

# Ocean data assimilation and coupling

DA training course 2025

Hao Zuo & Phil Browne

With inputs from M Chrust, M A Balmaseda, K Mogensen, E de Boisseson, R Buizza and many others

[hao.zuo@ecmwf.int](mailto:hao.zuo@ecmwf.int)

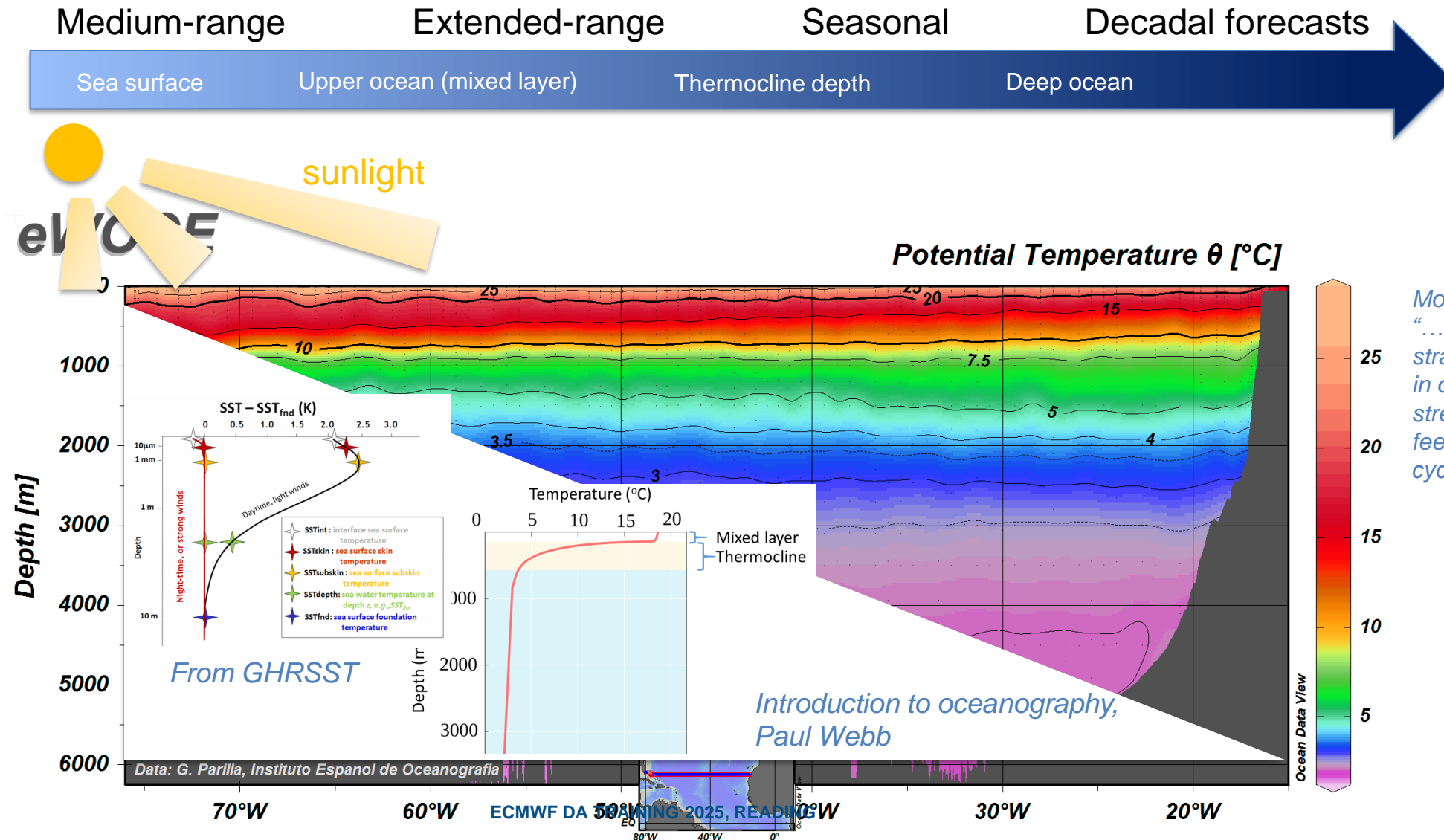
# Outline

- Ocean system
- Ocean Data Assimilation system - NEMOVAR
- Assimilation of ocean observations (In-situ and satellite data)
- Bias correction methods
- Coupled Data Assimilation

# Why do we need Ocean DA?

- Forecasting: initialization of coupled forecasts
  - NWP, monthly, seasonal, decadal
  - Calibration and reforecasts
- Verification/evaluation/co-design of Global Ocean observing network (OSE/OSSE)
- Climate applications
  - reconstruct & monitor the ocean (ECV/EOV);
  - study EEI and energy/water cycle;
- Towards coupled DA system (weakly -> quasi-strong -> strong ...)
- Other Commercial applications (oil rigs, ship route ...), safety and rescue, environmental (algii blooms, spills)

## Ocean for coupled forecasts



Mogensen et al., 2017:  
“... the upper ocean stratification is the key in determining the strength of the coupled feedback (of tropical cyclone forecasts).”

# Ocean vs Atmosphere

**Spatial/time scales** The radius of deformation in the ocean is small (~30km) compared to the atmosphere (~3000km). Time scales varies from hours (mixing) to decades (overturning circulations).

**Ocean is a data sparse system**, in-situ observation is limited and mostly covers upper ocean only, satellite observation only covers ocean surface and is only available for a relatively short period.

**The ocean is forced at the surface and land boundary**, by the wind/waves, heating/cooling and fresh-water fluxes

Uncertainty in forcing fluxes contributes to uncertainty in model results.

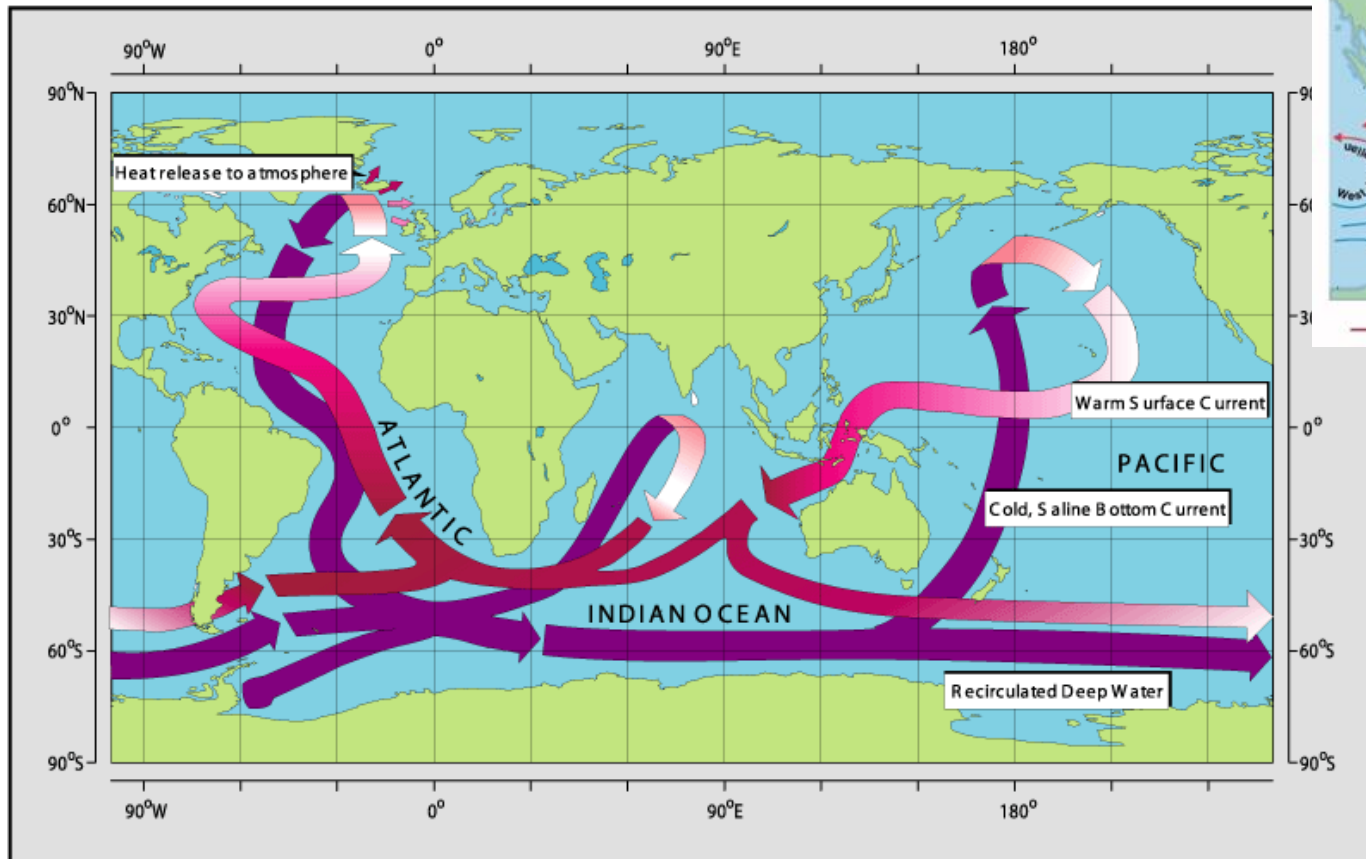
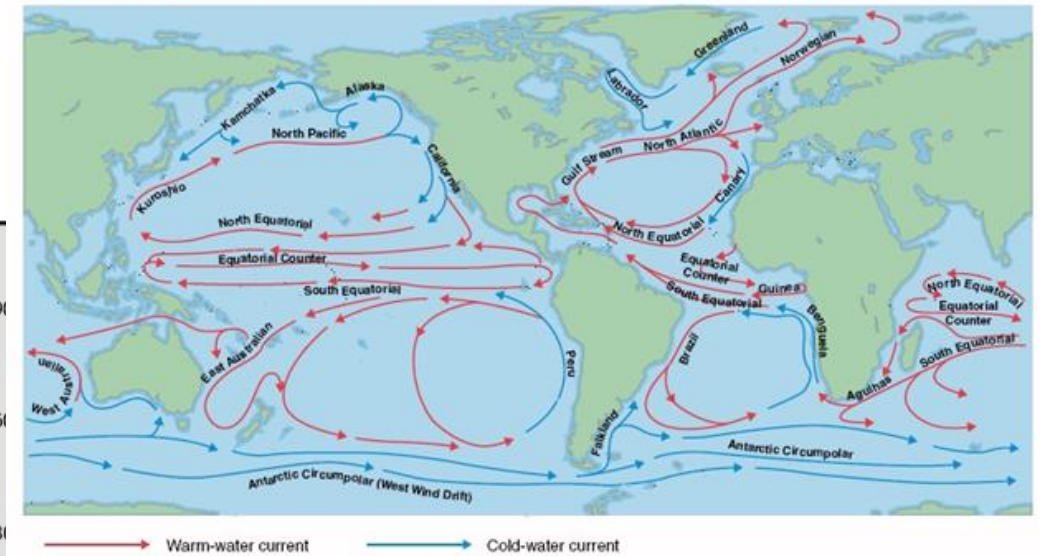
**The ocean is strongly stratified in the vertical**, especially near the surface. Although deep convection also occurs

Density is determined by Temperature and Salinity

**The ocean has continental boundaries**; dealing with them is not trivial in data assimilation

# Ocean time scales: from hours to centuries

Wind Driven: Gyres, Western Boundary Currents, Upwelling regions (coastal, equatorial), Ekman pumping and subduction



