Numerical Weather Prediction
Parametrization of diabatic processes

Convection I: General circulation and concepts

https://www.ecmwf.int/en/learning/education-material/lecture-notes
(Atmospheric moist convection, Atmospheric Thermodynamics)
https://www.ecmwf.int/en/learning/education-material/elearning-online-resources

Peter Bechtold

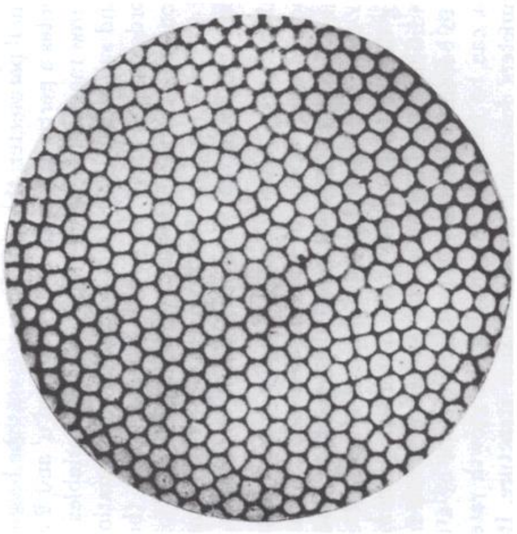
Convection Parametrisation and Dynamics - Text Books

- Yano&Plant (Editors), 2015: Parameterization of atmospheric convection. *World scientific, Imperial College Press*
- Lin, J., T. Qian, P. Bechtold et al. : Atmospheric Convection https://doi.org/10.1080/07055900.2022.2082915
- Emanuel, 1994: Atmospheric convection, *OUP*
- Houze R., 1993: Cloud dynamics, *AP*
- Holton, 2004: An introduction to Dynamic Meteorology, *AP*
- Bluestein, 1993: Synoptic-Dynamic meteorology in midlatitudes, Vol II. *OUP*
- Peixoto and Ort, 1992: The physics of climate. *American Institute of Physics*
- Emanuel and Raymond, 1993: The representation of cumulus convection in numerical models. *AMS Meteor*. *Monogr*.
- Smith, 1997: The physics and parametrization of moist atmospheric convection. *Kluwer*
- Dufour et v. Mieghem: Thermodynamique de l’Atmosphère, 1975: *Institut Royal météorologique de Belgique*
- Anbaum, 2010: Thermal Physics of the atmosphere. J *Wiley Publishers*

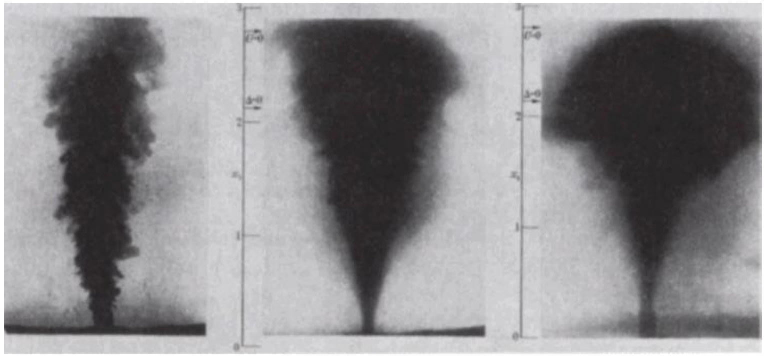
AP=Academic Press; OUP=Oxford University Press

Convection=heat the bottom&cool the top

Pre-frontal deep convection July 2010 near Baden-Baden Germany



Rayleigh-Benard cellular convection



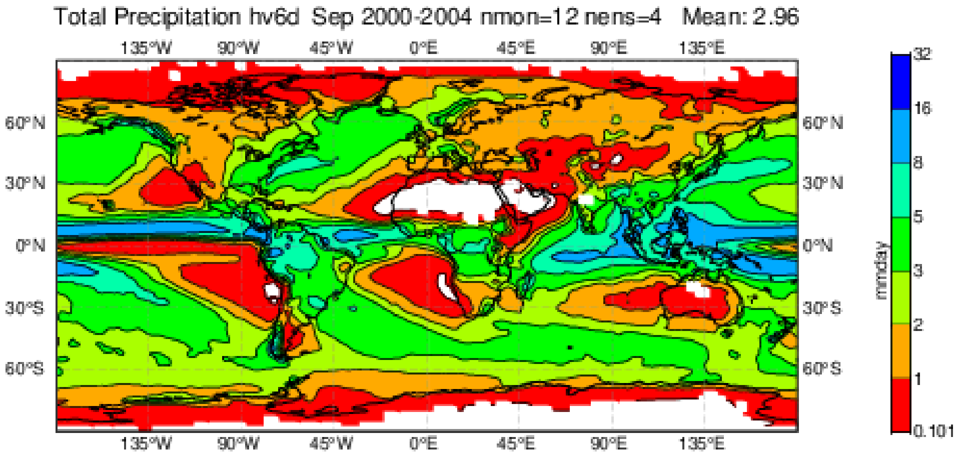
Classic plume experiment

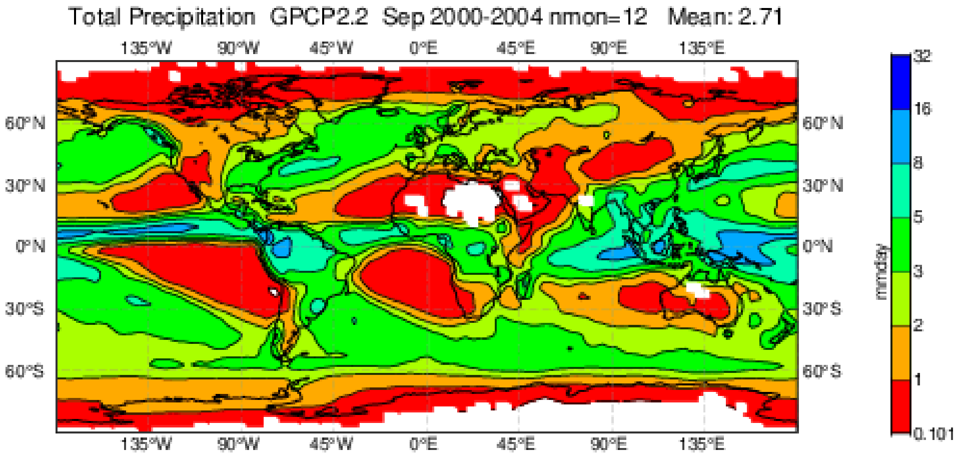
Outline

General:

- Convection and tropical circulations
- Useful concepts and tools:
- Buoyancy
- Convective Available Potential Energy
- Soundings and thermodynamic diagrams
- Convective quasi-equilibrium
- Apparent heating from large-scale observational budget
- Tropical waves and convective organisation:
- Tropical waves
- Middle latitude Convection

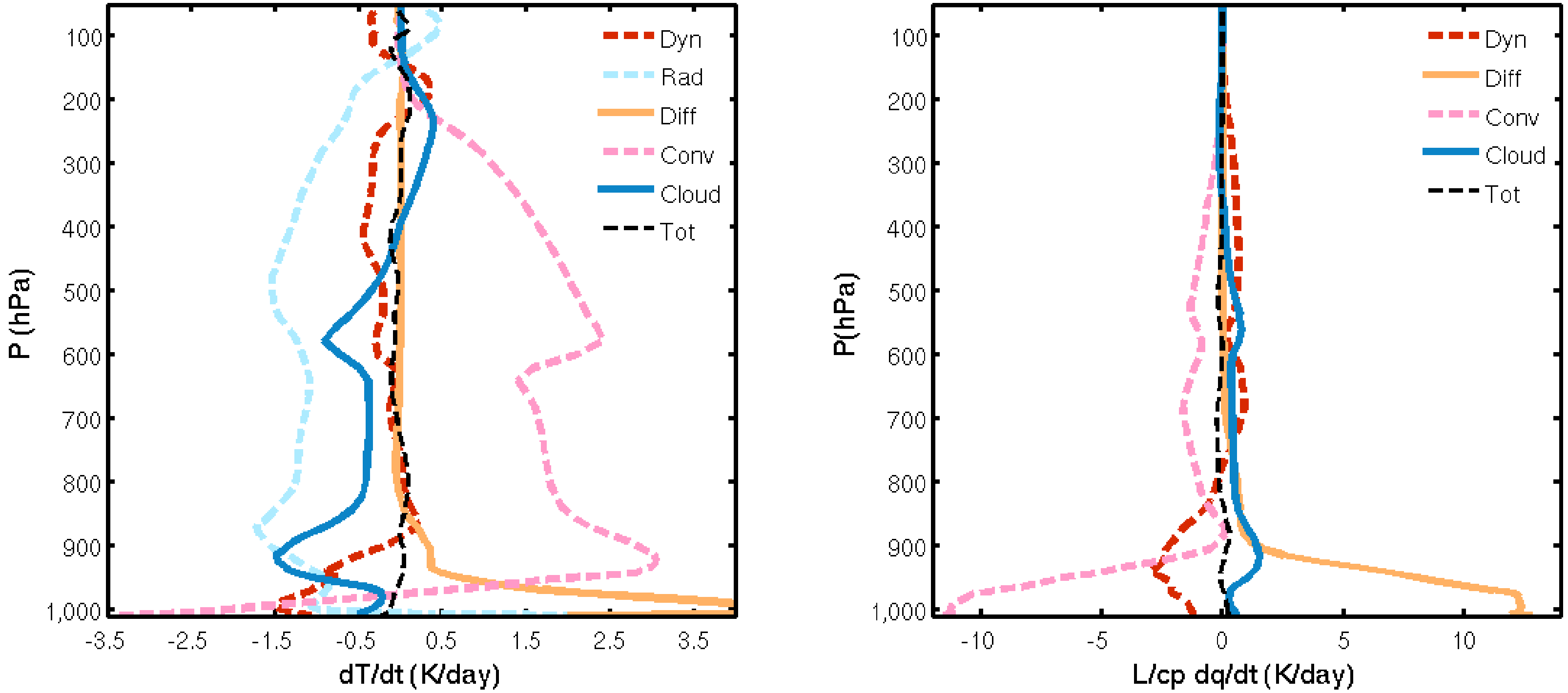
It’s raining again… 2000-2003 annual mean daily precipitation from IFS Cy48r1 (2023) coupled and GPCP2.2 dataset





about 2.7-2.8 mm/day is falling globally, but most i.e. 5-7 mm/day in the Tropics

Model Tendencies – Tropical Equilibria

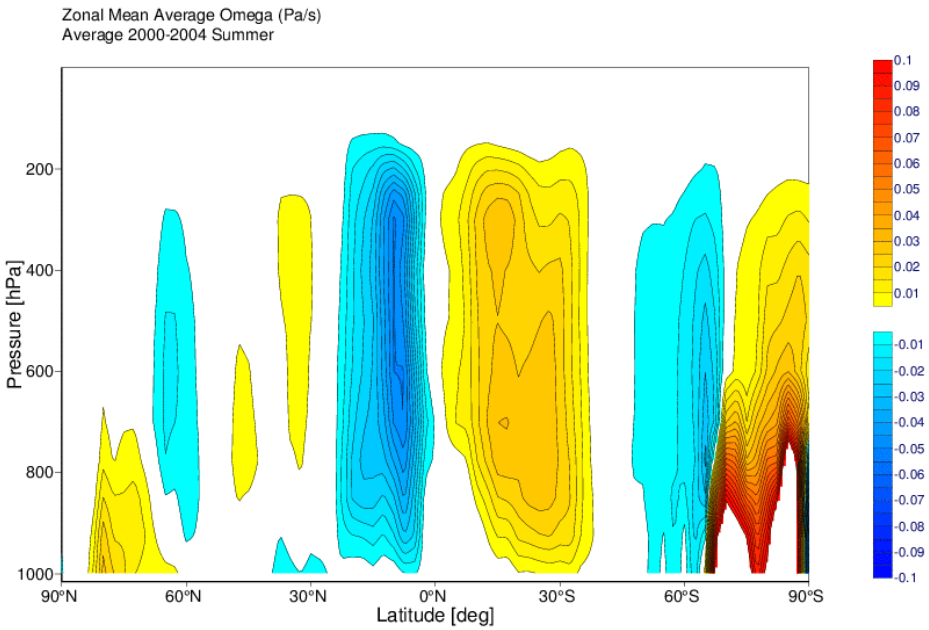


Above the boundary layer, for Temperature there is on average radiative-convective equilibrium; and convective-dynamic equilibrium over the large-scale disturbance, whereas for moisture there is roughly an equilibrium between dynamical transport (moistening) and convective drying. - *Global Budgets are very* similar

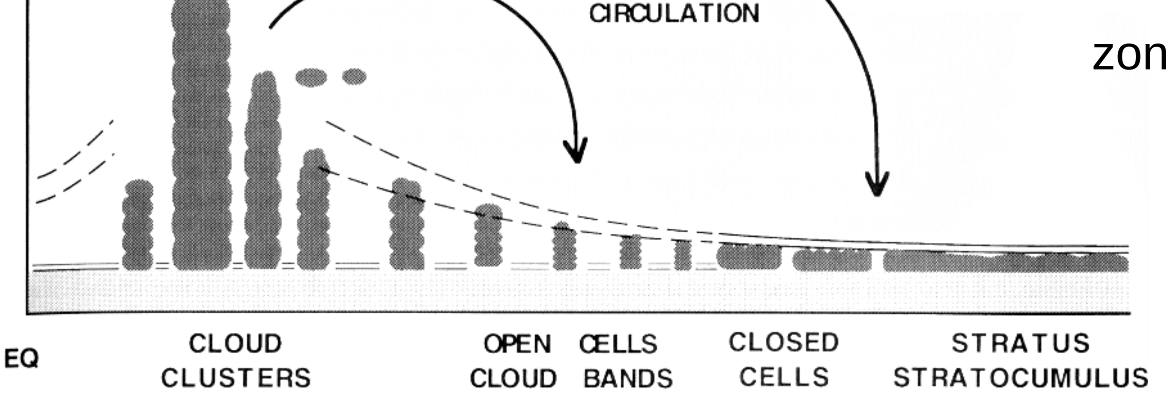
The driving force for atmospheric convection is the radiation

