means we can get profiles of $\overline{\phi}$ from flux

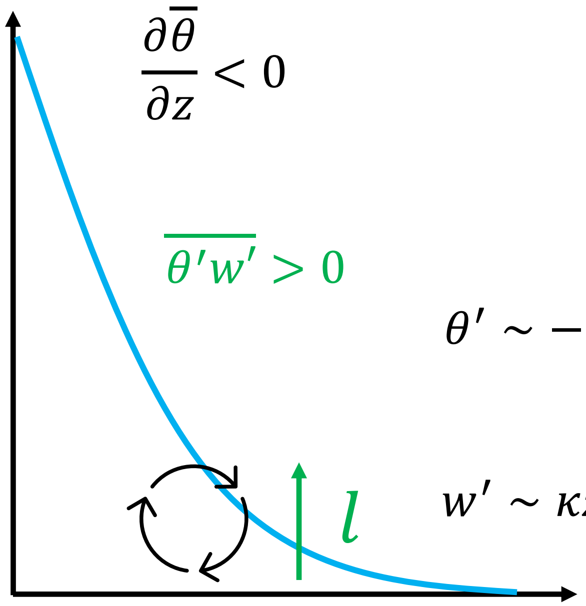
Any quantity ϕ:

$$\kappa z \frac{\partial \overline{\phi}}{\partial z} = \phi_*$$

Integrate

$$\overline{\phi}_z - \overline{\phi}_s = \frac{\phi_*}{\kappa} \int_{z_{0\phi}}^{z+z_{0M}} \frac{1}{z} dz$$

Using $u_* = \sqrt{\left(\overline{u'w'}\right)_s} = \kappa z \left|\frac{\partial \overline{u}}{\partial z}\right|$



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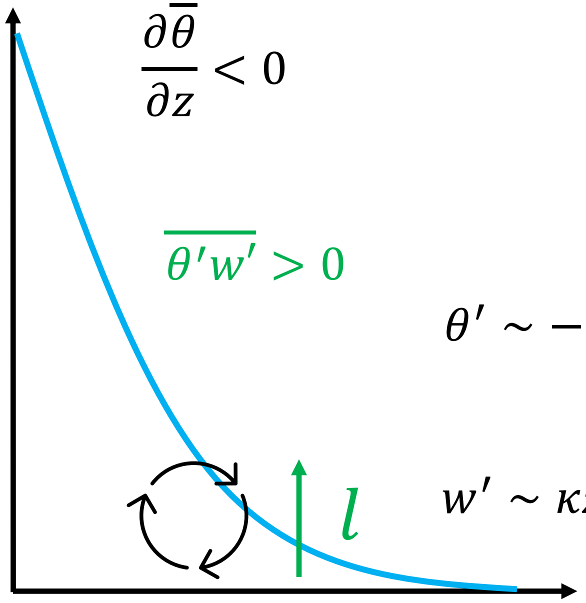
Any quantity ϕ:

$$\kappa z \frac{\partial \overline{\phi}}{\partial z} = \phi_*$$

Integrate

$$\overline{\phi}_z - \overline{\phi}_s = \frac{\phi_*}{\kappa} \log\left(\frac{z + z_{0\mathrm{M}}}{z_{0\phi}}\right)$$

Using $u_* = \sqrt{\left(\overline{u'w'}\right)_s} = \kappa z \left|\frac{\partial \overline{u}}{\partial z}\right|$



$$\theta' \sim -\kappa z \frac{\partial \overline{\theta}}{\partial z}$$

$$w' \sim \kappa z \left|\frac{\partial \overline{u}}{\partial z}\right|$$

This means we can get profiles of $\overline{u}$ and $\overline{\theta}$ from flux

Momentum

$$\kappa z \frac{\partial \overline{u}}{\partial z} = u_*$$

Thermodynamics

$$\kappa z \frac{\partial \overline{\theta}}{\partial z} = \theta_*$$

Where $u_* = \sqrt{\left(\overline{u'w'}\right)_s} = \kappa z \left|\frac{\partial \overline{u}}{\partial z}\right|$ is the surface frictional velocity

θ_* is the temperature scaling,
similarly, q_* is the moisture scaling

This means we can get profiles of $\overline{u}$ and $\overline{\theta}$ from flux

Momentum

$$\kappa z \frac{\partial \overline{u}}{\partial z} = u_*$$

Integrate:

$$\overline{u}_z = \frac{u_*}{\kappa} \int_{z_{0M}}^{z+z_{0M}} \frac{1}{z} dz$$

Thermodynamics

$$\kappa z \frac{\partial \overline{\theta}}{\partial z} = \theta_*$$

Integrate:

$$\overline{\theta}_z - \overline{\theta}_s = \frac{\theta_*}{\kappa} \int_{z_{0H}}^{z+z_{0M}} \frac{1}{z} dz$$

Where $u_* = \sqrt{\left(\overline{u'w'}\right)_s} = \kappa z \left|\frac{\partial \overline{u}}{\partial z}\right|$ is the surface frictional velocity

θ_* is the temperature scaling,
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This means we can get profiles of $\overline{u}$ and $\overline{\theta}$ from flux

Momentum

$$\kappa z \frac{\partial \overline{u}}{\partial z} = u_*$$

Integrate:

$$\overline{u}_z = \frac{u_*}{\kappa} \log\left(\frac{z + z_{0\mathrm{M}}}{z_{0\mathrm{H}}}\right)$$

Thermodynamics

$$\kappa z \frac{\partial \overline{\theta}}{\partial z} = \theta_*$$

Integrate:

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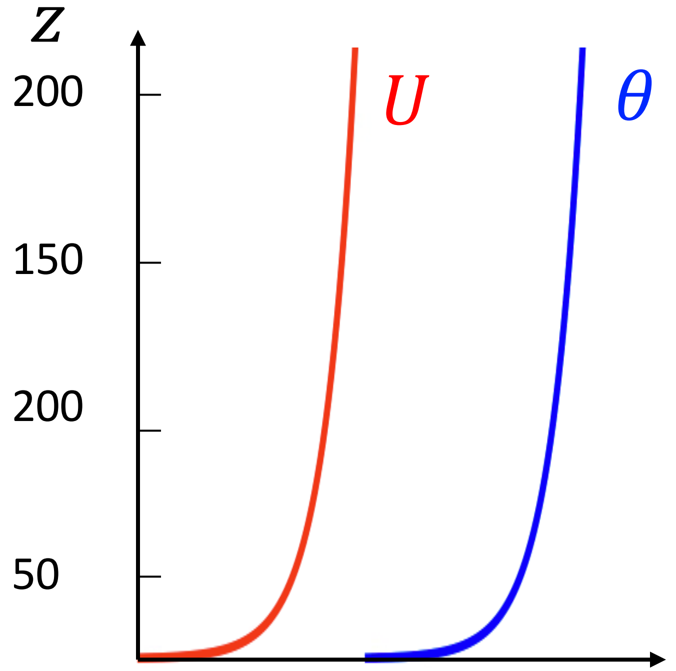
θ_* is the temperature scaling, similarly, q_* is the moisture scaling

Gives log profile for winds and potential temperature



Similarly, can get surface fluxes from profiles of $\overline{u}$ and $\overline{\theta}$

Momentum

$$\overline{u'w'} = u_*^2$$

Rearrange:

$$\overline{u'w'} = \frac{\kappa^2}{\log^2\left(\frac{z+z_0}{z_0}\right)} |\overline{u}_z| \overline{u}_z$$

Where $u_* = \sqrt{\left(\overline{u'w'}\right)_s} = \kappa z \left|\frac{\partial \overline{u}}{\partial z}\right|$ is the surface frictional velocity

θ_* is the temperature scaling, similarly, q_* is the moisture scaling

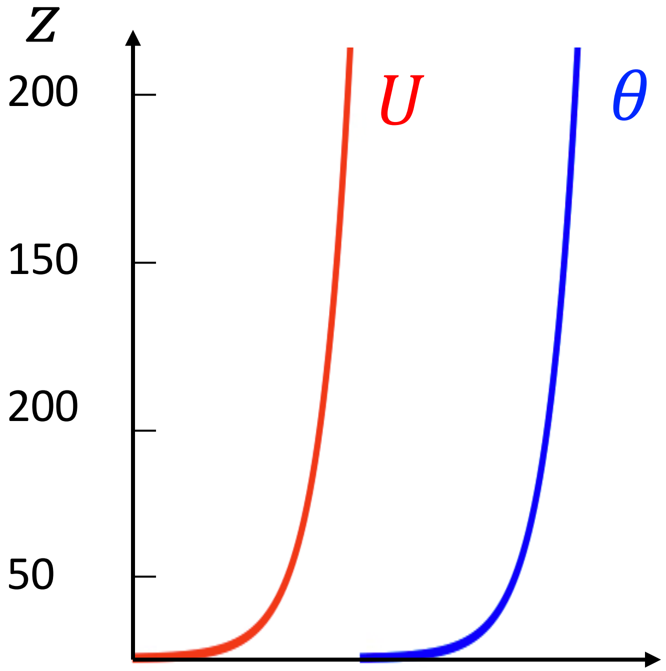
Thermodynamics

$$\overline{\theta' w'} = \theta_* u_*$$

Rearrange:

$$\overline{\theta' w'} = \frac{\kappa^2}{\log\left(\frac{z+z_{0M}}{z_{0H}}\right)\log\left(\frac{z+z_{0M}}{z_{0M}}\right)} |\overline{u}_z| (\overline{\theta}_z - \overline{\theta}_s)$$

Gives log profile for winds and potential temperature



Similarly, can get surface fluxes from profiles of $\overline{u}$ and $\overline{\theta}$

Momentum

$$\overline{u'w'} = u_*^2$$

Rearrange:

$$\rho\overline{u'w'} = C_M|\overline{u}_z|\overline{u}_z$$

Where $u_* = \sqrt{\left(\overline{u'w'}\right)_s} = \kappa z\left|\frac{\partial\overline{u}}{\partial z}\right|$ is the surface frictional velocity

θ_* is the temperature scaling,
similarly, q_* is the moisture scaling

Thermodynamics

$$\overline{\theta'w'} = \theta_* u_*$$

Rearrange:

$$\rho\overline{\theta'w'} = C_H|\overline{u}_z|(\overline{\theta}_z - \overline{\theta}_s)$$

Define surface exchange coefficients:

$$C_M = \frac{\kappa^2}{\log^2\left(\frac{z+z_0}{z_0}\right)}$$

$$C_H = \frac{\kappa^2}{\log\left(\frac{z+z_{0M}}{z_{0\mathrm{H}}}\right)\log\left(\frac{z+z_{0M}}{z_{0M}}\right)}$$

What is the roughness length z_0?