**Density Driven:**  
Thermohaline Circulation

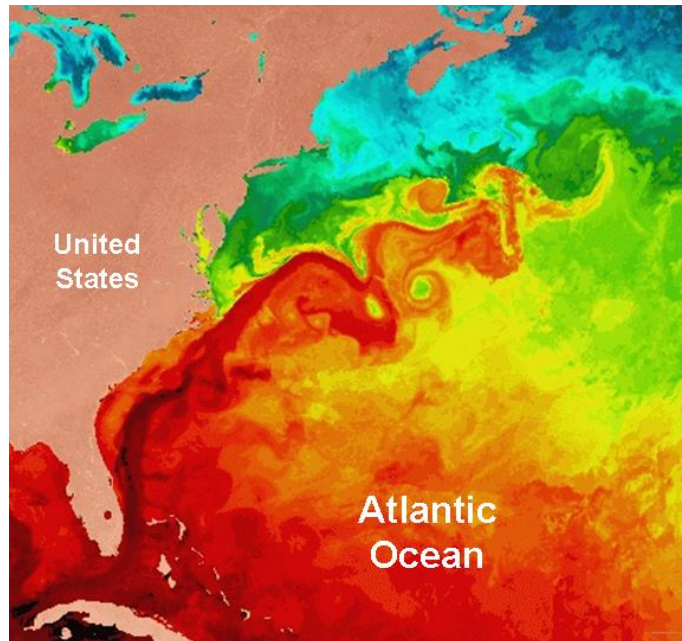
Ocean is a system with much longer memory but slow response compared to the Atmosphere



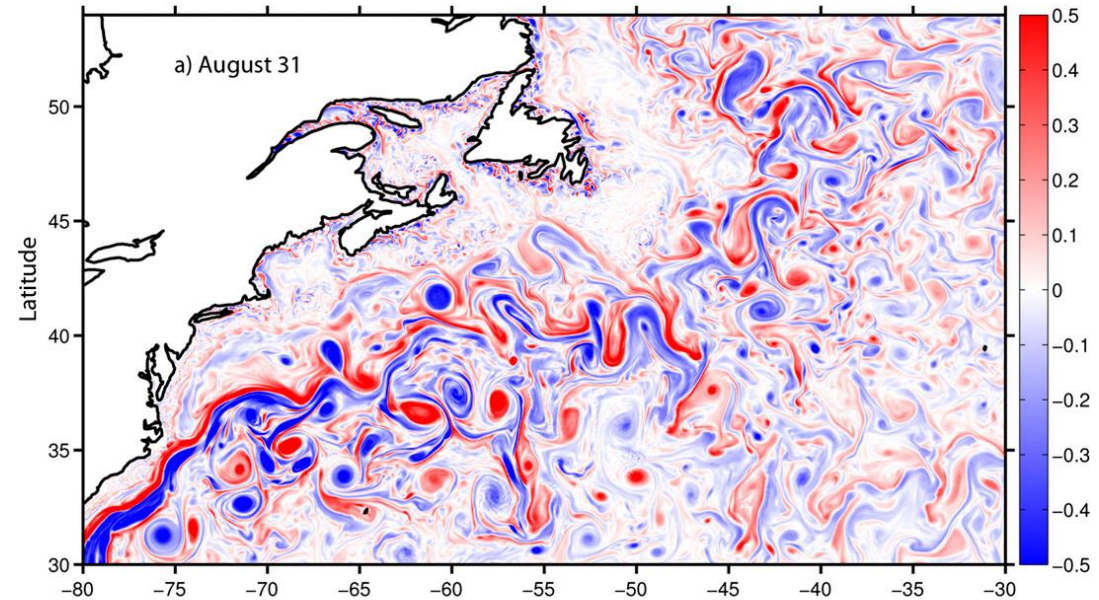
# Ocean spatial scales

The radius of deformation in the ocean is small ( $\sim 30\text{km}$ ) compared to the atmosphere ( $\sim 3000\text{km}$ ).

Satellite image of SST in the North Atlantic Ocean (from NOAA)



1/50 degree Ocean surface relative vorticity (CHASSIGNET and Xu, 2017)



mesoscale and sub-mesoscale eddies

Ocean variables with various spatial scales: from hundred meters to hundreds of km

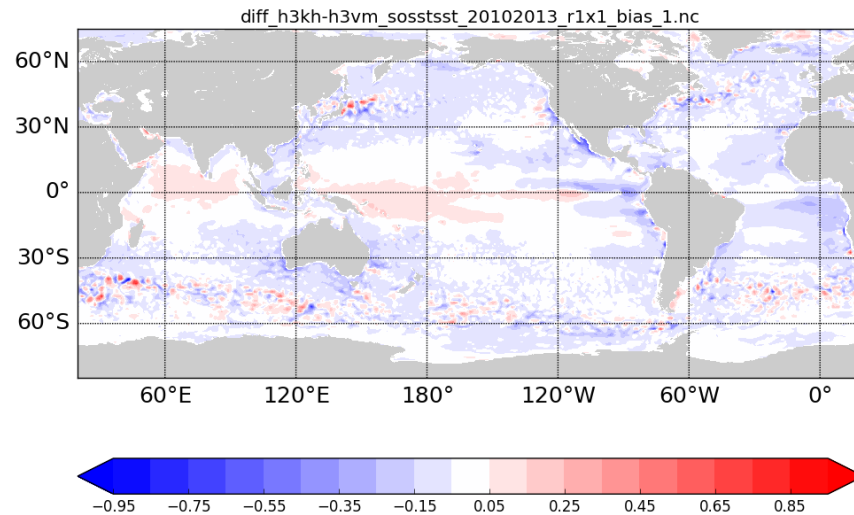
# Ocean boundary conditions

The ocean is forced at the surface and land boundary, by the wind/waves, heating/cooling and fresh-water fluxes. Uncertainty in forcing fluxes contributes to uncertainty in model results.

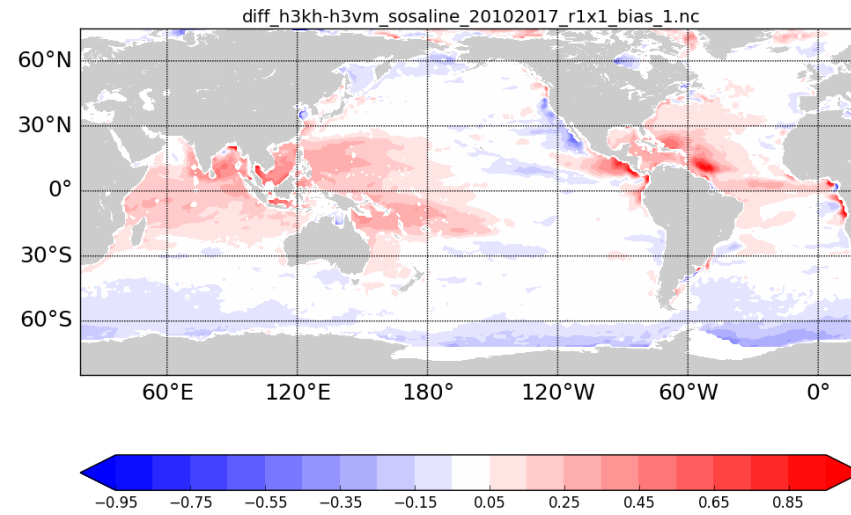
## ERA5 vs ERAInt

Same model version, no data assimilation

Mean SST difference



Mean SSS difference



- SST difference is mostly due to changes in ERA5 shortwave/longwave radiations.
- SSS difference is directly related with precipitation changes in ERA5

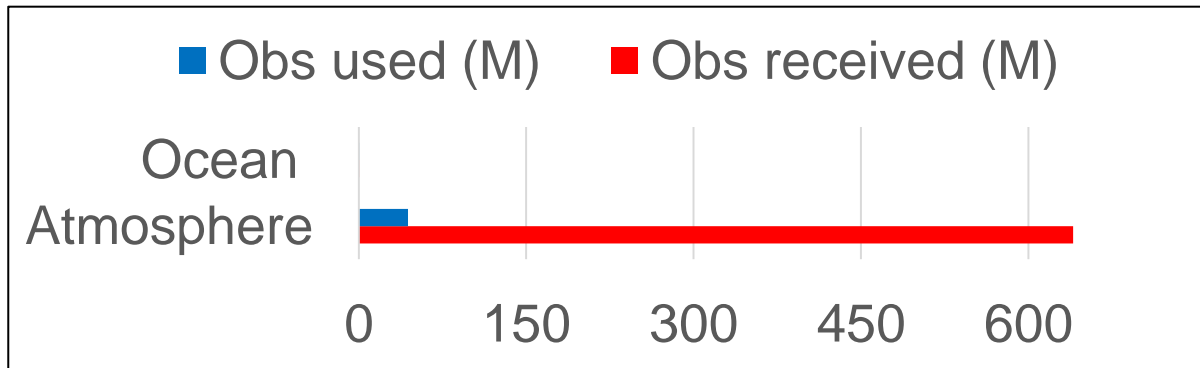


# Ocean observing system

Ocean observation is about 1/1000 to 1/10000 smaller than Atmospheric observation

- Ocean in-situ observation is limited and mostly covers upper ocean only.
- Satellite observation only covers ocean surface and is only available for a relatively short period.