Convection and tropical circulations (1)

The ITCZ and Hadley meridional circulation

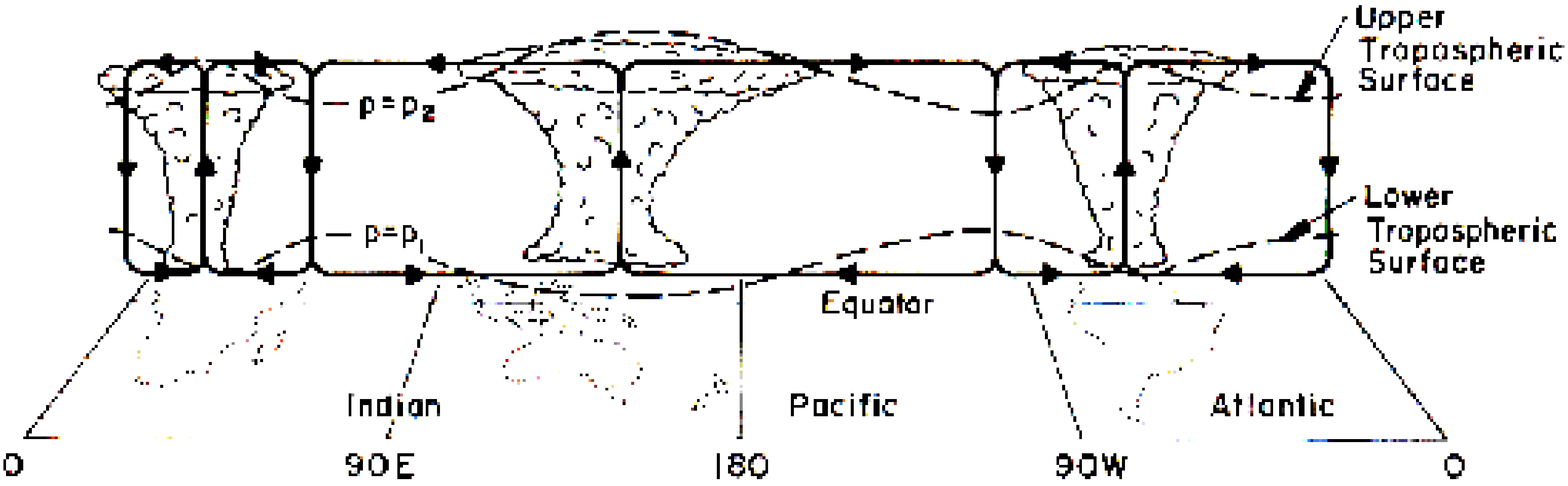


zonal mean omega (Pa/s)



Convection and tropical circulations (2)

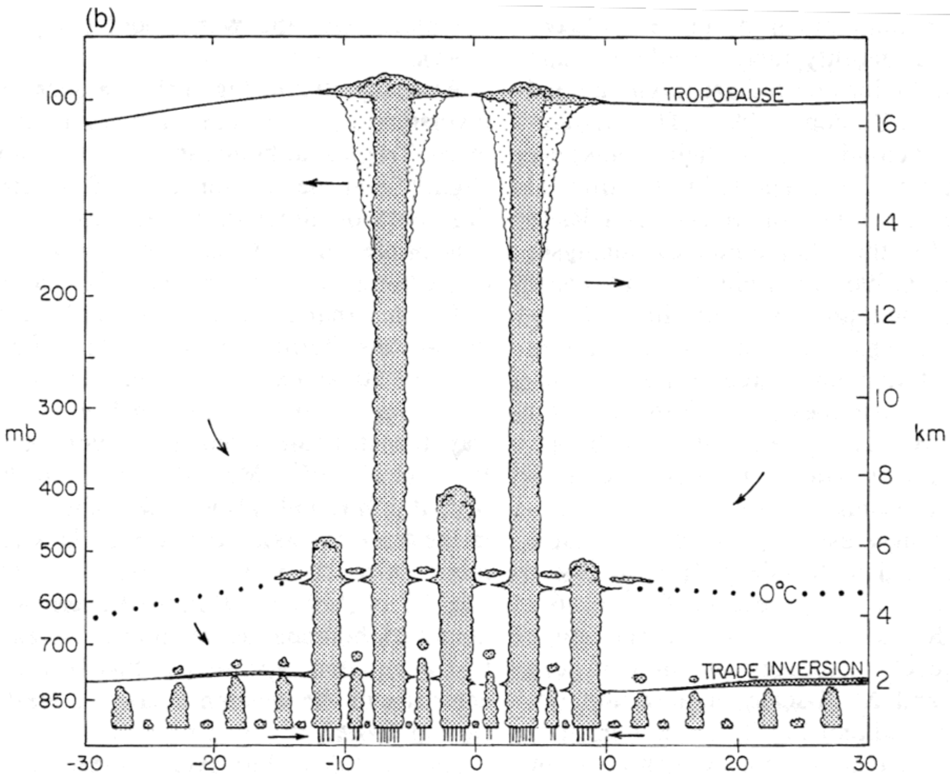
The Walker zonal Circulation and SST coupling



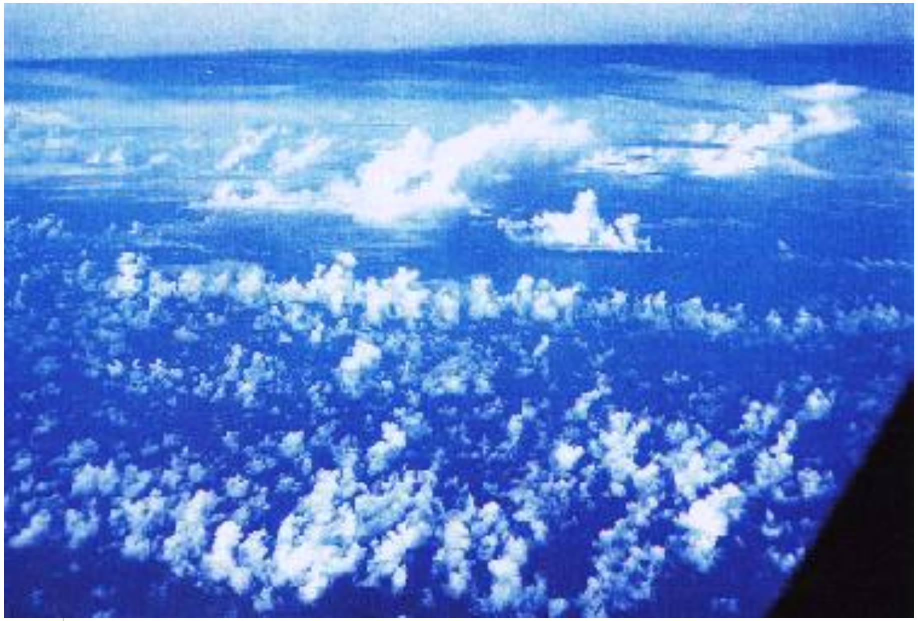
Nota: the Hadley and Walker cells are coupled

From Salby (1996)

Vertical distribution of convective clouds



Johnson et al., 1999, JCL

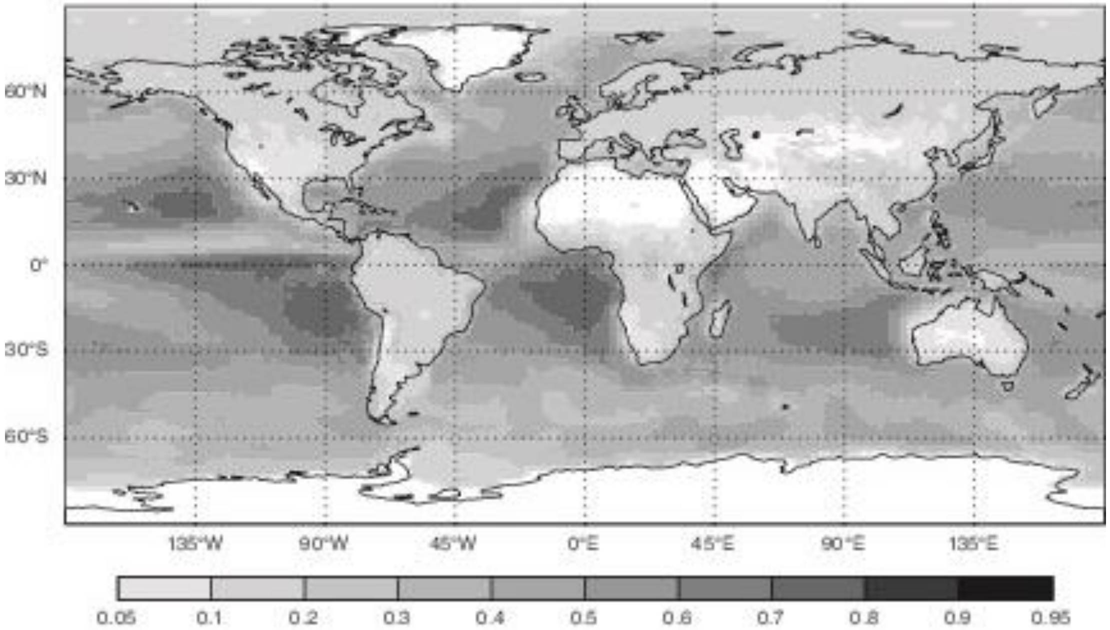


Tri-modal distribution: Shallow cumulus, Congestus attaining the melting level, Deep penetrating convection

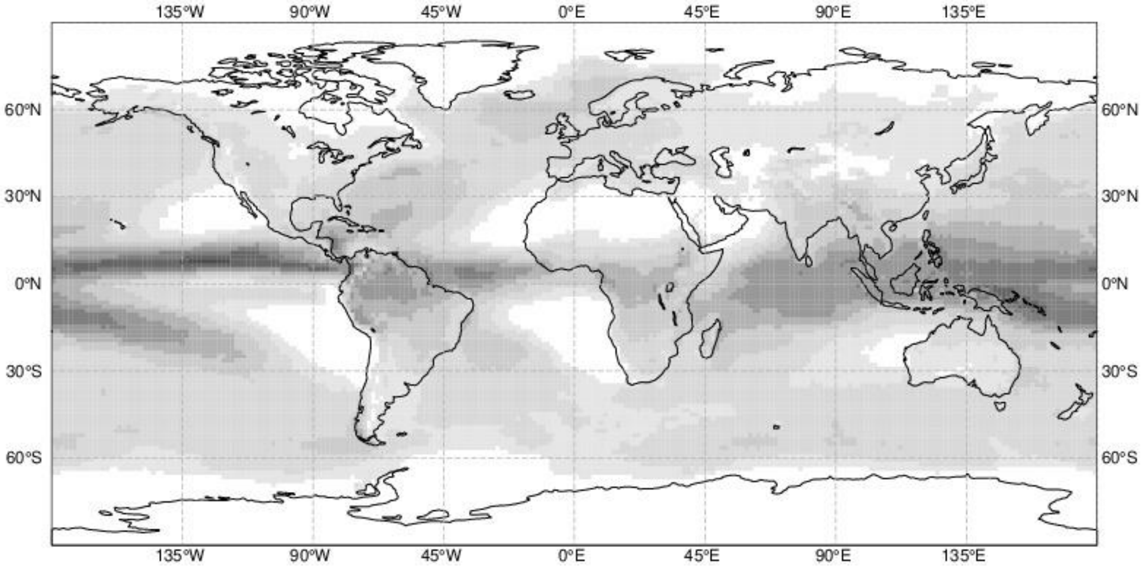
Frequency distribution of shallow and deep in IFS

Cy46r1 (2019)

Shallow convection



Deep convection including congestus



Summary: the weather and thermal equilibria

- Suppose we have a series of nice clear sky anticyclonic days, then above the boundary-layer

$$\frac{d\theta}{dt} \approx 0 \;\Rightarrow w\frac{d\theta}{dz} = \frac{d\theta}{dt}\bigg|_{rad} = -\frac{2K}{86400s} \Rightarrow$$

w ~ -0.5 cm/s
subsidence

~0.5 K/100 m

- But what happens if we have a thunderstorm day with Pr=100 mm/day

$$\frac{c_p}{g}\int_{Ptop=200hPa}^{Psurf=1000hPa} \frac{\partial T}{\partial t}dp = L_V \rho_{water} \Pr(m/s)$$

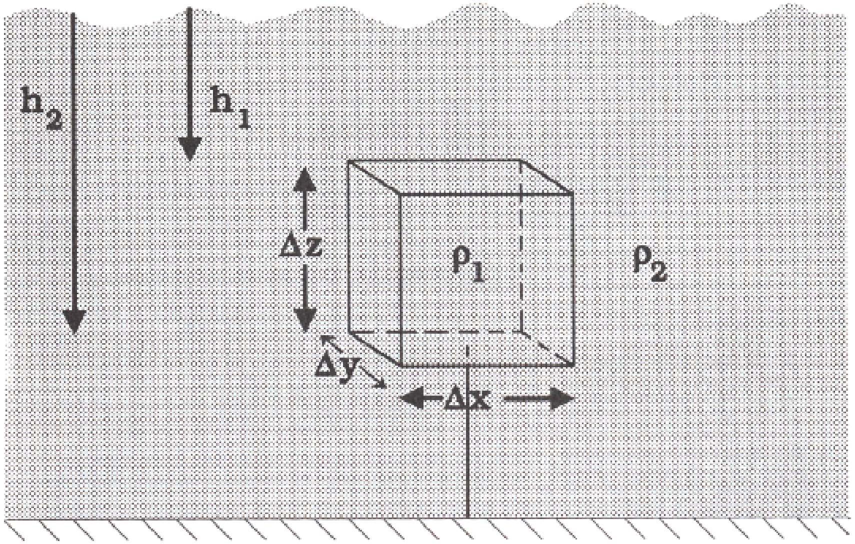
$$c_p = 1004Jkg^{-1}K^{-1} \quad \rho_{water} = 1000kgm^{-3} \quad L_v = 2.5x10^6 Jkg^{-1}$$

$$g = 9.81ms^{-2} \;\; \Pr = 100\,mm/day = 1.16\times10^{-6}ms^{-1}$$

100 mm/day precipitation heats the atmospheric column by 2893 W/m2 or by 30 K/day on average. This heating must be compensated by uplifting of w ~ 10 cm/s ➔ heavy precip/convection requires large-scale perturbations.

