

# Numerical Weather Prediction

## Parameterization of diabatic processes

### Convection II: The mass flux approach and the IFS scheme

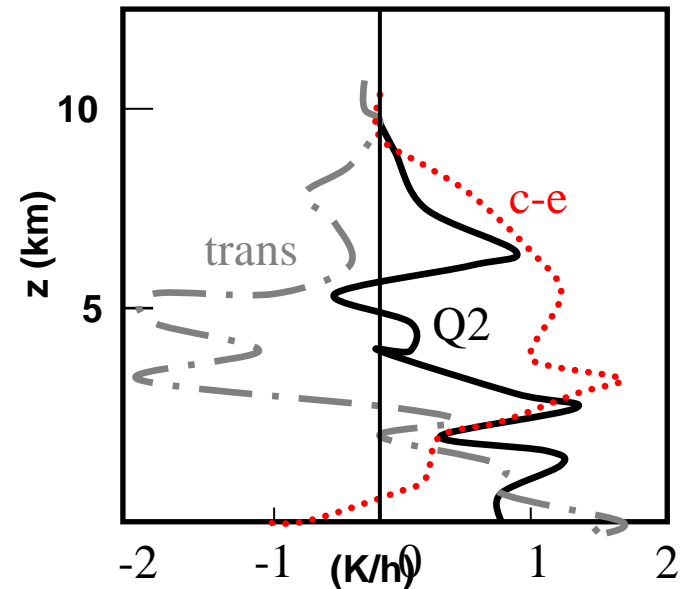
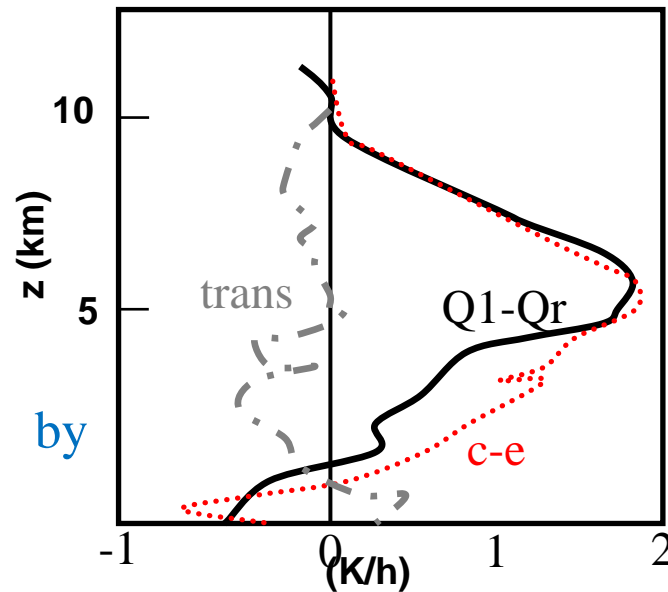


Peter Bechtold

# Task of convection parametrization: Q1 and Q2

To calculate the collective effects of an ensemble of convective clouds in a model column as a function of grid-scale variables. Hence parameterization needs to describe Condensation/Evaporation and Transport

$$Q_{1c} \equiv Q_1 - Q_R \equiv L(\bar{c} - \bar{e}) - \frac{\overline{\partial \omega' s'}}{\partial p}$$



$Q_{1c}$  is dominated by condensation term


but for  $Q_2$  the transport and condensation terms are equally important

Caniaux, Redelsperger, Lafore, JAS 1994

# Types of convection schemes

- Schemes based on **moisture budgets**
  - Kuo, 1965, 1974, *J. Atmos. Sci.*
- **Adjustment** schemes
  - moist convective adjustment, Manabe, 1965, *Mon. Wea. Rev.*
  - penetrative adjustment scheme, Betts and Miller, 1986, *Quart. J. Roy. Met. Soc.*, Betts-Miller-Janic
- **Mass-flux** schemes (bulk+spectral)
  - entraining plume - spectral model, Arakawa and Schubert, 1974, Fraedrich (1973,1976), Neggers et al (2002), Cheinet (2004), *all J. Atmos. Sci.* ,
  - Entraining/detraining plume - bulk model, e.g., Bougeault, 1985, *Mon. Wea. Rev.*, Tiedtke, 1989, *Mon. Wea. Rev.*; Gregory and Rowntree, 1990, *Mon. Wea. Rev.*; Kain and Fritsch, 1990, *J. Atmos. Sci.*, Donner , 1993 *J. Atmos. Sci.*; Bechtold et al 2001, *Quart. J. Roy. Met. Soc.*; Park, 2014. *J. Atmos. Sci.*
  - episodic mixing, Emanuel, 1991, *J. Atmos. Sci.*

# The mass-flux approach

$$Q_{1c} \equiv L(\bar{c} - \bar{e}) - \frac{\overline{\partial \omega' s'}}{\partial p}$$


Condensation term

Eddy transport term

**Aim:** Look for a simple expression of the eddy transport term

$$\overline{\omega' \Phi'} = ?$$

# The mass-flux approach

Reminder:

$$\Phi = \bar{\Phi} + \Phi' \quad \text{with} \quad \overline{\Phi'} = 0$$

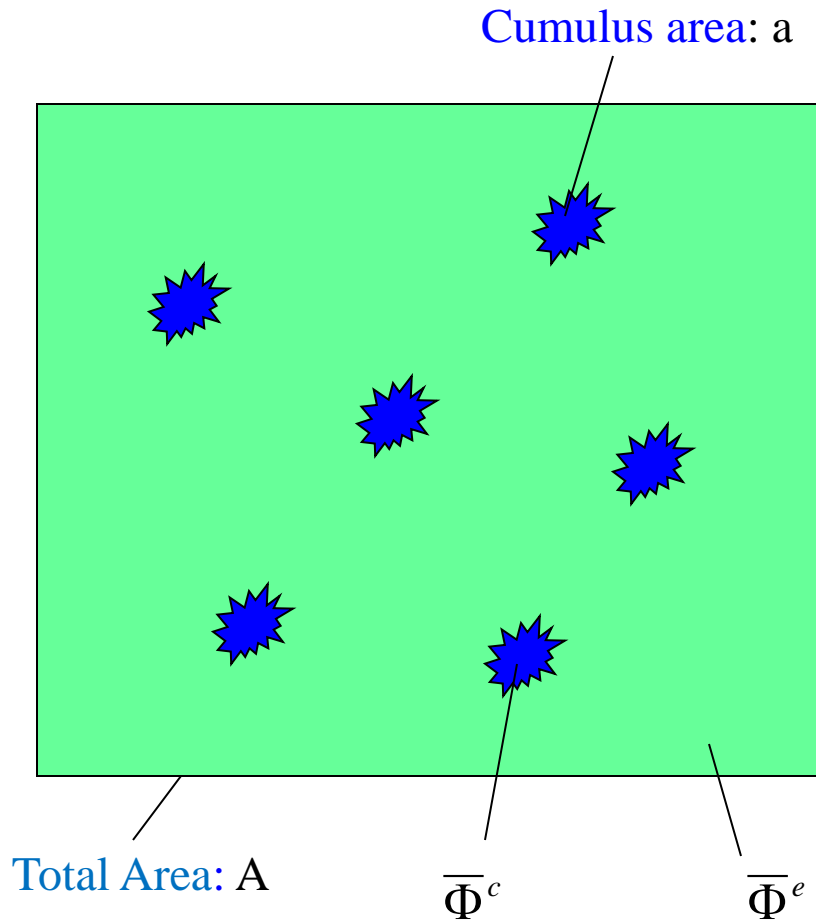
Hence

$$\begin{aligned} \overline{\omega\Phi} &= \overline{(\bar{\omega} + \omega')(\bar{\Phi} + \Phi')} \\ &= \overline{\bar{\omega}\bar{\Phi}} + \underbrace{\overline{\bar{\omega}\Phi'}}_{0} + \underbrace{\overline{\omega'\bar{\Phi}}}_{0} + \overline{\omega'\Phi'} \end{aligned}$$

and therefore

$$\begin{aligned} \overline{\omega\Phi} &= \bar{\omega}\bar{\Phi} + \overline{\omega'\Phi'} \\ \overline{\omega'\Phi'} &= \overline{\omega\Phi} - \bar{\omega}\bar{\Phi} \end{aligned}$$

# The mass-flux approach: Cloud – Environment decomposition



Fractional coverage with  
cumulus elements:

$$\sigma = \frac{a}{A}$$

Define area average:

$$\overline{\Phi} = \sigma \overline{\Phi}^c + (1 - \sigma) \overline{\Phi}^e$$

# The mass-flux approach: Cloud-Environment decomposition

With the above:

$$\overline{\omega\Phi} = \sigma \overline{\omega\Phi}^c + (1 - \sigma) \overline{\omega\Phi}^e$$

Average over cumulus elements

Average over environment

$$\overline{\omega\Phi}^c = \bar{\omega}^c \bar{\Phi}^c + \cancel{\overline{\omega''\Phi''}^c} \quad \text{and} \quad \overline{\omega\Phi}^e = \bar{\omega}^e \bar{\Phi}^e + \cancel{\overline{\omega''\Phi''}^e}$$

Neglect subplume variations : (1) The top hat assumption



(see also Siebesma and Cuijpers, JAS 1995 for a discussion of the validity of the top-hat assumption)

# The mass-flux approach:

$$\overline{\omega'\Phi'} = \overline{\omega\Phi} - \bar{\omega}\bar{\Phi} = \sigma\overline{\omega\Phi}^c + (1-\sigma)\overline{\omega\Phi}^e - \bar{\omega}\bar{\Phi}$$

Use Reynolds averaging again for cumulus elements and environment separately:

$$= \sigma\overline{\omega\Phi}^c + (1-\sigma)\overline{\omega\Phi}^e - (\sigma\bar{\omega}^c + (1-\sigma)\bar{\omega}^e)\bar{\Phi}$$

(1) top hat approximation

$$= \sigma\bar{\omega}^c\bar{\Phi}^c + (1-\sigma)\bar{\omega}^e\bar{\Phi}^e - (\sigma\bar{\omega}^c + (1-\sigma)\bar{\omega}^e)\bar{\Phi}$$

$$= \sigma\bar{\omega}^c(\bar{\Phi}^c - \bar{\Phi}) + (1-\sigma)\bar{\omega}^e(\bar{\Phi}^e - \bar{\Phi})$$

Either drop this term (small area approximation) or

further expanding, for your exercise!