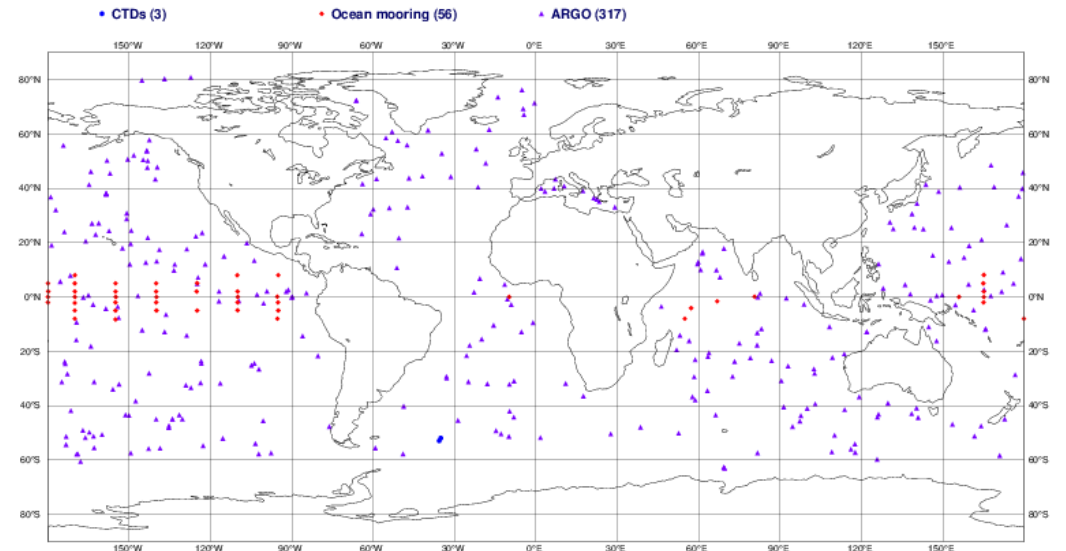
## Daily obs



ECMWF data coverage (used observations) - SALINITY

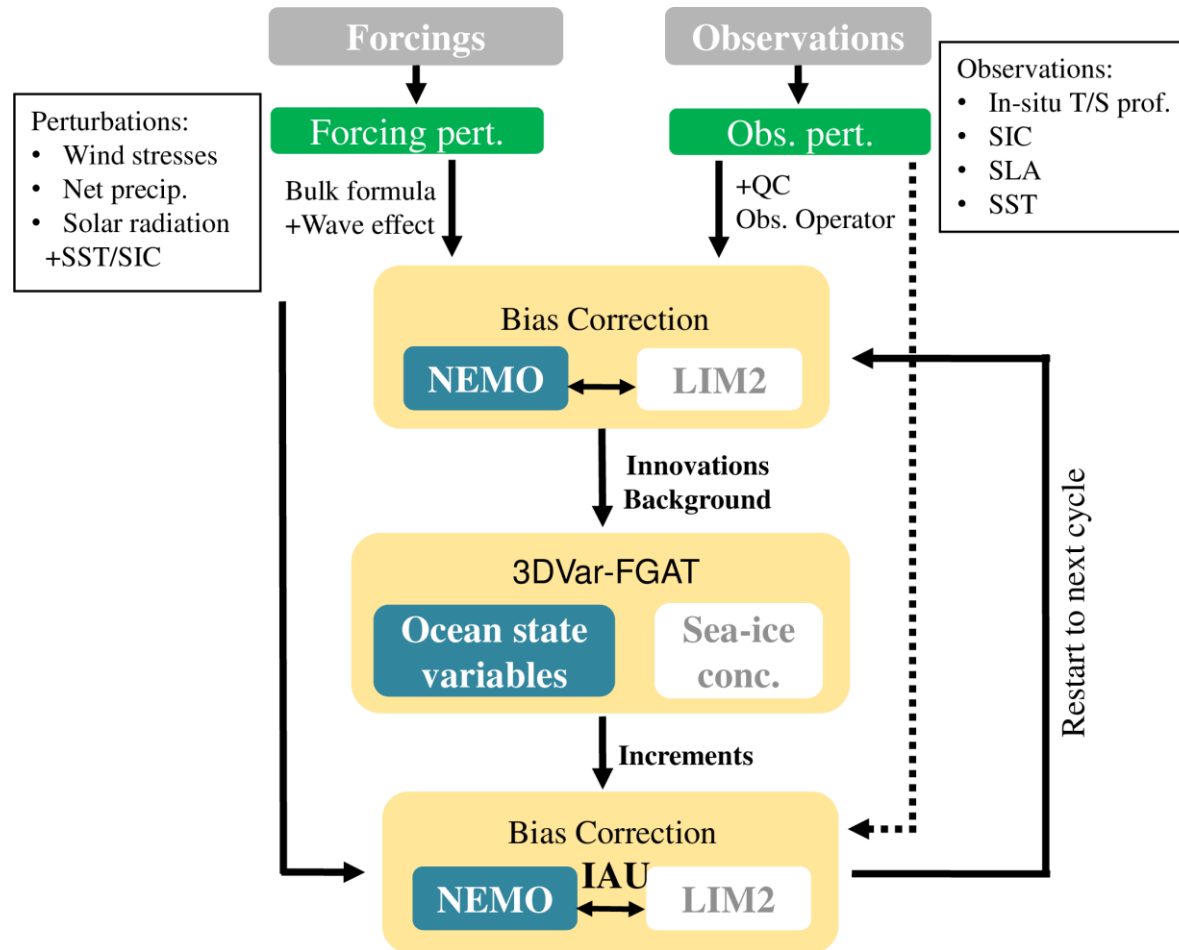
20171030 00

Total number of obs = 376



- Ocean system
- Ocean Data Assimilation system - NEMOVAR
- Assimilation of ocean observations (In-situ and satellite data)
- Bias correction methods
- Coupled Data Assimilation

# Ocean DA system at ECMWF



OCEAN5 is the operational ocean and sea-ice analysis system (Zuo et al., 2019).

- Ocean: NEMOv3.4
- Sea-ice: LIM2
- Resolution:  $\frac{1}{4}$  degree with 75 levels
- Assimilation: 3DVAR-FGAT
- 5 ensemble member

Overview of the OCEAN5 setup

# Ocean DA at ECMWF: NEMOVAR

## NEMOVAR (CERFACS/ECMWF/INRIA/Met Office)

- En Variational DA system for **NEMO** ocean model.
  - Solves a linearized version of the full non-linear cost function.
  - Incremental **3D-Var FGAT** running operational, 4D-Var in research model
- Background correlation model based **diffusion operators**
- Background errors are correlated between different variables through **balance operator**

## 3DVar-FGAT as in Daget et al 2009

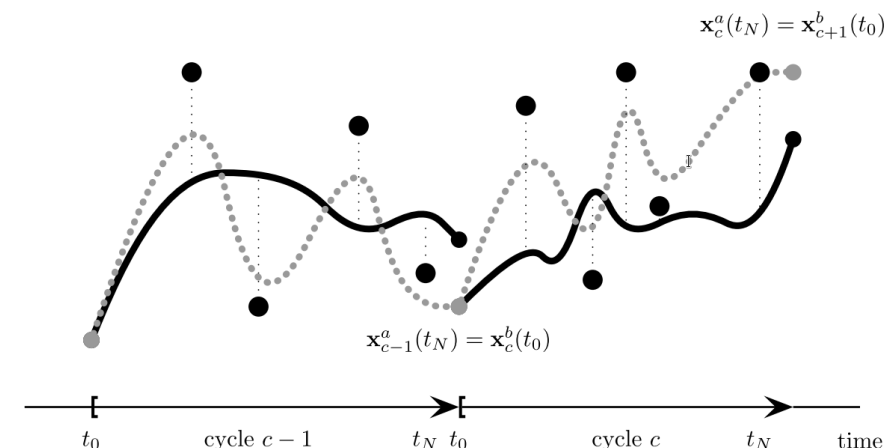


Figure 1: Schematic illustration of the procedure used to cycle 3D-Var. On each cycle  $c$ , the model is integrated from  $t_0$  to  $t_N$  starting from a background initial condition  $\mathbf{x}_c^b(t_0)$  (grey dots) to produce the background trajectory  $\mathbf{x}_c^b(t_i)$  (black solid curve). The difference between the observations  $\mathbf{y}_{c,i}^o$  (black dots) and their background counterpart ( $\mathbf{H}_{c,i}\mathbf{x}_c^b(t_i)$ ) is computed (represented by the vertical thin dotted lines) for use in the 3D-Var FGAT minimization. After minimization, the model integration is repeated from the same initial condition ( $\mathbf{x}_c^b(t_0)$ ) but with the analysis increment applied using IAU. This produces the analysis trajectory  $\mathbf{x}_c^a(t_i)$  (grey dashed curve). The updated model state  $\mathbf{x}_c^a(t_N)$  at the end of cycle  $c$  is then used as the background initial condition for the next cycle  $c+1$  (grey dots).

Weaver et al 2003,2005;  
Balmaseda et al 2013;

Daget et al 2009;  
Chrut et al., 2021

Mogensen et al 2012;

# NEMOVAR: Linearized Cost function

$$J[\delta \mathbf{w}] = \frac{1}{2} \delta \mathbf{w}^T \mathbf{B}^{-1} \delta \mathbf{w} + \frac{1}{2} (\mathbf{G} \delta \mathbf{w} - \mathbf{d})^T \mathbf{R}^{-1} (\mathbf{G} \delta \mathbf{w} - \mathbf{d})$$

$\mathbf{y}^o = \{(\mathbf{y}_0^o)^T \dots (\mathbf{y}_i^o)^T \dots (\mathbf{y}_N^o)^T\}^T \rightarrow$  4D observation array

$\delta \mathbf{w} = \mathbf{w} - \mathbf{w}^b \rightarrow$   $\mathbf{w}$  is the control vector

$\mathbf{d} = \mathbf{y}^o - \mathbf{G}(\mathbf{w}^b) \rightarrow$  Departure vector

$$\mathbf{G}(\mathbf{w}) = \begin{pmatrix} \vdots \\ G_i(\mathbf{w}) \\ \vdots \end{pmatrix} = \begin{pmatrix} \vdots \\ H_i[M(t_i, t_0)\{K(\mathbf{w})\}] \\ \vdots \end{pmatrix}$$

- **Balance operator:** convert to  $\mathbf{w}$  space,  $\mathbf{B}$  becomes block diagonal, representing the spatial covariance model.
- **Diffusion operator:** The spatial covariances is specified by diffusion operator (*Weaver and Courtier 2001*)

- *Weaver et al 2003,2005*
- *Daget et al 2009*
- *Mogensen et al 2012*
- *Balmaseda et al 2013*

**Balance operator**

$\mathbf{w}^b \equiv K^{-1}\{\mathbf{x}^b(t_0)\}$

$(T, S_U, \eta_U, u_U, v_U)^T \leftarrow (T, S, \eta, u, v)^T$

linearly independent

## Solution:

$$\delta \mathbf{w}^a \approx \mathbf{B} \mathbf{G}^T (\mathbf{G} \mathbf{B} \mathbf{G}^T + \mathbf{R})^{-1} \mathbf{d}.$$

$$\delta \mathbf{x}^a = K(\mathbf{w}^b + \delta \mathbf{w}^a) - K(\mathbf{w}^b) \approx K \delta \mathbf{w}^a$$

$$\mathbf{x}^a(t_i) = M(t_i, t_{i-1})[\mathbf{x}^a(t_{i-1}), F_i \delta \mathbf{x}^a]$$

*IAU, Bloom et al 1996*

# NEMOVAR: Linearized Balance Operator

Define the balance operator symbolically by the sequence of equations

Temperature	$\delta T = \delta T$	
Salinity	$\delta S = K_{S,T}^b \delta T + \delta S_U = \boxed{\delta S_B} + \delta S_U$	$\left. \begin{array}{l} \delta S_U \\ \delta \eta_U \\ \delta u_U \\ \delta v_U \end{array} \right\} \text{Treated as approximately mutually independent without cross correlations}$
SSH	$\delta \eta = K_{\eta,\rho} \delta \rho + \delta \eta_U = \delta \eta_B + \delta \eta_U$	
u-velocity	$\delta u = K_{u,p} \delta p + \delta u_U = \delta u_B + \delta u_U$	
v-velocity	$\delta v = K_{v,p} \delta p + \delta v_U = \delta v_B + \delta v_U$	

Density	$\delta \rho = K_{\rho,T}^b \delta T + K_{\rho,S}^b \delta S$	$\left. \begin{array}{l} \delta \rho \\ \delta p \end{array} \right\}$
Pressure	$\delta p = K_{p,\rho} \delta \rho + K_{p,\eta} \delta \eta$	

