Radiative transfer in numerical models of the atmosphere

Robin Hogan

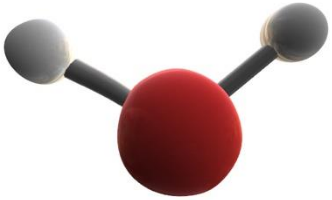
Slides contain contributions from Jean-Jacques Morcrette, Alessio Bozzo,Tony Slingo and Piers Forster

Outline

- Lecture 1
    1. Global context
    2. From Maxwell to the two-stream equations

- Lecture 2
    3. Gaseous absorption and emission
    4. Representing cloud structure
    5. Some remaining challenges

- Lecture 3 (Mark Fielding)
    - The ECMWF radiation scheme



Further reading

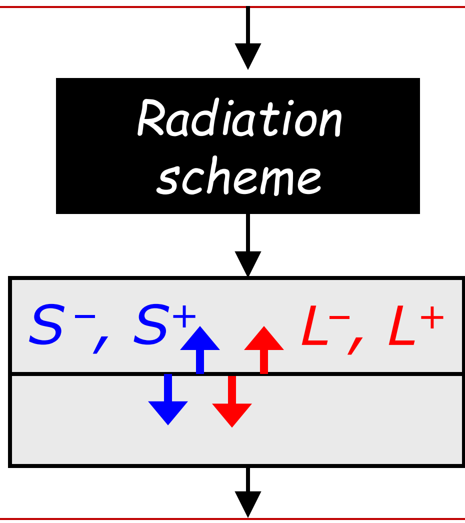
- Petty, G., 2006: *A first course on atmospheric radiation*
- Randall, D. A., 2000: *General circulation model development*
- Hogan, R. J., and J. K. P. Shonk, 2009: Radiation parametrization and clouds. Proc. ECMWF Seminar 1-4 Sept 2008.
- Hogan, R. J., and A. Bozzo, 2018: A flexible and efficient radiation scheme for the ECMWF model. *J. Adv. Modeling Earth Sys.,* **10,** 1990-2008.
- Scattering animations http://www.met.rdg.ac.uk/clouds/maxwell

Part 1: Global context

- What does a radiation scheme do?
- How does radiation determine global temperature?
- What is the role of radiation in the global circulation?
- How do we evaluate radiation schemes globally?

What does a radiation scheme do?

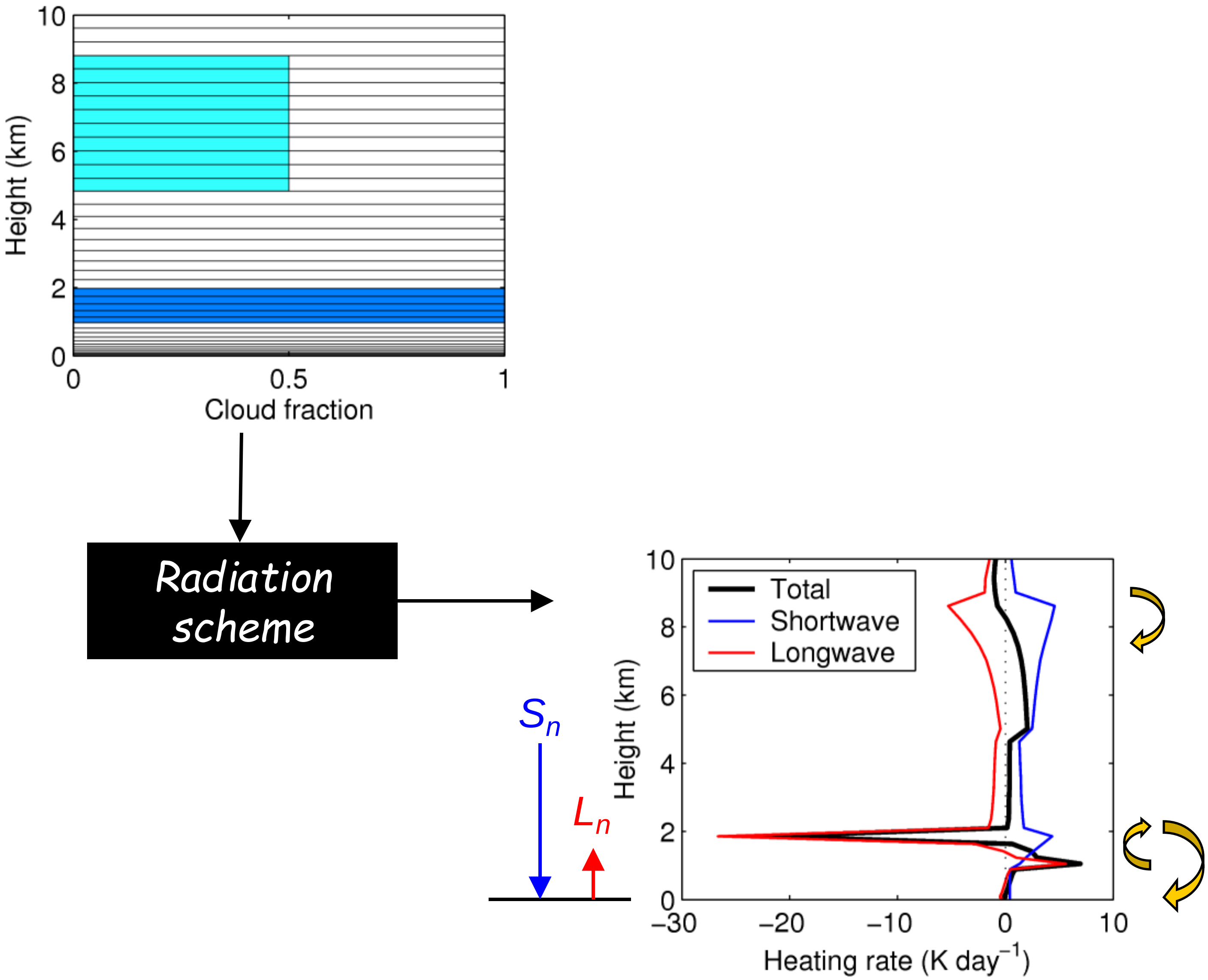
- *Prognostic variables:* temperature, humidity, cloud fraction, liquid and ice mixing ratios, surface temperature
- *Diagnostic variables:* sun angle, surface albedo, pressure, O_3, aerosol; well-mixed gases: CO_2, O_2, CH_4, N_2O, CFC-11 and CFC-12
- *CAMS project can provide prognostic aerosols, CO_2 and CH_4*



- Fluxes / irradiances between model levels in W m^{-2}
- Net flux $R_n = S^- - S^+ + L^- - L^+$

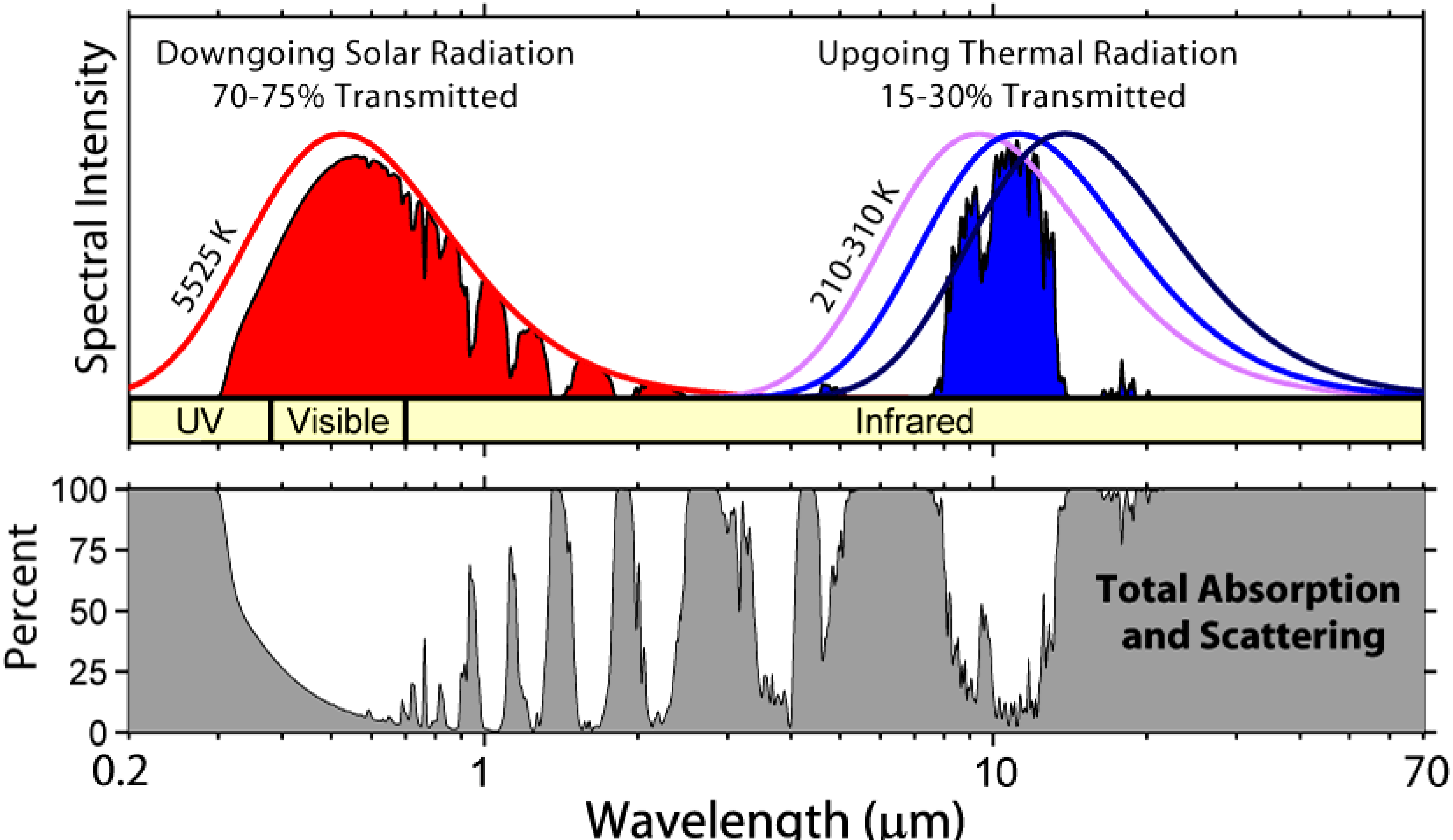
- Thermodynamic equation: $$\frac{D\bar{\theta}}{Dt} = \frac{1}{\rho C_p}\frac{\partial R_n}{\partial z} + latent + \cdots$$
- Radiation terms in surface energy balance: soil & sea temperatures