# The mass-flux approach:

$$\begin{aligned}\overline{\omega'\Phi'} &= \overline{\omega\Phi} - \bar{\omega}\bar{\Phi} \\ &= \sigma(1-\sigma)(\bar{\omega}^c - \bar{\omega}^e)(\bar{\Phi}^c - \bar{\Phi}^e) \\ &= \sigma(\bar{\omega}^c - \bar{\omega}^e)(\bar{\Phi}^c - \bar{\Phi}) = \underbrace{\sigma\bar{\omega}^c \left(1 - \frac{\bar{\omega}^e}{\bar{\omega}^c}\right)}_{f(dx)} (\bar{\Phi}^c - \bar{\Phi})\end{aligned}$$

see Malardel&Bechtold,  
2019 QJRMS

## (2) The small area approximation

$$\sigma \ll 1 \Rightarrow (1 - \sigma) \approx 1; \quad \bar{\omega}^c \gg \bar{\omega}^e$$

$$\overline{\omega'\Phi'} = \sigma\bar{\omega}^c (\bar{\Phi}^c - \bar{\Phi})$$

$$-\overline{\omega'\Phi'} = gM_c (\bar{\Phi}^c - \bar{\Phi})$$

$$M_c = \frac{-\sigma\bar{\omega}^c}{g} = \rho\sigma\bar{w}^c$$

# The mass-flux approach

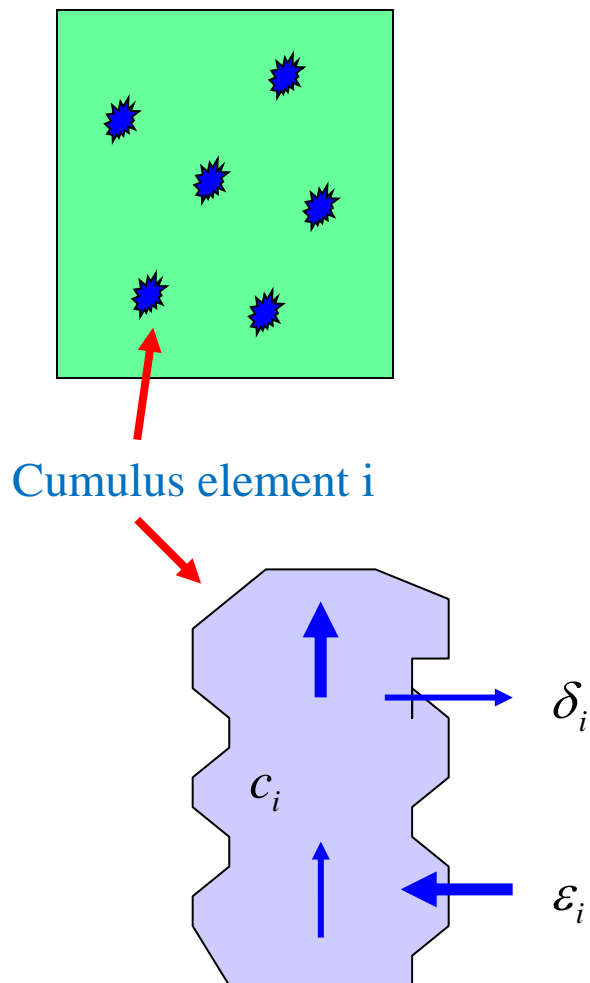
With the above we can rewrite:

$$Q_{1c} \equiv L(\bar{c} - \bar{e}) + g \frac{\partial [M_c (\bar{s}^c - \bar{s})]}{\partial p}$$

$$Q_2 \equiv L(\bar{c} - \bar{e}) - Lg \frac{\partial [M_c (\bar{q}^c - \bar{q})]}{\partial p}$$

To predict the **influence of convection** on the large-scale we now need to describe **the convective mass-flux, the values (s, q, u, v) inside the convective elements** and **the condensation/evaporation term**. This requires, as usual, a **cloud model** and a **closure** to determine the absolute (scaled) value of the mass flux.

# Mass-flux entraining plume models



## Entraining plume model

Continuity:

$$\frac{\partial \sigma_i}{\partial t} + D_i - E_i - g \frac{\partial M_i}{\partial p} = 0$$

Heat: see Gregory & Miller 1989

$$\frac{\partial (\sigma_i s_i)}{\partial t} + D_i s_i - E_i \bar{s} - g \frac{\partial (M_i s_i)}{\partial p} = L c_i$$

Specific humidity:

$$\frac{\partial (\sigma_i q_i)}{\partial t} + D_i q_i - E_i \bar{q} - g \frac{\partial (M_i q_i)}{\partial p} = -c_i$$

# Mass-flux entraining plume models

## Simplifications

1. **Steady state plumes**, i.e.,  $\frac{\partial X}{\partial t} = 0$

most mass-flux convection parametrizations make that assumption, some (e.g. Gerard&Geleyn) are prognostic

2. Instead of spectral (Arakawa Schubert 1974) use **one representative updraught=bulk scheme** with entrainment/detrainment written as

$$\frac{1}{M} \frac{dM}{dz} = \varepsilon - \delta \Rightarrow -g \frac{\partial M_c}{\partial p} = E - D$$

$\varepsilon, \delta$  [ $\text{m}^{-1}$ ] denote fractional entrainment/detrainment,  
 $E, D$  [ $\text{s}^{-1}$ ] entrainment/detrainment rates

# Large-scale cumulus effects deduced from mass-flux models

$$-g \frac{\partial M_c}{\partial p} = E - D$$

$$-g \frac{\partial (M_c \bar{s}^c)}{\partial p} = E\bar{s} - D\bar{s}^c + Lc$$

$$Q_{1c} \equiv L(c - e) + g \frac{\partial [M_c (\bar{s}^c - \bar{s})]}{\partial p}$$

Flux form

Combine:

$$Q_{1c} \equiv -gM_c \frac{\partial \bar{s}}{\partial p} + D(\bar{s}^c - \bar{s}) - Le$$

Advective form

# Large-scale cumulus effects deduced using mass-flux models: Interpretation

$$Q_{1c} \equiv -gM_c \frac{\partial \bar{s}}{\partial p} + D(\bar{s}^c - \bar{s}) - Le$$

Convection affects the large scales by

Heating through **compensating subsidence** between cumulus elements (term 1)

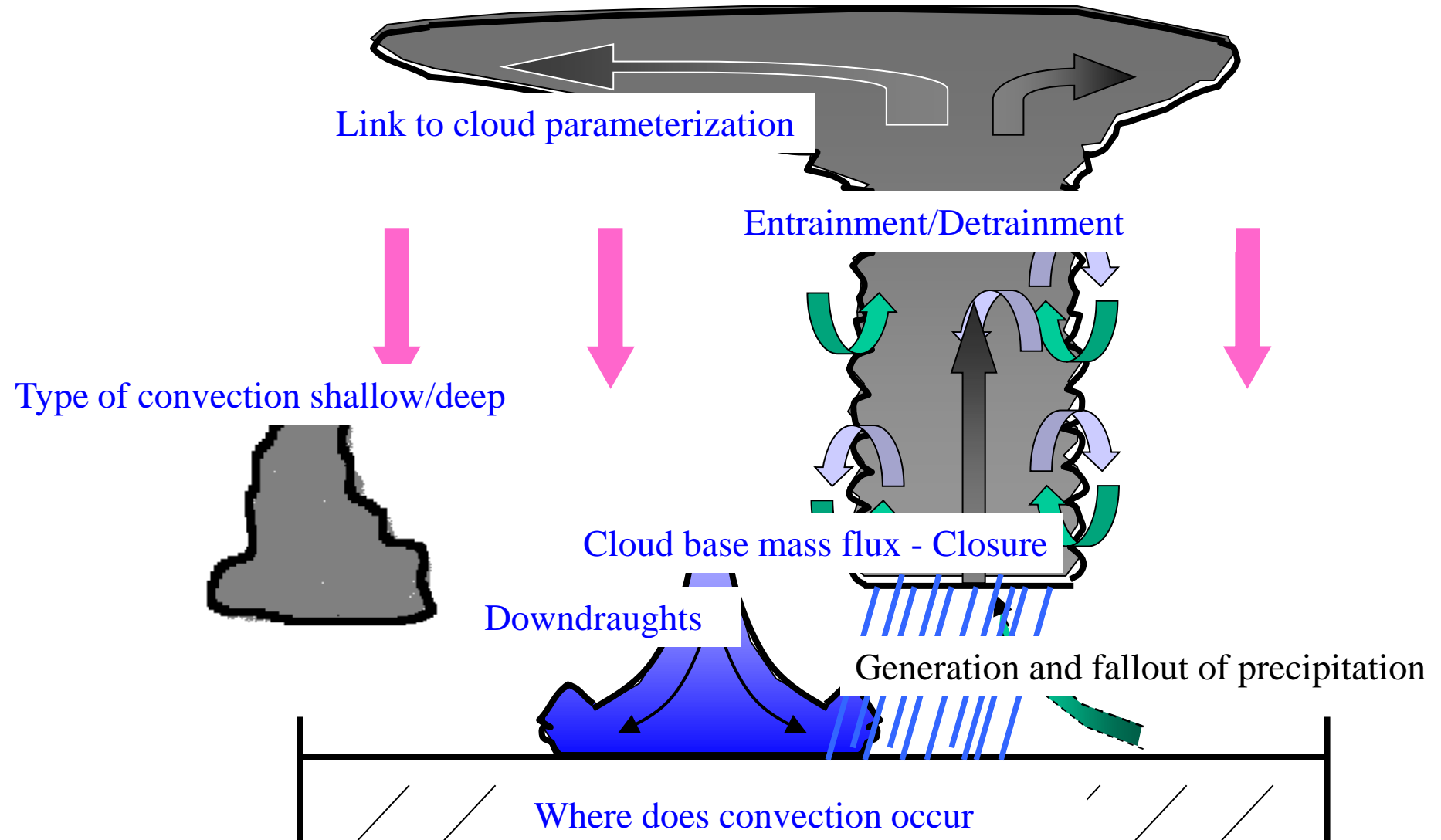
The **detrainment of cloud air** into the environment (term 2)