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Momentum

$$\kappa z \frac{\partial \overline{u}}{\partial z} = u_*$$

Integrate:

$$\overline{u}_z = \frac{u_*}{\kappa} \log\left(\frac{z + z_{0M}}{z_{0H}}\right)$$

Thermodynamics

$$\kappa z \frac{\partial \overline{\theta}}{\partial z} = \theta_*$$

Integrate:

$$\overline{\theta}_z - \overline{\theta}_s = \frac{\theta_*}{\kappa} \log\left(\frac{z + z_{0M}}{z_{0H}}\right)$$

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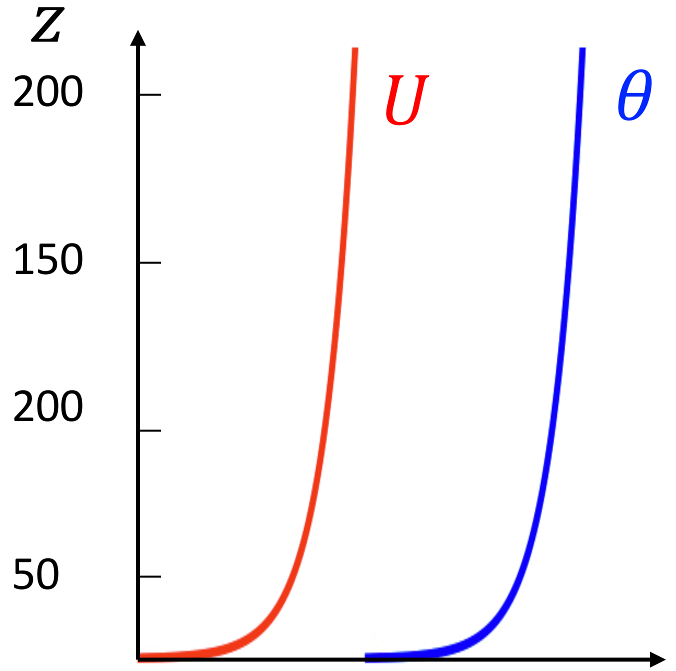
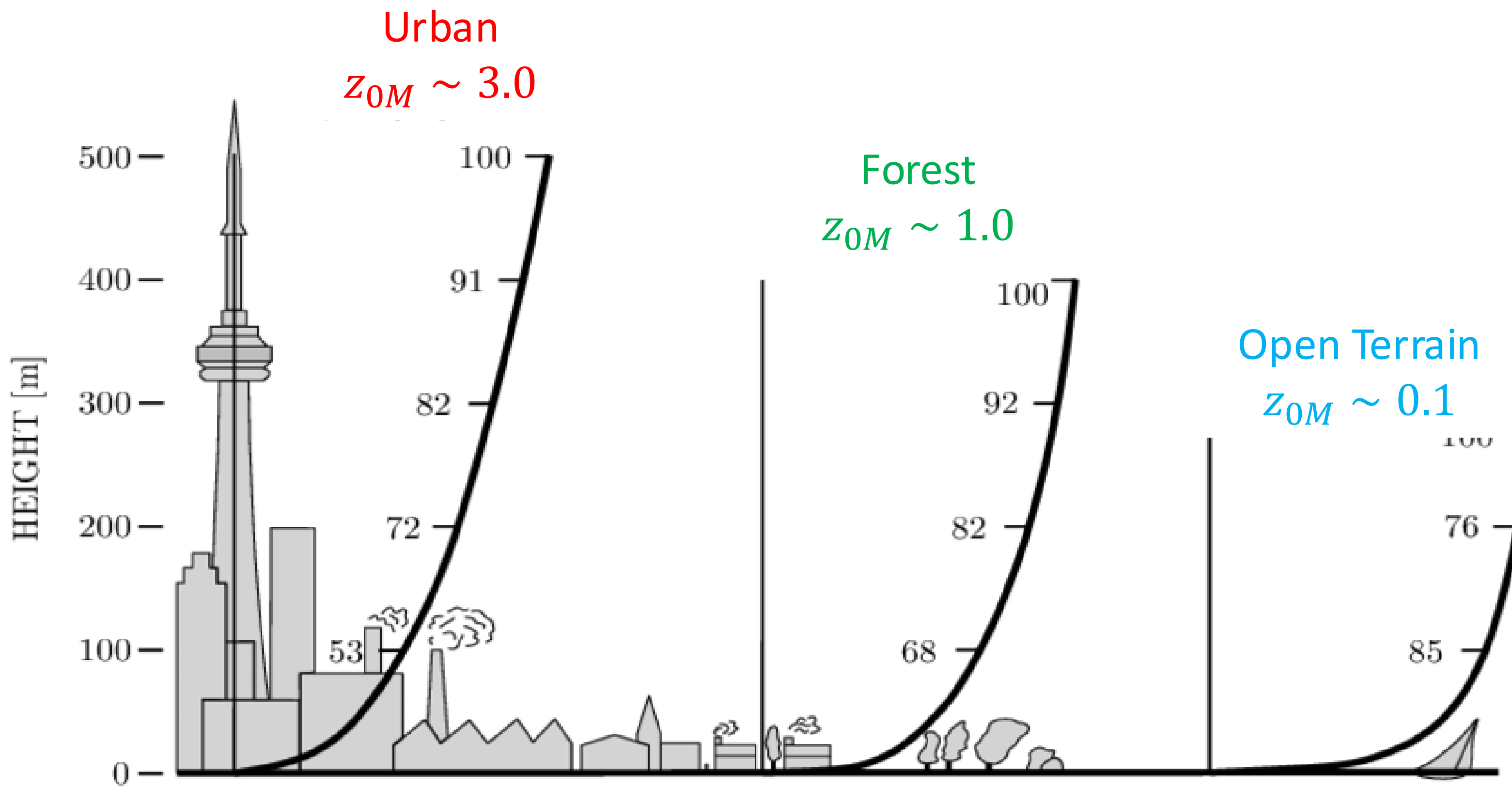
Gives log profile for winds and potential temperature



What is the roughness length z_0?

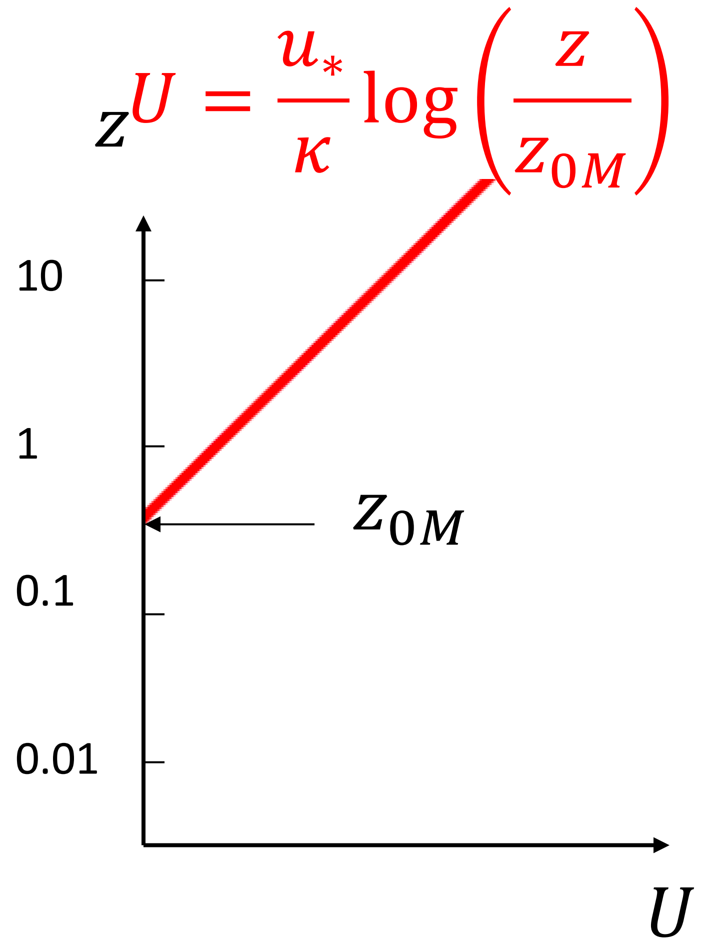
- Roughness length for momentum z_{0M} is not the same as for heat z_{0H}
- z_{0M} and z_{0H} determines the shape of the wind and temperature profiles
- They are a property of the underlying surface and are (assumed) to be a function of the height of the roughness elements



What is the roughness length z_0?

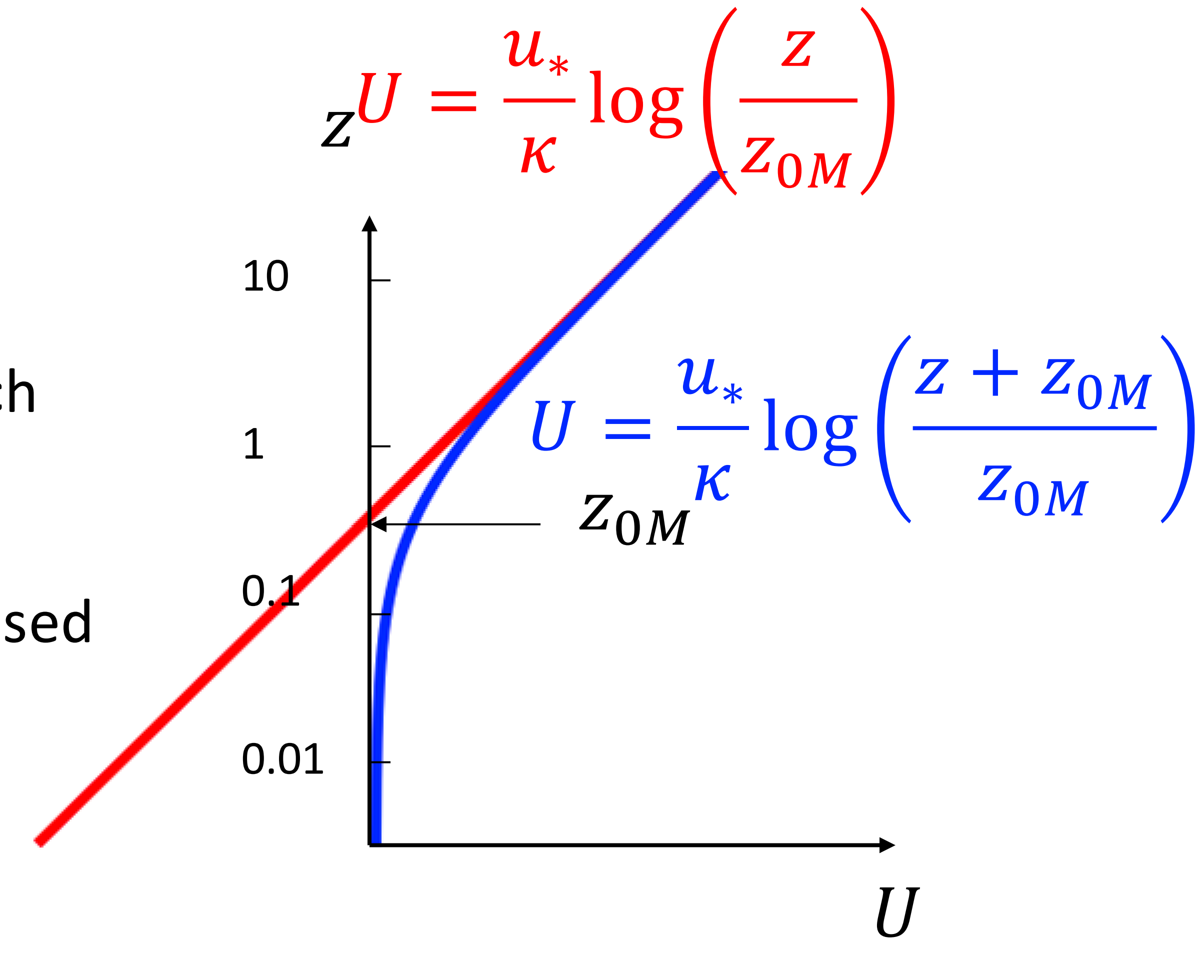
- Surface aerodynamic roughness length is defined from the logarithmic wind profile

- The roughness length is the height at which the winds become zero



What is the roughness length z_0?