*Weaver et al., 2005, QJRMS*

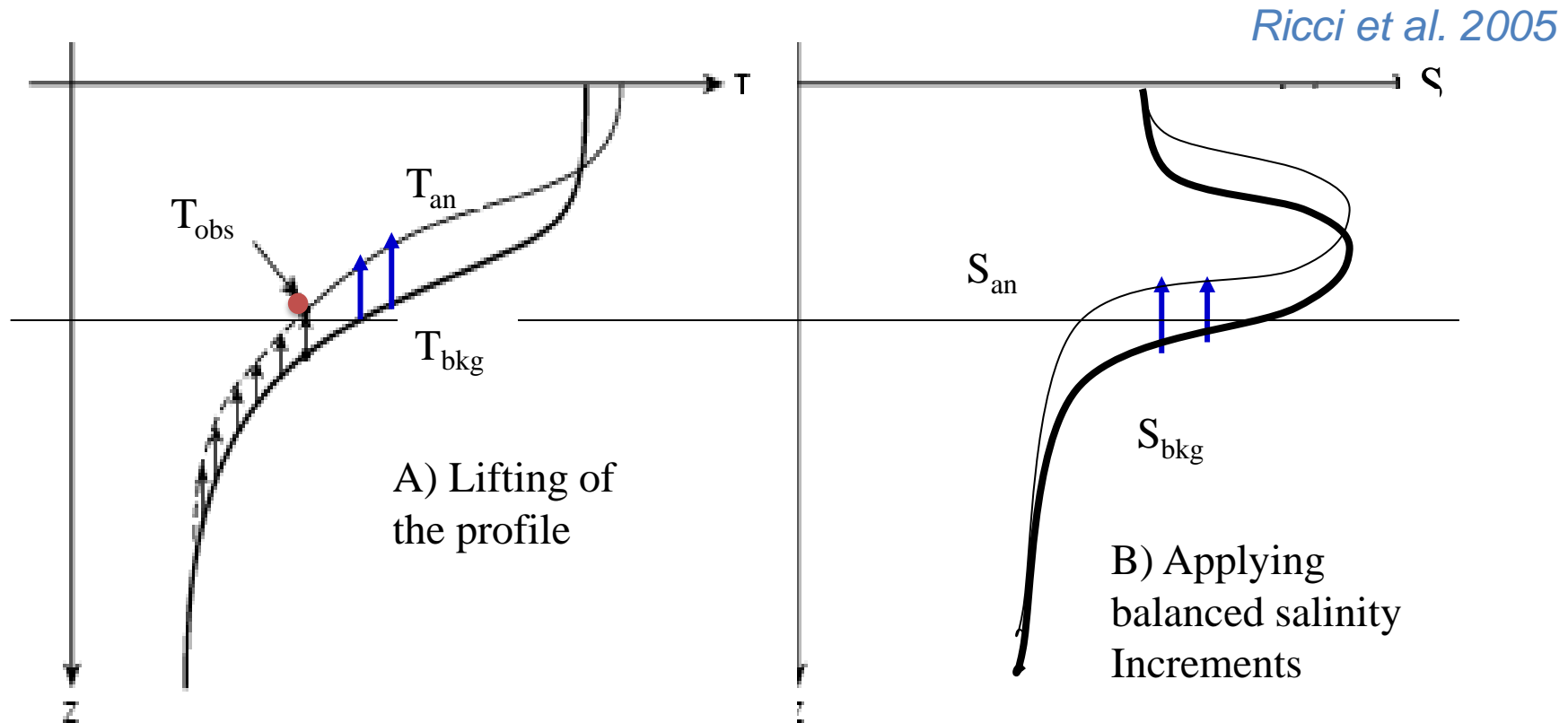


# NEMOVAR: balance operator

## Salinity balance

(approx. T-S conservation)  
To preserve the water mass properties  
following *Troccoli and Haines (1999)*)

$$\delta S_B = \gamma_S^b \left( \frac{\partial S}{\partial z} \right)^b \delta z \quad \delta z = \left( \frac{\partial z}{\partial T} \right)^b \delta T. \quad \gamma_S^b \text{ is 0 unless T-S is weakly correlated}$$



**T/S balance: vertical displacement of the water profile.**

# NEMOVAR: background error model

## General B formulation in NEMOVAR

$$\mathbf{B} = \beta_m^2 (\mathbf{B}_{m_1} + \mathbf{B}_{m_2} + \dots) + \beta_e^2 \mathbf{B}_e + \beta_E^2 \mathbf{B}_{EOF}$$

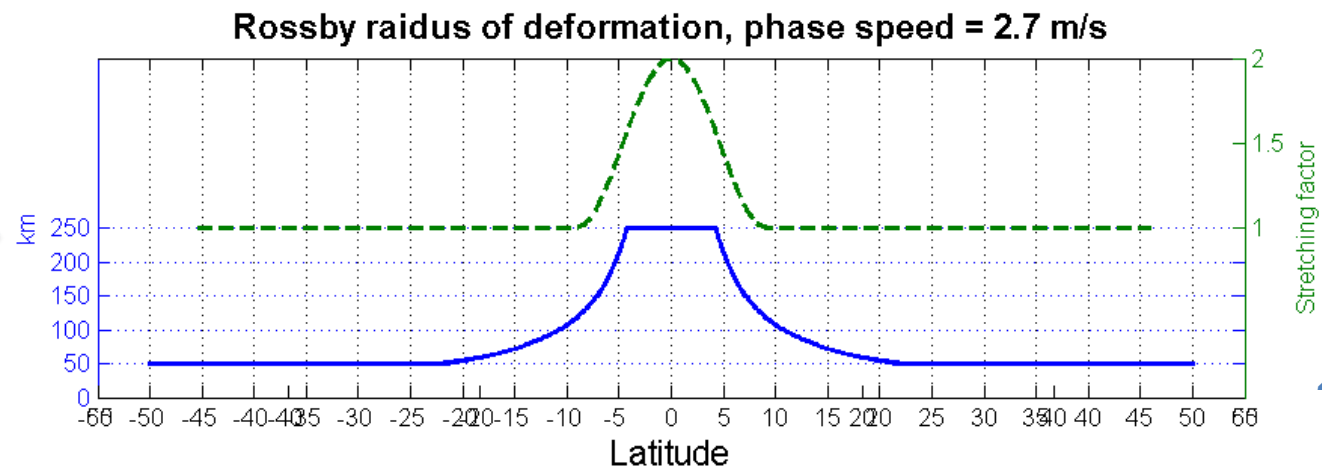
$$\mathbf{B}_{m_i} = \mathbf{K}_b \mathbf{D}_i^{1/2} \mathbf{C}_{m_i} \mathbf{D}_i^{1/2} \mathbf{K}_b^T$$

- $\mathbf{B}_m$  is modelled covariance matrix (can use multiple model to represent different scales)
- $\mathbf{B}_e$  is a localized ensemble-based covariance matrix
- $\mathbf{B}_{EOF}$  is a EOF-based covariance matrix
- $\mathbf{C}_m$  is correlation matrix (including diffusion operator)
- $\mathbf{D}_m$  is a diagonal matrix of variances (block-diagonal).

$$\mathbf{C}_X^{1/2} = \mathbf{\Gamma}_X^{1/2} \mathbf{L}_X^{1/2} \mathbf{W}_X^{-1/2}$$

diffusion operator use  
diffusion tensor  $\kappa_m$  to  
represent a particular de-  
correlation length-scales

## Horizontal correlation length-scales used in ORAP5



## Ensemble Var DA with Hybrid-B

A cost-effective way is to use only a single **modelled covariance** matrix  $\mathbf{B}_m$ , but use ensemble to estimate both variances ( $\mathbf{D}_m \rightarrow \mathbf{D}_e$ ) and the local correlation tensor ( $\boldsymbol{\kappa}_m \rightarrow \boldsymbol{\kappa}_e$  in  $\mathbf{C}_m$ )

$$\mathbf{B}_m = \mathbf{K}_b \mathbf{D}_e^{1/2} \mathbf{C}_e \mathbf{D}_e^{1/2} \mathbf{K}_b^T$$

Or even take a hybrid approach when estimate  $\mathbf{D}_h$  and  $\boldsymbol{\kappa}_h$  in  $\mathbf{B}_m$

$$\mathbf{D}_h = \alpha_m^2 \mathbf{D}_m + \alpha_e^2 \mathbf{D}_e$$

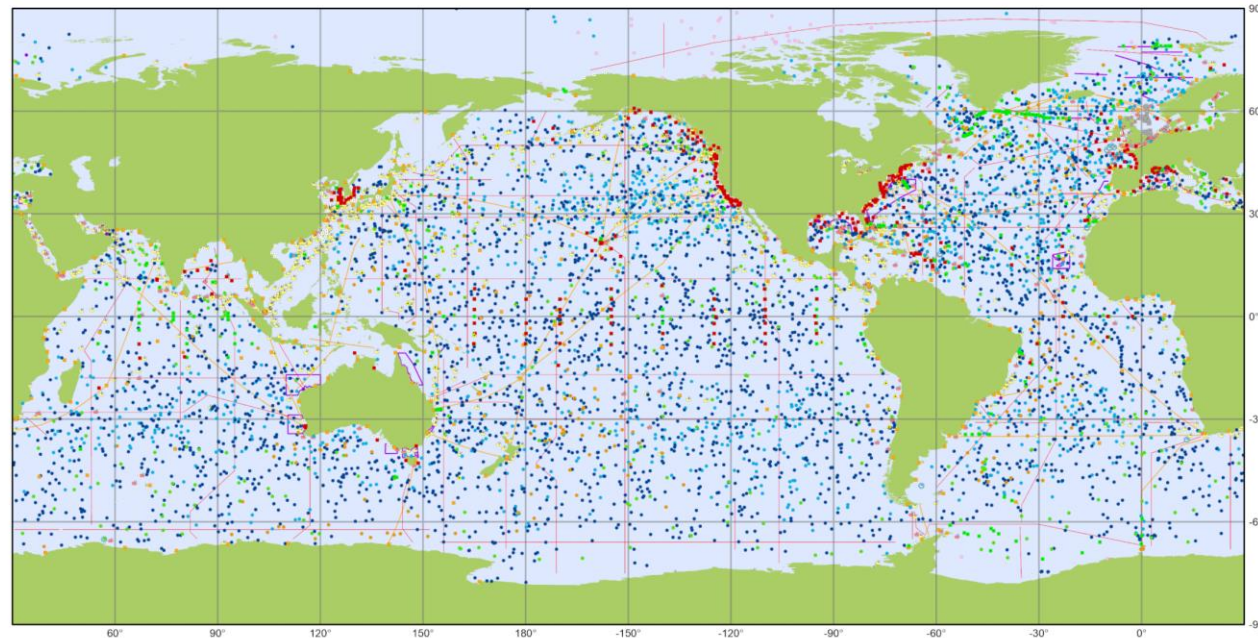
$$\boldsymbol{\kappa}_h = \gamma_m^2 \boldsymbol{\kappa}_m + \gamma_e^2 \boldsymbol{\kappa}_e$$

Where  $\mathbf{D}_m$  and  $\boldsymbol{\kappa}_m$  are modelled (static) components, and  $\mathbf{D}_e$  and  $\boldsymbol{\kappa}_e$  are ensemble (flow-dependent) estimations,  $\alpha_{m/e}$  and  $\gamma_{m/e}$  are constant weights.

- Ocean system
- Ocean Data Assimilation system - NEMOVAR
- Assimilation of ocean observations (In-situ and satellite data)
- Bias correction method
- Coupled Data Assimilation

# Ocean in-situ observations

New observations types are emerging: ALAMO, gliders, Deep Argo, BioArgo, drifter, saildrone ...