Buoyancy (1)- Archimedes said ‘Heureka!’

Body in a fluid



Assume fluid to be in hydrostatic equlibrium

$$\frac{dp_2}{dz} = -\rho_2 g$$

$$\rho_2 = const. \longrightarrow p_2 = \rho_2 g h$$

Forces:

Top $F_{top} = -\rho_2 g h_1 \Delta x \Delta y$

Bottom $F_{bot} = \rho_2 g h_2 \Delta x \Delta y$

Gravity $F_{grav} = -\rho_1 g \Delta x \Delta y \Delta z$

Net Force: $F = F_{top} + F_{bot} + F_{grav} = \rho_2 g (h_2 - h_1) \Delta x \Delta y - \rho_1 g \Delta x \Delta y \Delta z = g(\rho_2 - \rho_1) \Delta x \Delta y \Delta z$

Acceleration: $A = F / M_{body} = F / \rho_1 \Delta x \Delta y \Delta z = g \dfrac{(\rho_2 - \rho_1)}{\rho_1}$

Emanuel, 1994

Buoyancy (2)

Vertical momentum equation:

$$\frac{dw}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g$$

$$p = \overline{p} + p' \qquad \rho = \overline{\rho} + \rho' \qquad \frac{\partial \overline{p}}{\partial z} = -\overline{\rho} g$$

$$\frac{dw}{dt} = -\frac{1}{\overline{\rho} + \rho'}\frac{\partial(\overline{p} + p')}{\partial z} - g$$

$$\frac{1}{\overline{\rho} + \rho'} = \frac{1}{\overline{\rho}}\left(\frac{1}{1 + \rho'/\overline{\rho}}\right) = \frac{1}{\overline{\rho}}\left[1 - \frac{\rho'}{\overline{\rho}} + \left(\frac{\rho'}{\overline{\rho}}\right)^2 + \ldots\right]$$

$\rho' << \overline{\rho}$ $\longrightarrow$ Neglect second order terms

Buoyancy (3)

$$\frac{dw}{dt} = \underset{\substack{\| \\ g}}{-\frac{1}{\bar{\rho}}\frac{\partial \bar{p}}{\partial z}} - \frac{1}{\bar{\rho}}\frac{\partial p'}{\partial z} - g + \frac{\rho'}{\bar{\rho}}\underset{\substack{\| \\ -g}}{\frac{1}{\bar{\rho}}\frac{\partial \bar{p}}{\partial z}} + \xcancel{\frac{\rho'}{\bar{\rho}}\frac{1}{\bar{\rho}}\frac{\partial p'}{\partial z}}$$

$$\boxed{\frac{dw}{dt} = -\frac{1}{\bar{\rho}}\frac{\partial p'}{dz} - \underset{\|}{\frac{\rho'}{\bar{\rho}}g}}$$

B - buoyancy acceleration

Buoyancy (4) T and P contributions

Buoyancy

$$\boxed{B = -\frac{\rho'}{\overline{\rho}}\, g}$$

Dry air:

$$\rho = \frac{p}{RT} \rightarrow \rho' = \frac{p'}{R\overline{T}} - \frac{\overline{p}T'}{R\overline{T}^{2}} \rightarrow \frac{\rho'}{\overline{\rho}} = \frac{p'}{\overline{p}} - \frac{T'}{\overline{T}}$$

$$\frac{p'}{\overline{p}} << \frac{T'}{\overline{T}} \text{ and } B \approx g\frac{T'}{\overline{T}}$$

$$\boxed{\frac{dw}{dt} \approx g\frac{T'}{\overline{T}} - \frac{1}{\overline{\rho}}\frac{\partial p'}{\partial z}}$$

T’>0 (warm parcel) => upward acceleration

Buoyancy (5) moist atmosphere

effects of humidity and condensate need to be taken into account

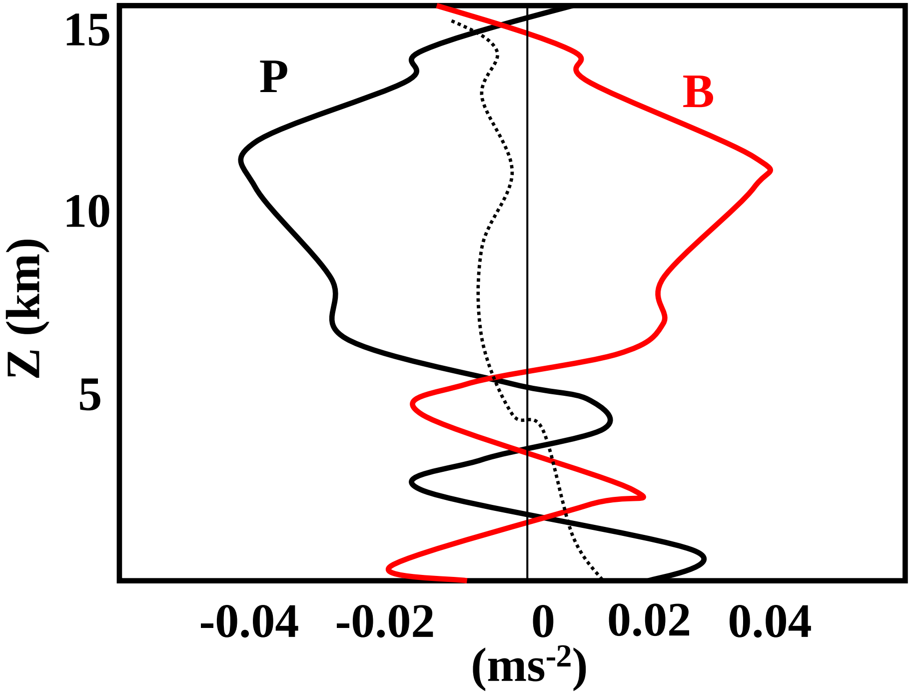
via virtual temperature

$$B = -g\frac{\rho'}{\bar{\rho}} \approx -g\left(\frac{T'}{\bar{T}} + 0.608q' - q_l\right)$$

In general all 3 terms are important. 1 K perturbation in T is equivalent to 5 g/kg perturbation in water vapor or 3 g/kg in condensate

Non-hydrostat. Pressure gradient effects

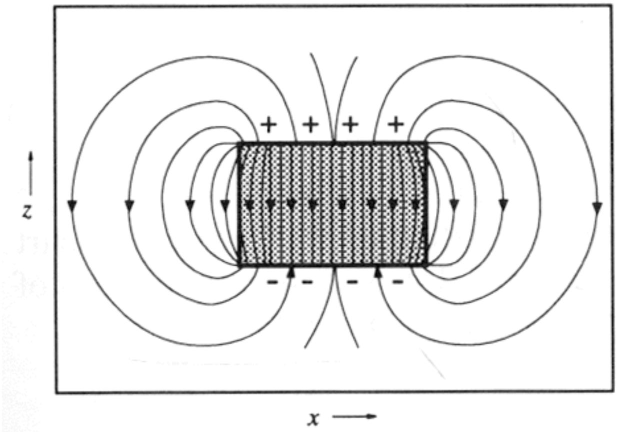
$$\frac{dw}{dt} = -\frac{1}{\rho}\frac{\partial p'}{\partial z} - \frac{\rho'}{\overline{\rho}}g$$



CRM analysis of the terms

by F. Guichard and D. Gregory

Physics:



Vector field of the buoyancy pressure-gradient force for a uniformly buoyant parcel of finite dimensions in the x-z-plane. (Houze, 1993, Textbook)

Convective available potential energy (CAPE)

$$CAPE = \int \vec{F} \cdot d\vec{l} = \int_{base}^{top} B\,dz$$

$$CAPE \approx \int_{base}^{top} g \frac{T_{cld} - T_{env}}{T_{env}} dz$$

$$\frac{dw}{dt} = w\frac{dw}{dz} = \frac{1}{2}\frac{dw^2}{dz} \approx g\frac{T'}{\overline{T}}$$

$$w^2(z) = 2\int_0^z g\frac{T'}{\overline{T}}dz = 2 \cdot CAPE$$

$$w = \sqrt{2 \cdot CAPE}$$

CAPE represents the amount of potential energy of a parcel lifted to its level of neutral buoyancy. This energy can potentially be released as kinetic energy in convection.

Example:

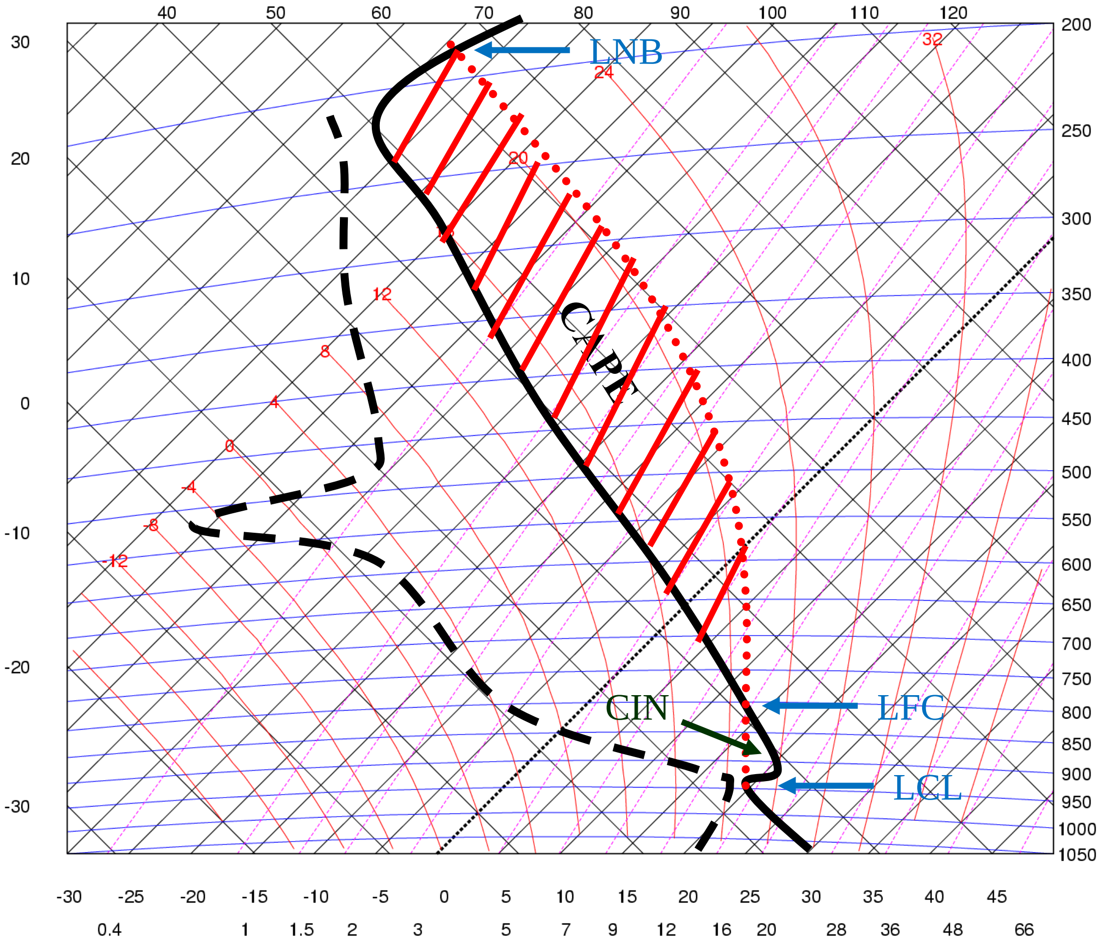
$T'=5$ K, $T=250$ K, cloud depth=10 km

$$w \approx 60 ms^{-1}$$

Much larger than observed - what’s going on ?