- Surface aerodynamic roughness length is defined from the logarithmic wind profile
- The roughness length is the height at which the winds become zero
- In the model, the displacement height is used to obtain $U = 0$ at $\mathrm{z} = 0$.

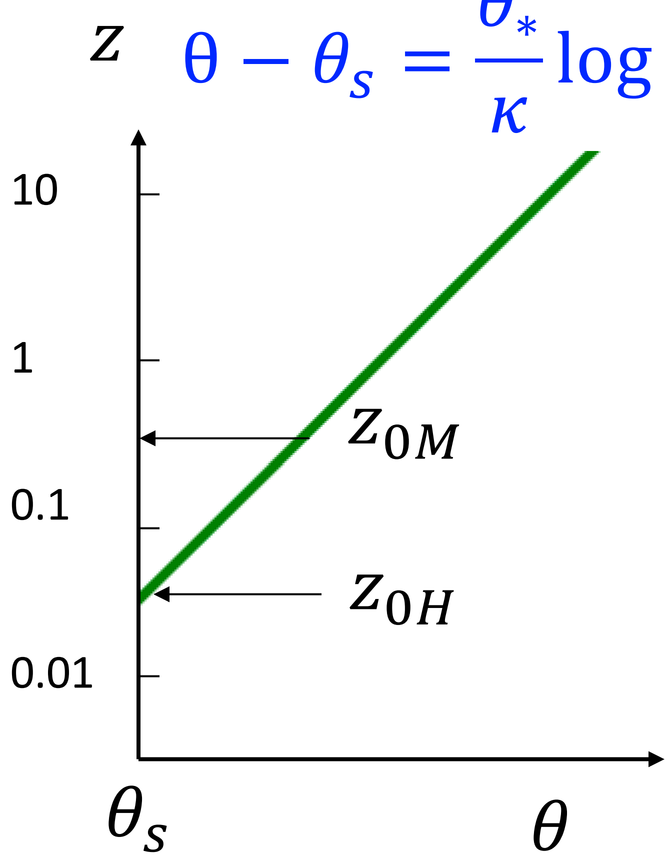


What is the roughness length z_0?

- Surface thermal roughness length is defined from the logarithmic temperature profile

- The thermal roughness length is the height at which the potential temperature becomes the surface temperature

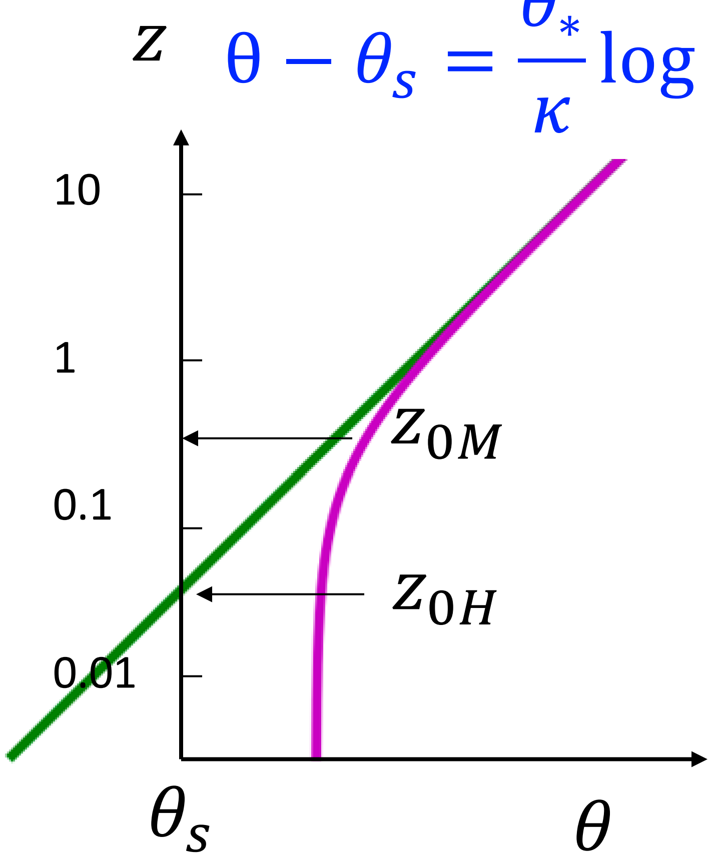
$$\theta - \theta_s = \frac{\theta_*}{\kappa} \log\left(\frac{z}{z_{0H}}\right)$$



What is the roughness length z_0?

- Surface thermal roughness length is defined from the logarithmic temperature profile
- The thermal roughness length is the height at which the potential temperature becomes the surface temperature
- In the model, the displacement height is used to obtain the temperature above the roughness elements, $\theta = \theta_s$ at $z = 0$ only when $z_{0M} = z_{0H}$

$$\theta - \theta_s = \frac{\theta_*}{\kappa} \log\left(\frac{z}{z_{0H}}\right)$$



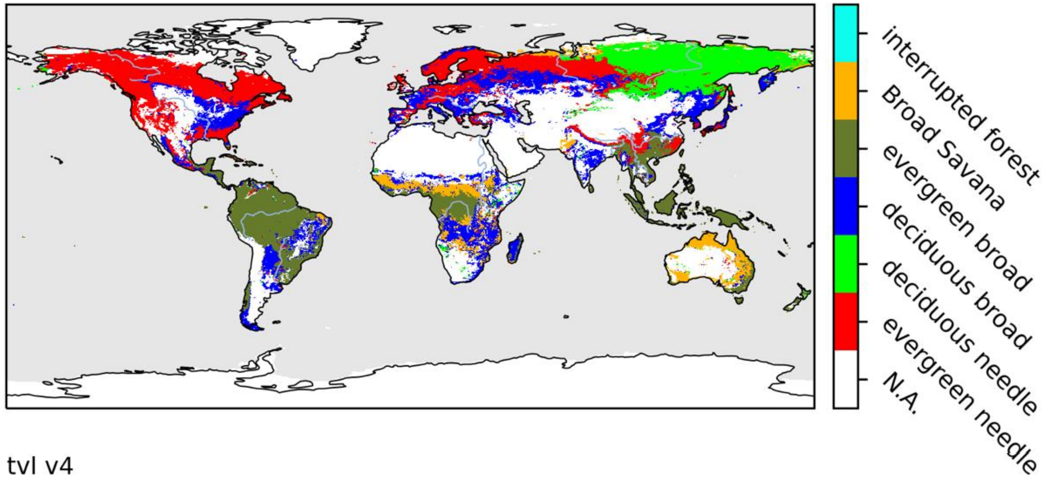
$$\theta - \theta_s = \frac{\theta_*}{\kappa} \log\left(\frac{z + z_{0M}}{z_{0H}}\right)$$

What is the roughness length z_0 over land?

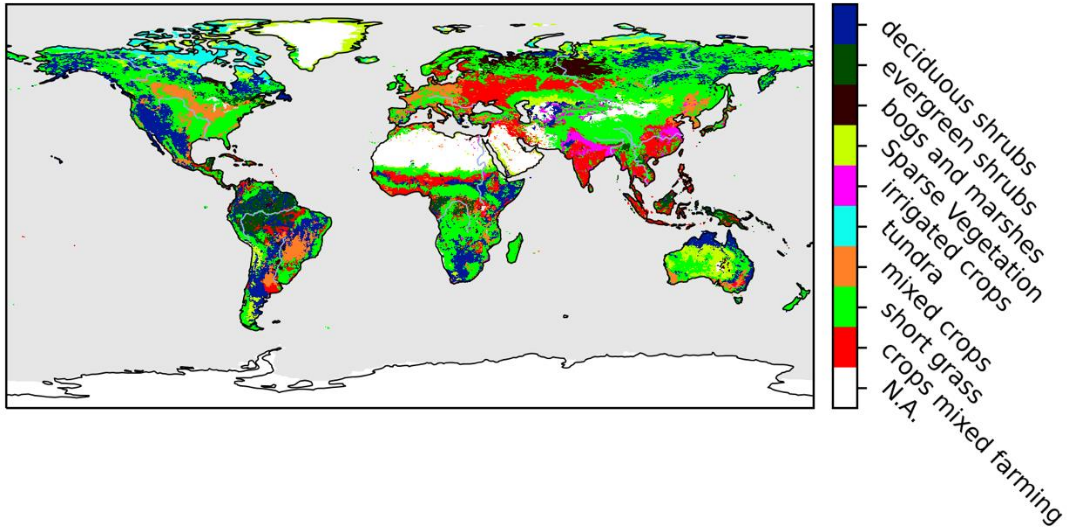
Note that $z_{0H} = \frac{z_{0M}}{10}$

Index	Vegetation type	H/L veg	z_{0m}	z_{0h}
1	Crops, mixed farming	L	0.25	$0.25\ 10^{-2}$
2	Short grass	L	0.1	$0.1\ 10^{-2}$
3	Evergreen needleleaf trees	H	2.0	2.0
4	Deciduous needleleaf trees	H	2.0	2.0
5	Deciduous broadleaf trees	H	2.0	2.0
6	Evergreen broadleaf trees	H	2.0	2.0
7	Tall grass	L	0.47	$0.47\ 10^{-2}$
8	Desert	–	0.013	$0.013\ 10^{-2}$
9	Tundra	L	0.034	$0.034\ 10^{-2}$
10	Irrigated crops	L	0.5	$0.5\ 10^{-2}$
11	Semidesert	L	0.17	$0.17\ 10^{-2}$
12	Ice caps and glaciers	–	$1.3\ 10^{-3}$	$1.3\ 10^{-4}$
13	Bogs and marshes	L	0.5	$0.5\ 10^{-2}$
14	Inland water	–	–	–
15	Ocean	–	–	–
16	Evergreen shrubs	L	0.100	$0.1\ 10^{-2}$
17	Deciduous shrubs	L	0.25	$0.25\ 10^{-2}$
18	Mixed forest/woodland	H	2.0	2.0
19	Interrupted forest	H	1.1	1.1
20	Water and land mixtures	L	–	–