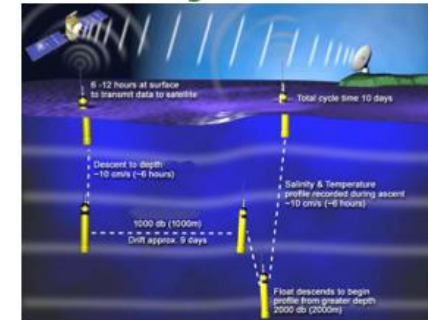
Global ocean observing system  
In situ operational platforms monitored by OceanOPS

January 2022

- |  |  |  |   |
|--|--|--|---|
| <b>Mobile systems</b><br><ul style="list-style-type: none"> <li>Core floats - Argo</li> <li>Deep floats - Argo</li> <li>Biogeochemistry floats - Argo</li> <li>Underwater gliders - OceanGliders</li> <li>Drifting buoys - DBCP</li> </ul> | <ul style="list-style-type: none"> <li>Polar buoys - DBCP</li> <li>Animal borne sensors</li> <li>Tsunameters - DBCP</li> <li>Offshore platforms - DBCP</li> <li>Moored buoys - DBCP</li> </ul> | <ul style="list-style-type: none"> <li>Ocean reference stations - OceansITES</li> <li>Sea level gauges - GLOSS</li> <li>High Frequency radars</li> <li>Ship based measurements</li> <li>Manned weather stations - SOT/VOS</li> <li>Automated weather stations - SOT/VOS</li> </ul> | <b>Reference lines and areas</b><br><ul style="list-style-type: none"> <li>Radiosondes - SOT/ASAP</li> <li>Repeat hydrography - GO-SHIP</li> <li>eXpendable BathyThermographs - SOT/SOOP</li> <li>Sampled sites - OceanGliders</li> </ul> |
|--|--|--|---|

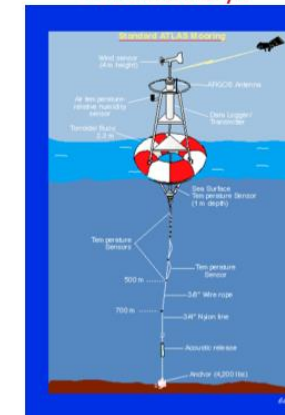
Generated by ocean-ops.org, 2022-02-06

## Argo floats



Argo operational cycle.  
[Argo 2018]

## Moored buoys



[PMEL 2018]

## Mammals!

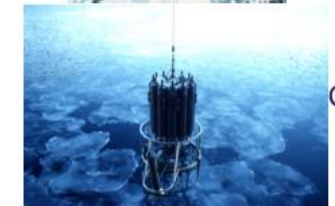


[MEOP et al. 2015]

## Ship based observations



XBT



CTD

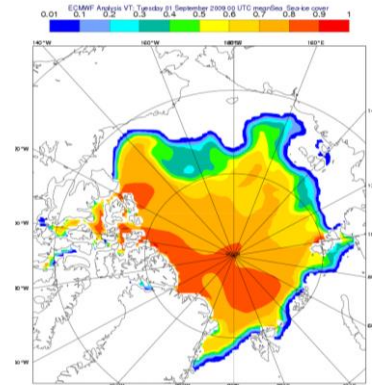
[CSIRO 2001]



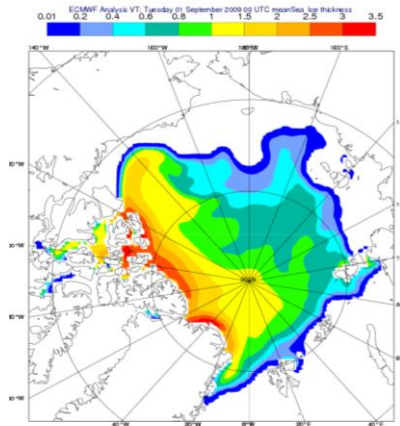
# Satellite sea surface observations

- Satellite provide important observations on monitoring sea surface states (SST, SSS, sea-ice states, sea surface height, surface currents, ocean color, etc).
- These sea surface observations are essential input for ocean and sea-ice reanalysis system and works as complementary data sources to the ocean in-situ observing networks.
- Challenge to deal with various data densities among different in-situ types, and between in-situ and satellite observations.

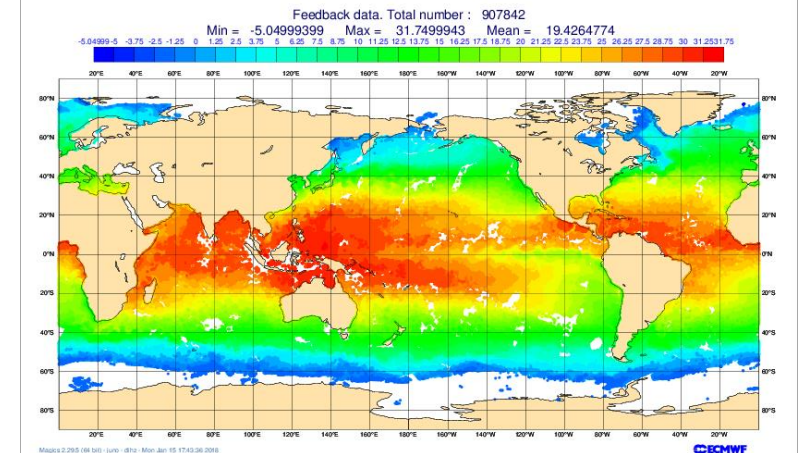
## Sea-ice concentration



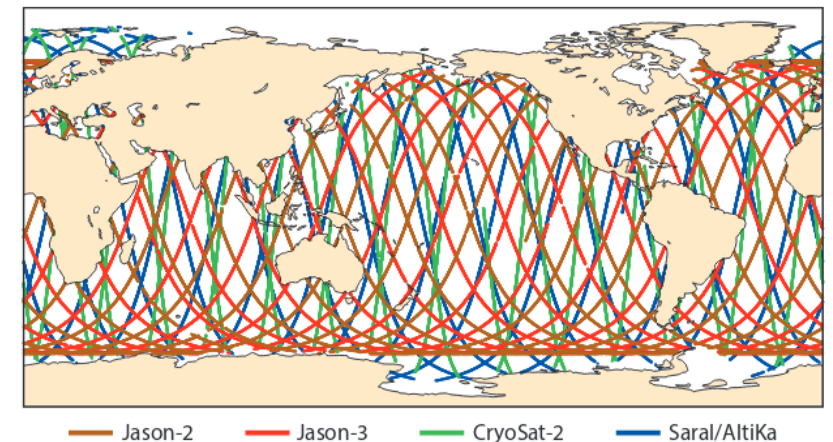
## Sea-ice thickness



## SST (IR, PMW)



## Sea-Level Anomaly (Altimeter)

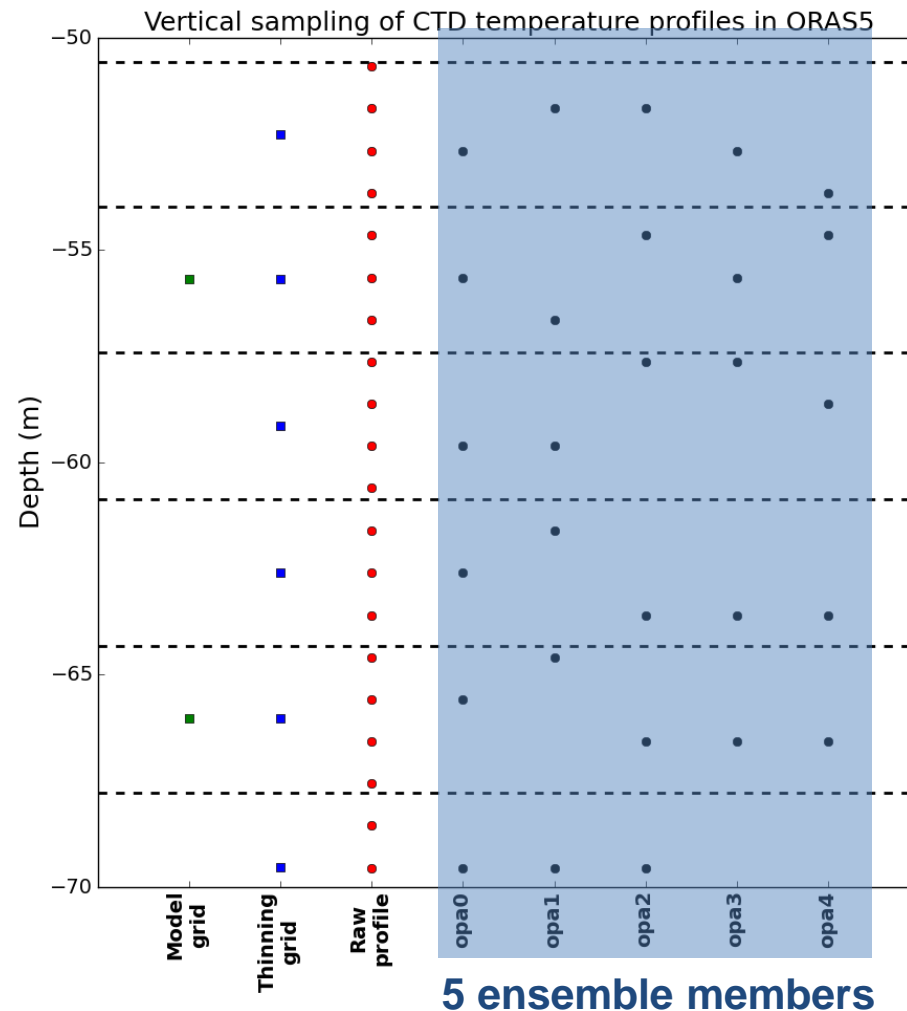




# Assimilation of in-situ observations

The in-situ observations are perturbed for ensemble generation.

- perturbing the position of the whole profile within the scales that the model resolution permits to resolve
- applying stratified random sampling vertically



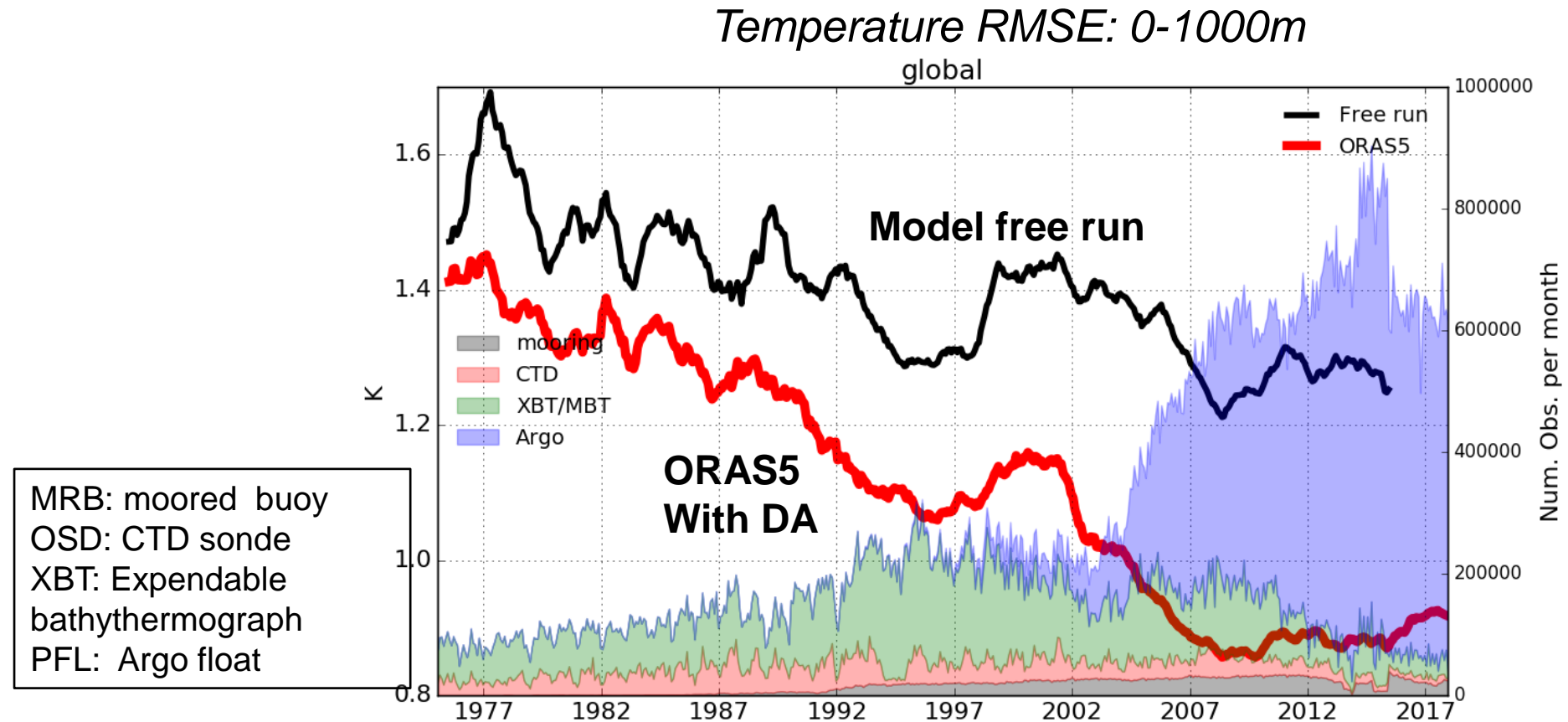
Each ensemble member assimilates slightly different in-situ observations with stratified random sampling approach

Example of a CTD profile used in 5 Ens. Mem.