Convection in thermodynamic diagrams (1)

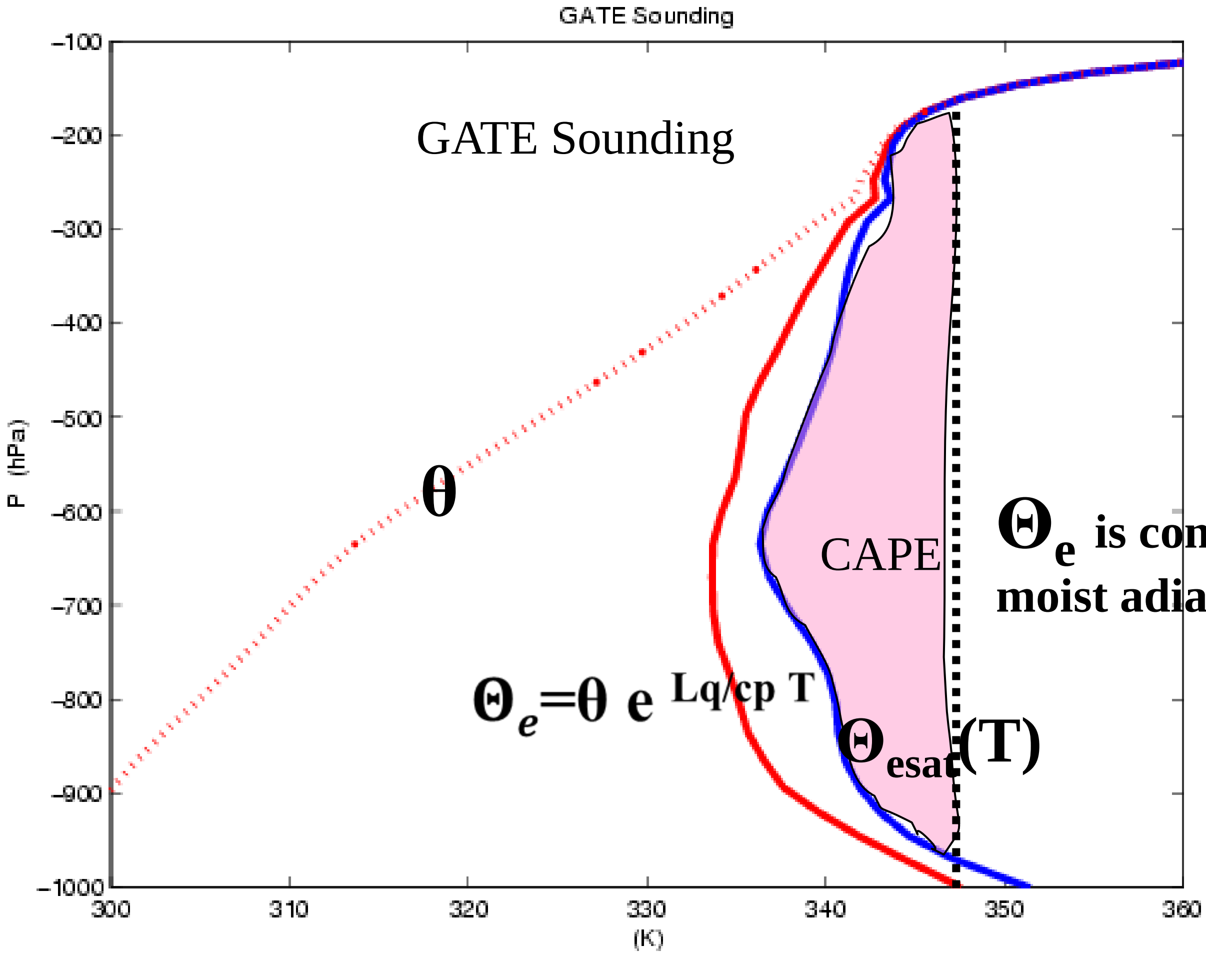
using Tephigram/Emagram



Idealised Profile

Convection in thermodynamic diagrams (2)

using equivalent Potential Temperatures



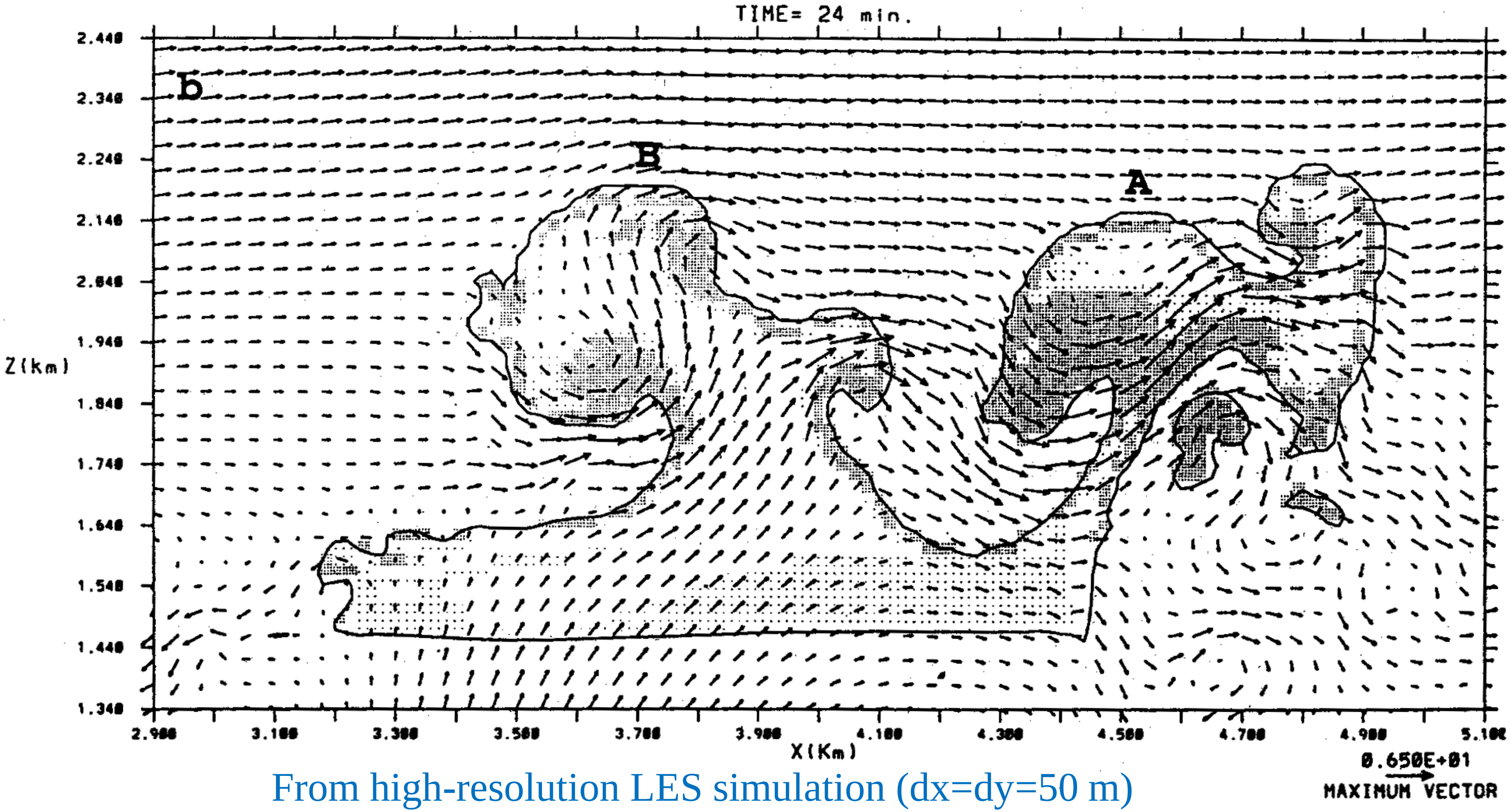
$\Theta_e = \theta e^{Lq/c_p T}$

Θ_e is conserved during moist adiabatic ascent

Note that no CAPE is available for parcels ascending above 900 hPa and that the tropical atmosphere is stable above 600 hPa ($\boldsymbol{\theta}_e$ increases) – downdrafts often originate at the minimum level of $\boldsymbol{\theta}_e$ in the mid-troposphere.

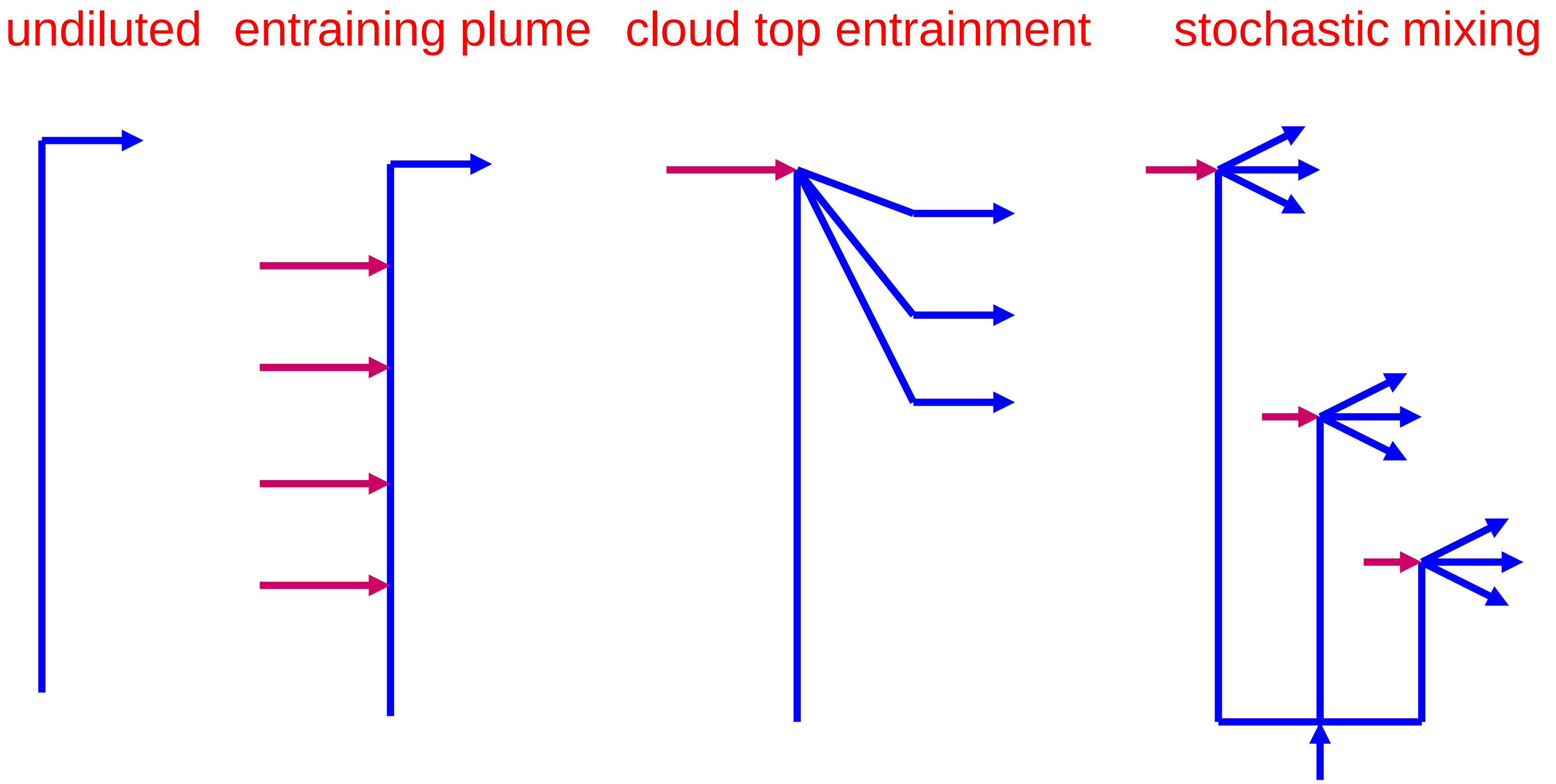
Mixing and 3D flow

subcloud and cloud-layer Circulations



From high-resolution LES simulation (dx=dy=50 m)
Vaillancourt, You, Grabowski, JAS 1997

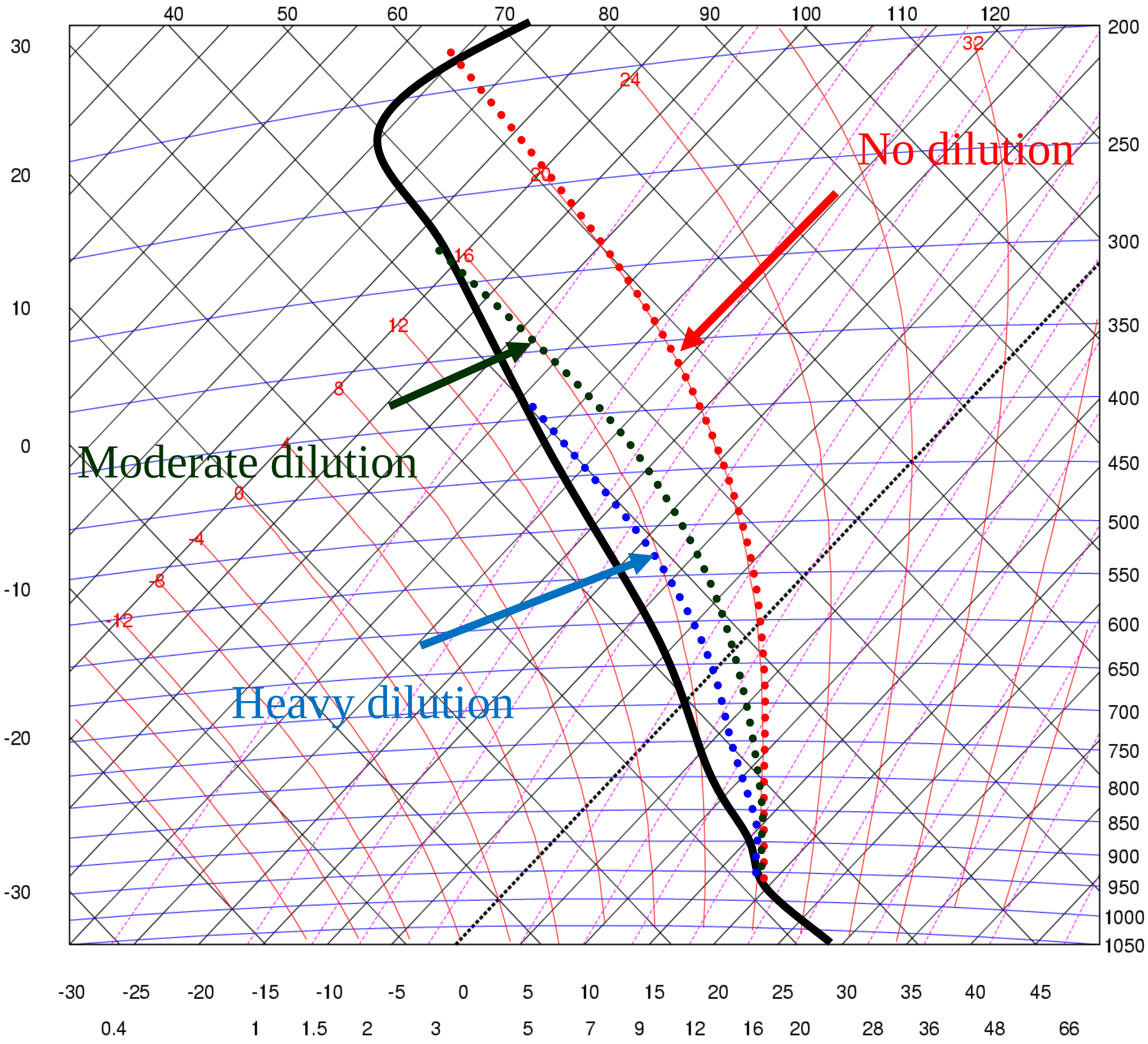
Mixing models



undiluted entraining plume cloud top entrainment stochastic mixing

after Raymond,1993

Effect of mixing on parcel ascent



Mixing affects both cloud top height and virtual temperature excess (CAPE)

Large-scale effects of convection (1) Q_1 and Q_2

Thermodynamic equation (dry static energy) :

$$\frac{\partial s}{\partial t} + \nabla \vec{v}_h s + \frac{\partial \omega s}{\partial p} = Q_R + L(c-e)$$

Why use s or θ, not T ?