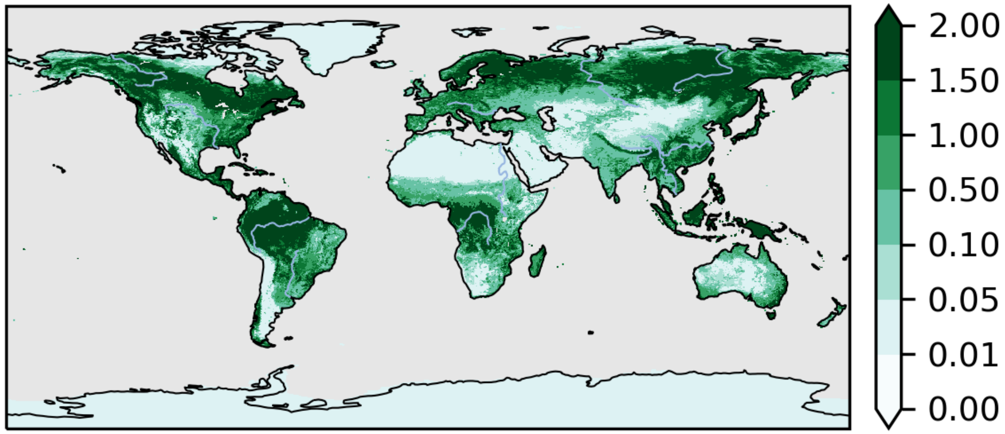
tvh v4



tvl v4



z0m None v4



What is the roughness length z_0 over ocean?

$$z_{0M} = \alpha_M \frac{\nu}{u_*} + \alpha_{Ch} \frac{u_*^2}{g}$$

$$z_{0H} = \alpha_H \frac{\nu}{u_*}$$

$$z_{0Q} = \alpha_Q \frac{\nu}{u_*}$$

ν = kinematic viscosity

u_* =surface frictional velocity

$\alpha_M, \alpha_H, \alpha_Q$ = constants

α_{Ch}= Charnock coefficient,
provided by the wave model

Bidlot et al, 2020

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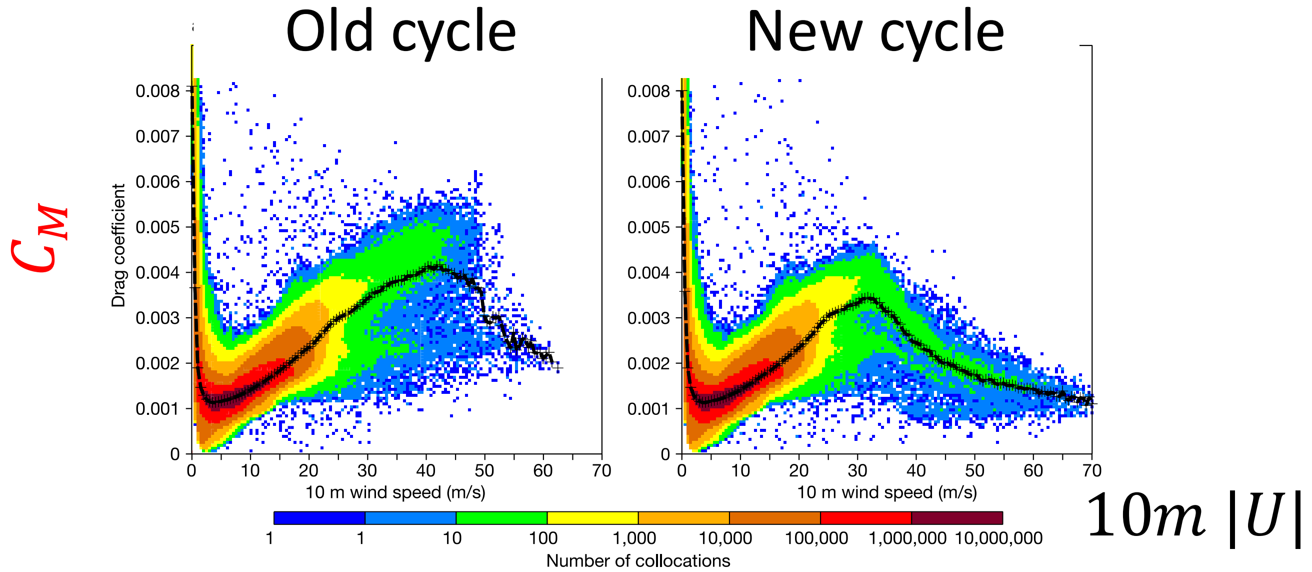

$$C_M = \frac{\kappa^2}{\left[\log\left(\frac{z + z_{0m}}{z_{0m}}\right) - \Psi_M\left(\frac{z + z_{0m}}{L}\right)\right]^2}$$

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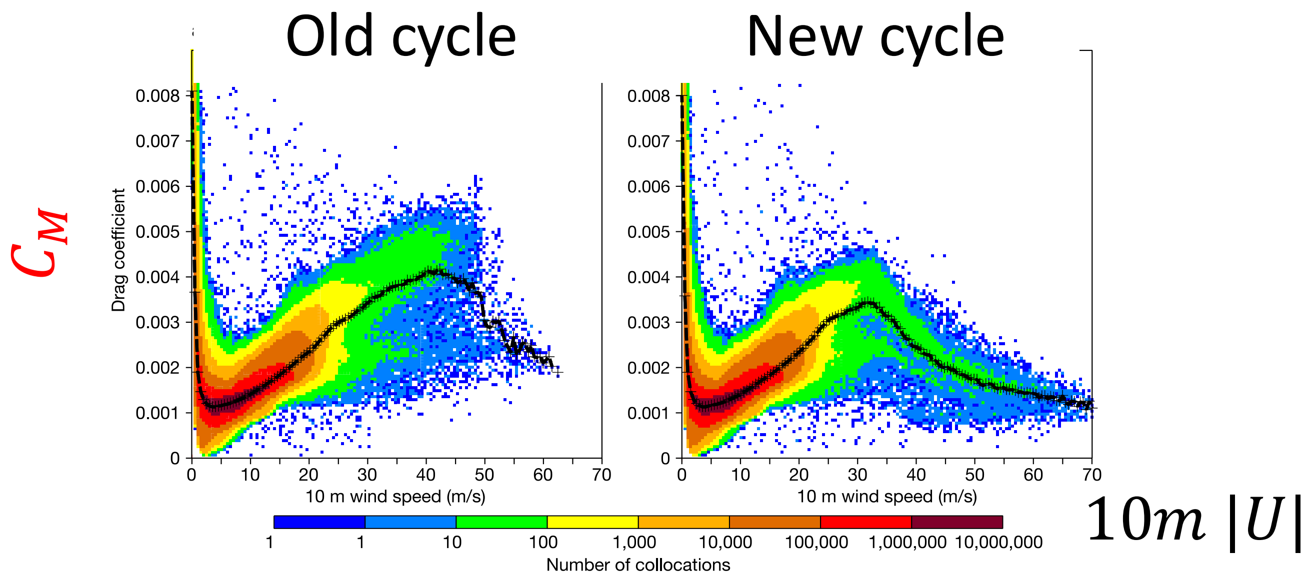
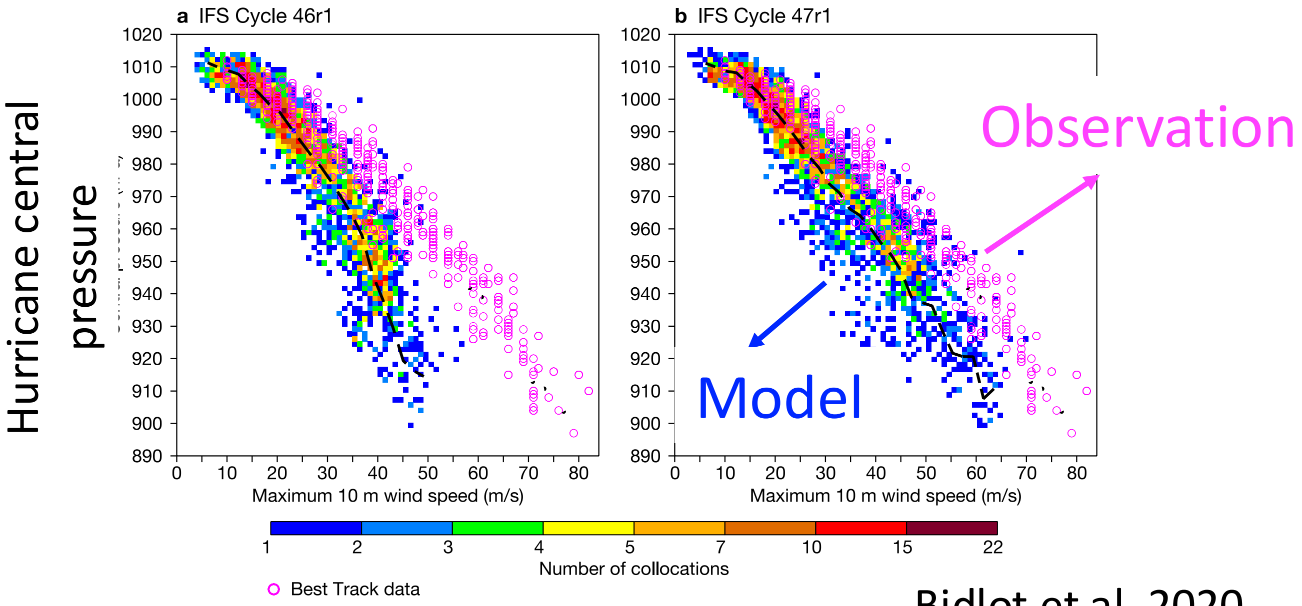




Bidlot et al, 2020

What is the roughness length z_0 over sea-ice?