**Evaporation** of cloud and precipitation (term 3)

**Note:** In the **advective form** the **condensation heating** does **not** appear **directly** in  $Q_1$ . It is however the dominant term using the **flux form** and is a **crucial part of the cloud model**, where this heat is transformed in kinetic energy of the updrafts.

# The IFS bulk mass flux scheme

## *What needs to be considered*



# Basic Features

- Bulk mass-flux scheme
- Entraining/detraining plume cloud model
- 3 types of convection: deep, shallow and mid-level - mutually exclusive
- saturated downdraughts
- simple microphysics scheme
- closure dependent on type of convection
  - deep: CAPE adjustment
  - shallow: PBL equilibrium
- strong link to cloud parameterization - convection provides source for cloud condensate

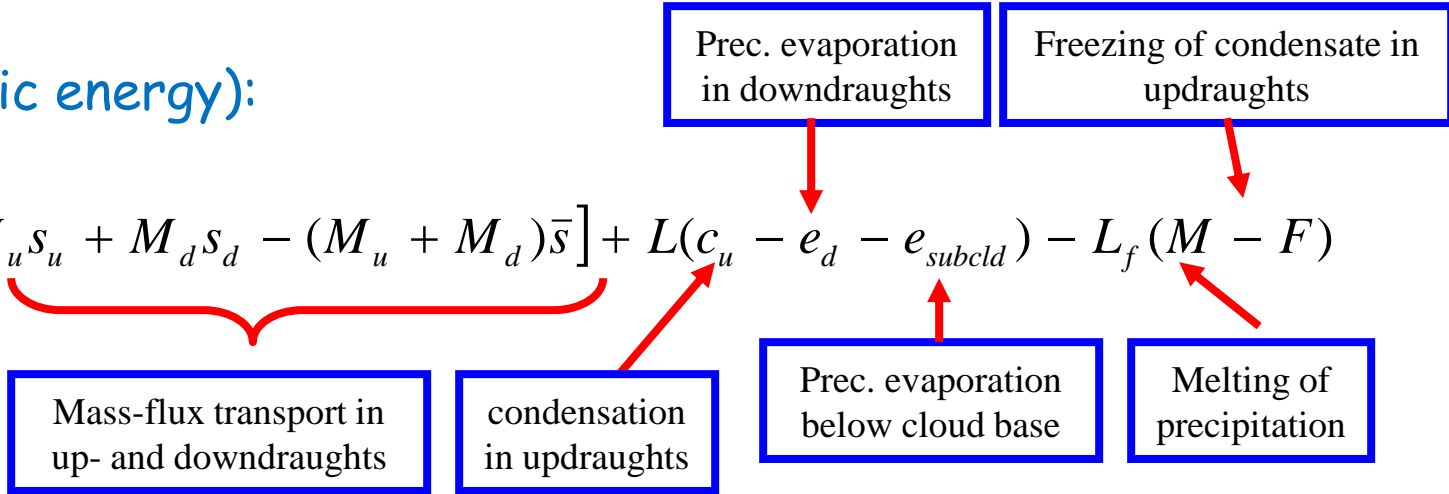


# Large-scale budget equations: Heat & moisture

$$M = \rho w; \quad M_u > 0; \quad M_d < 0$$

Heat (dry static energy):

$$\left( \frac{\partial s}{\partial t} \right)_{cu} = g \frac{\partial}{\partial p} \left[ M_u s_u + M_d s_d - (M_u + M_d) \bar{s} \right] + L(c_u - e_d - e_{subcl}) - L_f(M - F)$$



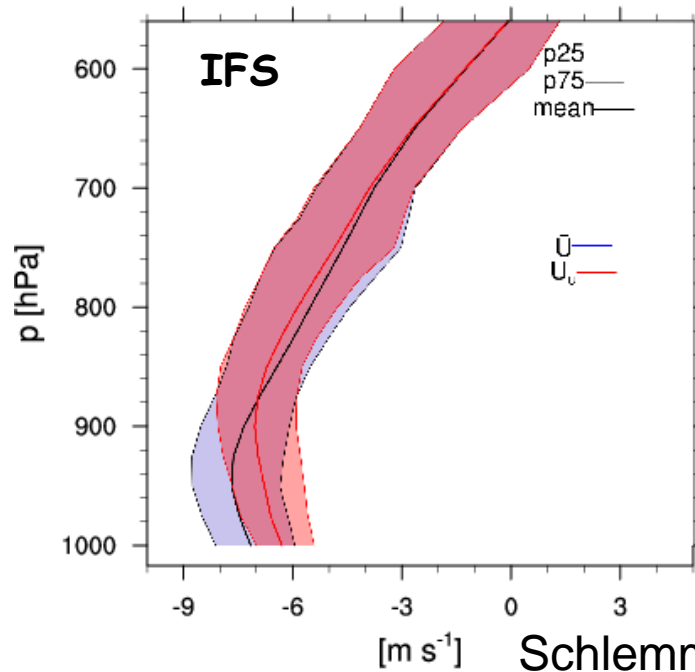
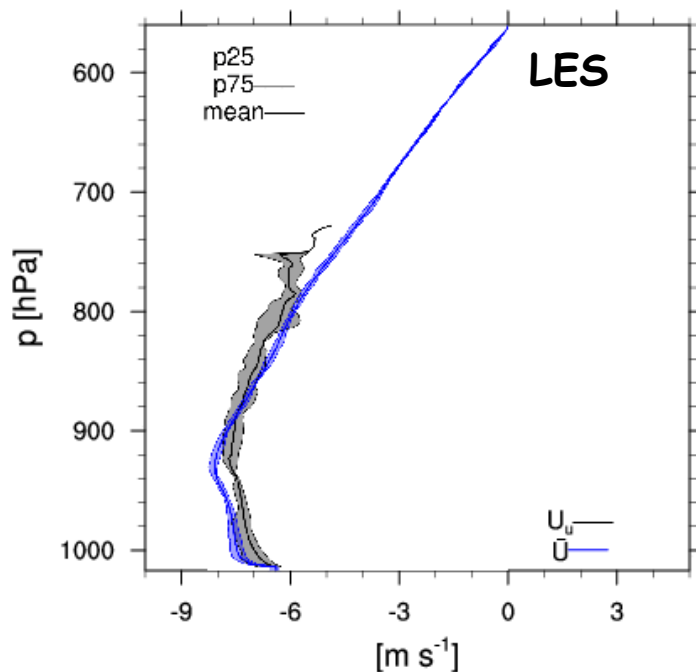
Humidity: 
$$\left( \frac{\partial q}{\partial t} \right)_{cu} = g \frac{\partial}{\partial p} \left[ M_u q_u + M_d q_d - (M_u + M_d) \bar{q} \right] - (c_u - e_d - e_{subcl})$$

Detained Condensate: 
$$\left( \frac{\partial l}{\partial t} \right)_{cu} = -g \frac{\partial}{\partial p} \left[ (M^u + M^d) \bar{l} \right] + D_u l_u$$

# Large-scale budget equations: Momentum

$$\left(\frac{\partial u}{\partial t}\right)_{cu} = g \frac{\partial}{\partial p} \left[ M_u u_u + M_d u_d - (M_u + M_d) \bar{u} \right]$$