$s = c_p T + gz$

$ds/dz = C_p dT/dz + g$

If $dT/dz = -g/c_p$ (dry adiabatic lapse rate), then $ds = d\theta = 0$

Define averaging operator over area A such that:

$$\overline{\Phi} = \frac{1}{A}\int_A \Phi dA \qquad \text{and} \qquad \Phi = \overline{\Phi} + \Phi'$$

Apply to thermodynamic equation, neglect horizontal second order terms, use averaged continuity equation:

$$\underbrace{\frac{\partial \bar{s}}{\partial t} + \bar{\vec{v}}_h \nabla \bar{s} + \bar{\omega}\frac{\partial \bar{s}}{\partial p}}_{\text{“large-scale observable” terms}} = \underbrace{\overline{Q}_R + \boxed{L(\bar{c} - \bar{e}) - \frac{\partial \overline{\omega' s'}}{\partial p}}}_{\text{“sub-grid” terms}}$$

In convective regions these terms will be dominated by convection

ECMWF

Large-scale effects of convection Q_1, Q_2 and Q_3

Define: $$Q_1 \equiv Q_R + L(\bar{c} - \bar{e}) - \frac{\partial \overline{\omega' s'}}{\partial p}$$ Apparent heat source

Analogous: $$Q_2 \equiv L(\bar{c} - \bar{e}) + L\frac{\partial \overline{\omega' q'}}{\partial p}$$ Apparent moisture sink

$$\vec{Q}_3 \equiv \frac{\partial \overline{\omega' \vec{v}_h'}}{\partial p}$$ Apparent momentum source

This quantity can be derived from observations of the “large-scale” terms on the l.h.s. of the area-averaged equations and describe the influence of the “sub-grid” processes on the atmosphere.

Note that:

$$Q_1 - Q_2 - Q_R \equiv -\frac{\partial \overline{\omega' h'}}{\partial p}$$ with $h = s + Lq$ Moist static energy

Large-scale effects of convection (2)

vertical integrals of Q_1 and Q_2

$$\int_{Pt}^{Ps} Q_1 \frac{dp}{g} \equiv \int_{Pt}^{Ps} Q_R \frac{dp}{g} + L\,\mathrm{Pr} + \rho C_p (\overline{w'T'})_{P=Ps} = \int_{Pt}^{Ps} Q_R \frac{dp}{g} + L\,\mathrm{Pr} + HS$$

Surface Precipitation flux

Surface sensible Heat flux

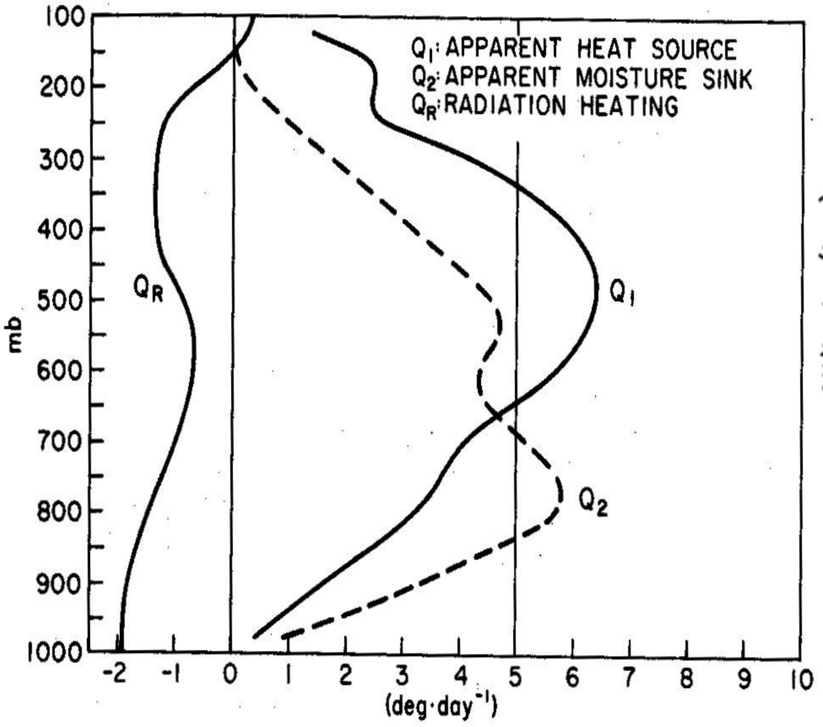
$$\int_{Pt}^{Ps} Q_2 \frac{dp}{g} \equiv L\,\mathrm{Pr} - \rho L(\overline{w'q'})_{P=Ps} = L\,\mathrm{Pr} - HL$$

Surface Precipitation

Surface latent Heat flux

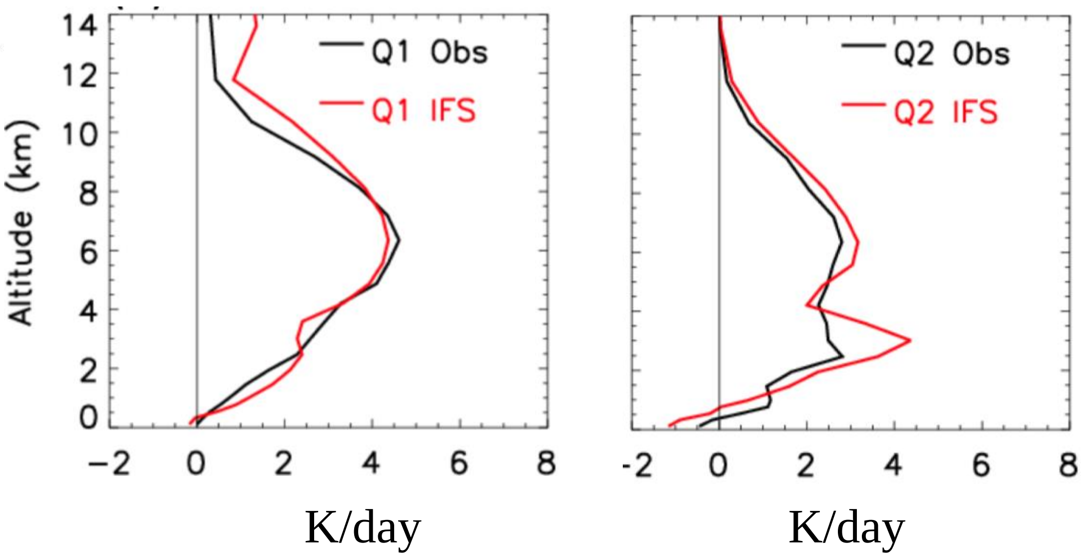
Large-scale effects of convection (3)

Budgets from Obs: Tropical Pacific



Yanai et al., 1973, JAS

Budgets from Obs and IFS : Indian Ocean

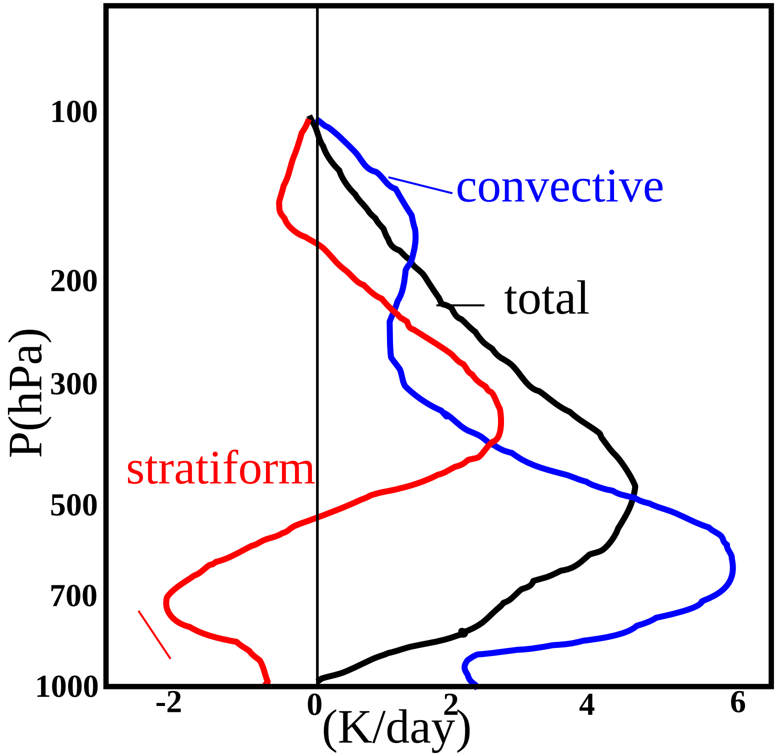


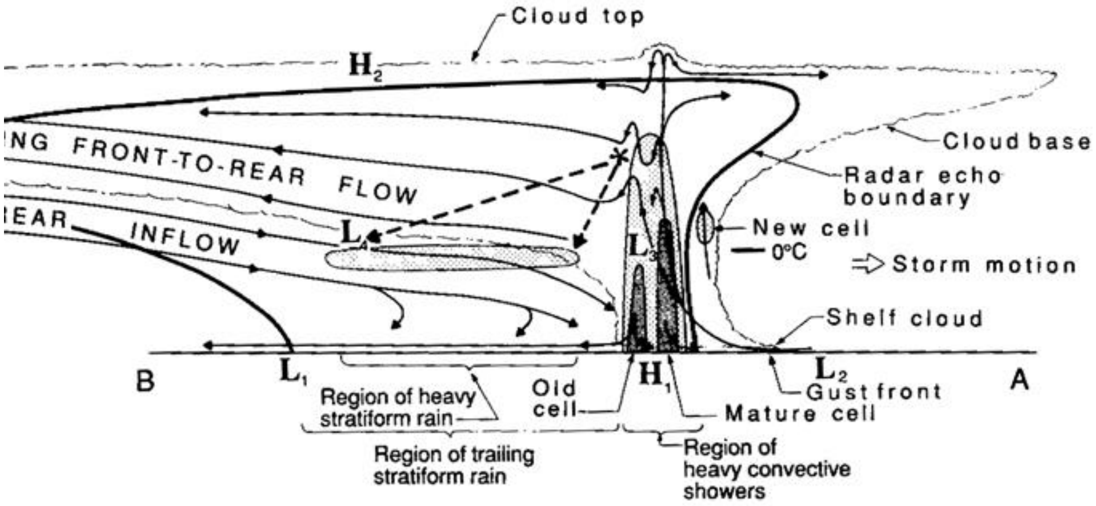
J.-E Kim et al. 2017, JAS

Note the typical tropical maximum of Q1 at 500 hPa, Q2 maximum is lower and typically around 700 -800 hPa