Heating rate profiles



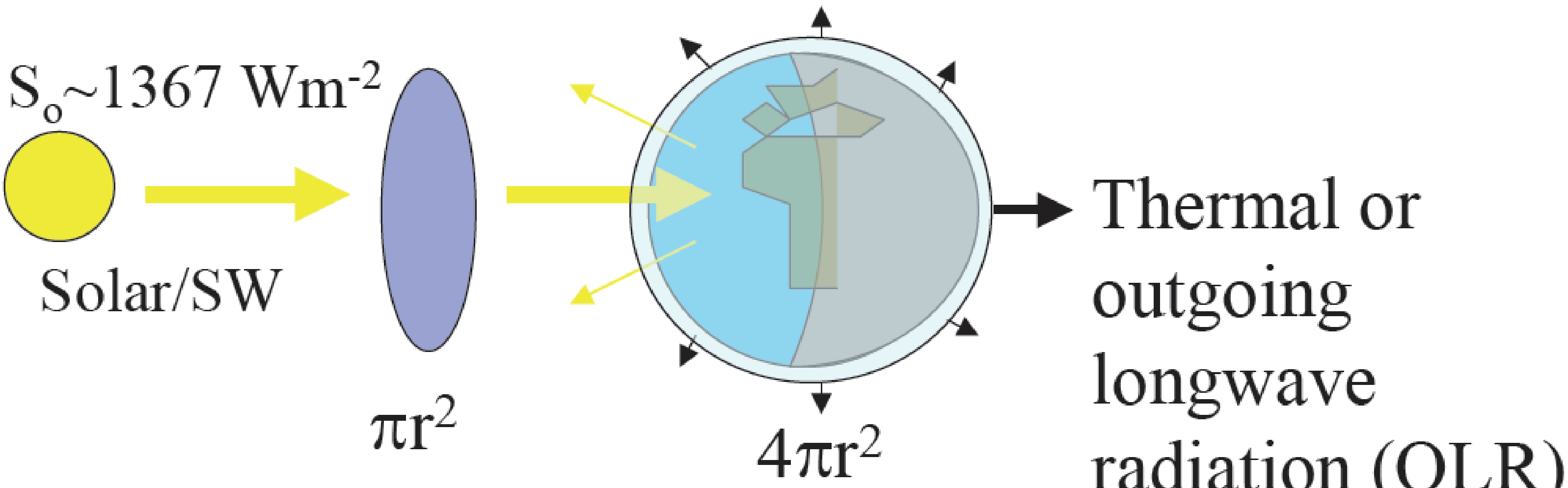
Radiation in the presence of clouds tends to destabilize the atmosphere

Spectral distribution of radiation



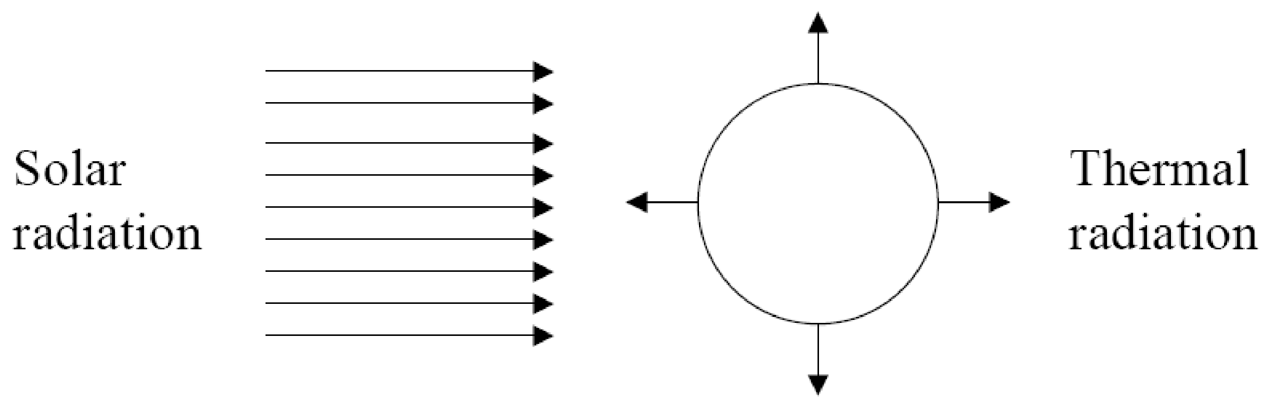
- **Shortwave: atmosphere is mostly transparent**
- **Longwave: atmosphere is mostly opaque**

Earth's radiation balance



- In equilibrium, the net absorption of solar radiation is balanced by the emission of thermal radiation back to space
- The thermal emission is controlled by the strength of the greenhouse effect
- If there is an increase in the concentrations of greenhouse gases, such as carbon dioxide, then the system warms as it tries to reach a new equilibrium

Overall energy balance of the Earth



$$(1-\alpha)\, S_o\, \pi\, r^2 = 4\, \pi\, r^2\, \sigma\, T^4_{eff}$$

Simplifying, we find that;

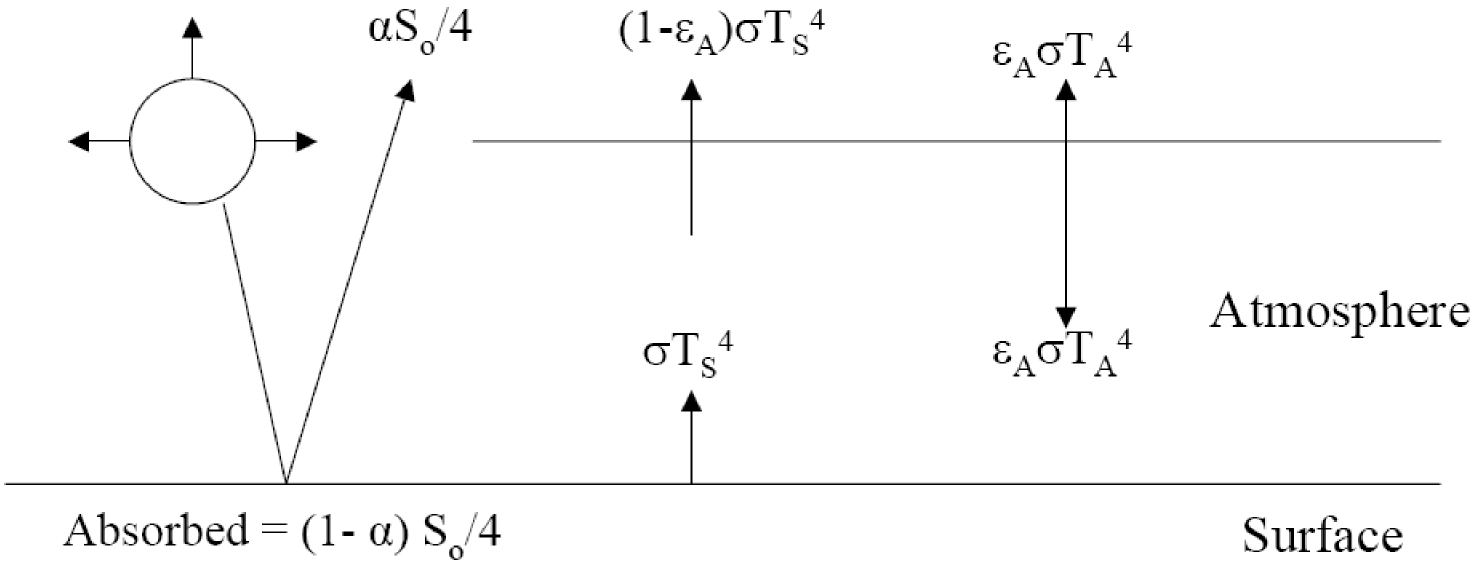
$$\sigma\, T^4_{eff} = (1-\alpha)\, S_o/4$$

and hence

$$T_{eff} \approx 255\ \mathrm{K}$$

If the Earth was black (α=0), T_{eff} = 278 K, still lower than observed 288 K

Overall energy balance including the greenhouse effect



Consider the equilibrium of the atmosphere and then of the surface;

$$\varepsilon_A \sigma T_S^4 = 2\varepsilon_A \sigma T_A^4 \qquad (4)$$

$$(1-\alpha)S_o/4 + \varepsilon_A \sigma T_A^4 = \sigma T_S^4 \qquad (5)$$

Hence

$$\sigma T_S^4 = \{(1- \alpha)S_o/4\} / (1 - \varepsilon_A/2) \quad (6)$$

and

$$T_A = T_S/2^{1/4} \quad (7)$$

Note that T_S is larger than T_{eff} given by (2), because of the additional downward thermal emission from the atmosphere. So, the greenhouse effect ensures that the surface is warmer with an atmosphere than without. Secondly, the atmosphere is colder than the surface and slightly colder than T_{eff}.

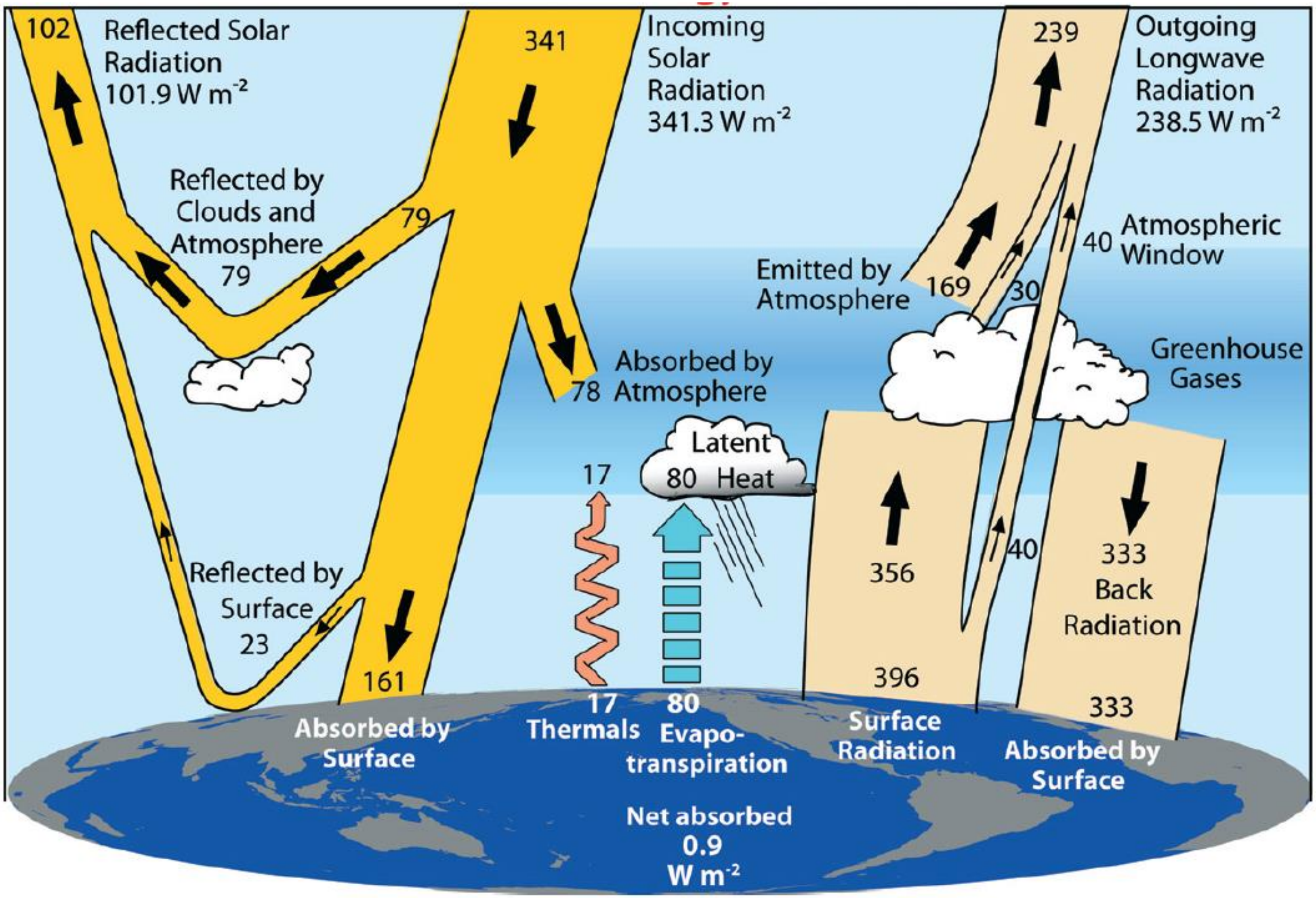
If we assume that $\alpha = 0.3$ and $\varepsilon_A = 0.8$ then we find that;