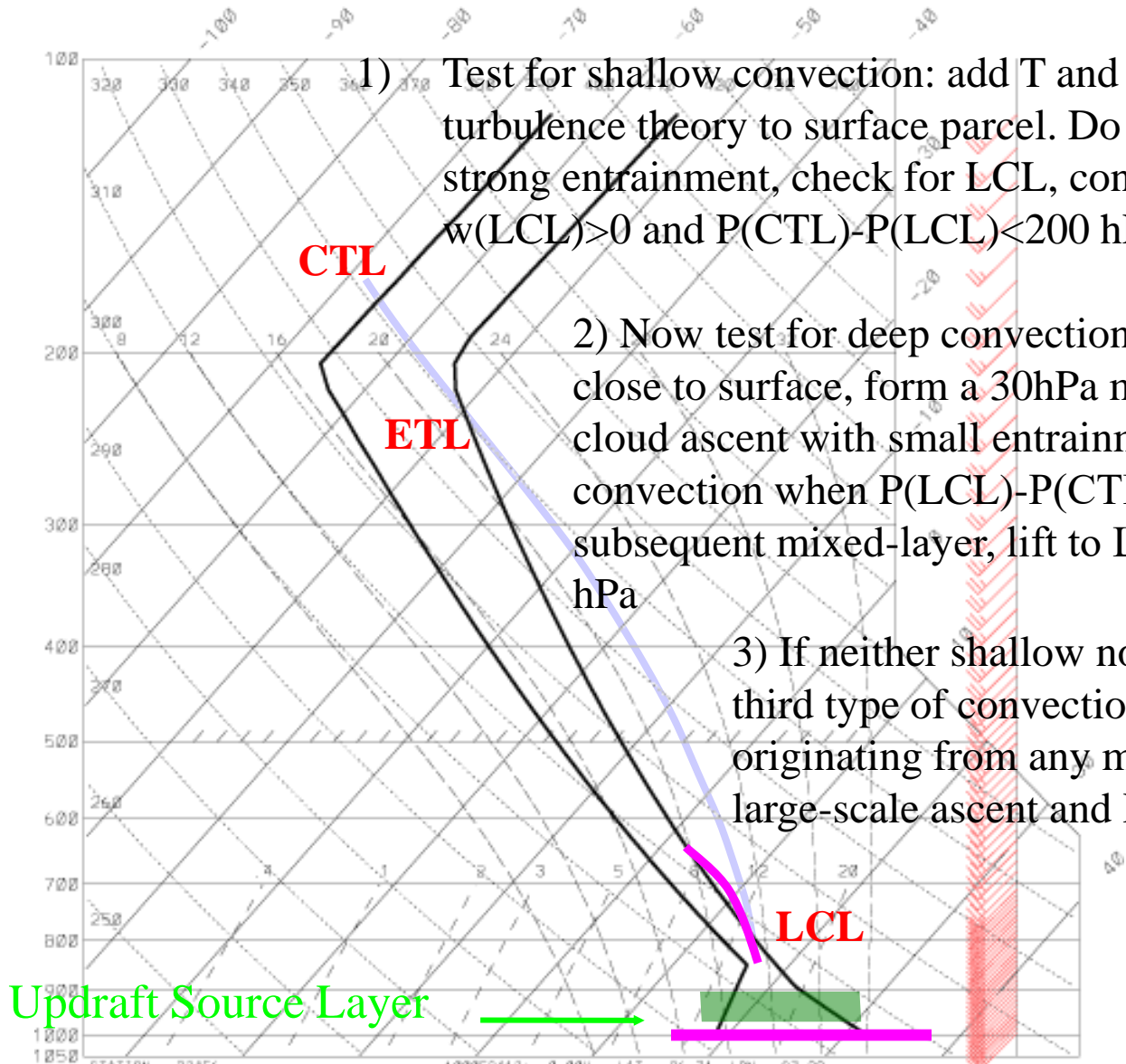
$$\left(\frac{\partial v}{\partial t}\right)_{cu} = g \frac{\partial}{\partial p} \left[ M_u v_u + M_d v_d - (M_u + M_d) \bar{v} \right]$$



Schlemmer et al. 2017

Shallow cumulus: convective momentum transport reduces on average the shear

# Occurrence of convection: *make a first-guess parcel ascent*



1) Test for shallow convection: add T and q perturbation based on turbulence theory to surface parcel. Do ascent with w-equation and strong entrainment, check for LCL, continue ascent until  $w < 0$ . If  $w(LCL) > 0$  and  $P(CTL) - P(LCL) < 200$  hPa : shallow convection

2) Now test for deep convection with similar procedure. Start close to surface, form a 30hPa mixed-layer, lift to LCL, do cloud ascent with small entrainment+water fallout. Deep convection when  $P(LCL) - P(CTL) > 200$  hPa. If not .... test subsequent mixed-layer, lift to LCL etc. ... and so on until 300 hPa

3) If neither shallow nor deep convection is found a third type of convection – “midlevel” – is activated, originating from any model level below 10 km if large-scale ascent and  $RH > 80\%$ .

Updraft Source Layer

# Cloud model equations – updraughts

E and D are positive by definition

Mass (Continuity)

$$-g \frac{\partial M_u}{\partial p} = E_u - D_u$$

Heat

$$-g \frac{\partial M_u s_u}{\partial p} = E_u \bar{s} - D_u s_u + L c_u$$

Humidity

$$-g \frac{\partial M_u q_u}{\partial p} = E_u \bar{q} - D_u q_u - c_u$$

Liquid+Ice

$$-g \frac{\partial M_u l_u}{\partial p} = -D_u l_u + c_u - G_{P,u}$$

Precip

$$-g \frac{\partial M_u r_u}{\partial p} = -D_u r_u + G_{P,u} - Sf_{out}$$

Momentum

$$-g \frac{\partial M_u u_u}{\partial p} = E_u \bar{u} - D_u u_u$$

$$-g \frac{\partial M_u v_u}{\partial p} = E_u \bar{v} - D_u v_u$$

Kinetic Energy (vertical velocity) - use height coordinates

$$\frac{\partial K_u}{\partial z} = -\frac{E_u}{M_u} (1 + \beta C_d) 2K_u + \frac{1}{f(1 + \gamma)} g \frac{T_{v,u} - \bar{T}_v}{\bar{T}_v}, \quad K_u = \frac{w_u^2}{2}$$

# Downdraughts

1. Find level of free sinking (LFS)

highest model level for which an equal saturated mixture of cloud and environmental air becomes negatively buoyant

2. Closure  $M_{d,LFS} = -\alpha M_{u,b} \quad \alpha = 0.3$

# Cloud model equations – downdraughts

**E and D are defined positive**

$$g \frac{\partial M_d}{\partial p} = E_d - D_d$$

Mass

$$g \frac{\partial M_d s_d}{\partial p} = E_d \bar{s} - D_d s_d + L e_d$$

Heat

$$g \frac{\partial M_d q_d}{\partial p} = E_d \bar{q} - D_d q_d + e_d$$

Humidity

$$g \frac{\partial M_d u_d}{\partial p} = E_d \bar{u} - D_d u_d$$

$$g \frac{\partial M_d v_d}{\partial p} = E_d \bar{v} - D_d v_d$$

Momentum

# Entrainment/Detrainment (1)

$$-g \frac{\partial M_u}{\partial p} = E_u - D_u = \frac{M_u}{\rho} (\varepsilon - \delta) = \frac{M_u}{\rho} (\varepsilon_{turb} - \delta_{turb} - \delta_{org})$$

$\varepsilon$  and  $\delta$  are generally given in units ( $m^{-1}$ )

$$\varepsilon = c_1 \underbrace{(1.3 - RH)}_{buoy > 0} F_\varepsilon; \quad RH = \frac{\bar{q}}{\bar{q}_s}; \quad \delta_{turb} = c_2$$