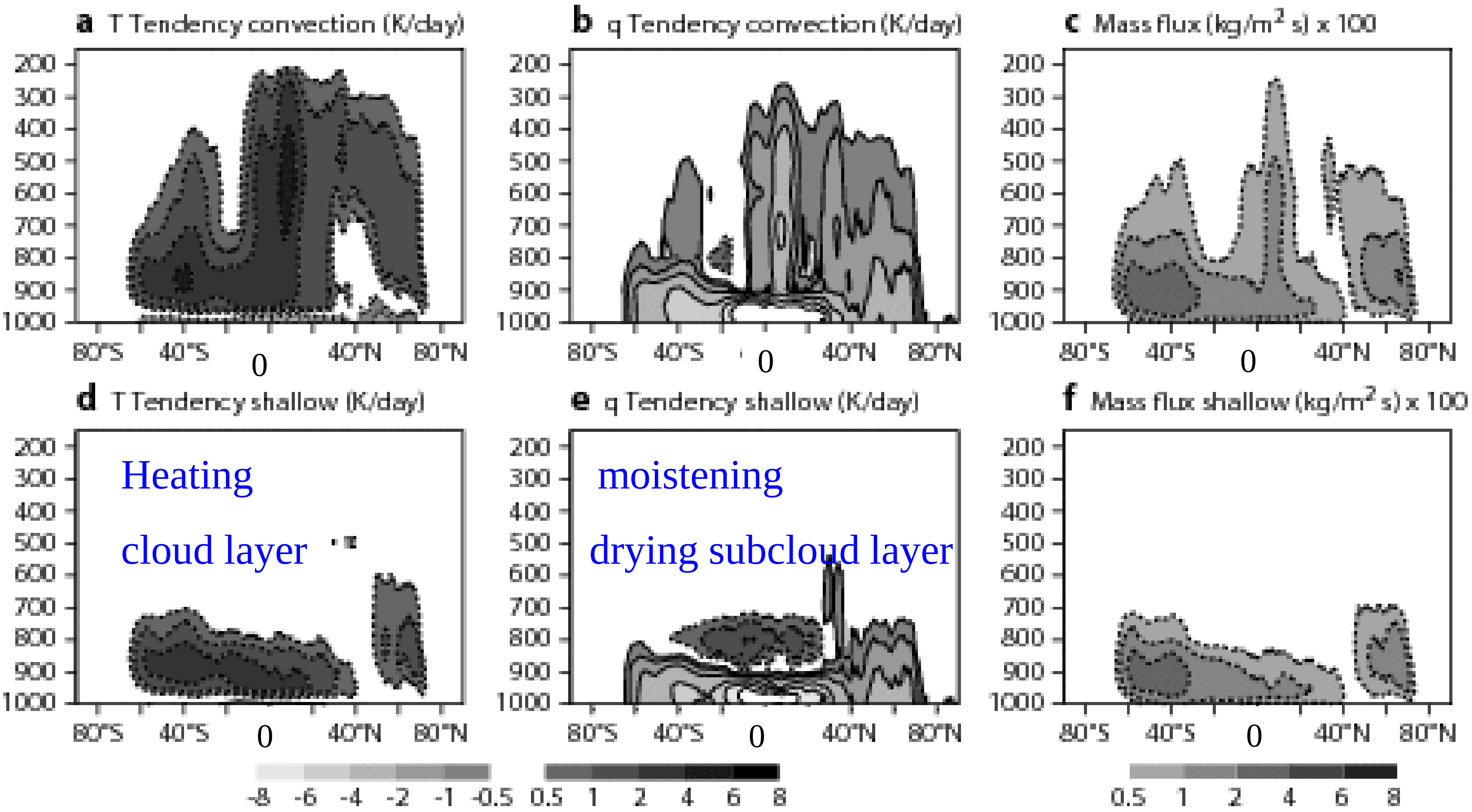
Effects of mesoscale organization

convective and stratiform heating modes





Zonal mean convective tendencies (deep & shallow) July 2013 and mass flux in IFS



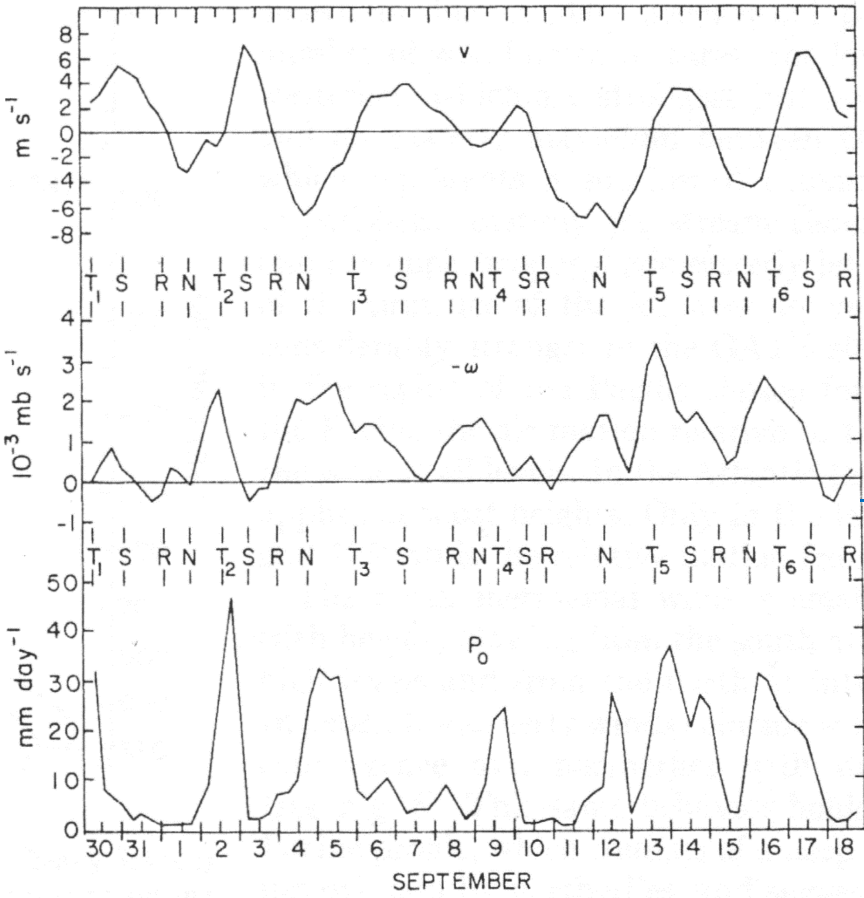
Convective quasi-equilibrium

Arakawa and Schubert (1974) postulated that the level of activity of convection is such that their stabilizing effect balances the destabilization by large-scale processes.

Observational evidence:

GARP Atlantic Tropical Experiment (1974)

Thompson et al., JAS, 1979



v (700 hPa)

−ω (700 hPa)

Precipitation

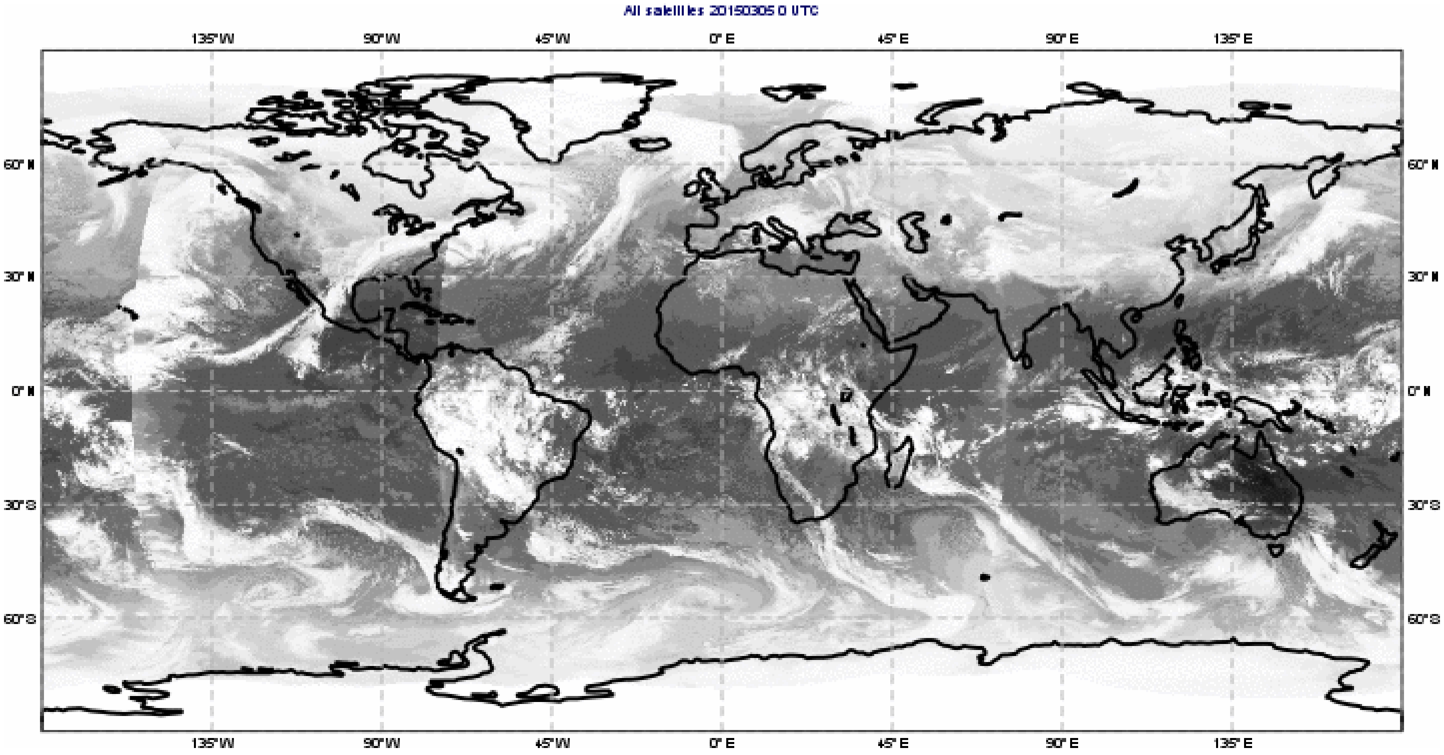
Summary

- Convection affects the atmosphere through condensation / evaporation and eddy transports
- To first order convection stabilizes the environment and on large horizontal scales convection is in quasi-equilibrium with the large-scale forcing
- Q1, Q2 and Q3 are quantities that reflect the time and space average effect of convection (“unresolved scale”) and stratiform heating/drying (“resolved scale”)
- An important parameter for the strength of convection is CAPE
- Shallow convection is present over very large (oceanic) areas, it determines the non-local heat and momentum fluxes into the cloud layer-> the horizontal transport of vapor and momentum from the subtropics to the ITCZ

- The effect of convection (local heat source) is fundamentally different in the middle latitudes and the Tropics. In the Tropics the Rossby radius of deformation R=N H/f (N=Brunt Väisäla Freq, f=Coriolis parameter, H=tropopause height) is infinite, and therefore the effects are not locally bounded, but spread globally via gravity waves – “throwing a stone in a lake”

Convectively coupled waves:

Rossby, Kelvin, MJO and African easterly Waves

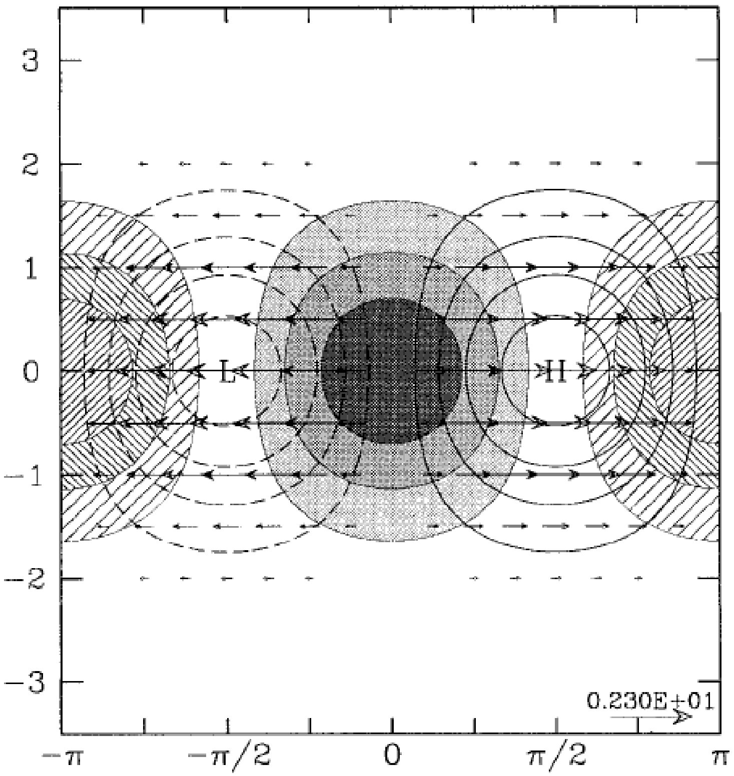


Analytical: solve DRY shallow water equations (see Lecture Note)

$$u = u_0 f(y)\, e^{i(kx-\omega t)}; \quad f(y) = e^{-y^2/2}$$

$$v = \hat{v}(y) f(y)\, e^{i(kx-\omega t)}; \quad \hat{v}(y) = Hermite\ Polynomials$$