$$T_S = 289 \text{ K}$$

$$T_A = 243 \text{ K}$$

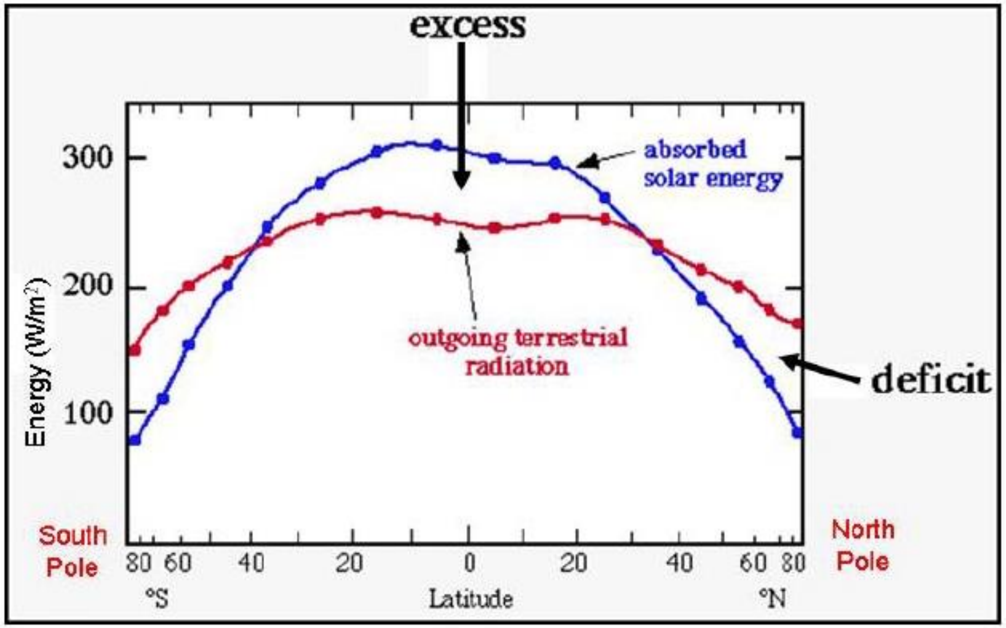
Which are reasonable values for the global mean surface and atmospheric temperatures.

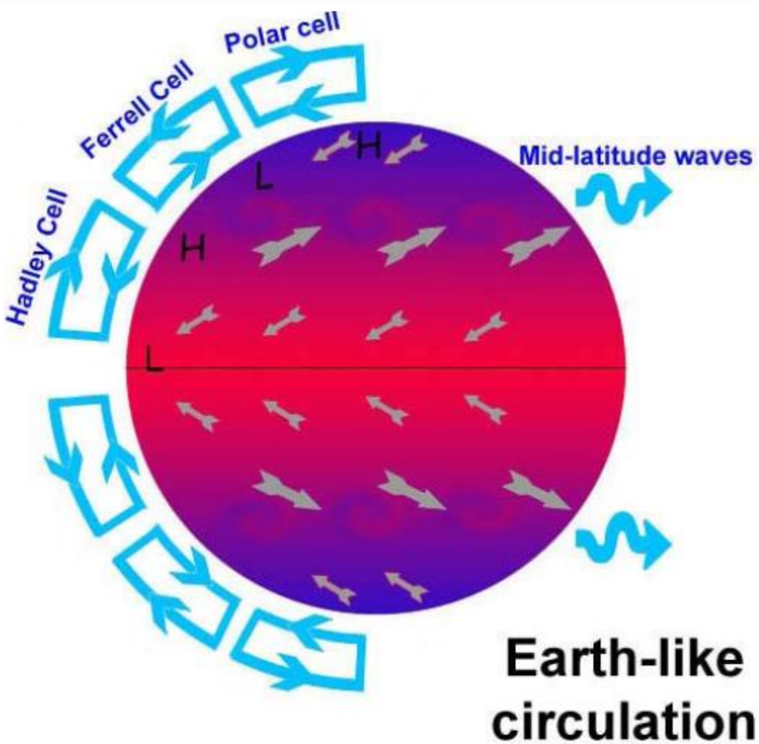
Global energy flows



- Trenberth et al. (2009); modification of Kiehl & Trenberth (1997)

Global circulation

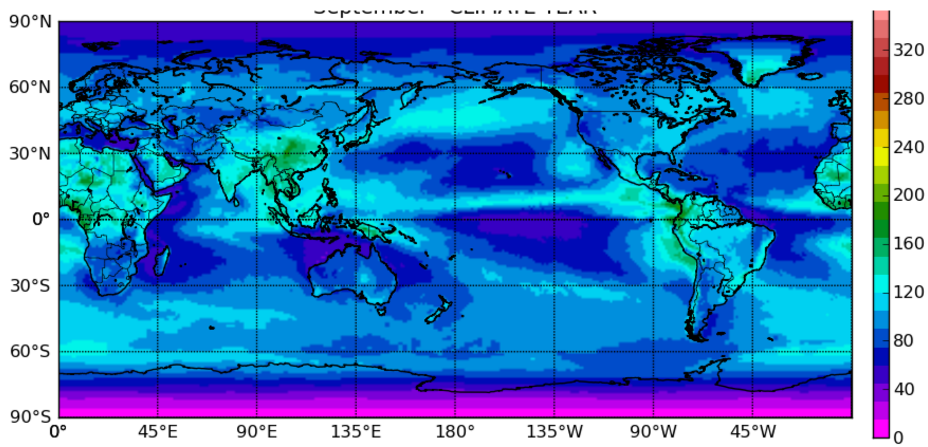


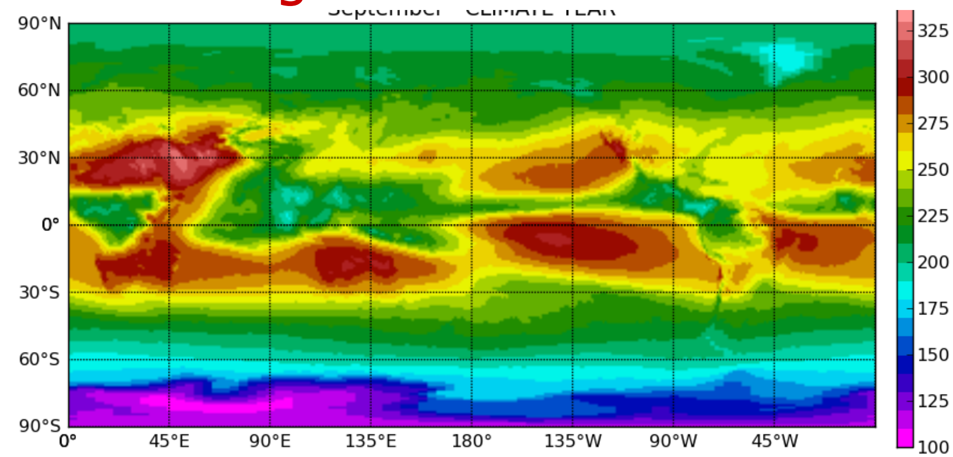


- Warmer tropics means same pressure layers are thicker at equator
- By thermal wind balance there must be westerlies
- Excess heat transported polewards by
  - Disturbances in these westerlies
  - Oceanic transport

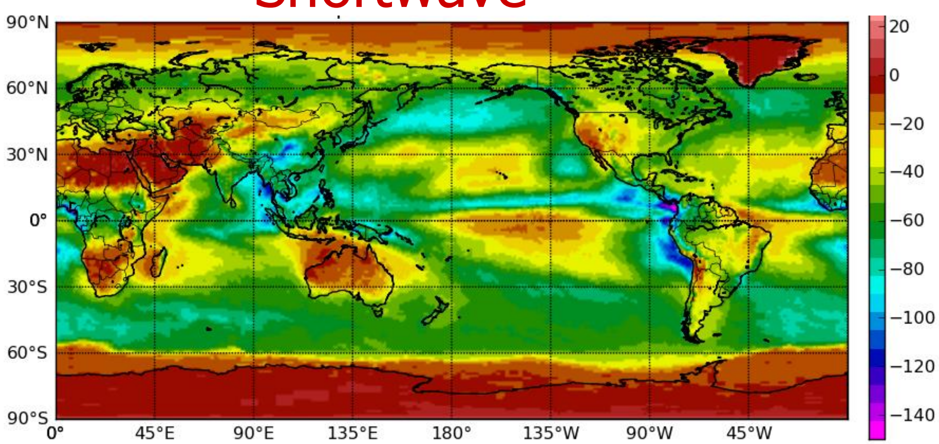
CERES radiometer (Sept)

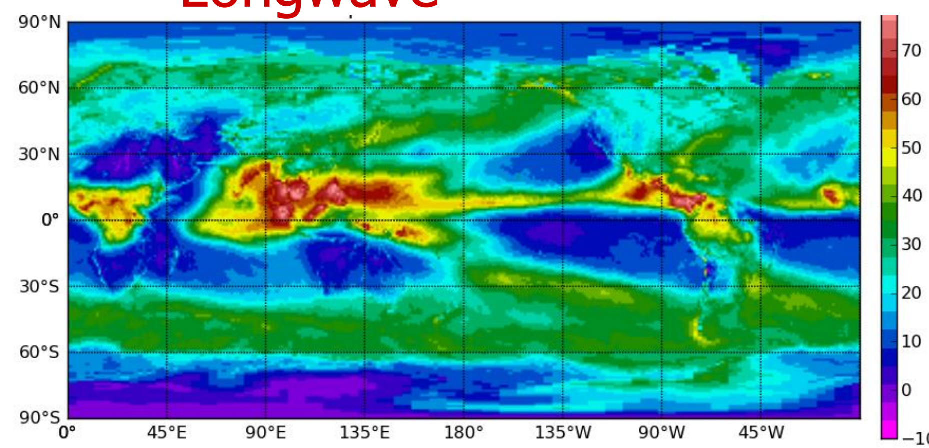
- TOA total upwelling irradiance
  - Shortwave
  - Longwave