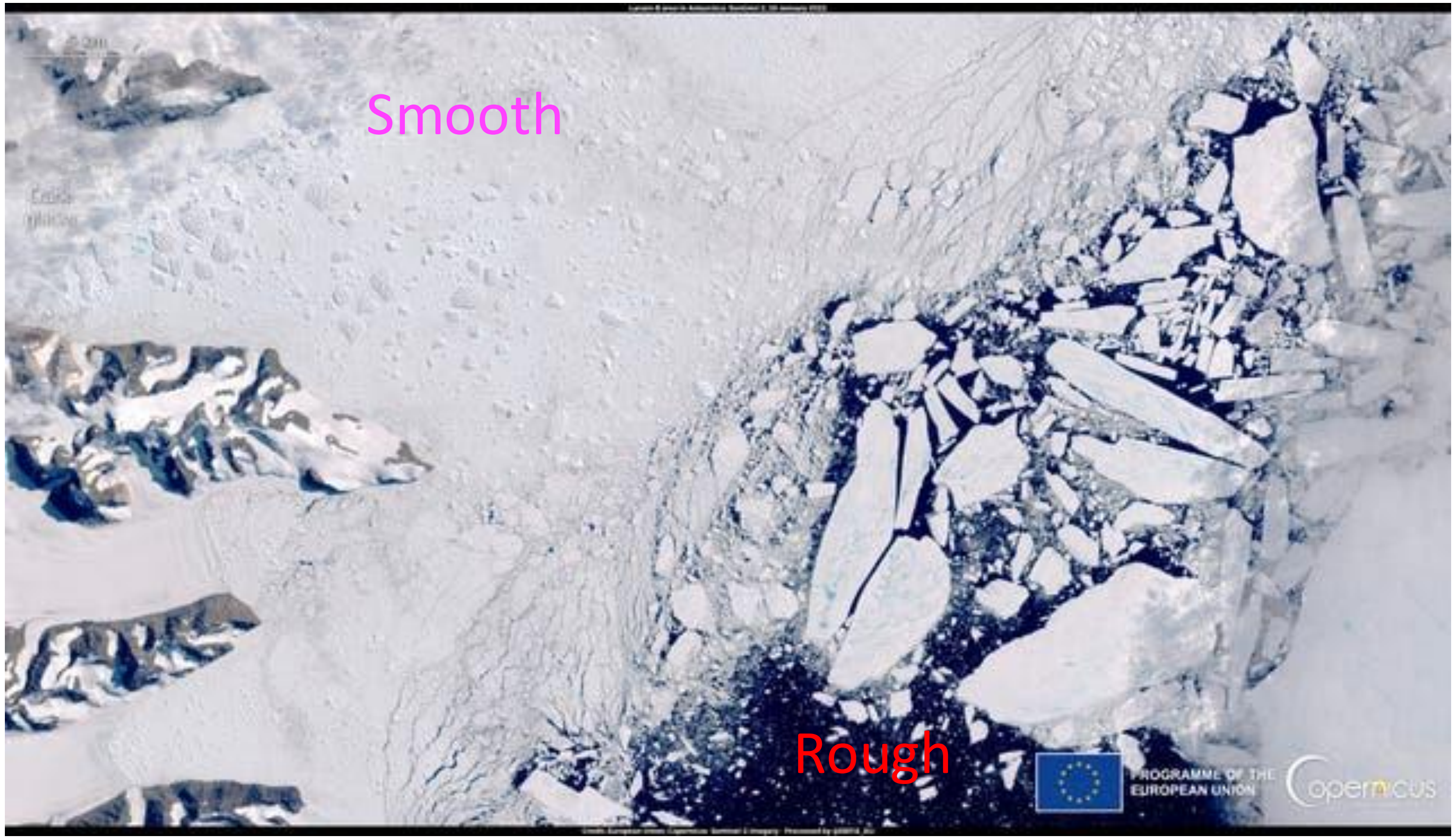
$$z_{0M} = \max(10^{-3}, f(c_i))$$

$$z_{0H} = 10^{-3}$$

$$z_{0Q} = 10^{-3}$$

c_i = sea ice concentration

$f(c_i)$: The dependence on sea-ice concentration reflects observation that partial ice-cover leads to more broken up sea ice and therefore increased drag



How does stability affect surface exchange?

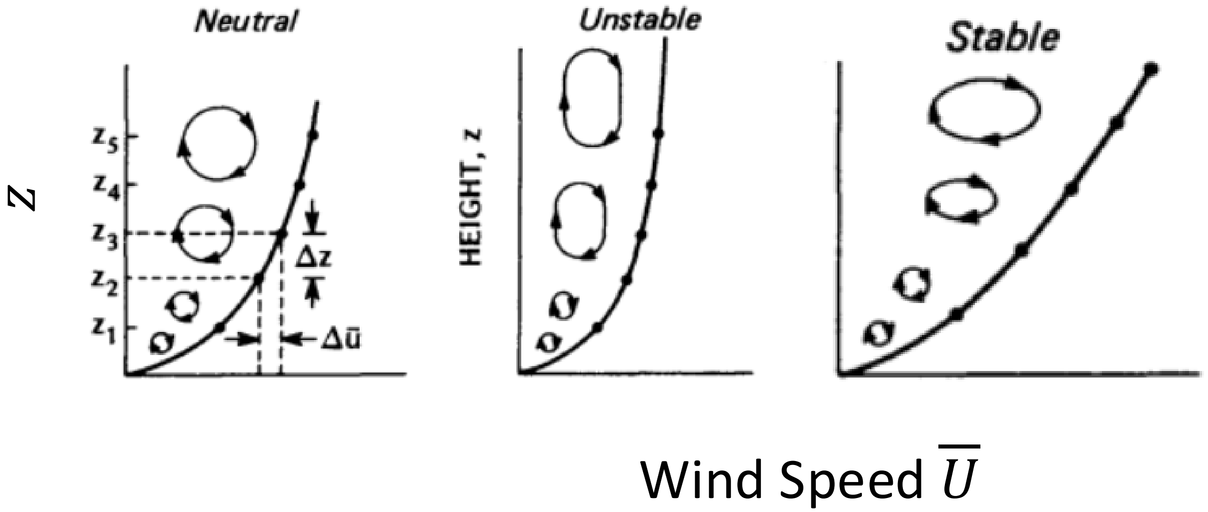
Adding stability dependence

Any quantity ϕ:

$$\kappa z \frac{\partial \overline{\phi}}{\partial z} = \phi_*$$

Integrate →

$$\overline{\phi}_z - \overline{\phi}_s = \frac{\phi_*}{\kappa} \log\left(\frac{z + z_0}{z_0}\right)$$



Log profiles
are only valid
in neutral flow
conditions….

Adding stability dependence

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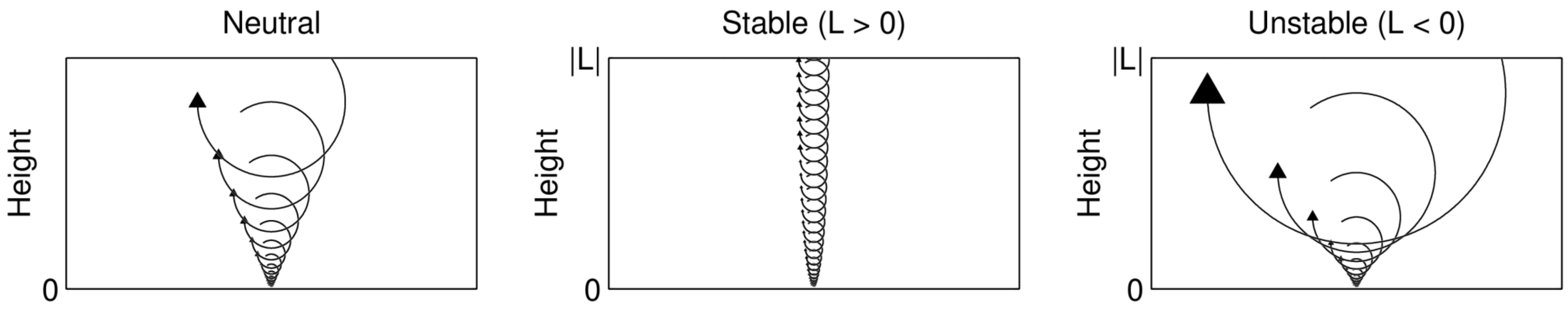


Mixing length modified to account for stability using a function ϕ :

$$l = \frac{\kappa z}{\Phi(stability)}$$

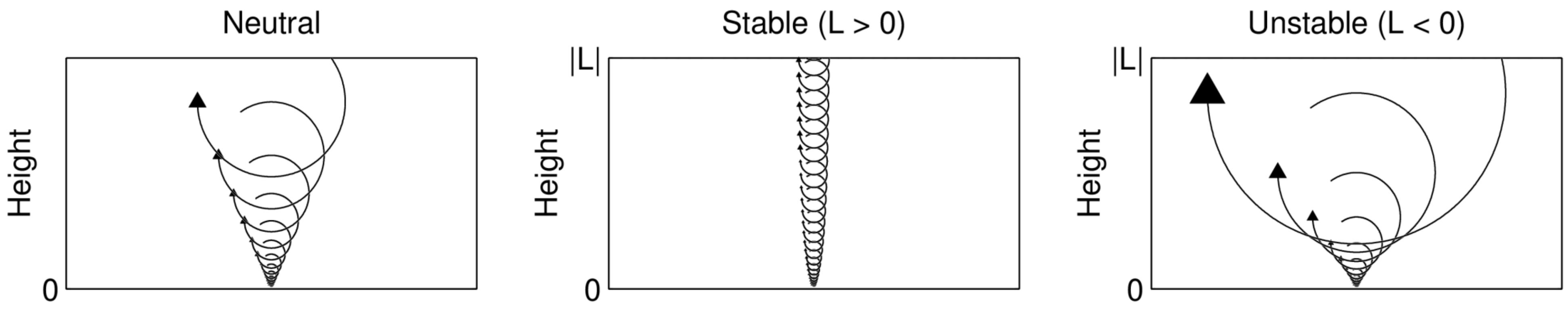
Adding stability dependence

Any quantity ϕ:

$$\frac{\kappa z}{\Phi_\phi}\frac{\partial\overline{\phi}}{\partial z} = \phi_*$$

Integrate

$$\overline{\phi}_z - \overline{\phi}_s = \frac{\phi_*}{\kappa}\int_{z_0}^{z+z_0}\frac{1}{z}\Phi_\phi\, dz$$



Mixing length is modified to account for stability using a function ϕ :

$$l = \frac{\kappa z}{\Phi(\zeta)}, \qquad \zeta = \frac{z}{L}$$

$L =$ Obukhov length (will come back to this)

Getting vertical profiles with stability dependence

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$$d\zeta = \frac{1}{L}dz$$

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Ψ_ϕ is the integral of $\Phi_\phi(\zeta)$

What is the Obukhov-length?

- Derived from scaling arguments - Reduces degrees of freedom so that ‘universal’ relations (they work for all situations) can be derived

$\zeta > 0$ Stable

$\zeta < 0$ Unstable

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- Height above the surface at which:

buoyant production > shear production of turbulence

Buoyancy production : $\frac{g}{\theta}\overline{\theta' w'} = \frac{g}{\theta}\theta_* u_*$

$\div$

Shear production: $-\overline{u' w'}\frac{\partial u}{\partial z} = u_*^2 \frac{\partial u}{\partial z}$

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$$\zeta = \frac{z}{L} = \frac{g}{\theta}\frac{\theta_* u_*}{u_*^2 \frac{\partial u}{\partial z}}$$

Why the Obukhov-length?