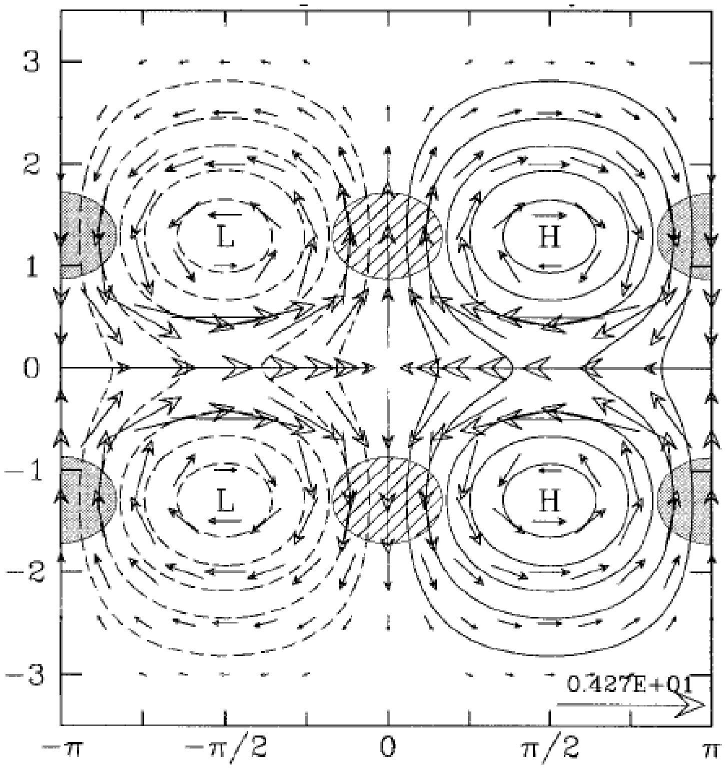
The Kelvin wave



V=0, eastward moving ~18 m/s

sym. around equator

The n=1 Rossby wave

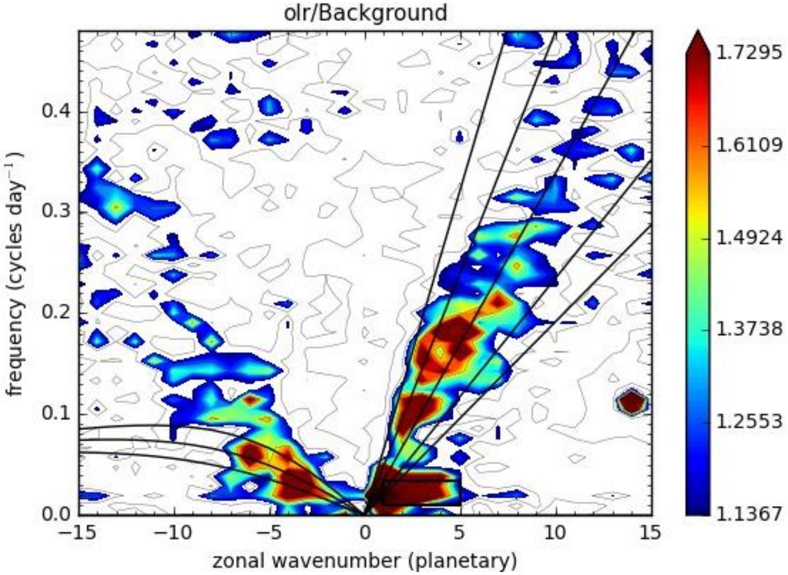


westward moving ~5 m/s

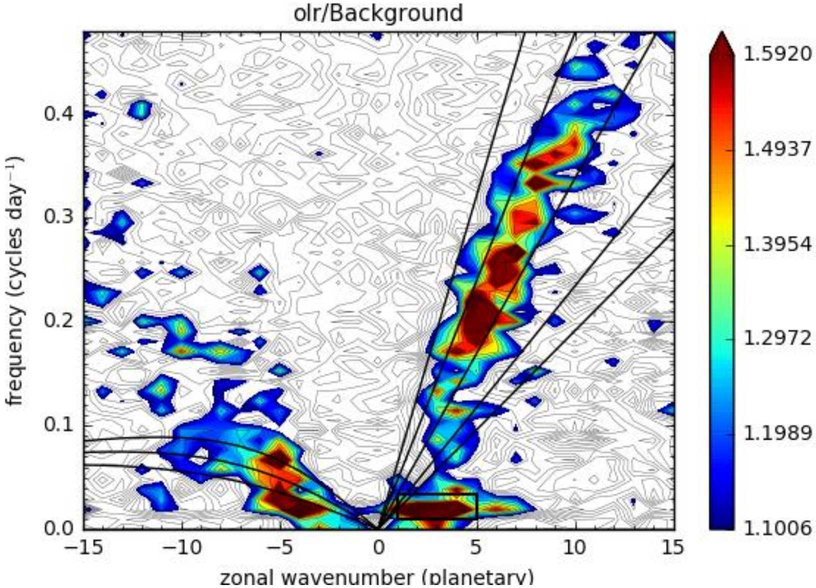
sym. around equator

OLR anomaly shaded, winds max at equator

Wavenumber frequency Diagrams of OLR



NOAA Satellite

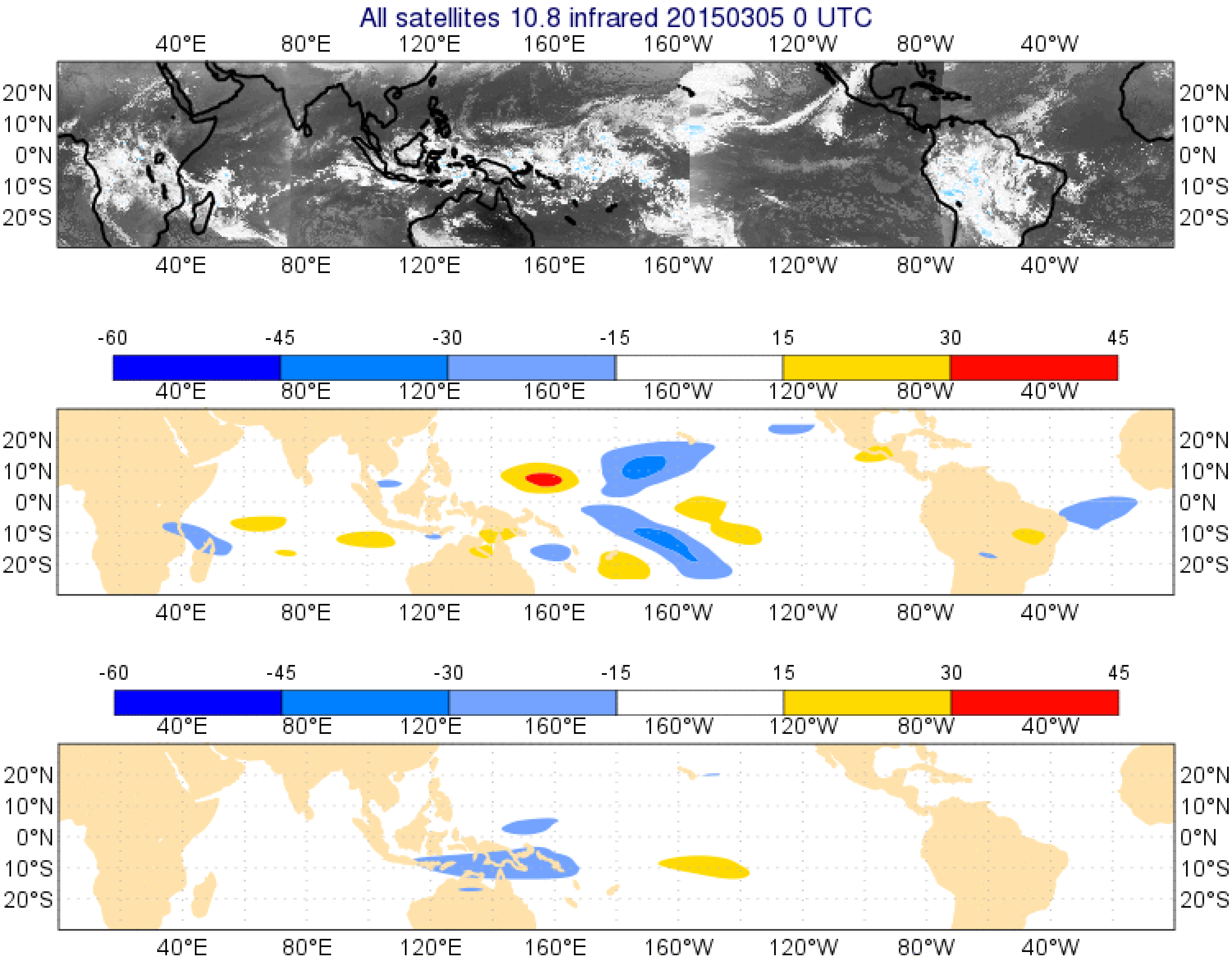


Cy46r1 6y (2019)

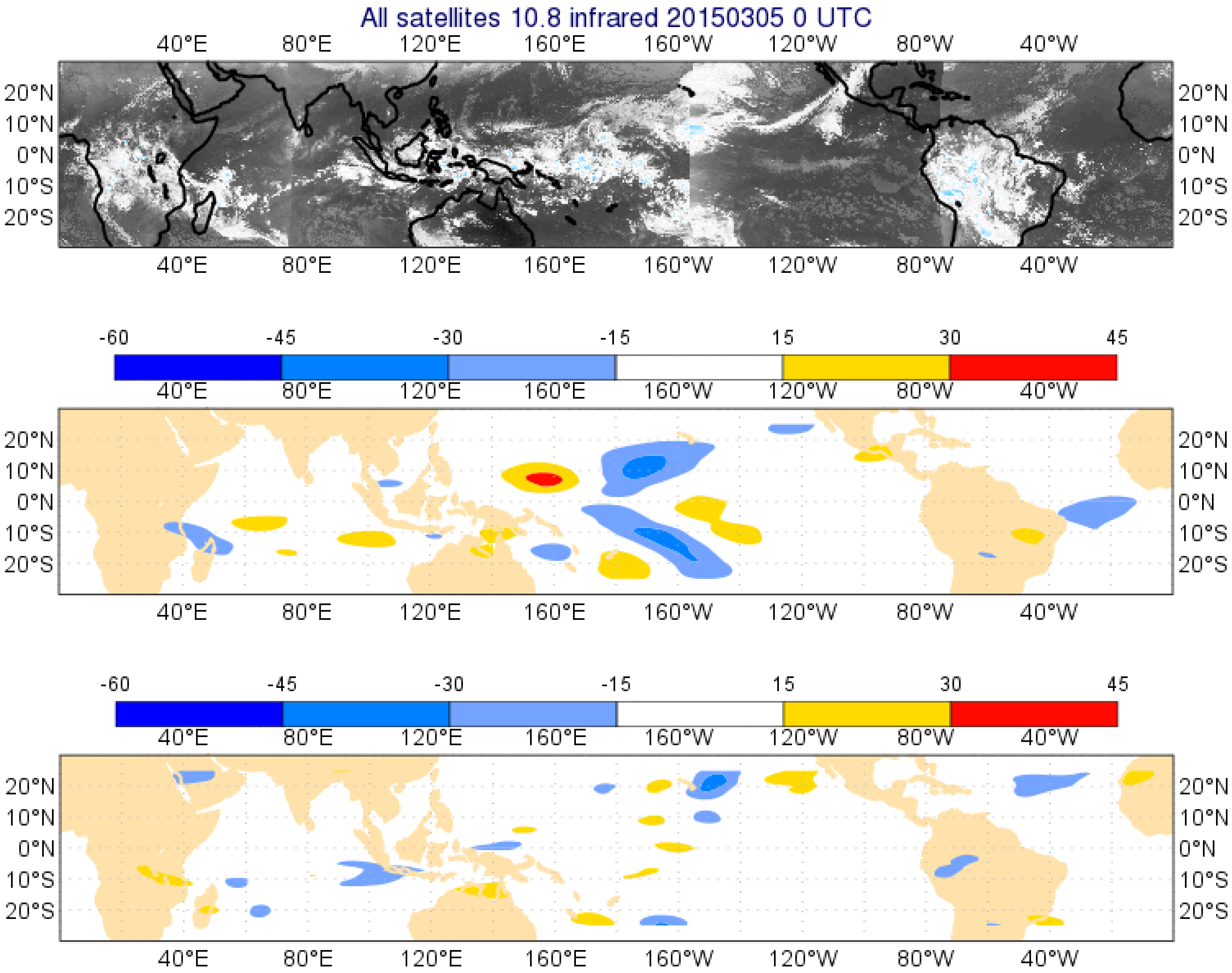
software courtesy Michael Herman (New Mexico Institute)

(all spectra have been divided by their own= smoothed background)

Rossby & MJO 5.3.2015-18.3 2015



Rossby & Kelvin 5.3.2015-16.3 2015

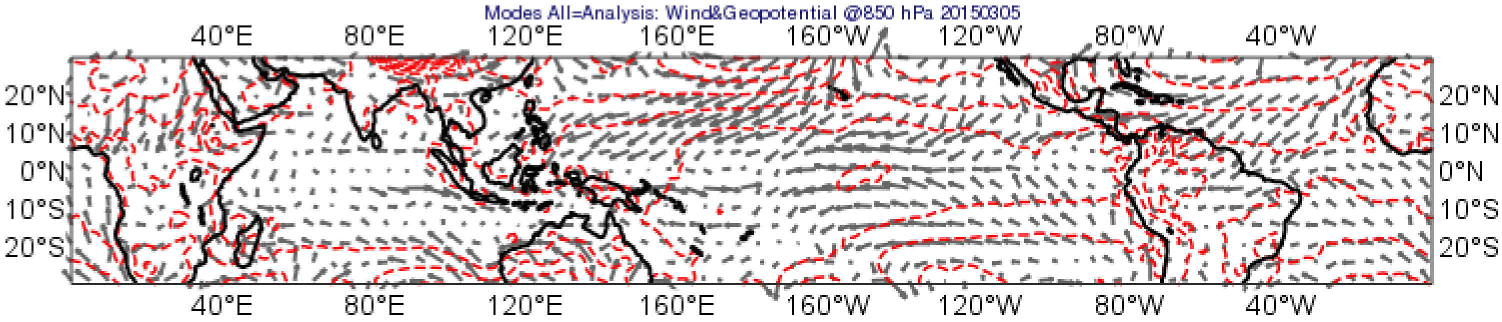


All satellites 10.8 infrared 20150305 0 UTC

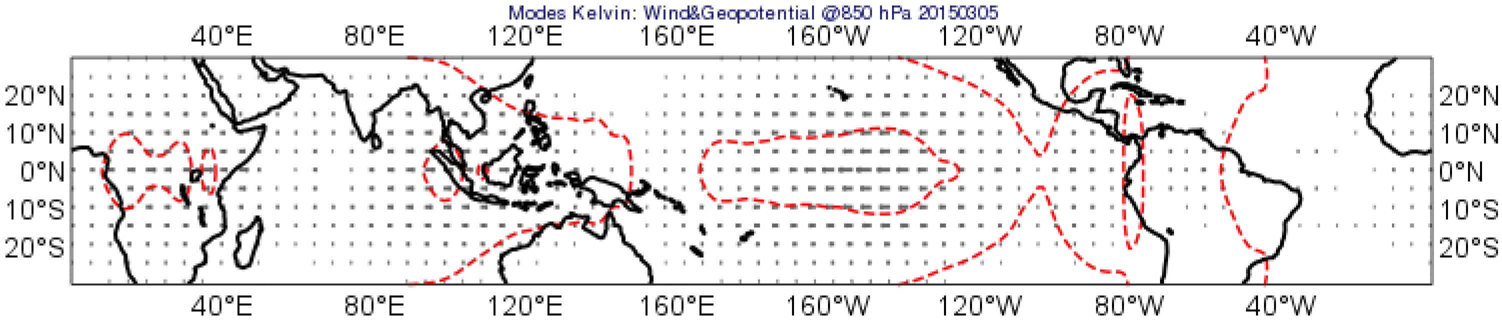
Normal mode projection and filtering

Žagar et al. (Geosc. Mod. Dev. 2015)