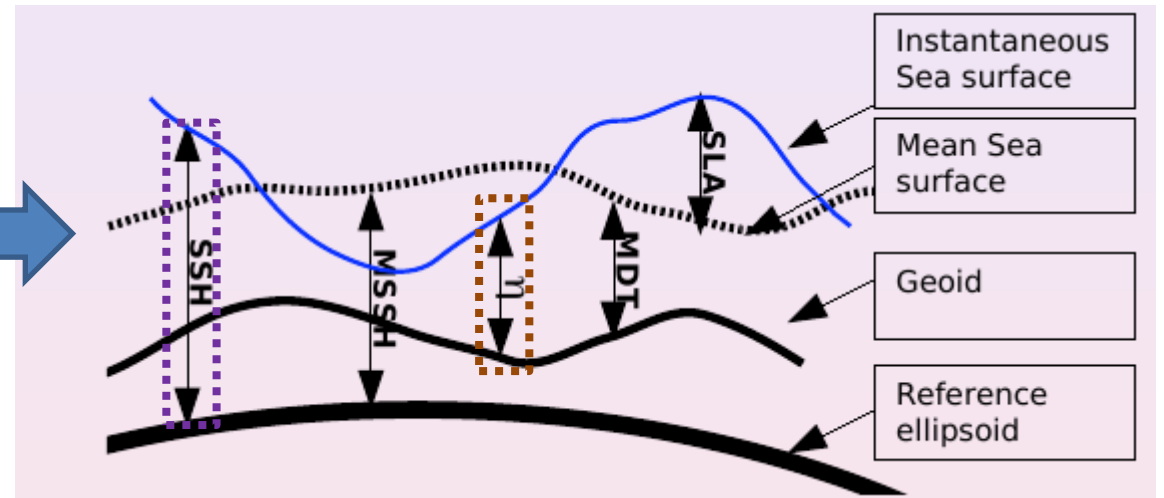
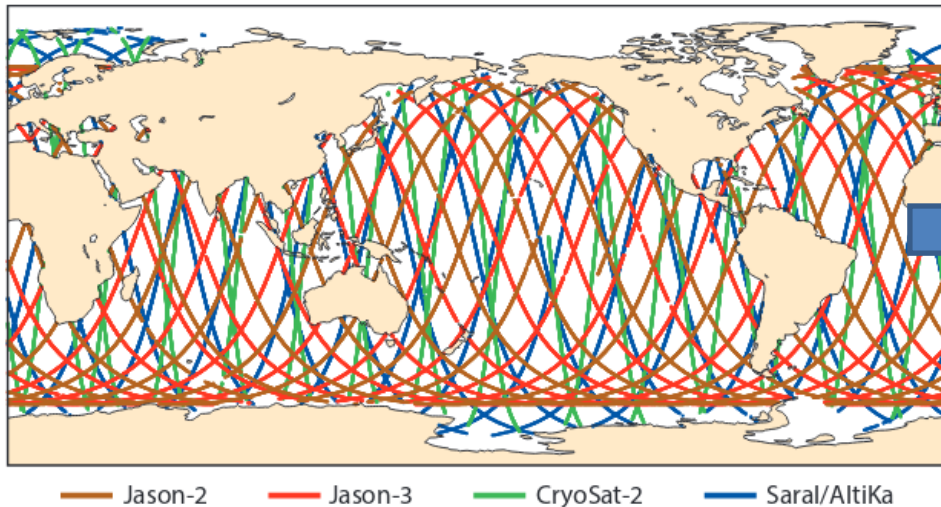
*Zuo et al., 2017*

# Assimilation of in-situ observations

Assimilation of ocean in-situ observations helps to constrain the 3D ocean, therefore providing better estimation of the ocean initial condition for the coupled forecasting system



# Assimilation of Sea Surface Height (SSH) data



**Altimeter measures SSH** (respect reference ellipsoide)

**Model represents  $\eta$**  (ssh referred to the Geoid)

$$\text{SSH-Geoid} = \eta$$

Geoid was poorly known (until recent years)

Alternative: Assimilate Sea Level Anomalies (SLA) respect a time mean

$$\text{Obs: SSH anomalies} = \text{SSH} - \text{MSSH} = \text{Obs SLA}$$

$$\text{Mod: } \eta \text{ anomalies} = \eta - \text{MDT} = \text{Mod SLA}$$

Where: MSSH= Temporal Mean SSH ;

MDT = Temporal Mean of model SL Mean

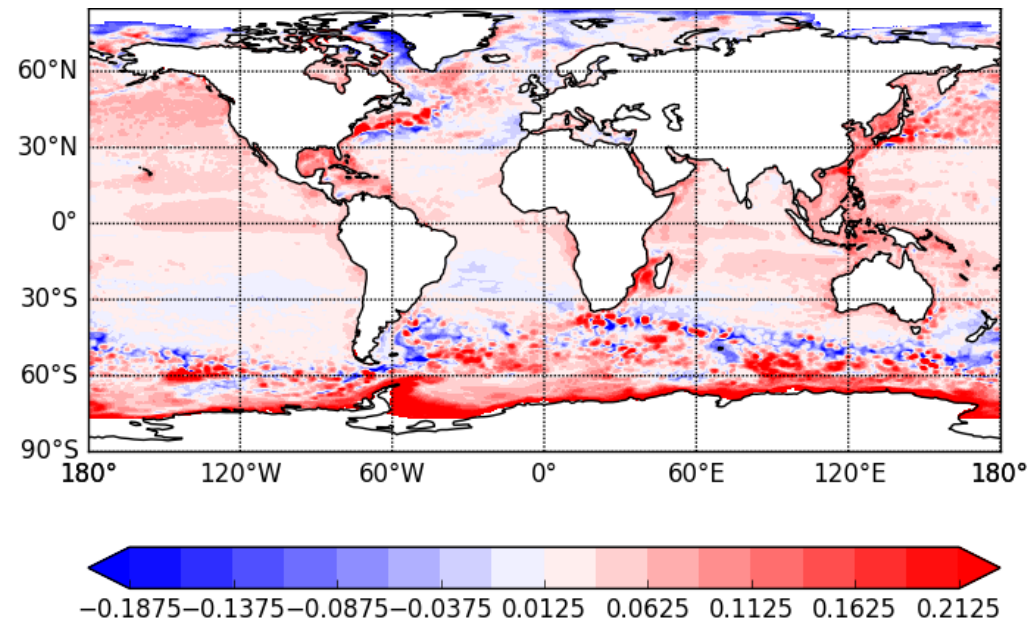
Dynamic Topography

$$\text{MSSH} - \text{Geoid} = \text{MDT}$$

# Assimilation of SSH: MDT

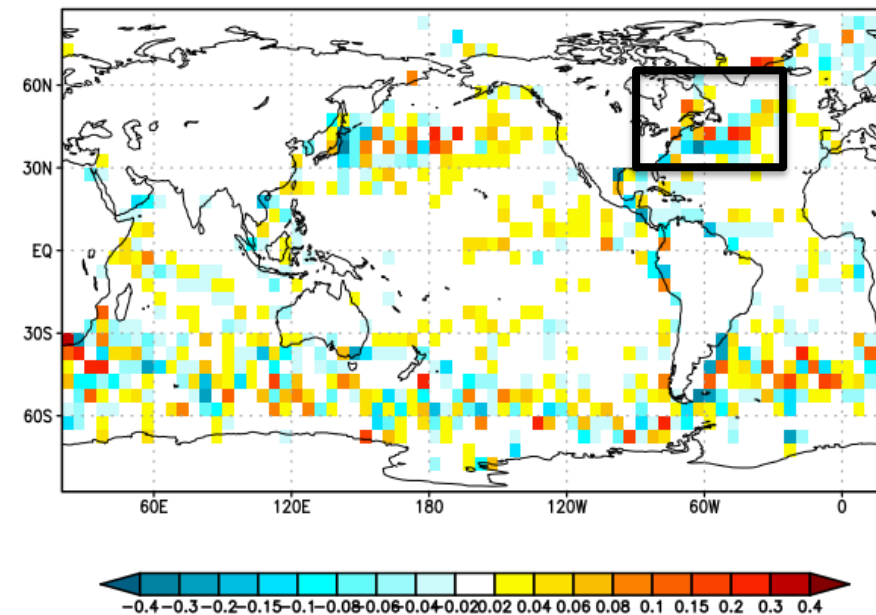
- $MDT_m$  : model MDT as  $\text{mean}(SSH_m)$ , mean model biases not corrected (Balmaseda et al., 2013)
- $MDT_o$  : observation MDT as  $\text{mean}(SSH_o)$ , observation bias not corrected (Waters et al., 2015 and Lellouche et al., 2018)
- $\text{bias corr.}MDT_o$  : observation biases corrected (Lea et al., 2008)

$MDT_m - MDT_o$  (in m)



$\Delta \text{RMSE (O-B): } \text{bias corr.}MDT_o - MDT_m$

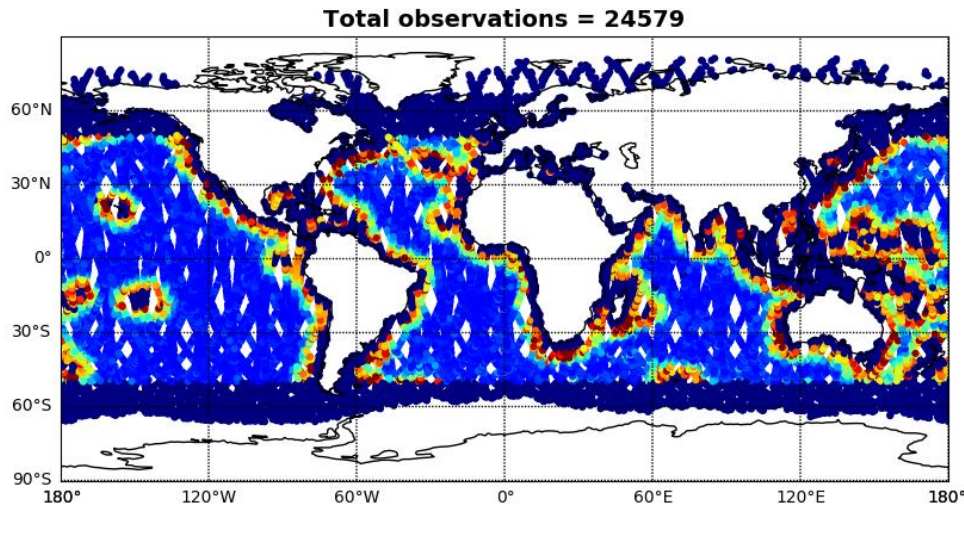
temperature RMS error 242 0–50m



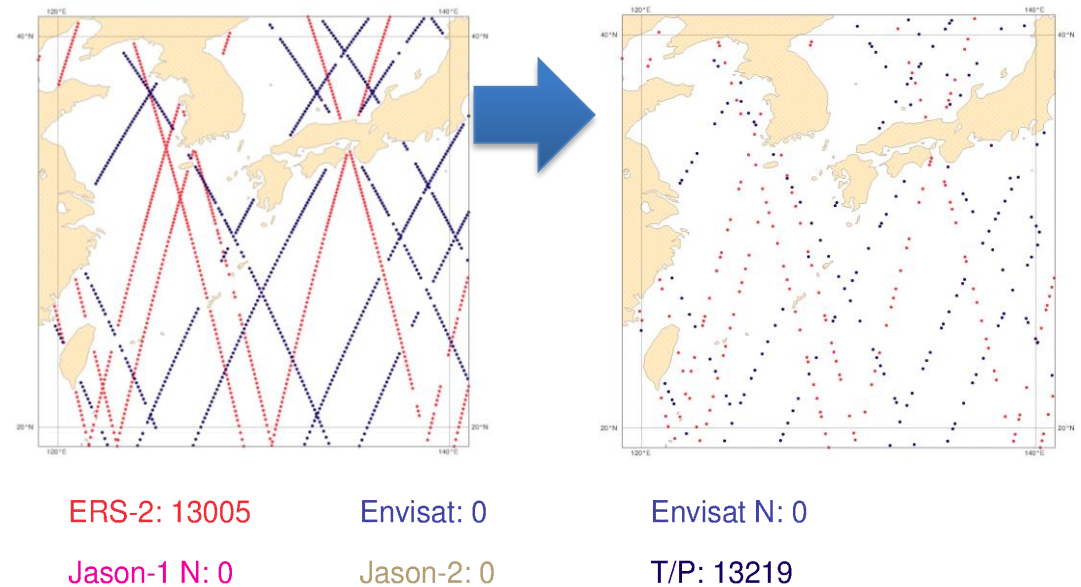
# Assimilation of SSH: thinning/perturbations