- Richardson number is also a measure of stability, but is locally defined and therefore can be noisy and can be highly variable with height

- Whereas, Obukhov length uses area averaged surface fluxes and so provide a more robust measure of surface layer stability

- L is directly related to surface fluxes, which allows for relation of fluxes to profiles from observations

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Getting surface fluxes from vertical profiles, with stability dependence

Momentum

$$\rho\overline{u'w'} = u_*^2$$

Rearrange:

$$\rho\overline{u'w'} = C_M|\overline{u}_z|\overline{u}_z$$

Thermodynamics

$$\overline{\theta' w'} = \theta_* u_*$$

Rearrange:

$$\rho\overline{\theta' w'} = C_H|\overline{u}_z|(\overline{\theta}_z - \overline{\theta}_s)$$

Surface exchange coefficient for heat:

$$\mathrm{C_H} = \frac{\kappa^2}{\left[\log\left(\frac{z+z_{0m}}{z_{0m}}\right) - \Psi_\mathrm{M}\left(\frac{z+z_{0m}}{L}\right)\right]\left[\log\left(\frac{z+z_{0m}}{z_{0H}}\right) - \Psi_\mathrm{H}\left(\frac{z+z_{0m}}{L}\right)\right]}$$

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Getting surface fluxes from vertical profiles, with stability dependence

Momentum

$$\rho\overline{u'w'} = u_*^2$$

Rear

Thermodynamics

$$\overline{\theta'w'} = \theta_* u_*$$

BUT… Exchange coefficients depend on $\zeta = \frac{z+z_{0m}}{L}$, which depends on surface fluxes

$$C_H \; \ldots \; \left[\log\left(\frac{z+z_{0m}}{z_{0m}}\right) - \Psi_M\left(\frac{z+z_{0m}}{L}\right)\right]\left[\log\left(\frac{z+z_{0m}}{z_{0H}}\right) - \Psi_H\left(\frac{z+z_{0m}}{L}\right)\right]$$

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How can we determine z/L and the surface fluxes?

1. Start with relationship between bulk Richardson number and z/L:

$$Ri_b = \frac{g}{\overline{\theta_z}} \frac{(\overline{\theta_z} - \overline{\theta_s})z}{|\overline{u_z}|^2}$$

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5. AND we can determine profiles of winds, temperature and humidity near the surface

Summary of Monin-Obukhov surface layer similarity theory

- The Obukhov-length is a measure of surface layer stability and can be thought of as the ratio of buoyancy / shear production of turbulence

- It is assumed that turbulent fluxes do not vary across the surface layer

- Functions that relate the Obukhov length (stability) to the vertical profiles of conserved quantities (e.g. wind and temperature) $\Phi_{\phi}(\zeta)$ in the surface layer can be derived from observations

- This is useful because we can relate Richardson number to z/L and get profiles and surface fluxes

Empirical surface layer stability functions

This means we can get profiles of $\overline{u}$ and $\overline{\theta}$ from flux

Momentum

$$\frac{\kappa z}{\Phi_M}\frac{\partial \overline{u}}{\partial z} = u_*$$

Integrate:

$$\overline{u}_z = \frac{u_*}{\kappa}\left[\log\left(\frac{z + z_{0m}}{z_{0m}}\right) - \Psi_{\mathrm{M}}\left(\frac{z + z_{0m}}{L}\right)\right]$$

Thermodynamics

$$\frac{\kappa z}{\Phi_H}\frac{\partial \overline{\theta}}{\partial z} = \theta_*$$

Integrate:

$$\overline{\theta}_z - \overline{\theta}_s = \frac{\theta_*}{\kappa}\left[\log\left(\frac{z+z_{0m}}{z_{0H}}\right) - \Psi_{\mathrm{H}}\left(\frac{z+z_{0M}}{L}\right)\right]$$

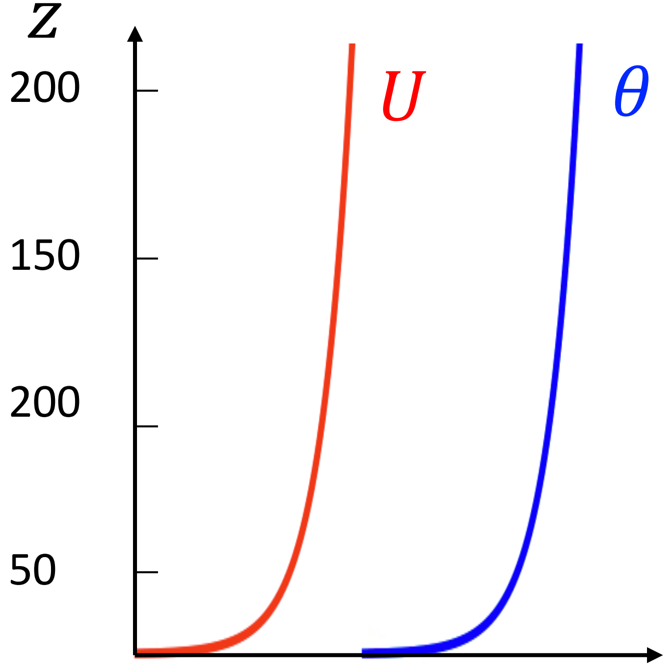
Recall that:

$$\overline{u'w'} = u_*^2$$

$$\overline{\theta'w'} = \theta_* u_*$$

Relationship between $\Phi_M(\zeta)$, $\Phi_H(\zeta)$ and ζ measured empirically and then integrated vertically

Ψ_{H}, Ψ_{M} are integrals of $\Phi_M(\zeta)$



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Empirical stability functions Cookbook

Ingredients:

- Accurate surface layer fluxes ($\overline{u'w'}, \overline{\theta'w'}$)
- Wind and temperature profiles at several heights
- Wide range of sampled stability

Mix well to form:

Richardson number:

$$Ri = \frac{g}{\theta}\frac{\frac{\partial\theta}{\partial z}}{\frac{\partial U}{\partial z}}$$

Dimensionless wind shear:

$$\Phi_M = \frac{\kappa z}{u_*}\frac{\partial U}{\partial z}$$

Dimensionless temperature gradient:

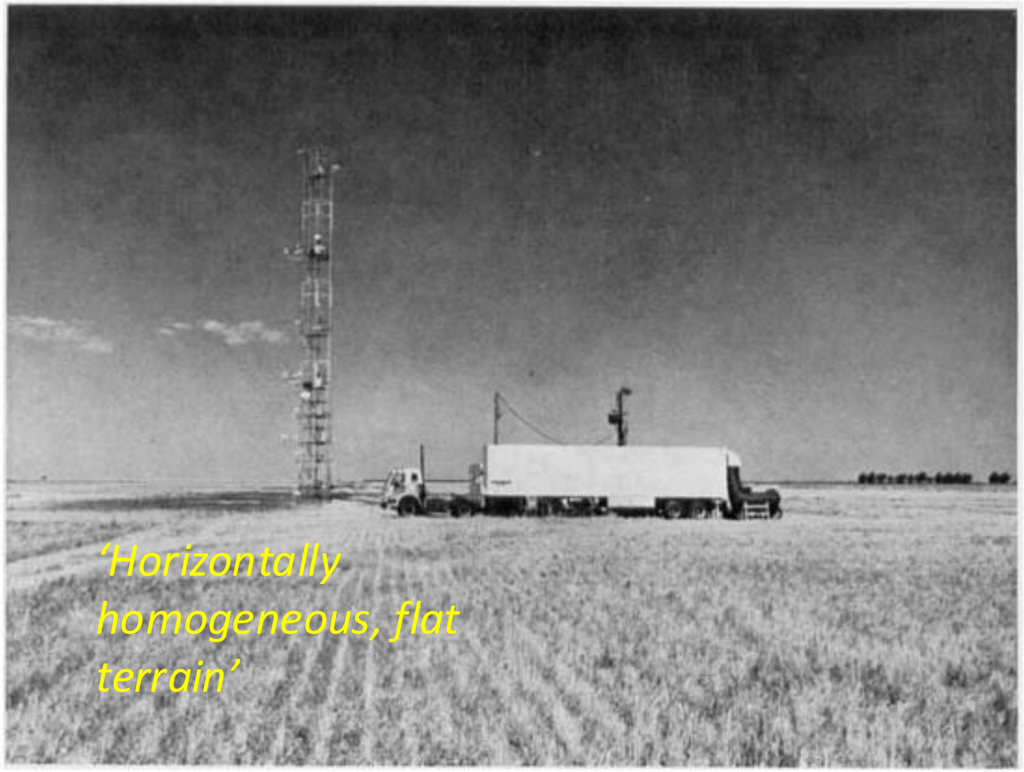
$$\Phi_H = \frac{\kappa z}{\theta_*}\frac{\partial \theta}{\partial z}$$

Dimensionless height:

$$\zeta = \frac{z}{L} = z\frac{\kappa g \overline{\theta'w'}}{\theta u_*^3}$$

Empirical stability functions – Businger et al (1970)