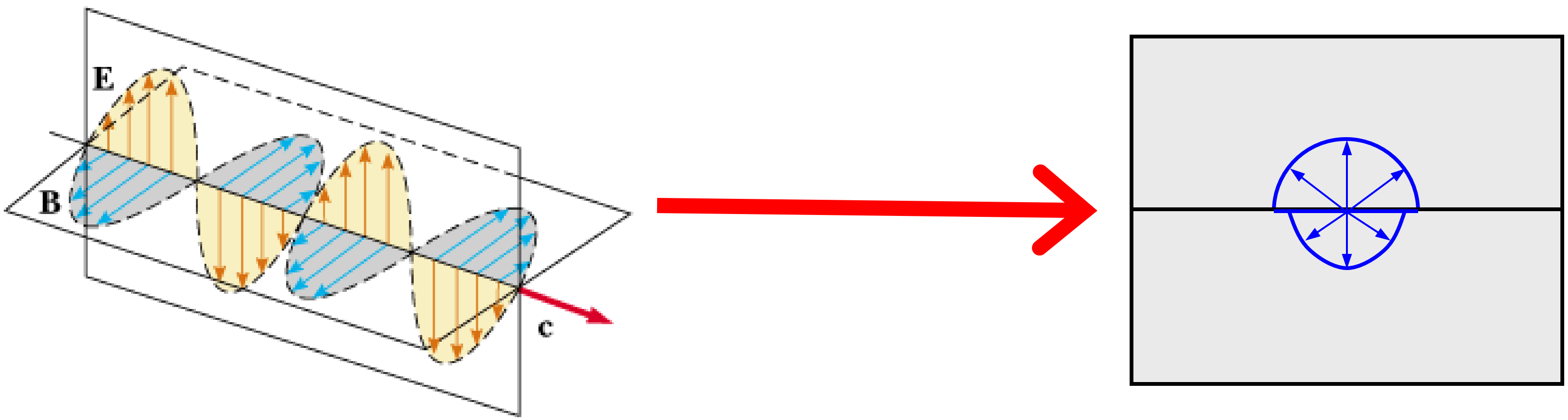


- TOA cloud radiative effect: $F_n^{\text{cloud}} - F_n^{\text{clear}}$
  - Shortwave
  - Longwave

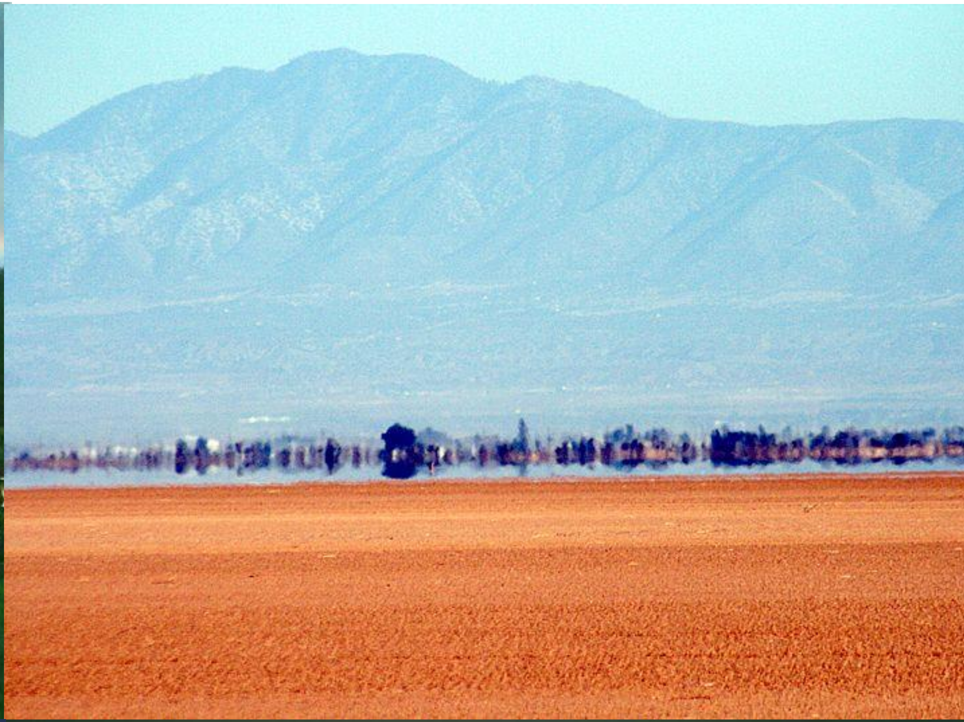




Part 2: Maxwell’s equations to the two-stream equations

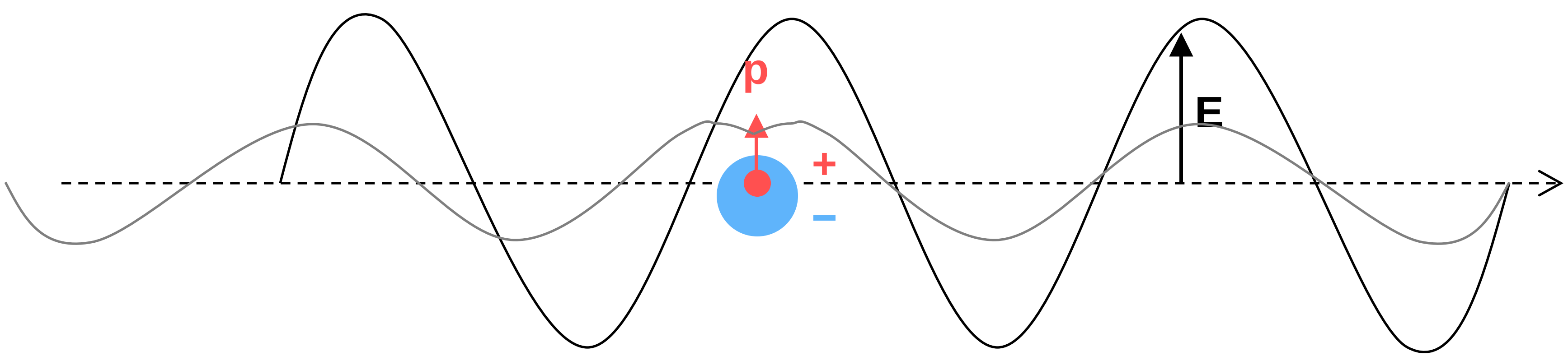


- How do Maxwell’s equations explain optical phenomena?
- How do we describe scattering by cloud particles, aerosols and molecules?
- How is radiative transfer implemented in models?

Building blocks of atmospheric radiation

1. Emission and absorption of quanta of radiative energy
   - Governed by quantum mechanics: the Planck function and the internal energy levels of the material
   - Responsible for complex gaseous absorption spectra
2. Electromagnetic waves interacting with a dielectric material
   - An oscillating dipole is excited, which then re-radiates
   - Governed by Maxwell's equations + Newton's 2nd law for bound charges
   - Responsible for *scattering, reflection* and *refraction*



Oscillating dipole **p** is induced, which is typically in phase with the incident electric field **E**

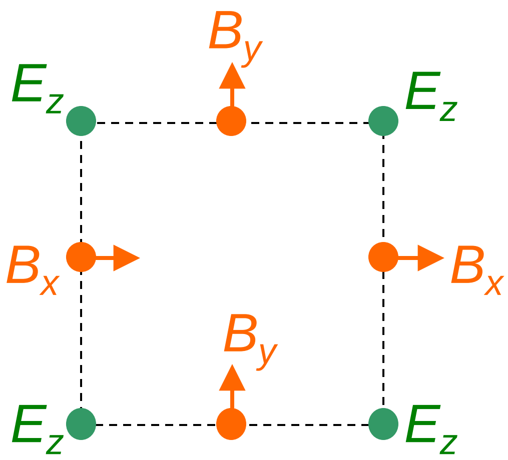
Dipole radiates in all directions (except directly parallel to its axis)

Maxwell's equations

- Almost all atmospheric radiative phenomena are due to this effect, described by the Maxwell curl equations:

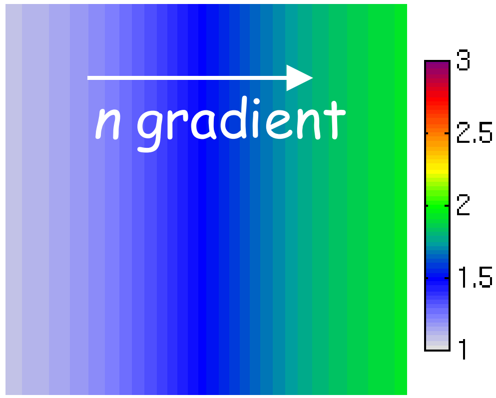
$$\frac{\partial \mathbf{E}}{\partial t} = \frac{c^2}{n^2} \nabla \times \mathbf{B} \qquad \frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

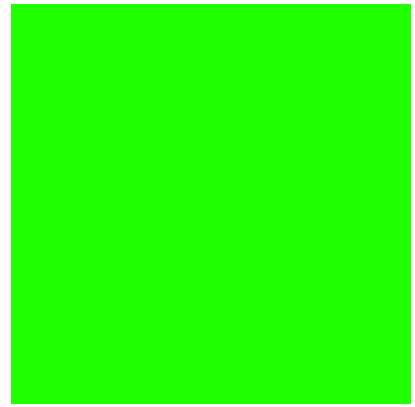
  - where c is the speed of light in vacuum, n is the complex refractive index (which varies with position), and **E** and **B** are the electric and magnetic fields (both functions of time and position);
- It is illuminating to discretize these equations directly
  - This is known as the Finite-Difference Time-Domain (FDTD) method
  - Use a staggered grid in time and space (Yee 1966)
  - Consider two dimensions only for simplicity
  - Need gridsize of ~0.02 μm and timestep of ~50 ps for atmospheric problems



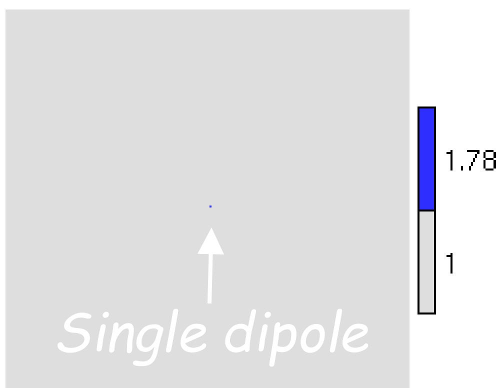
Simple examples

- Refraction (a mirage)





- Rayleigh scattering (blue sky)