- The SLA along track data has very high spatial (9-14km) resolution for the operational ocean assimilation systems.
  - Features in the data which the model can not represent
  - “Overfitting” to SLA obs
- This can be dealt with in different ways:
  - Inflate the observation error
  - Construction of “superobs” or thinning

## Inflate OBE std dev of SSH



## Thinning of SLA obs





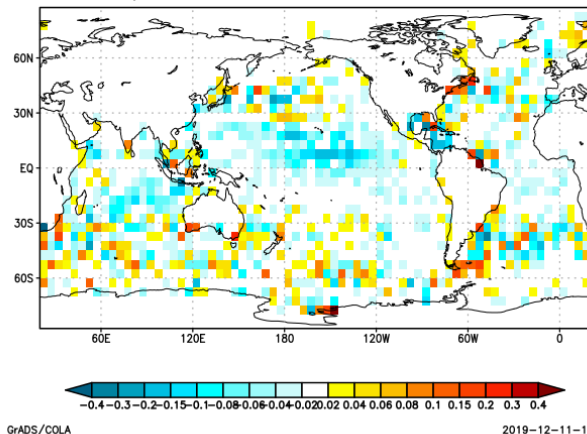
# Assimilation of SSH: impact on ocean states

Assimilation of SSH improves simulated ocean states

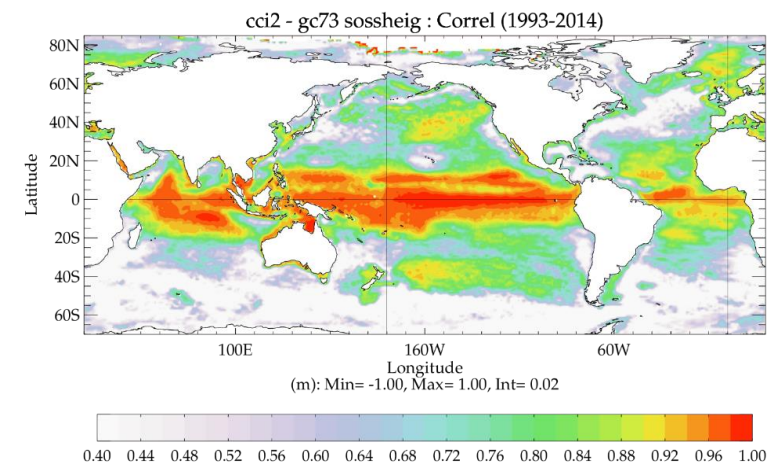
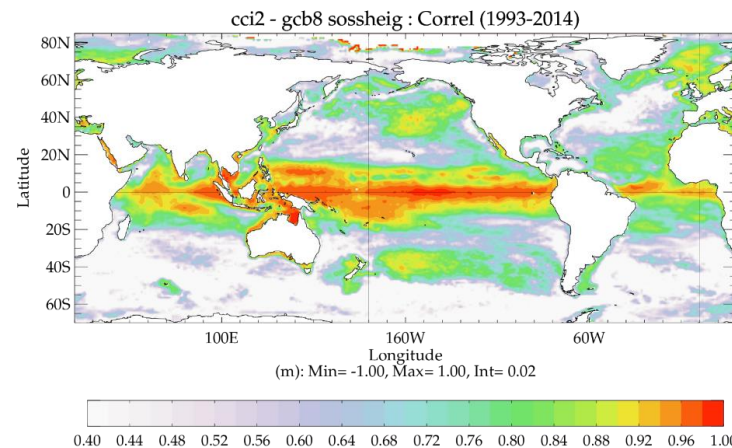
- Global mean sea-level changes
- Regional sea-level changes
- Subsurface temperature and salinity
- Large-scale ocean circulations

**T  $\Delta$ RMSE (O-B):**  
**assim. SSH – not assim. SSH**

temperature RMS error 242 50–200m



**Temporal correlation (monthly) to AVISO data**  
**ORAS5-NoAlti** **ORAS5**



*Zuo et al., 2018*



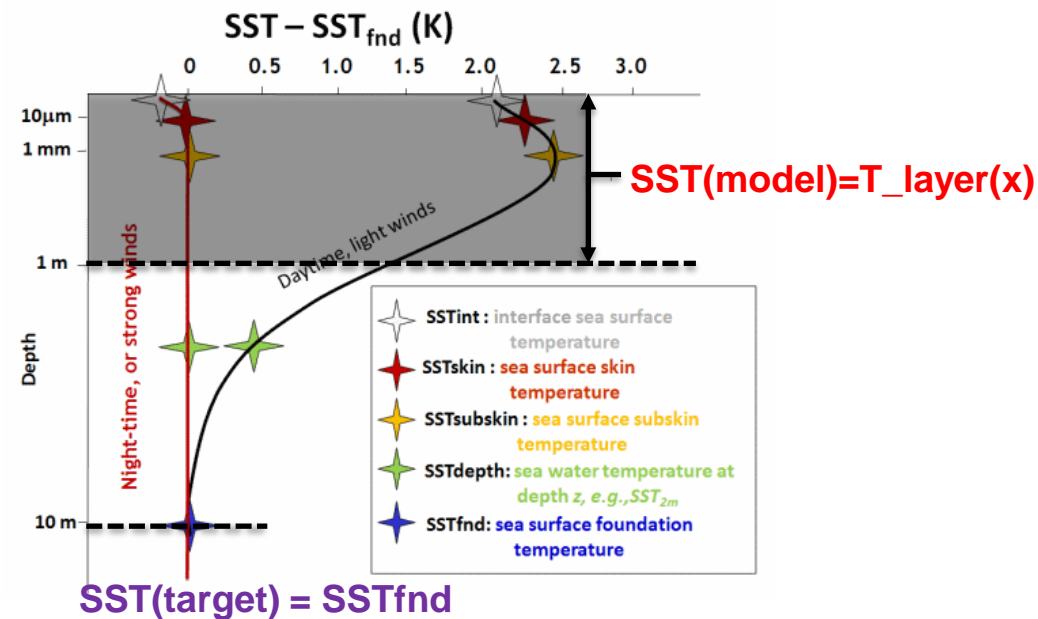
# Nudging SST

- Nudging SST towards L4 gridded data (e.g. OSTIA SST)
- Require flow-dependent nudging strengths

$$Q_{ns} = Q_{ns}^o + \frac{dQ}{dT} (SST_{MODEL} - SST_{TARGET})$$

*Haney 1917*

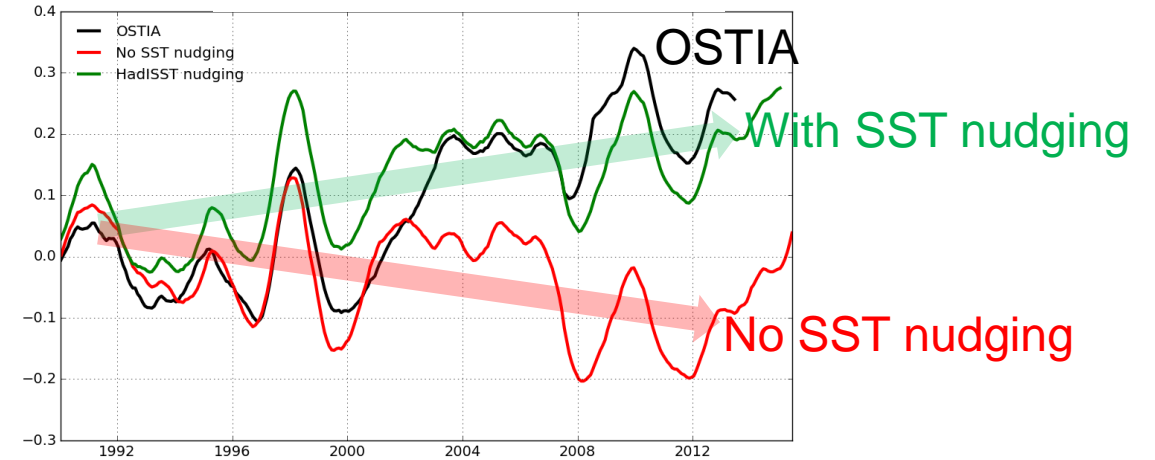
$Q_{ns}$  is non-solar total heat flux  
 $\frac{dQ}{dT}$  is Fixed negative feedback coefficient



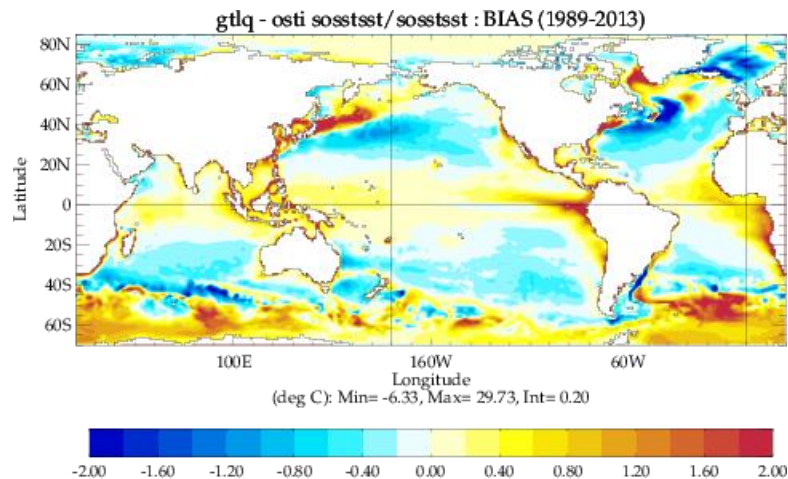
# Impact of SST nudging

- Overall, very effective except for some areas with weak vertical stratification
- Not accounting complicated error characteristics in the L4 SST analysis
- Not accounting vertical correlation when apply SST constrain in the surface