Haugen et al 1971



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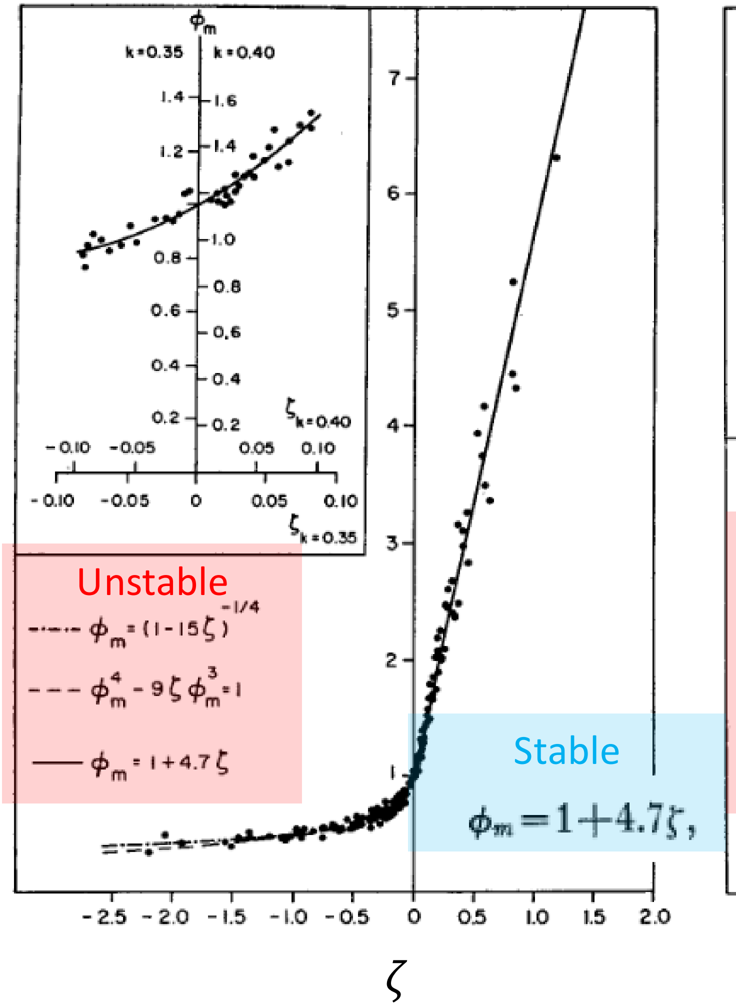
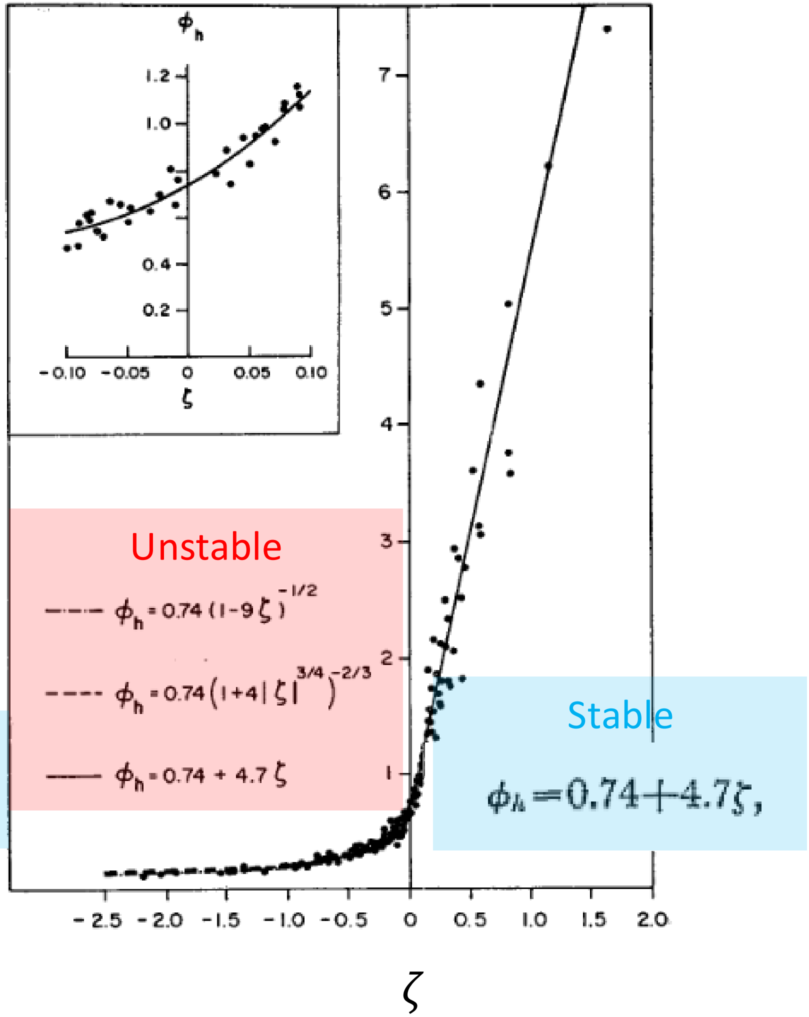
Momentum: Φ_M



Heat: Φ_H



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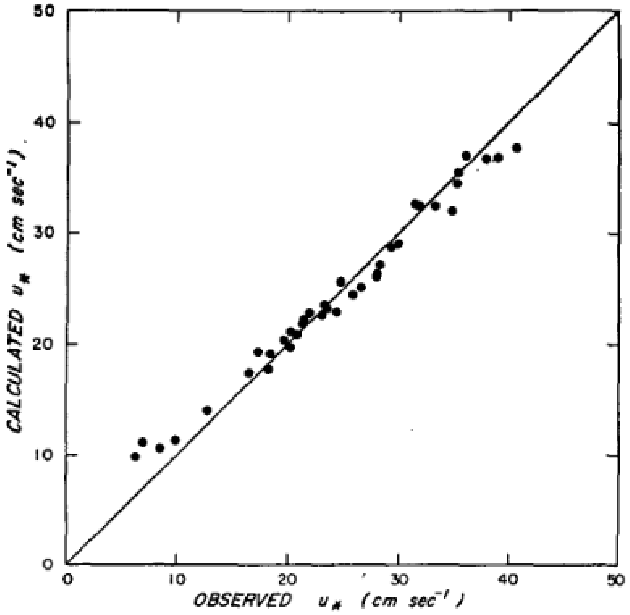
Empirical stability functions – Businger et al (1970)

Momentum flux

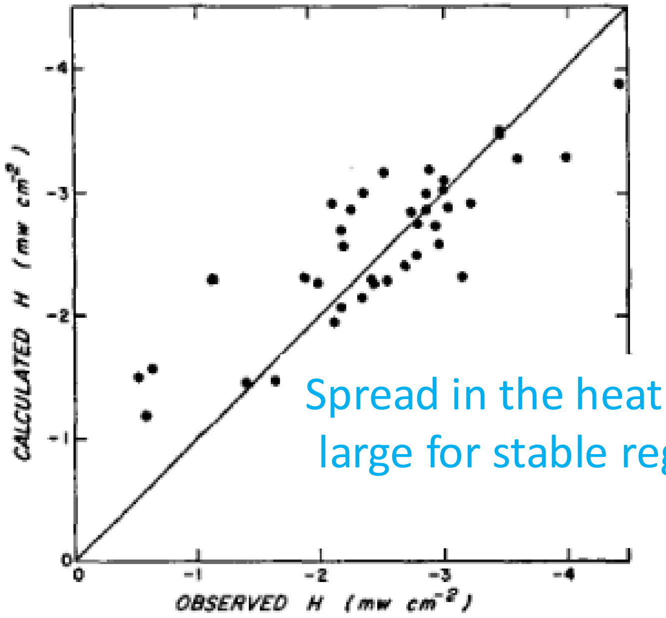
Calculated



observed

Heat flux

Calculated



observed

Spread in the heat fluxes large for stable regimes

d

Plots show observed vs calculated heat and momentum fluxes in stable situations

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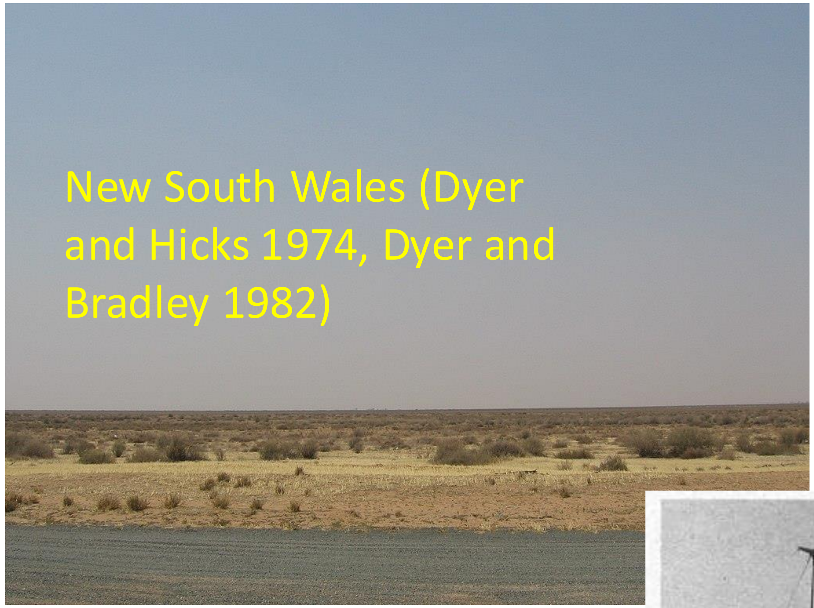
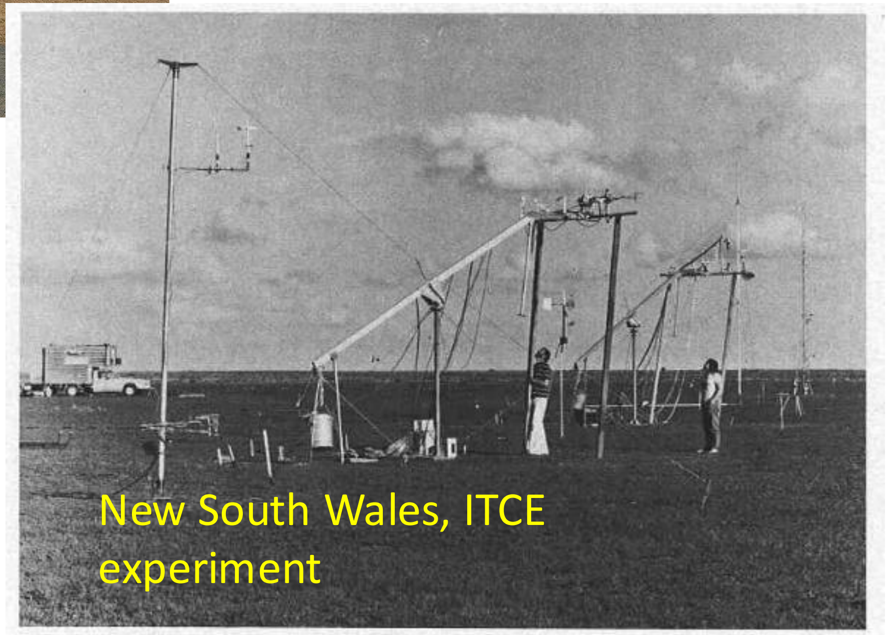
Empirical stability functions – Other locations





Also 'Horizontally homogeneous, flat terrain', …and mostly unstable conditions

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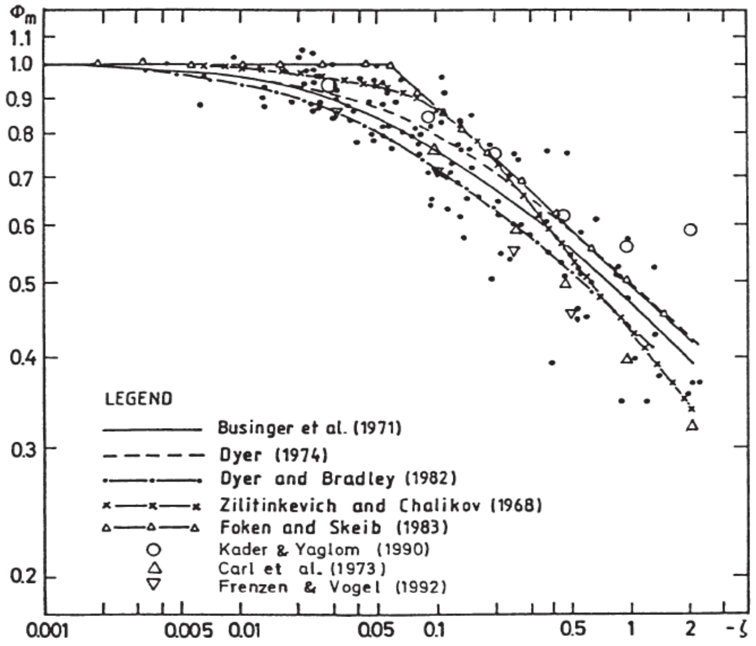
$$\zeta = \frac{z}{L} = z\frac{\kappa g\overline{\theta' w'}}{\theta u_*^3}$$