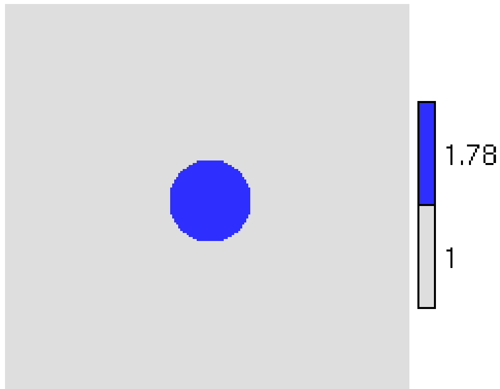
Refractive index

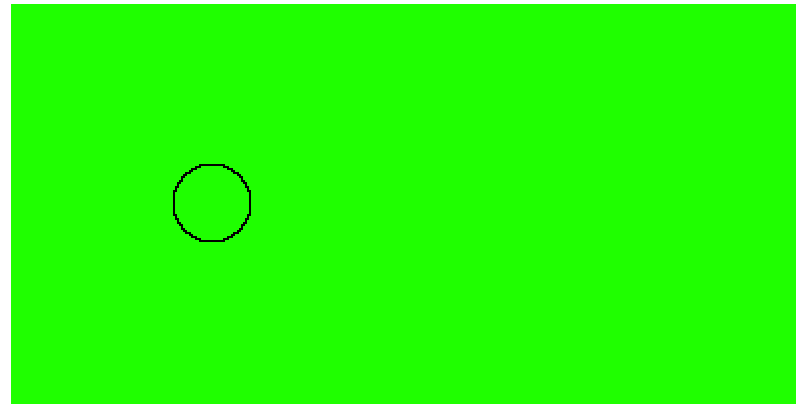
Total E_z field

Scattered field (total − incident)

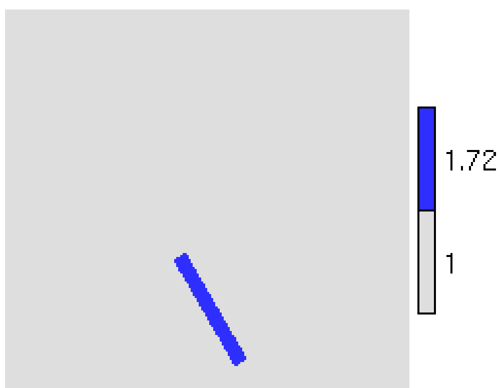
More complex examples

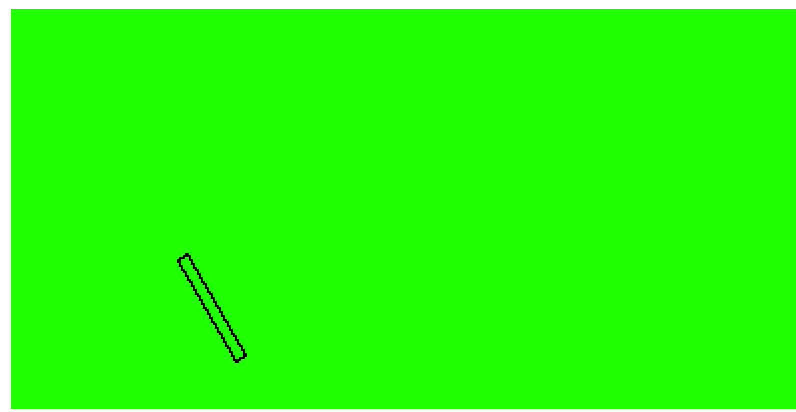
- A sphere (or circle in 2D)





- An ice column





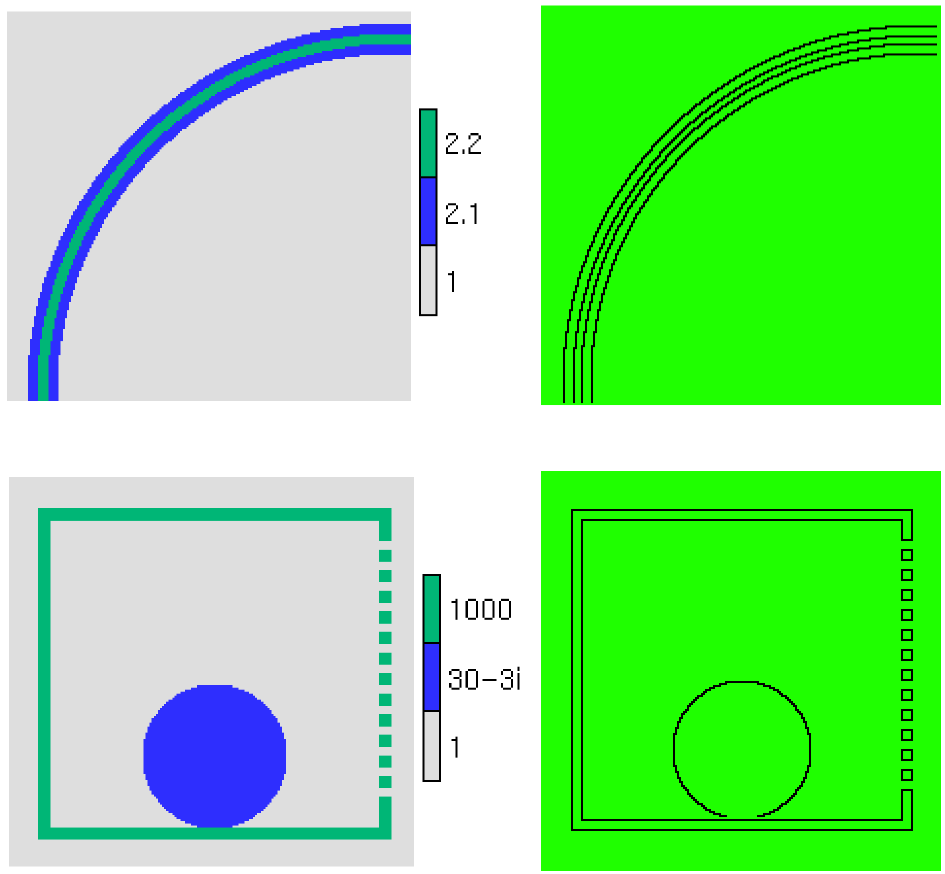
Refractive index

Total E_z field

Scattered field (total − incident)

Non-atmospheric examples

- Single-mode optic fibre
- Potato in a microwave oven

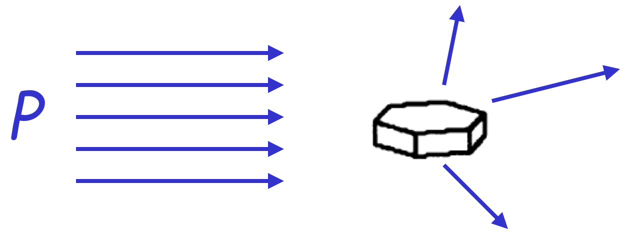


Refractive index   Total E_z field

Many more animations at www.met.rdg.ac.uk/clouds/maxwell
(interferometer, diffraction grating, dish antenna, clear-air radar, laser…)

Particle scattering

- Maxwell's equations used to obtain scattering properties
- Suppose we illuminate a single particle with monochromatic radiation of flux density *P* (in W/m^2)

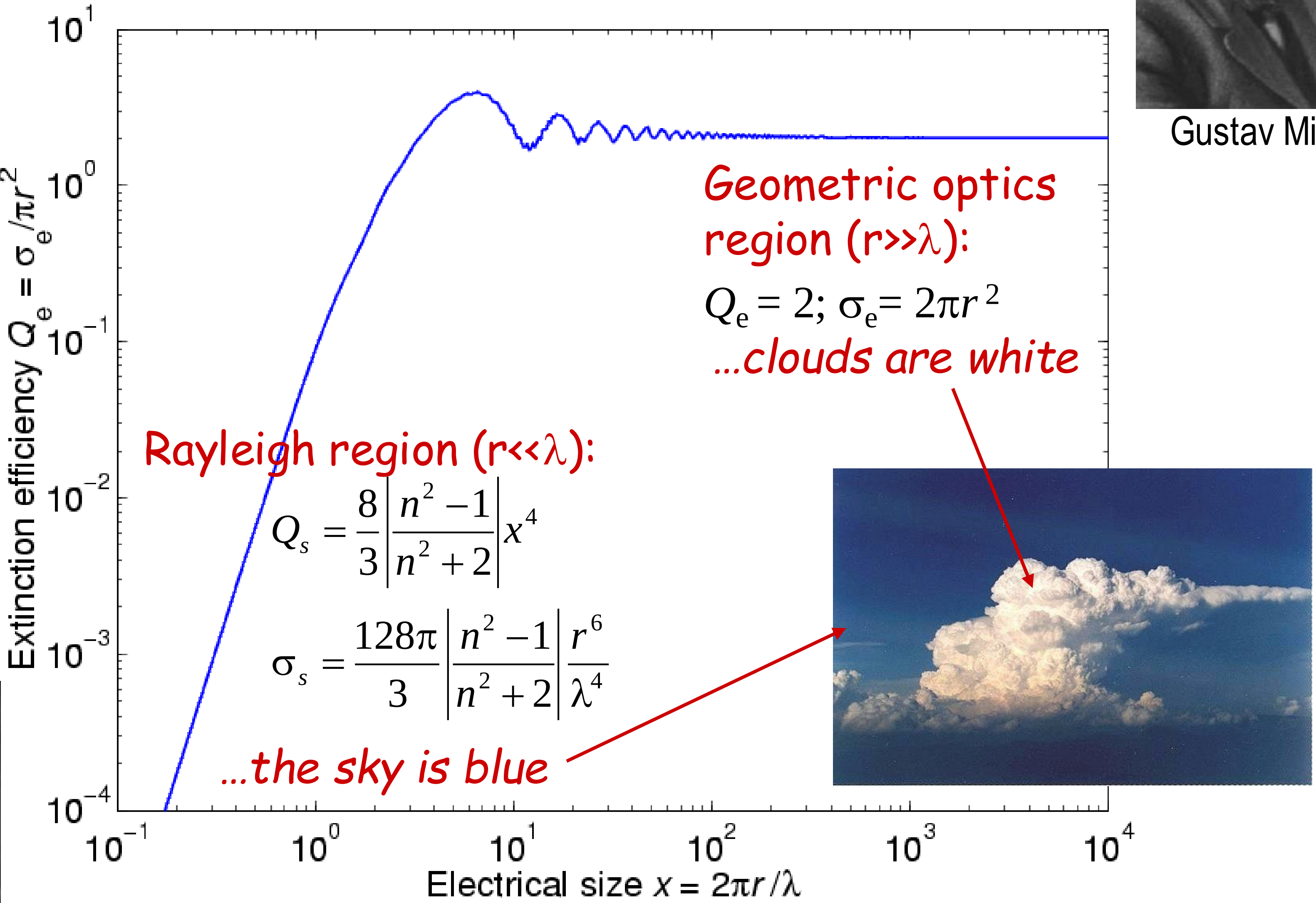


  - *Scattering cross-section* σ_s (in m^2) is defined such that the total scattered power (in W) is $P\sigma_s$
  - *Absorption cross-section* σ_a is the same but for absorbed power
  - *Extinction cross-section* $\sigma_e = \sigma_s + \sigma_a$ is the sum of the two
  - *Single scattering albedo* $\omega_0 = \sigma_s/\sigma_e$
- Directional scattering described by the *phase function* $p(\mathbf{\Omega})$
  - $\mathbf{\Omega}$ is the angle between incident and scattered directions
  - Phase function normalized such that

$$\int_{\mathbf{\Omega}} p(\mathbf{\Omega}) d\mathbf{\Omega} = 4\pi$$

The limits of Mie theory

Gustav Mie



Geometric optics region (r>>λ):