$$c_1 = 1.75 \times 10^{-3} m^{-1}; c_2 = 0.75 \times 10^{-4} m^{-1};$$

$$F_\varepsilon = \left( \frac{\bar{q}_s}{\bar{q}_{sbase}} \right)^3$$

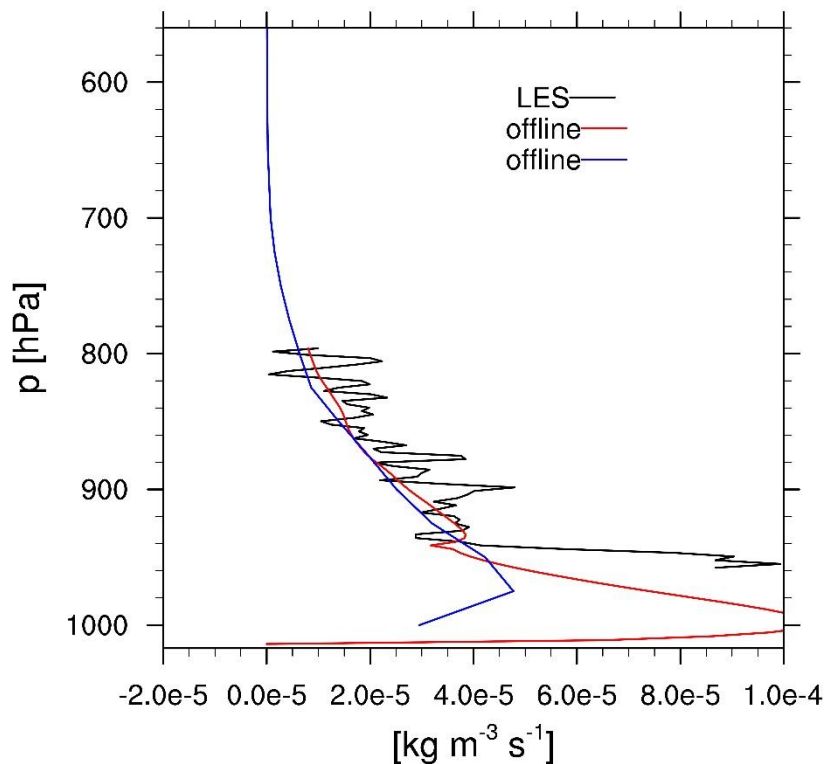
Constants

Scaling function to mimick a cloud ensemble

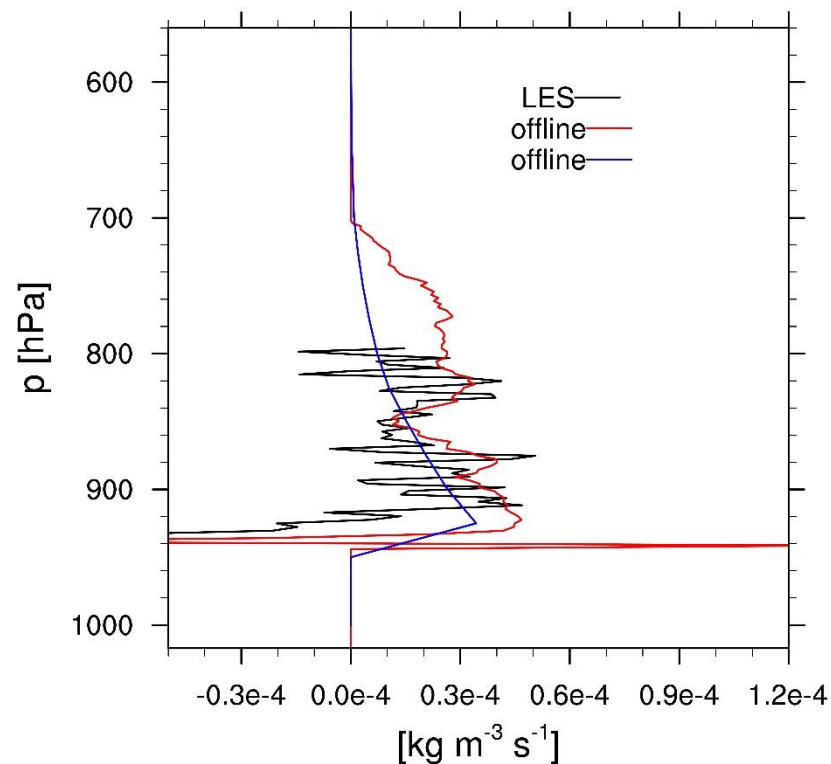
# Entrainment/Detrainment

Entrainment formulation sooo simple: so how does it compare to LES ?

RICO, Entrainment



RICO, Detrainment



LES (black)

IFS

IFS formula with LES data

Schlemmer et al. 2017

Nota: entrainment for shallow typically factor 2 larger than that for deep



# Entrainment/Detrainment (3)

## Organised Detrainment:

When updraught kinetic energy  $K$  decreases with height (negative buoyancy), compute mass flux at level  $z+\Delta z$  with following relation:

$$\frac{M_u(z)}{M_u(z+\Delta z)} \approx (1.6 - RH) \sqrt{\frac{K_u(z)}{K_u(z+\Delta z)}}; \quad \Rightarrow D_u = \frac{\Delta M_u}{\bar{\rho} \Delta z}$$

with 
$$K_u = \frac{w_u^2}{2}$$

# Precipitation

Liquid+solid precipitation fluxes:

$$P^{rain}(p) = \int_{P_{top}}^P (G^{rain} - e_{down}^{rain} - e_{subcl}^{rain} + Melt) dp / g$$

$$P^{snow}(p) = \int_{P_{top}}^P (G^{snow} - e_{down}^{snow} - e_{subcl}^{snow} - Melt) dp / g$$

Where  $P^{rain}$  and  $P^{snow}$  are the fluxes of precip in form of rain and snow at pressure level  $p$ .  $G^{rain}$  and  $G^{snow}$  are the conversion rates from cloud water into rain and cloud ice into snow. Evaporation occurs in the downdraughts  $e_{down}$ , and below cloud base  $e_{subcl}$ ,  $Melt$  denotes melting of snow.

Generation of precipitation in updraughts

$$\rho G_{P,u} = M_u \frac{c_0}{W_u} l_u \left[ 1 - e^{-\left(\frac{l_u}{l_{crit}}\right)^2} \right]$$

Simple representation of Bergeron process included in  $c_0$  and  $l_{crit}$

# Precipitation

## Fallout of precipitation from updraughts

$$\rho S_{fallout} = M_u \frac{V_{prec}}{w_u \Delta z} r_u$$

$$V_{prec,rain} = 5.32 r_u^{0.2} \quad V_{prec,ice} = 2.66 r_u^{0.2}$$

## Evaporation of precipitation

1. Precipitation evaporates to keep downdraughts saturated
2. Precipitation evaporates below cloud base

$$e_{subcl} = \sigma \alpha_1 (RH q_s - \bar{q}) \left( \frac{\sqrt{p/p_{surf}} \bar{P}}{\alpha_2 \sigma} \right)^{\alpha_3}, \quad \text{assume a cloud fraction } \sigma = 0.05$$



# Closure - Deep convection

$$CAPE = g \int_{cloud} \frac{T_{v,u} - \bar{T}_v}{\bar{T}_v} dz \approx g \int_{cloud} \frac{\theta_{e,u} - \bar{\theta}_{esat}}{\bar{\theta}_{esat}} dz$$

Use instead density scaling, time derivative then relates to mass flux:

$$PCAPE = - \int_{Pbase}^{Ptop} \frac{T_{v,u} - \bar{T}_v}{\bar{T}_v} dp$$

$$\begin{aligned} \frac{\partial PCAPE}{\partial t} &\approx - \underbrace{\int_{Pbase}^{Ptop} \frac{1}{\bar{T}_v} \frac{\partial \bar{T}_v}{\partial t} dp}_{LS+Cu} - \underbrace{\int_{Pbase}^{Ptop} \frac{1}{\bar{T}_v} \frac{\partial T_{v,u}}{\partial t} dp + \frac{T_{v,u} - \bar{T}_v}{\bar{T}_v} \bigg|_{base} \frac{\partial p_{base}}{\partial t}}_{BL+Cu} = \\ &= \frac{\partial PCAPE}{\partial t} \bigg|_{LS} + \frac{\partial PCAPE}{\partial t} \bigg|_{BL} + \frac{\partial PCAPE}{\partial t} \bigg|_{Cu=shal+deep} \end{aligned}$$

this is a prognostic CAPE closure: now try to determine the different terms and try to achieve balance

$$\frac{\partial PCAPE}{\partial t} \square \frac{\partial PCAPE}{\partial t} \bigg|_{cu}, \frac{\partial PCAPE}{\partial t} \bigg|_{LS}$$

# Closure - Deep convection

$$\overline{\tau} L \int \frac{dq}{dt} dp = \text{Advection}$$

1

$$\left. \frac{\partial \text{PCAPE}}{\partial t} \right|_{cu,1} = - \frac{\text{PCAPE} - \text{PCAPE}_{BL}}{\tau}; \quad \tau = \frac{H}{\overline{w}_u}$$

2

$$\begin{aligned} \left. \frac{\partial \text{PCAPE}}{\partial t} \right|_{cu,2} &= \int_{P_{base}}^{P_{top}} \frac{1}{\overline{T}_v} \left. \frac{\partial \overline{T}_v}{\partial t} \right|_{cu} dp = - \int_{z_{base}}^{z_{top}} \frac{g}{\overline{T}_v} M \left( \frac{\partial \overline{T}_v}{\partial z} + \frac{g}{c_p} \right) dz \\ &= - \frac{M_{u,b}}{M_{u,b}^*} \int_{z_{base}}^{z_{top}} \frac{g}{\overline{T}_v} M^* \left( \frac{\partial \overline{T}_v}{\partial z} + \frac{g}{c_p} \right) dz \end{aligned}$$

Nota: all the trick is in the  $\text{PCAPE}_{BL}$  term = PCAPE not available to deep convection but used for boundary-layer mixing (see Bechtold et al. 2014).

If  $\text{PCAPE}_{BL}=0$  then wrong diurnal cycle over land!

# Closure - Deep convection

Solve now for the cloud base mass flux by equating 1 and 2

$$M_{u,b} = M_{u,b}^* \frac{PCAPE - PCAPE_{BL} + L \int \frac{dq}{dt} dp}{\tau - \int_{cloud} M^* \frac{g}{\bar{T}_v} \frac{\partial \bar{T}_v}{\partial z} dz}; \quad M_{u,b} \geq 0$$

$$PCAPE_{BL} = -\tau_{BL} \frac{1}{T^*} \int_{psurf}^{pbase} \frac{\partial \bar{T}_v}{\partial t} \Big|_{BL} dp$$