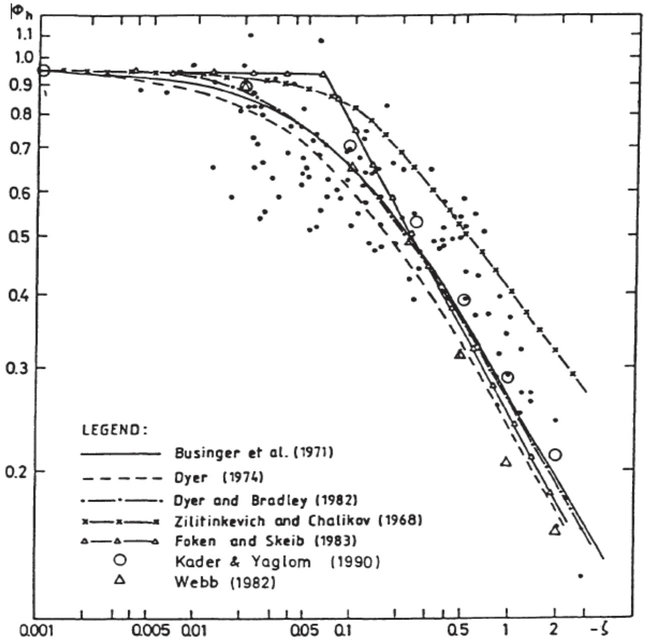
Empirical stability functions – Other locations

Φ_M



$-\zeta$

Φ_H



$-\zeta$

There is some disagreement in the functions, depending on where the measurements were taken

Mix well to form:

Richardson number:

$$Ri = \frac{g}{\theta}\frac{\frac{\partial\theta}{\partial z}}{\frac{\partial U}{\partial z}}$$

Dimensionless wind shear:

$$\Phi_M = \frac{\kappa z}{u_*}\frac{\partial U}{\partial z}$$

Dimensionless temperature gradient:

$$\Phi_H = \frac{\kappa z}{\theta_*}\frac{\partial \theta}{\partial z}$$

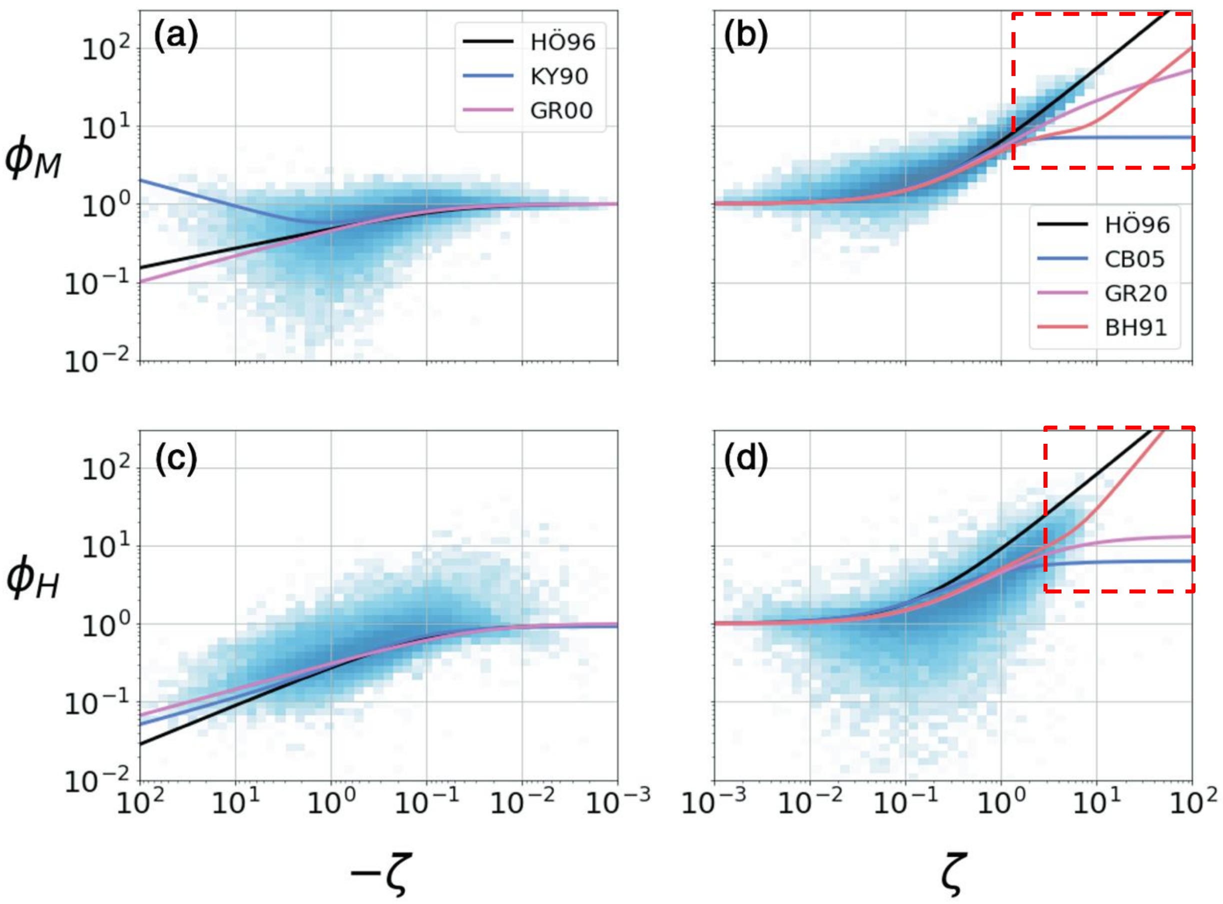
Dimensionless height:

$$\zeta = \frac{z}{L} = z\frac{\kappa g \overline{\theta' w'}}{\theta u_*^3}$$

Empirical stability functions – Other locations

Unstable cases

Stable cases



Mosso et al, 2023

There is large divergence in the commonly used functions in stable cases – fluxes are small and difficult to measure

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Empirical stability functions – SHEBA (very stable)

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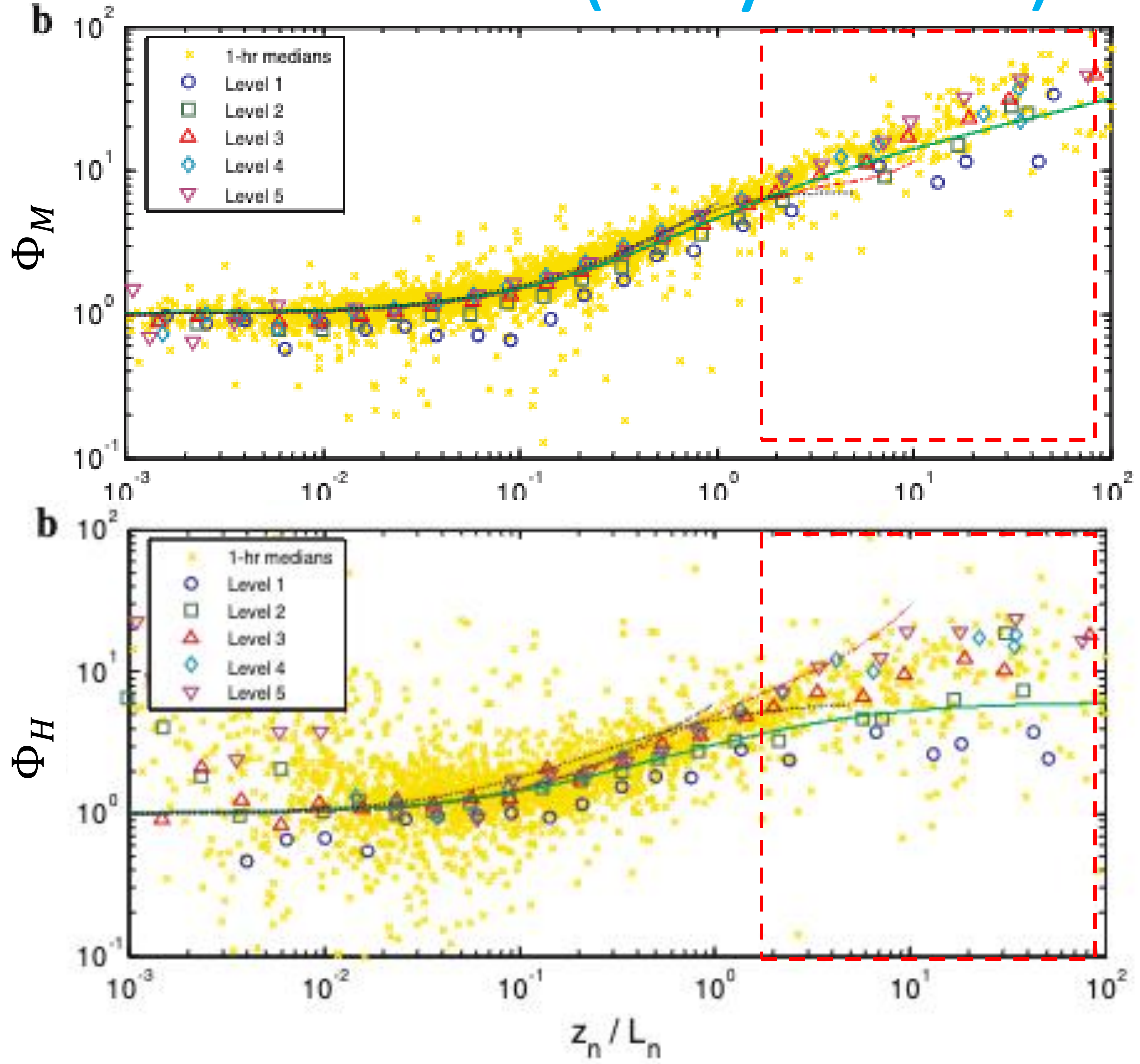

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Note that $\frac{z}{L}$ stopped at 2 in Businger et al (1970)

Summary of fundamental concepts

- **Local turbulence closure:**
  - Assumes local turbulent fluxes can be determined by a K-profile and the background gradients
  - Concept of an eddy lengthscale is used to determine the turbulent mixing
  - Lengthscale depends on height above the surface and the stability
- **MO surface layer similarity theory:**
  - Possible to relate the surface fluxes and near-surface gradients through universal functions
  - Functions depend on the Obukhov length (measure of surface stability)
- **Roughness length:**
  - Assumed to be a property of the surface roughness elements (e.g. vegetation / wave height)
- **Empirical stability functions:**
  - Most widely used stability functions are from flat terrain and relatively dated analysis
  - Recent campaigns over different regions and in stable conditions show disagreement from region to region – could be due to missing processes