$Q_e = 2; \sigma_e = 2\pi r^2$

…clouds are white

Rayleigh region (r<<λ):

$$Q_s = \frac{8}{3}\left|\frac{n^2-1}{n^2+2}\right| x^4$$

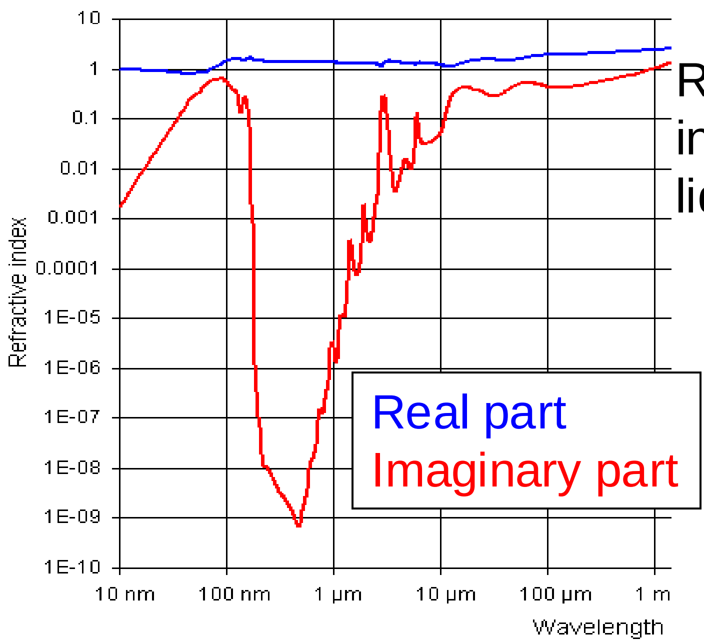
$$\sigma_s = \frac{128\pi}{3}\left|\frac{n^2-1}{n^2+2}\right| \frac{r^6}{\lambda^4}$$

…the sky is blue

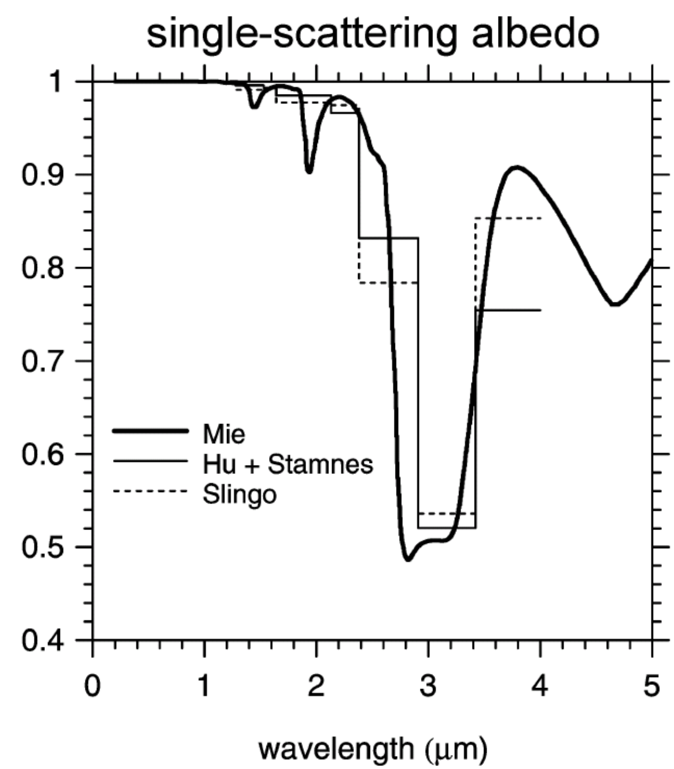
Lord Rayleigh

Single scattering albedo $\omega=\sigma_s/\sigma_e$

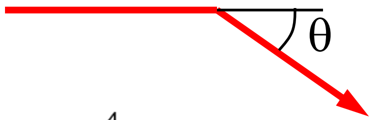
- Absorption related to imaginary part of refractive index m_i
- For liquid and ice
  - Visible: m_i is very small so ω is close to one (0.999…)
  - Longwave: m_i higher so $\omega \sim 0.5$



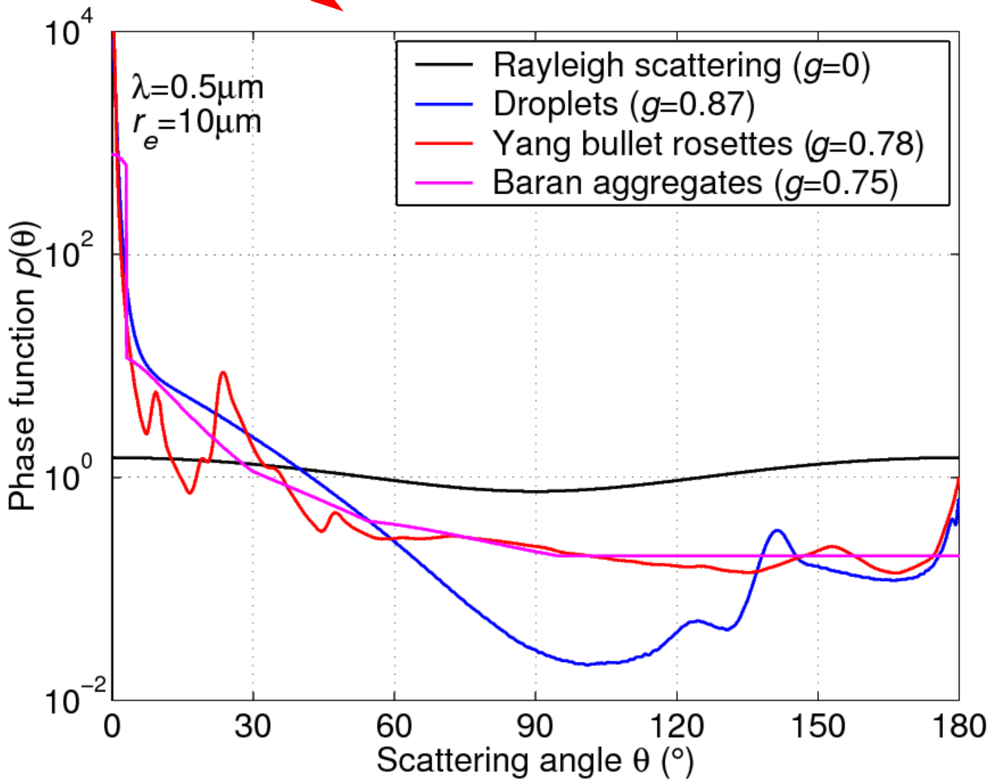
Refractive index of liquid water



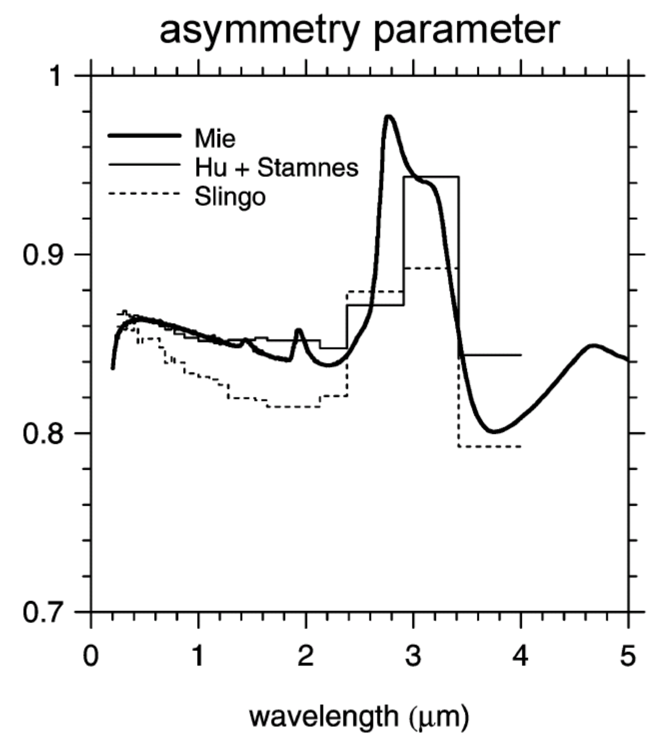
- Aerosols in the shortwave
  - Water soluble: 0.9-0.95; Black carbon ~ 0.3



The scattering phase function



- The distribution of scattered energy is known as the "scattering phase function"
- Different methods are suitable for different types of scatterer



- Radiation schemes can't use full phase function: approximate by *asymmetry factor* $g = \overline{\cos(\theta)}$
- Also apply delta-Eddington scaling: assume forward lobe not scattered at all

Size distributions

- We want *volume integral* of scattering properties
- Describe size distribution by $n(r)$ [m^{-4}], where $n(r)dr$ is number conc of particles with radius between r and $r+dr$
  - Extinction coefficient [m^{-1}] is integral of particle extinction cross-section [m^2] per unit vol: $\beta_e = \int n(r)\sigma_e(r)dr$
- In geometric optics region ($r \gg \lambda$) $\sigma_e(r) = 2\pi r^2$, so it is appropriate to characterize average particle size by
  - Effective radius $r_e = \frac{\int r^3 n(r)dr}{\int r^2 n(r)dr} = \frac{3\mathrm{LWC}}{2\rho_l \beta_{e,go}}$
- Can convert model's prognostic water content to extinction
- In each part of the spectrum, ω and g parameterized as a function of r_e

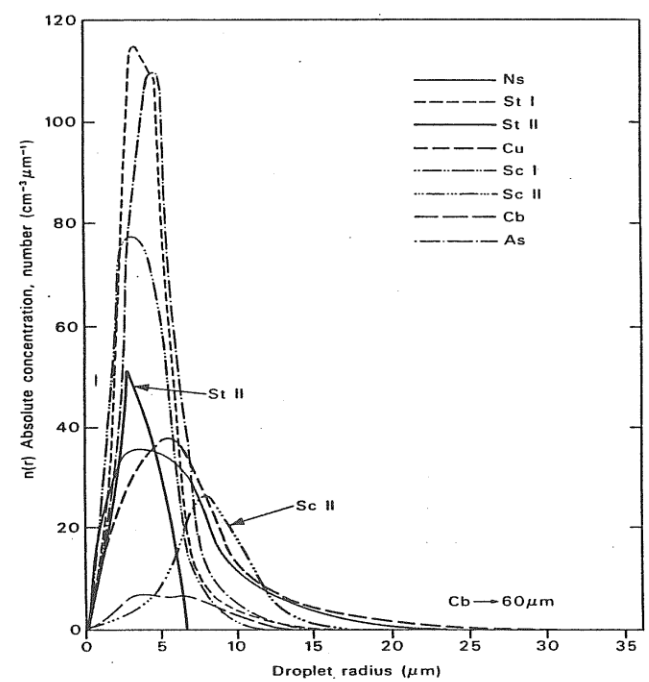


Fig. 3.6 The droplet distribution of eight cloud models. After Stephens (1979).