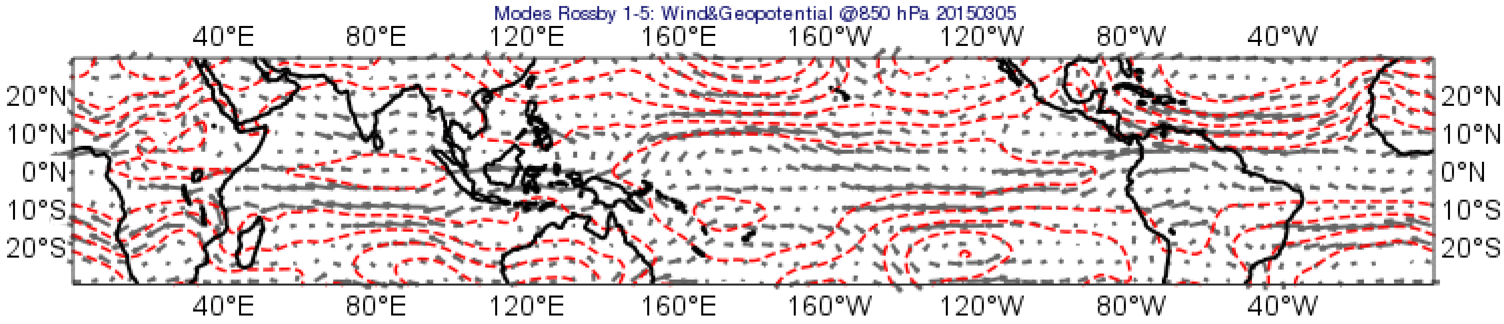
All =Analysis



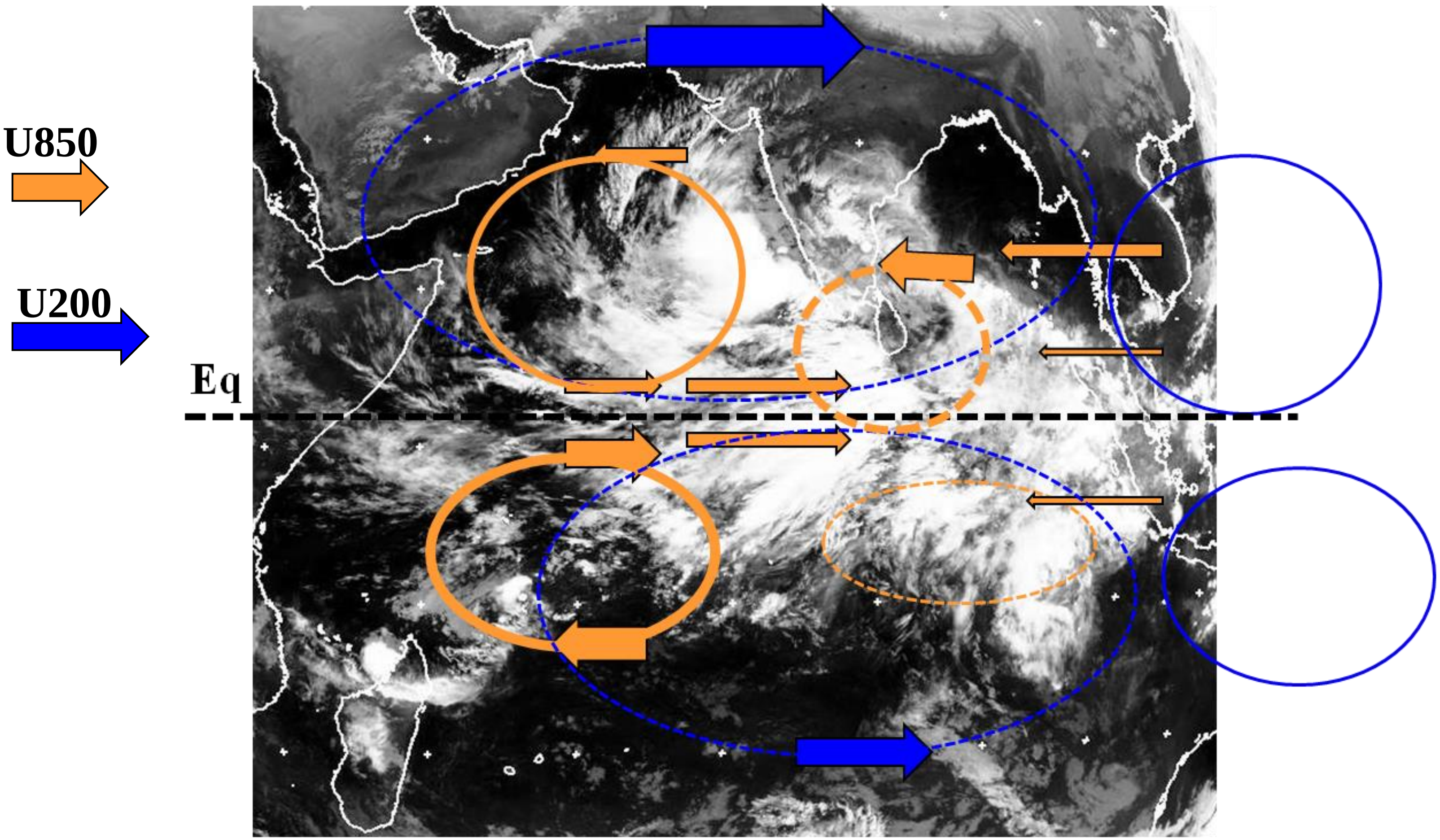
Kelvin



Rot 1-5

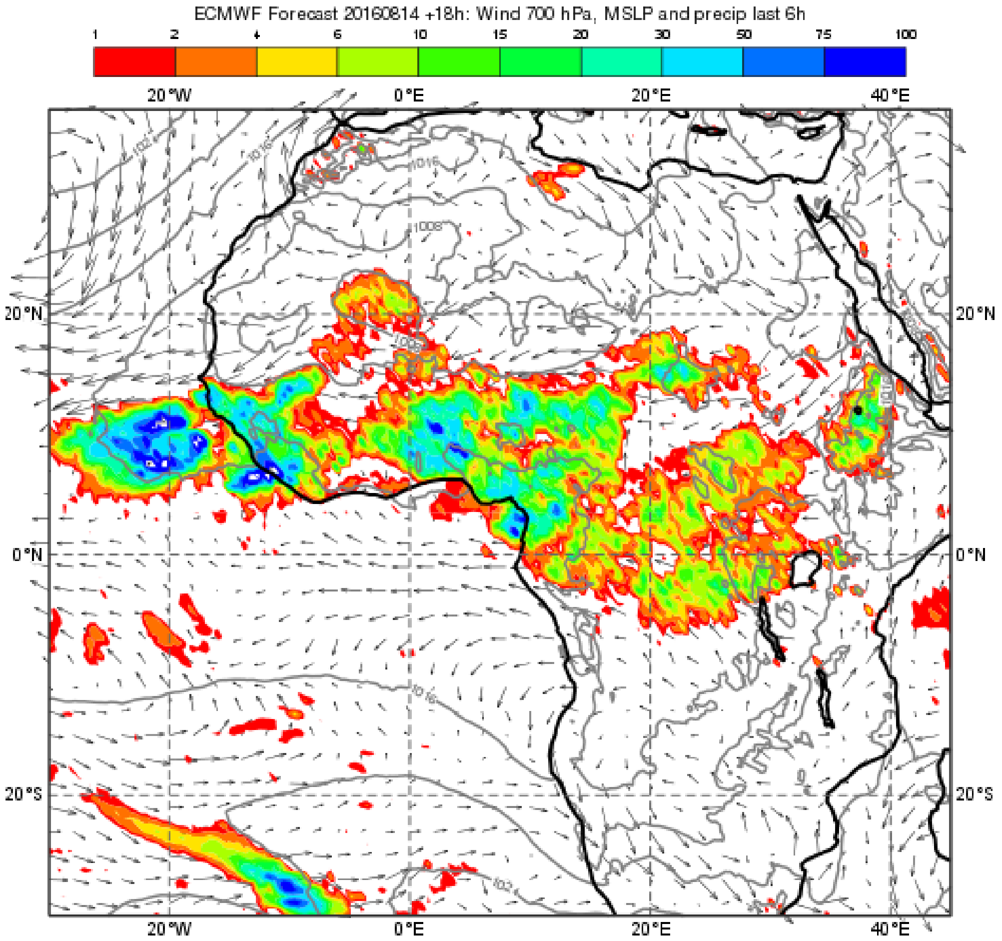


The MJO over Indian Ocean

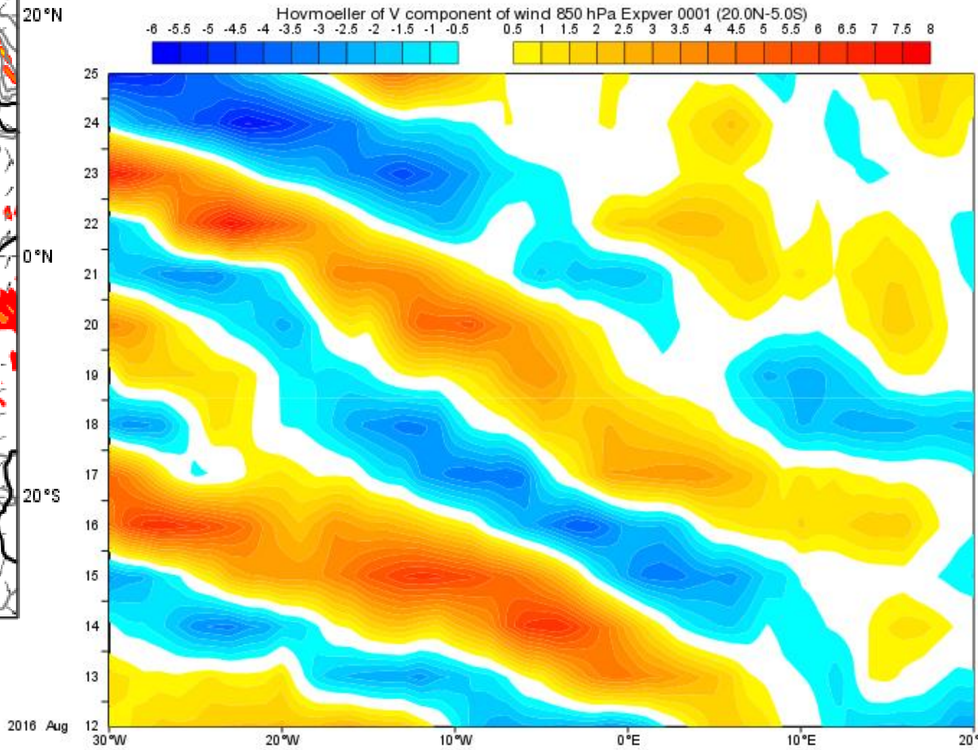


27 November 2011: Meteosat 7 + ECMWF Analysis

African Easterly waves



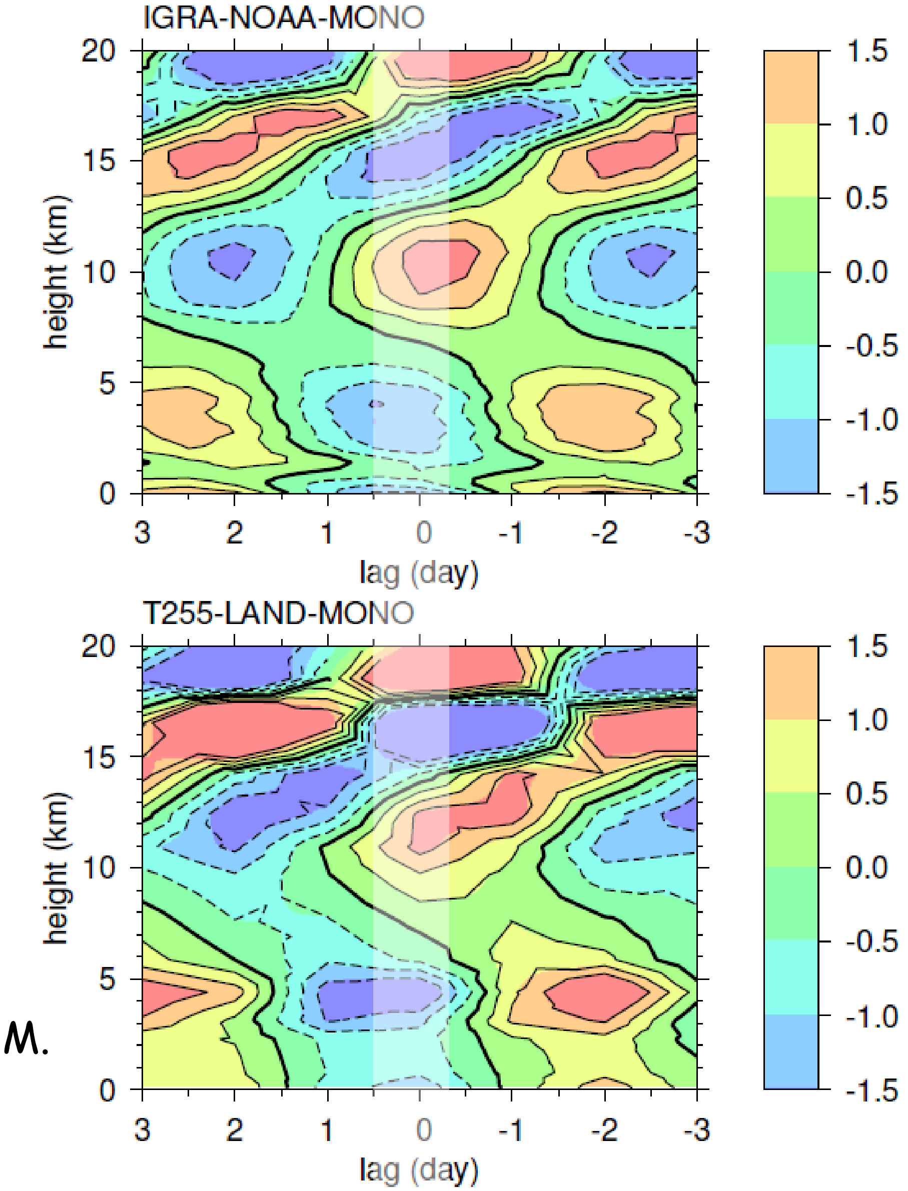
Hovmoeller diagrams as an easy way to plot waves (propagation + amplitude)



Kelvin waves: vertical T-anomalies

At z~10 km, warm anomaly and convective heating are in phase, leading to :

- ○ the conversion of potential in kinetic energy, given by specific density times omega= αω
- ○ The generation of potential energy, given by thermodynamic efficiency times heating = N Q



see also M. Herman et al. (2016, JAS) M. Steinheimer et al. (2008 Tellus) G. Shutts (2006, Dyn. Atmos. Oc.)