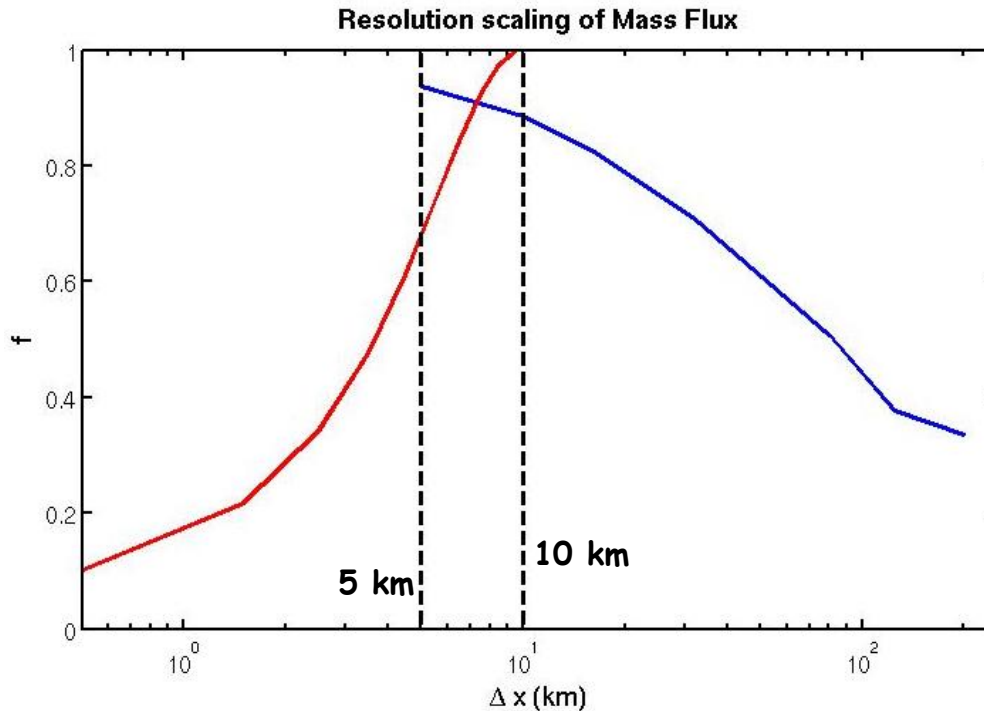
- $M^* = M_u + M_d$  Mass flux from the updraught/downdraught computation
- $M_{u,b}^*$  initial updraught mass flux at base, set proportional to  $0.1\Delta p$
- $PCAPE_{bl}$  contains the boundary-layer tendencies due to surface heat fluxes, radiation and advection

# Resolution scaling

$$\overline{\omega'\Phi'} = \overline{\omega\Phi} - \overline{\omega}\overline{\Phi}$$

$$= \underbrace{\sigma\overline{\omega}^c \left( 1 - \frac{\overline{\omega}^e}{\overline{\omega}^c} \right)}_{f(dx)} (\overline{\Phi}^c - \overline{\Phi})$$

Developed in collaboration with Deutsche Wetterdienst and ICON model (see also Malardel and Bechtold 2019, QJRMS)



Kwon and Hong, 2016 MWR independently developed very similar relations



# Closure - Shallow convection

Based on PBL equilibrium : what goes in must go out - including downdraughts

$$\int_{p_{surf}}^{p_{base}} \frac{\partial \bar{h}}{\partial t} dp = 0$$

$$\int_0^{cbase} \left[ g \frac{\partial (\overline{w'h'})}{\partial p} \Big|_{cu} + \left( \frac{\partial \bar{h}}{\partial t} \right)_{turb} + \left( \frac{\partial \bar{h}}{\partial t} \right)_{dyn} + \left( \frac{\partial \bar{h}}{\partial t} \right)_{rad} \right] dp = 0$$

$$\bar{\rho} (\overline{w'h'})_{cu,b} = M_{u,b} (h_u - \varepsilon h_d - (1 - \varepsilon) \bar{h})_{base} ; \quad \varepsilon = M_u / M_d ;$$

Assume 0 convective flux at surface, then it follows for cloud base flux

$$M_{u,b} = \frac{-\frac{1}{g} \int_{p_{surf}}^{p_{base}} \left[ \left( \frac{\partial \bar{h}}{\partial t} \right)_{turb} + \left( \frac{\partial \bar{h}}{\partial t} \right)_{dyn} + \left( \frac{\partial \bar{h}}{\partial t} \right)_{rad} \right] dp}{(h_u - \varepsilon h_d - (1 - \varepsilon) \bar{h})_{base}}$$

# Closure - Midlevel convection

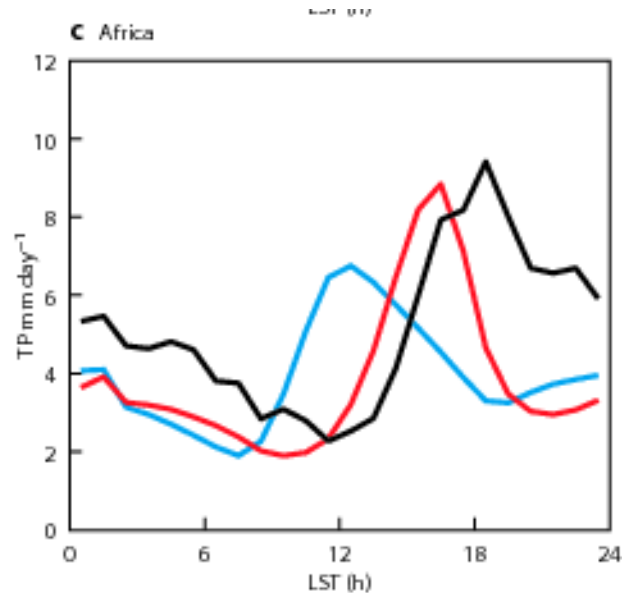
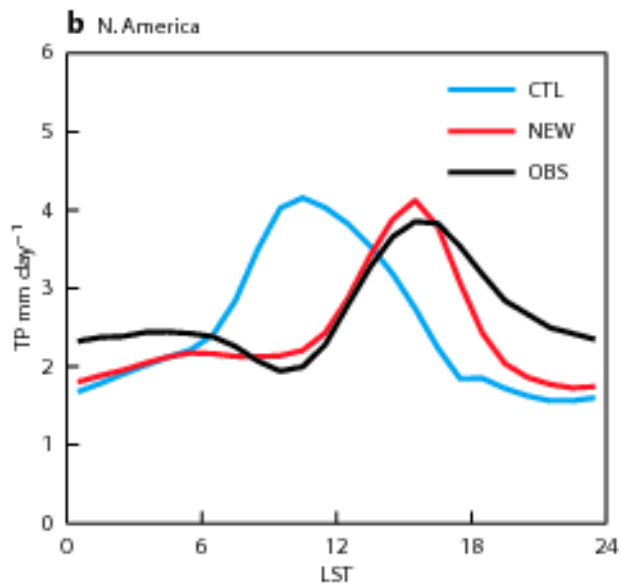
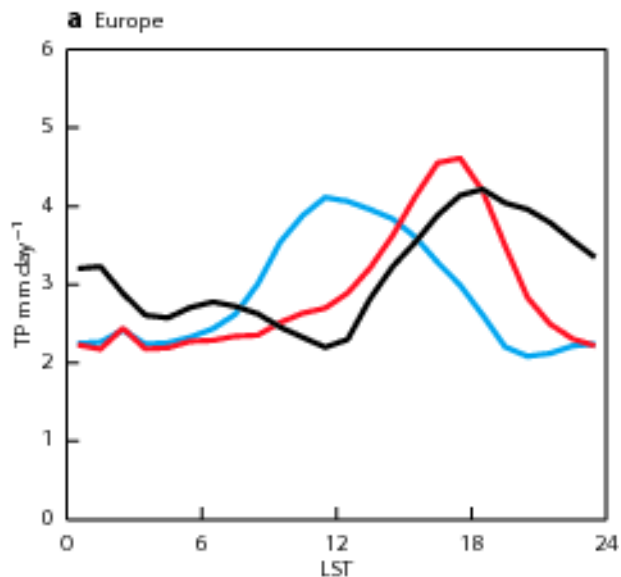
Roots of clouds originate outside PBL

assume midlevel convection exists if there is large-scale ascent,  
RH>80% and there is a convectively unstable layer

Closure:

$$M_{u,b} = \rho \overline{w}_b$$

# Impact of closure on diurnal cycle JJA 2011-2012 against Radar



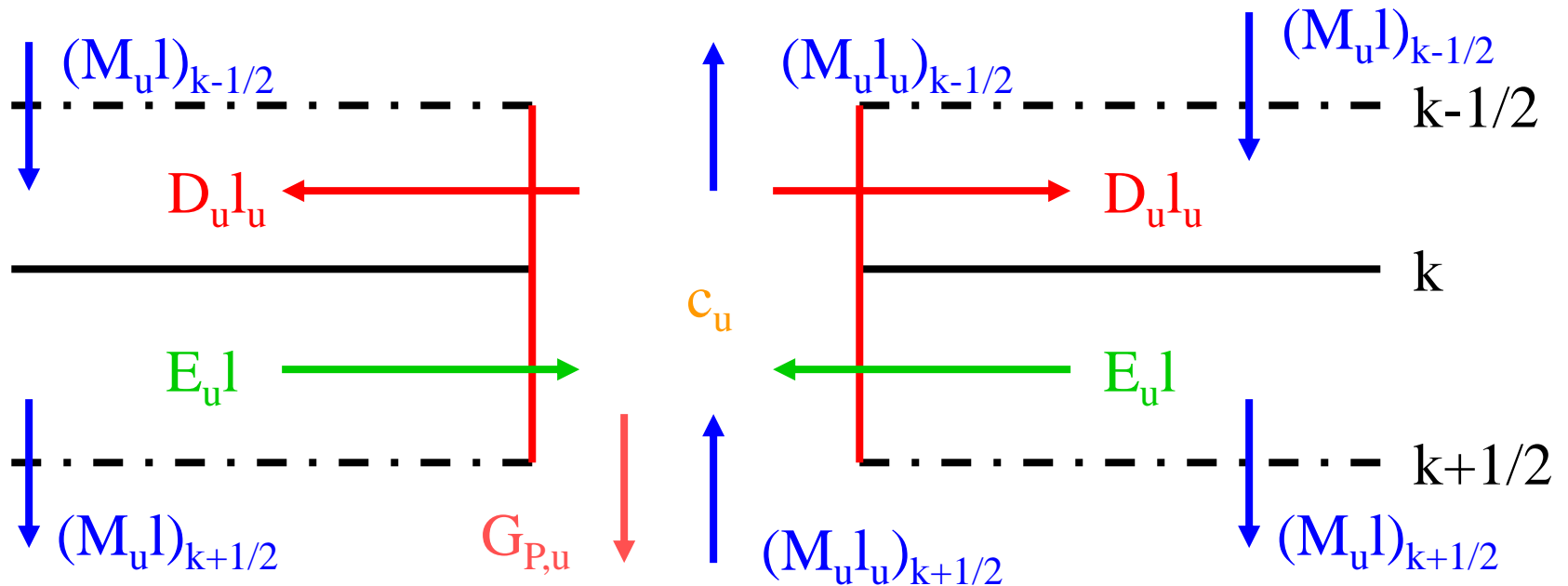
**Obs radar**

**NEW=with PCAPEBbl term**

Bechtold et al., 2014, J. Atmos. Sci.  
ECMWF Newsletter No 136 Summer 2013

# Vertical Discretisation

Fluxes on half-levels, state variable and tendencies on full levels



# Numerics: solving Tendency advection equation explicit solution

$$\left. \frac{\partial \bar{\psi}}{\partial t} \right|_{conv} = g \frac{\partial}{\partial p} \left[ M^u (\psi^u - \bar{\psi}) \right] + S; \quad \text{if } \psi = T, q \quad S = \frac{\partial}{\partial p} \text{Pr}$$

Use vertical discretisation with fluxes on half levels ( $k+1/2$ ), and tendencies on full levels  $k$ , so that

$$\Delta p = P_{k+1/2} - P_{k-1/2}$$

$$\left. \frac{\partial \bar{\psi}_k}{\partial t} \right|_{conv} = \frac{g}{\Delta p} \left[ M_{k+1/2}^u \psi_{k+1/2}^u - M_{k-1/2}^u \psi_{k-1/2}^u - M_{k+1/2}^u \bar{\psi}_{k+1/2} + M_{k-1/2}^u \bar{\psi}_{k-1/2} \right] + S_k$$

In order to obtain a better and more stable “upstream” solution (“compensating subsidence”, use shifted half-level values to obtain:

$$\bar{\psi}_{k-1/2} = \bar{\psi}_{k-1}$$

$$\left. \frac{\partial \bar{\psi}_k}{\partial t} \right|_{conv} = \frac{g}{\Delta p} \left[ M_{k+1/2}^u \psi_{k+1/2}^u - M_{k-1/2}^u \psi_{k-1/2}^u - M_{k+1/2}^u \bar{\psi}_k + M_{k-1/2}^u \bar{\psi}_{k-1} \right] + S$$

# Numerics: implicit solution

$$\left. \frac{\partial \bar{\psi}}{\partial t} \right|_{conv} = g \frac{\partial}{\partial p} \left[ M^u (\psi^u - \bar{\psi}) \right] + S; \quad \text{if } \psi = T, q \quad S = \frac{\partial}{\partial p} \text{Pr}$$

Use temporal discretisation with  $\bar{\psi}$  on RHS taken at future time  $\bar{\psi}^{n+1}$  and not at current time  $\bar{\psi}^n$

$$\Delta p = P_{k+1/2} - P_{k-1/2}$$

For “upstream” discretisation as before one obtains:

$$\bar{\psi}_{k-1/2} = \bar{\psi}_{k-1}$$

$$\bar{\psi}_k^{n+1} - \bar{\psi}_k^n = g \frac{\Delta t}{\Delta p} \left[ M_{k+1/2}^u \psi_{k+1/2}^u - M_{k-1/2}^u \psi_{k-1/2}^u - M_{k+1/2}^u \bar{\psi}_k^{n+1} + M_{k-1/2}^u \bar{\psi}_{k-1}^{n+1} \right] + \Delta t S_k^n$$

$$\left( 1 + \frac{g \Delta t}{\Delta p} M_{k+1/2}^u \right) \bar{\psi}_k^{n+1} - \frac{g \Delta t}{\Delta p} M_{k-1/2}^u \bar{\psi}_{k-1}^{n+1} = \bar{\psi}_k^n + \frac{g \Delta t}{\Delta p} \left[ M_{k+1/2}^u \psi_{k+1/2}^u - M_{k-1/2}^u \psi_{k-1/2}^u \right] + \Delta t S_k^n$$

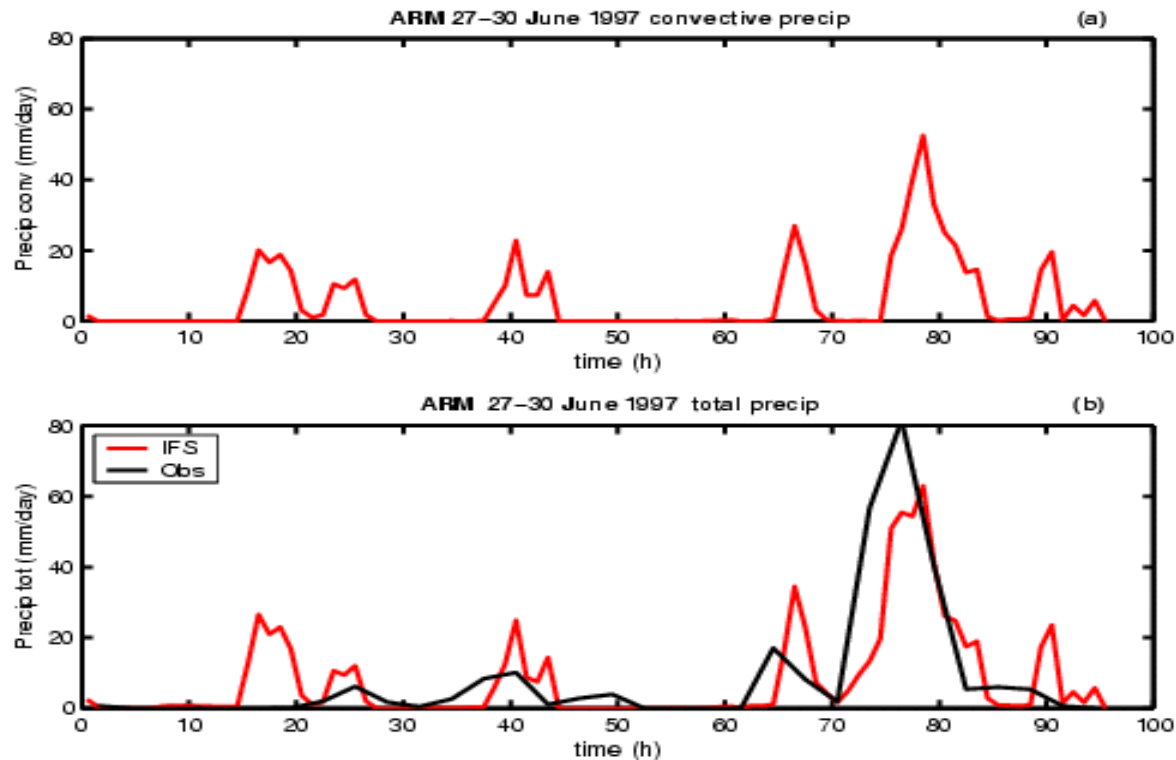
→ Only bi-diagonal linear system, and tendency is obtained as

$$\left. \frac{\partial \bar{\psi}_k}{\partial t} \right|_{conv} = \frac{\bar{\psi}_k^{n+1} - \bar{\psi}_k^n}{\Delta t}$$