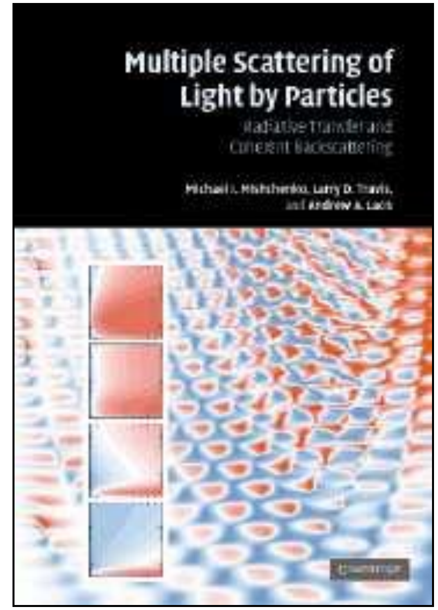
From Maxwell to radiative transfer

Maxwell's equations in terms of fields $\mathbf{E}(\mathbf{x},t)$, $\mathbf{B}(\mathbf{x},t)$

Reasonable assumptions:

- Ignore polarization
- Ignore time-dependence (sun is a continuous source)
- Particles are randomly separated so intensities add incoherently and phase is ignored
- Random orientation of particles so phase function doesn't depend on absolute orientation
- No diffraction around features larger than individual particles

Mishchenko et al. (2007)



3D radiative transfer in terms of monochromatic radiances $I(\mathbf{x},\Omega,\nu)$ in W m^{-2} sr^{-1} Hz^{-1}

The 3D radiative transfer equation

- This describes the radiance I in direction $\mathbf{\Omega}$ (where the $\mathbf{x}$ and ν dependence of all variables is implicit):

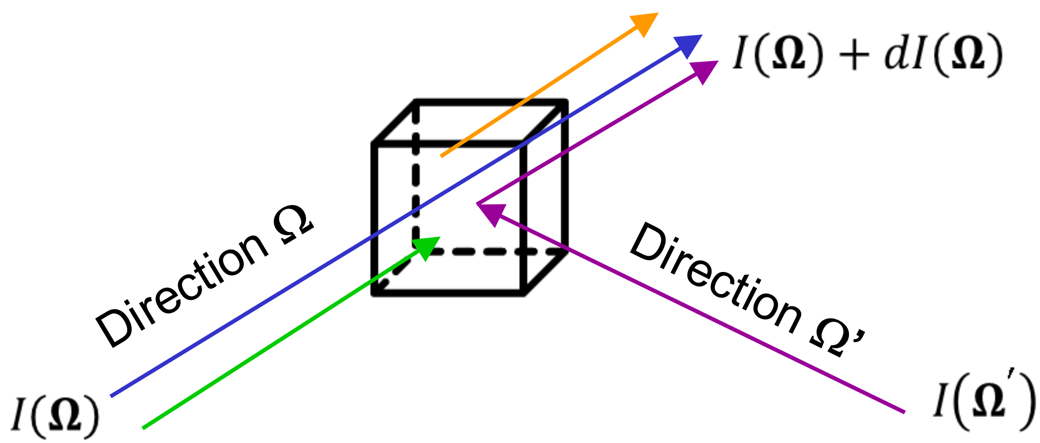
$$\mathbf{\Omega}\cdot\nabla I(\mathbf{\Omega}) = -\beta_e I(\mathbf{\Omega}) + \beta_s \int_{4\pi} p(\mathbf{\Omega},\mathbf{\Omega}')I(\mathbf{\Omega}')d\mathbf{\Omega}' + S(\mathbf{\Omega})$$

Spatial derivative representing how much radiation is upstream

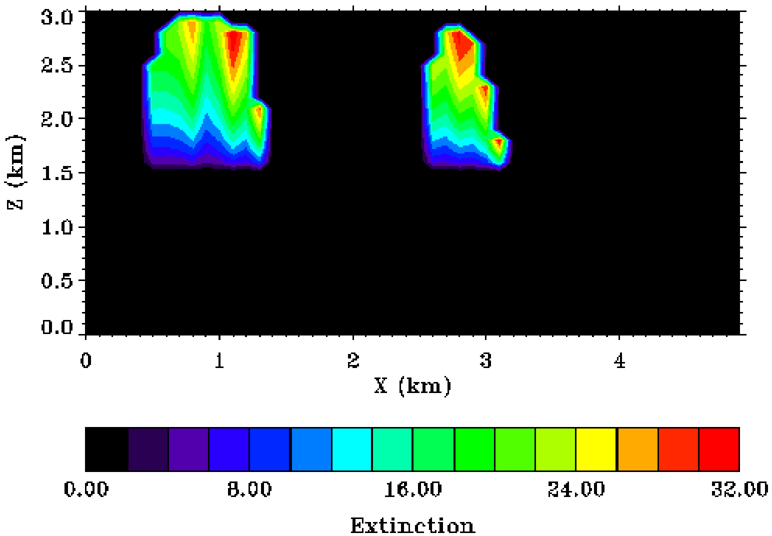
Loss by absorption or scattering

Gain by scattering Radiation scattered from all other directions

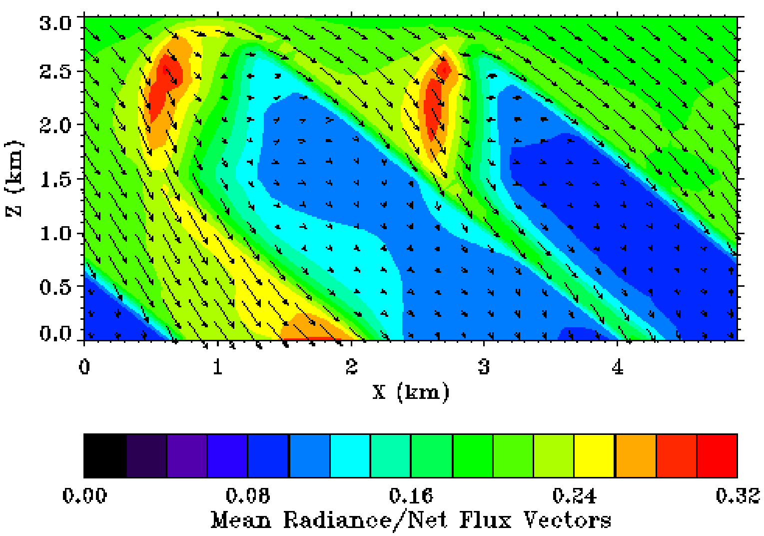
Source Such as thermal emission

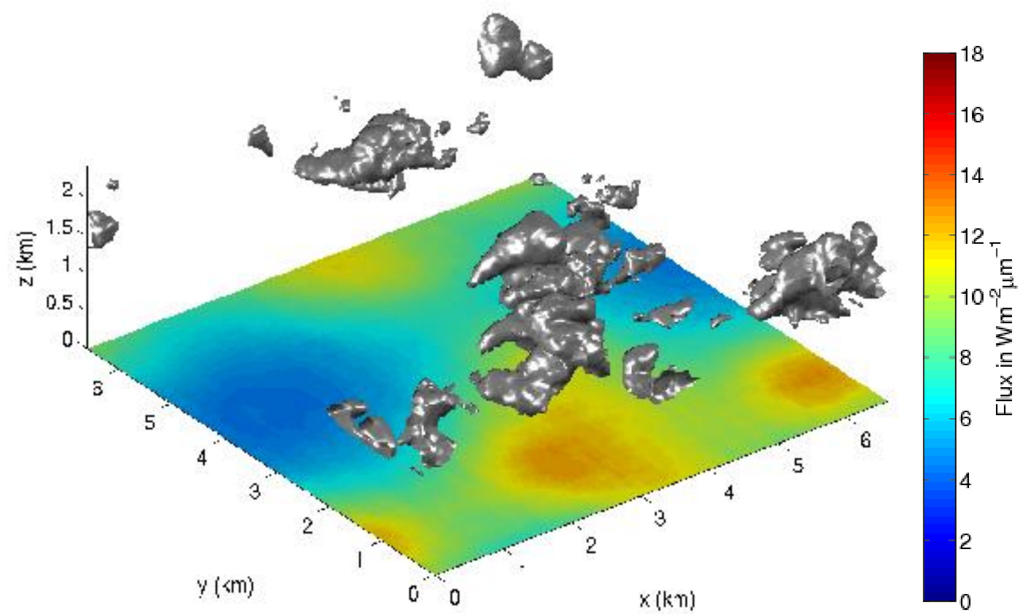


Explicit 3D radiation calculations



- Freely available Monte Carlo and SHDOM codes can compute radiance fields everywhere
- Very slow: 5D problem
- Need to approximate for GCMs

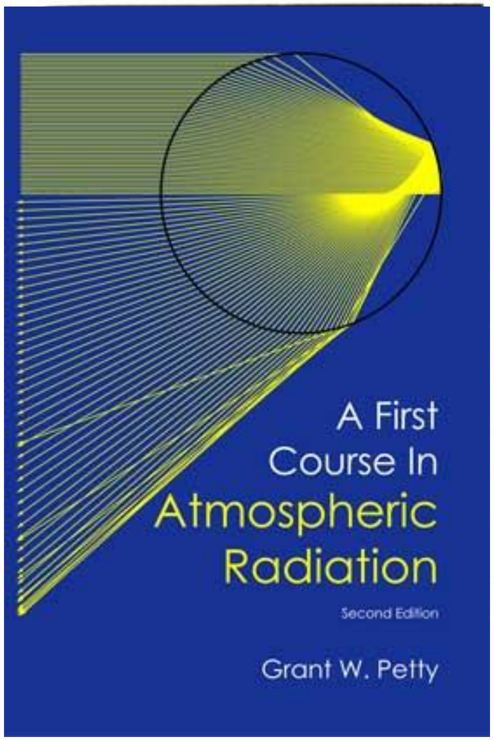




SW: Franklin Evans, University of Colorado LW: Sophia Schafer, University of Reading

Two-stream approximation

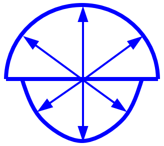
3D radiative transfer in terms of monochromatic radiances $I(\mathbf{x},\Omega,\nu)$



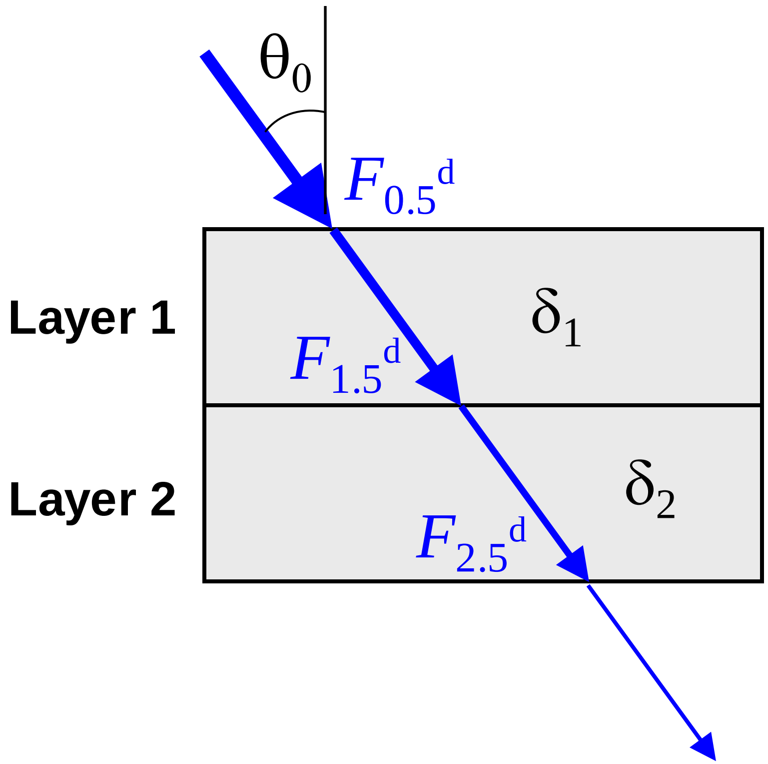
*Un*reasonable assumptions:

- Radiances in all directions represented by only 2 (or sometimes 4) discrete directions
- Atmosphere within a model gridbox is horizontally infinite and homogeneous
- Details of the phase functions represented by one number, the asymmetry factor $g = \overline{\cos\theta}$

1D radiative transfer in terms of two monochromatic fluxes $F^{\pm}(z,\nu)$ in W m^{-2} Hz^{-1}



Direct solar flux



- TOA flux:

  $F_{0.5}^{d} = S_0 \cos(\theta_0)$

- Zenith optical depth in layer i is δ_i, calculated simply as the vertical integral of extinction coefficient β_e across the layer
- Fluxes at layer interfaces are

  $F_{i+0.5}^{d} = F_{i-0.5}^{d} \exp(-\delta_i/\cos\theta_0)$

- For the moment we assume the model layers to be horizontally homogeneous and infinite: no representation of radiation transport between adjacent model columns

Two-stream equations for diffuse flux

Gradient of flux with height

Loss of flux by scattering or absorption

Gain in flux by scattering from other direction

- Upwelling flux: $$\frac{\partial F^+}{\partial z} = -\beta_e \left(\gamma_1 F^+ - \gamma_2 F^- \right) + S^+$$

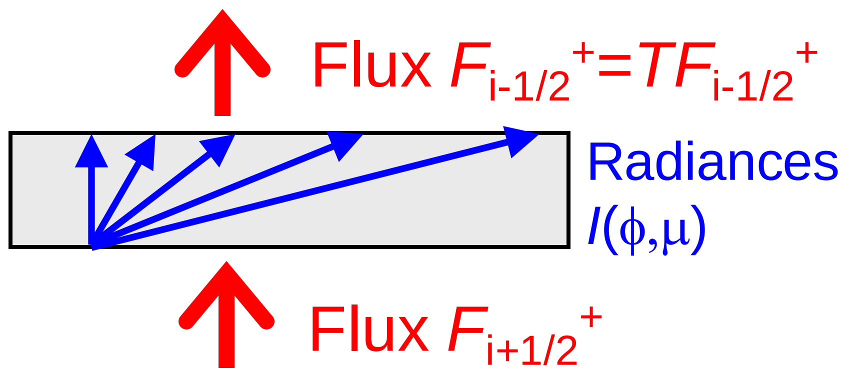
Source from scattering of the direct solar beam (shortwave) or emission (longwave)

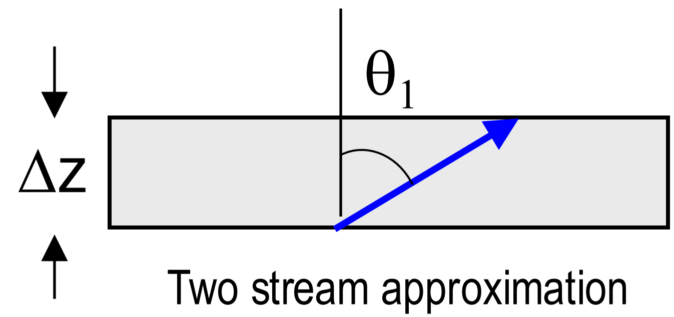
- Downwelling flux: $$-\frac{\partial F^-}{\partial z} = -\beta_e \left(\gamma_1 F^- - \gamma_2 F^+ \right) + S^-$$

- Where coefficients γ_1 and γ_2 are simple functions of asymmetry factor and single scattering albedo (after delta-Eddington scaling) and μ_1, the cosine of the effective zenith angle of diffuse radiation

Two-stream angle / diffusivity factor

- $\mu_1 = \cos(\theta_1)$ is the *effective* zenith angle that diffuse radiation travels at to get the right *transmittance T*



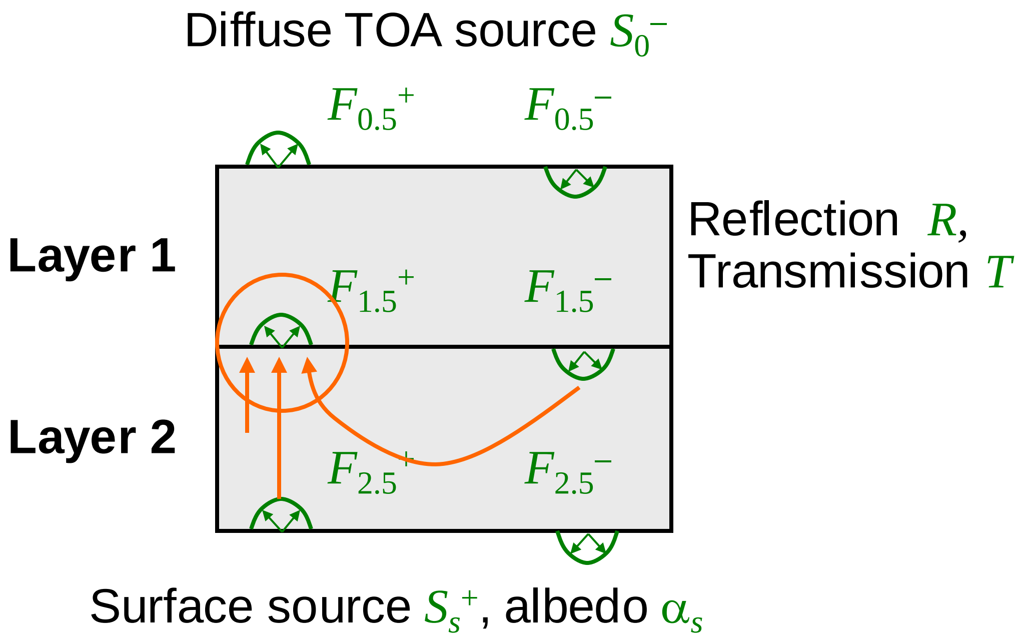


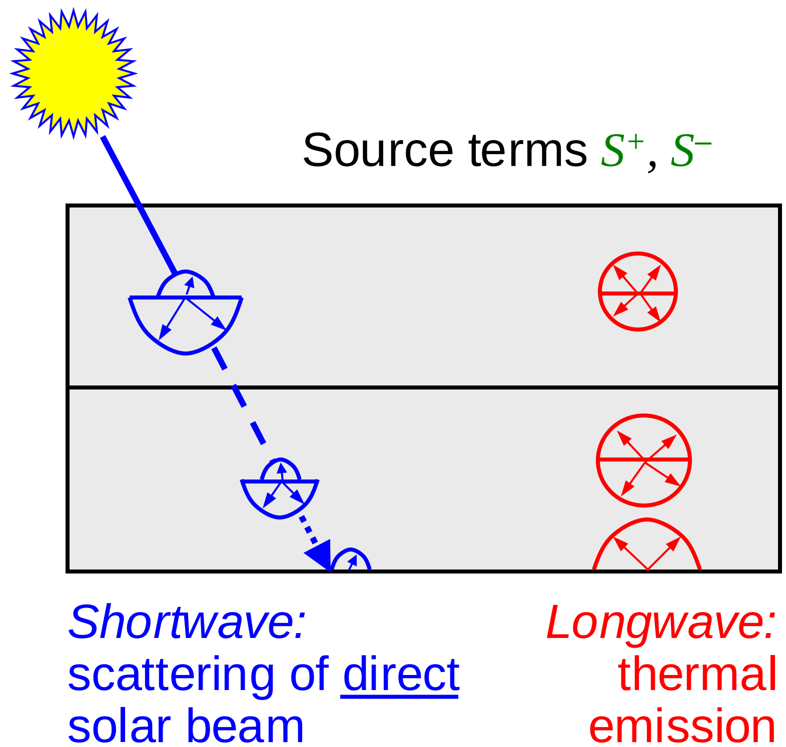
$$T = \frac{\int_0^{2\pi}\int_0^1 I(\phi,\mu)\exp(-\Delta z/\mu)\mu d\mu d\phi}{\int_0^{2\pi}\int_0^1 I(\phi,\mu)\mu d\mu d\phi}$$

$$T = \exp\left(-\frac{\Delta z}{\mu_1}\right)$$

- Most longwave schemes use Elsasser (1942) diffusivity factor of $1/\mu_1 = 1.66$, equivalent to $\theta_1 = 53°$

Discretized two-stream scheme





- Equations relating diffuse fluxes between levels take the form:

$$F_{i-0.5}^{+} = T_i F_{i+0.5}^{+} + R_i F_{i-0.5}^{-} + S_i^{+}$$

- Terms T, R and S found by solving two-stream equations for single homogeneous layers: solutions given by Meador and Weaver (1980)

Solution for two-level atmosphere

- Solve the following tri-diagonal system of equations

$$\begin{pmatrix} & 1 & & & & & \\ 1 & -R_1 & -T_1 & & & & \\ & -T_1 & -R_1 & 1 & & & \\ & & 1 & -R_2 & -T_2 & & \\ & & & -T_2 & -R_2 & 1 & \\ & & & & 1 & -\alpha_s & \end{pmatrix} \begin{pmatrix} F_{0.5}^+ \\ F_{0.5}^- \\ F_{1.5}^+ \\ F_{1.5}^- \\ F_{2.5}^+ \\ F_{2.5}^- \end{pmatrix} = \begin{pmatrix} S_0^- \\ S_1^+ \\ S_1^- \\ S_2^+ \\ S_2^- \\ S_S^+ \end{pmatrix}$$

- Efficient to solve and simple to extend to more layers
- Typical schemes also include separate regions at each height for cloud and clear-sky

Summary so far

- Radiation is the fundamental driver of the climate system
  - While a full radiative transfer model is quite complex, phenomenal such as anthropogenic climate change can be explained mathematically with very simple physical concepts
- The radiative transfer aspects of a radiation scheme can be traced back to Maxwell’s equations, including
  - Particle scattering
  - The two-stream equations

- *Next lecture: gas absorption spectra and clouds*