# Tracer transport experiments

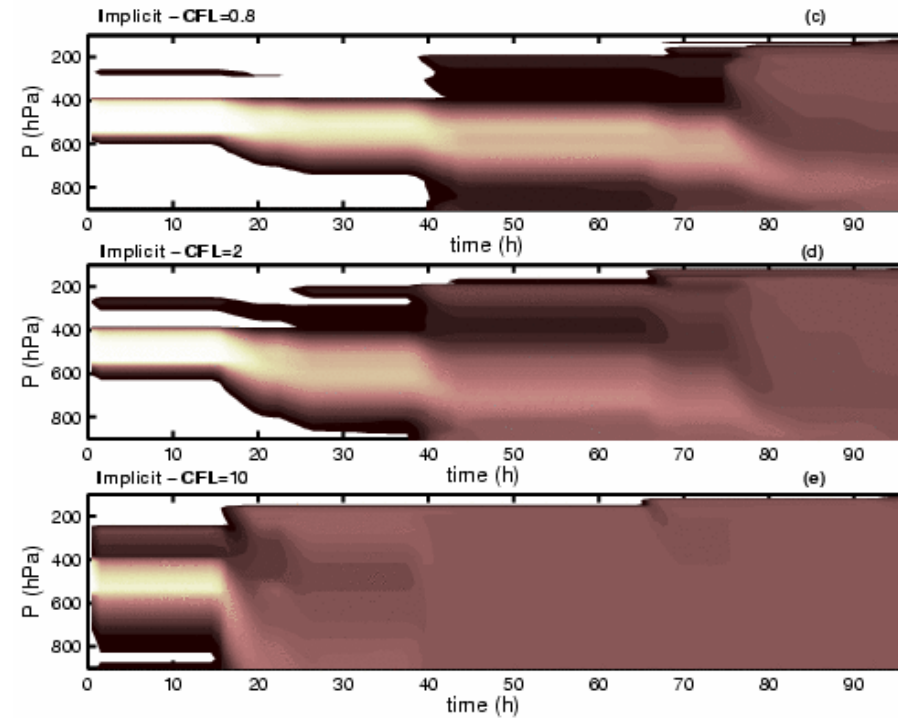
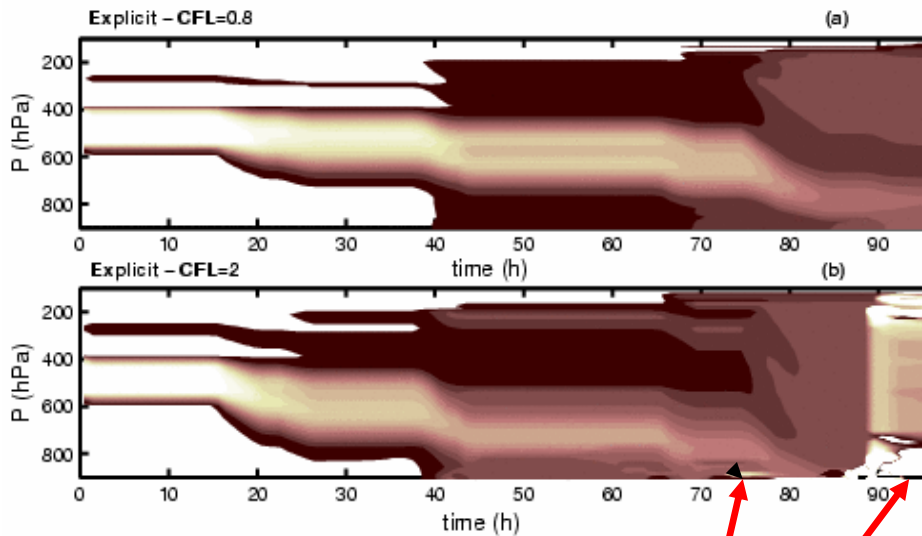
## Single-column simulations (SCM)

Surface precipitation; continental convection during ARM



# Tracer transport in SCM

## Stability in implicit and explicit advection



$$\text{CFL} = w \Delta t / \Delta z = M / g \Delta t / \Delta p \leq 1$$

instabilities

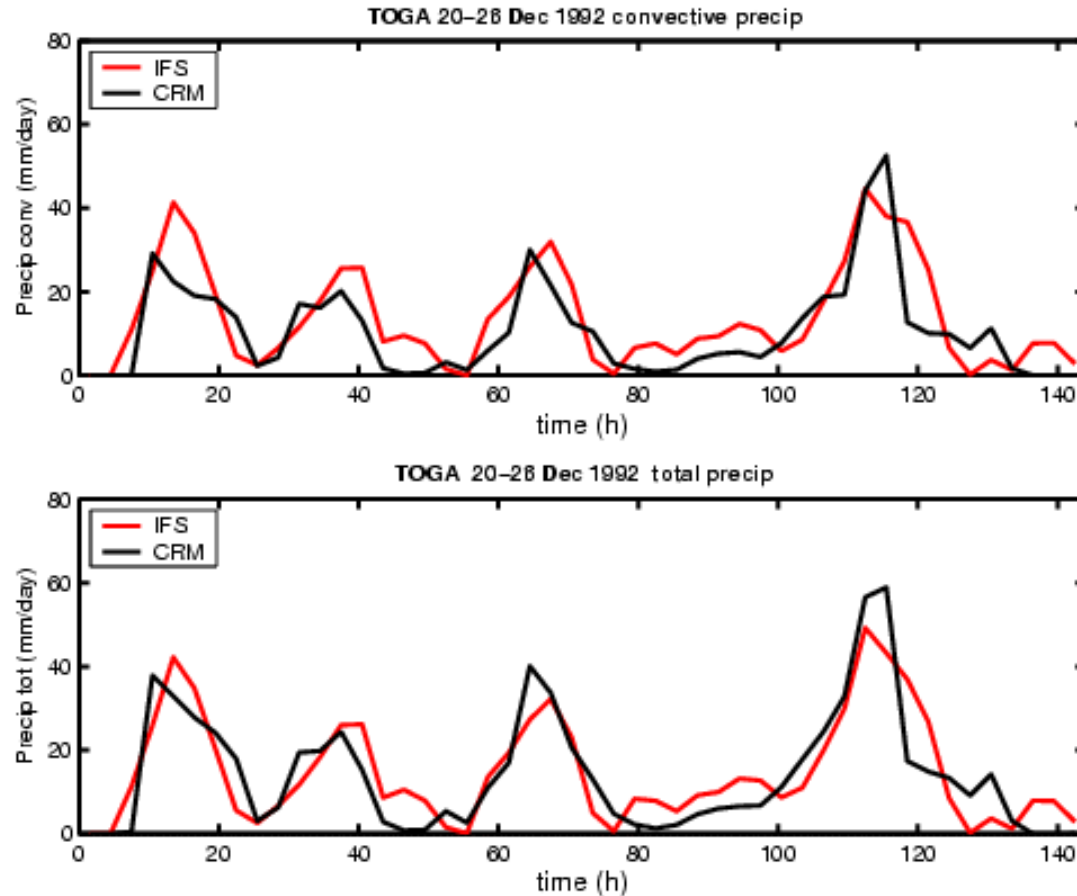
- Implicit solution is stable.
- If mass fluxes increases, mass flux scheme behaves like a diffusion scheme: well-mixed tracer in short time



# Tracer transport experiments (2)

## Single-column model against CRM

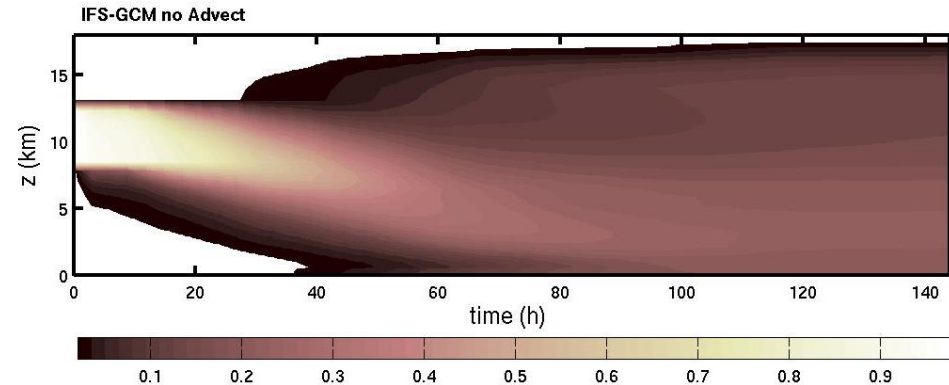
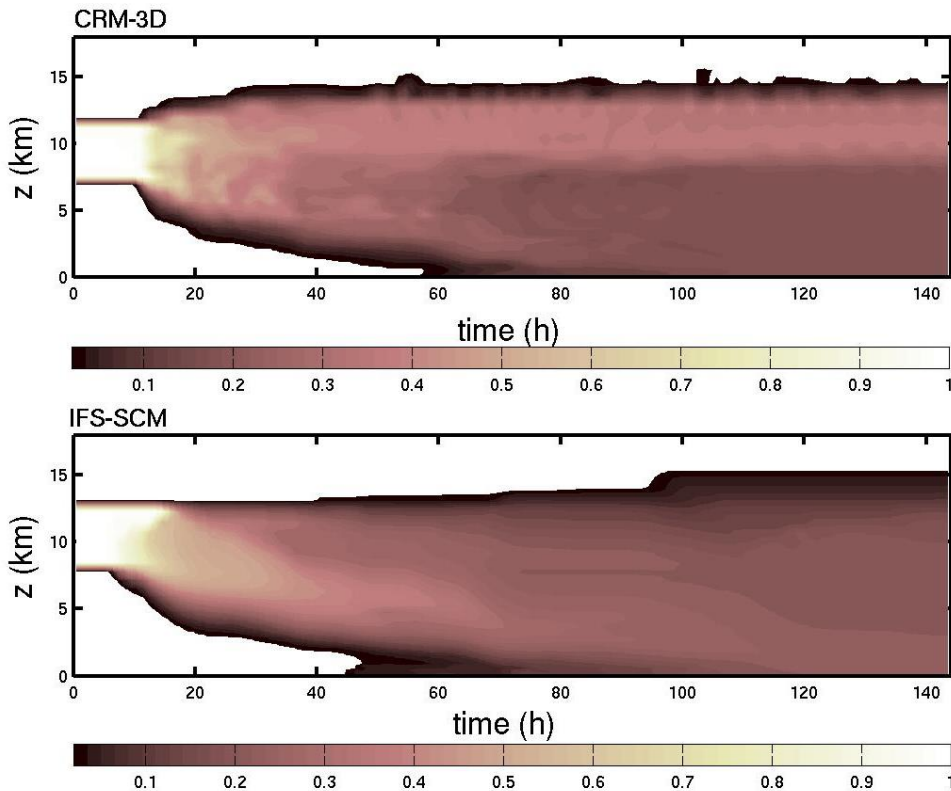
Surface precipitation; tropical oceanic convection during TOGA-COARE



# Tracer transport

## SCM and global model against CRM

### Mid-tropospheric Tracer



- Mid-tropospheric tracer is transported upward by convective draughts, but also slowly subsides due to cumulus induced environmental subsidence
- IFS SCM (convection parameterization) diffuses tracer somewhat more than CRM
- In GCM tropopause higher, normal, as forcing in other runs had errors in upper troposphere