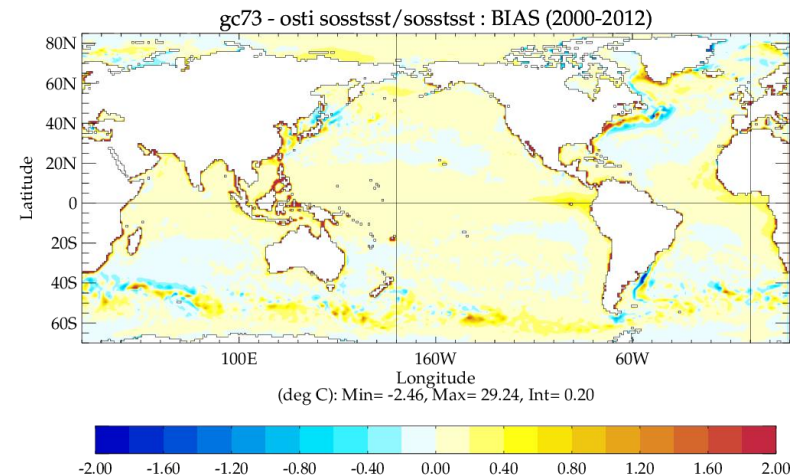
Global mean SST



SST bias: free run - OSTIA



SST bias: ORAS5 - OSTIA

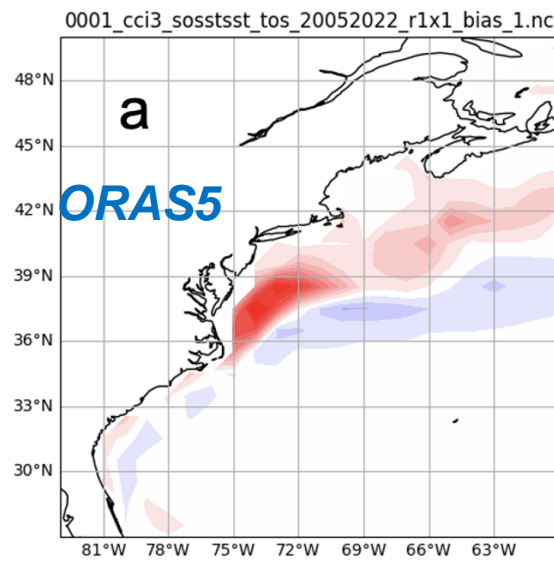


# Assimilation of SST with NEMOVAR

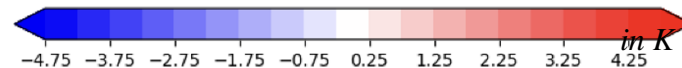
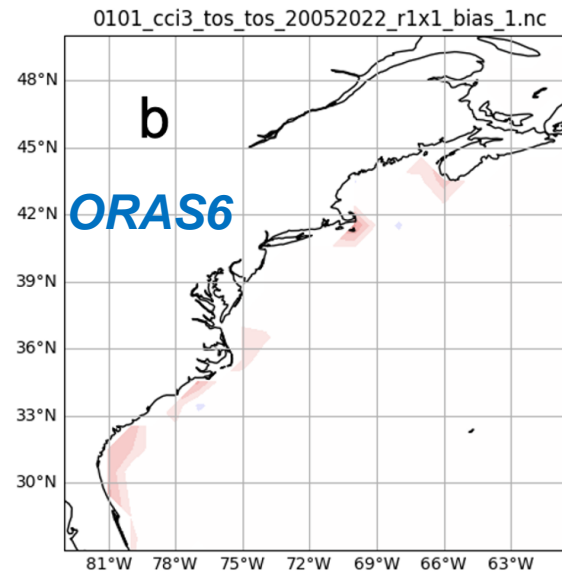
- Assimilation of L4 SST (OSTIA) with **Ens. 3DVar** and **hybrid-B** approach;
- Flow dependent vertical correlation scales is essential;
- SST DA leads to reduced SST biases on the Gulf Stream extensions w.r.t nudging method

## *Biases in SST*

Nudging SST



SST DA with Ens 3DVar

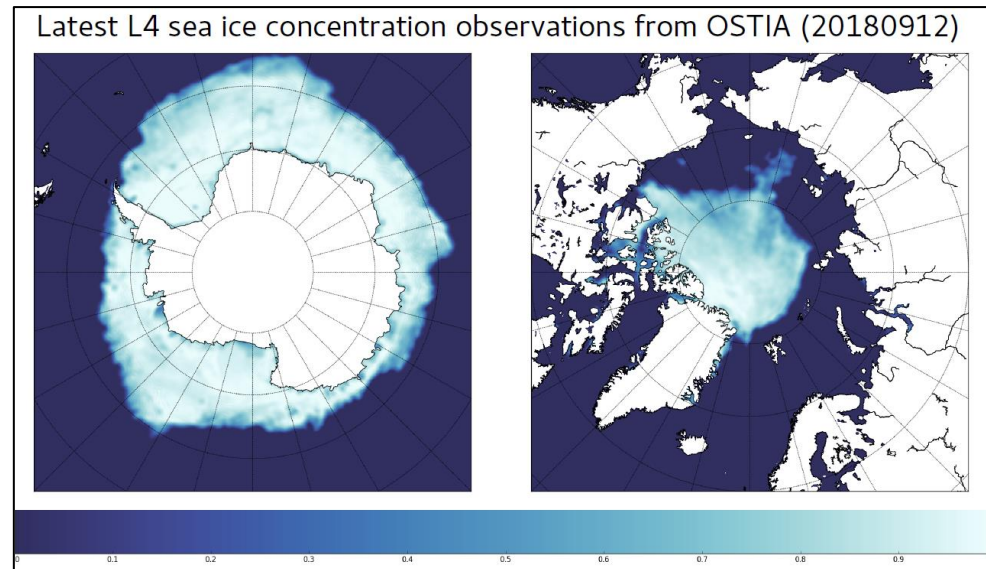


*Zuo et al., 2024, NL180*

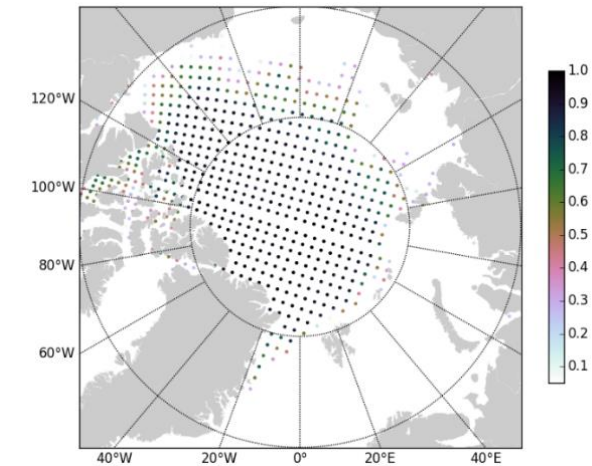
# Sea-ice DA with single-category LIM2 model

Sea-Ice Concentration data from L4 analysis is assimilated through 3DVar scheme in the OCEAN5 system

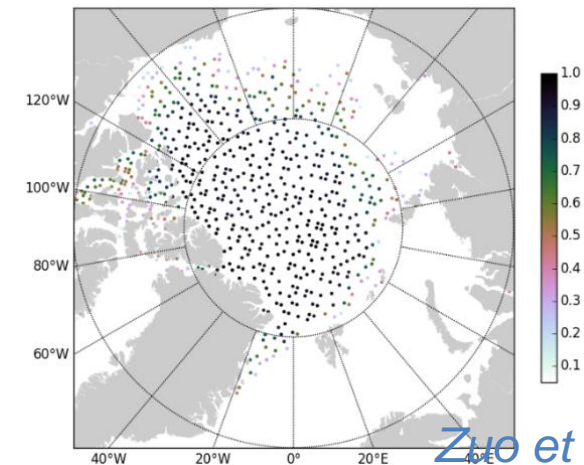
- Treated as univariate
- Pre-thinned via regular or stratified random sampling
- Assimilated through outer-loop coupling in NEMO-LIM2



Regular thinning



random thinning



Zuo et al., 2017

# Sea-ice DA with multi-category SI3 model

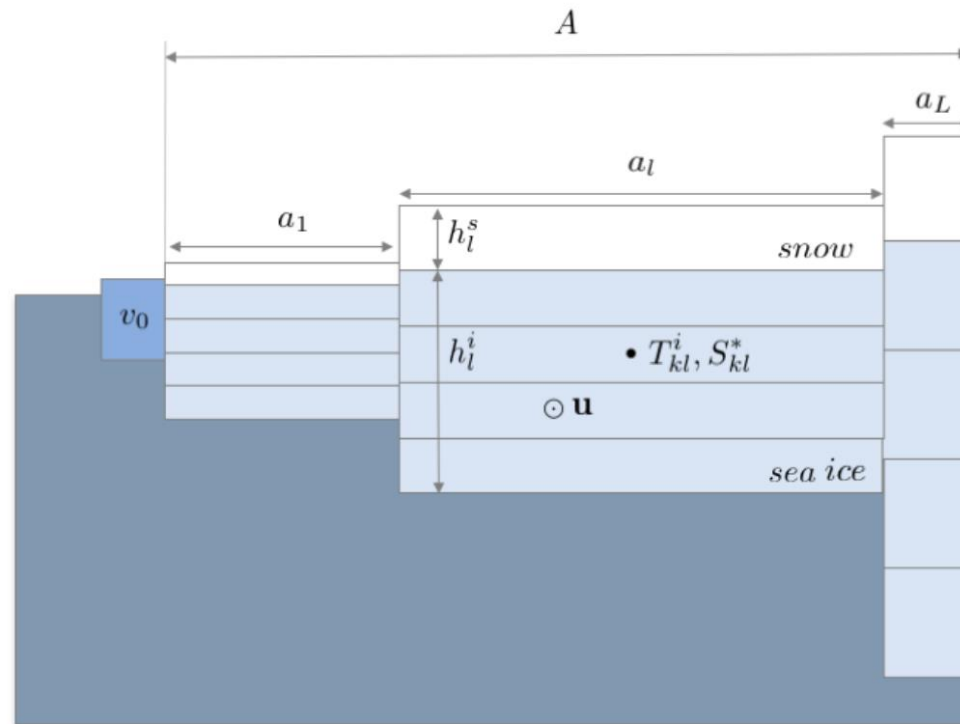


Figure 1.1.: Representation of the ice pack, using multiple categories with specific ice concentration ( $a_l, l = 1, 2, \dots, L$ ), thickness ( $h_l^i$ ), snow depth ( $h_l^s$ ), vertical temperature and salinity profiles ( $T_{kl}^i, S_{kl}^*$ ) and a single ice velocity vector ( $\mathbf{u}$ ).

Considerations with SIC DA in SI3 (multi-category sea-ice model with melt ponds)

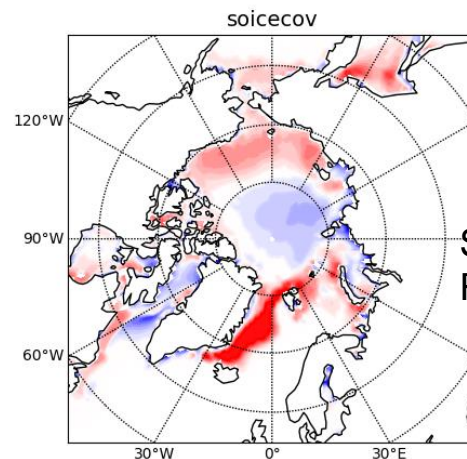
- How to **distribute increments** among different thickness categories
- Where to apply sea-ice increments in the **ice time-stepping scheme**
- Introduce **thermodynamic balance** between sea-ice and ocean state variables
- Grow sea-ice from **open water** with DA increment
- **Interaction** between sea-ice increment and ice advection



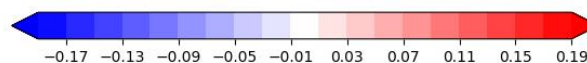
# Impact of Sea-ice DA

Assimilation of SIC data in ORAS5 leads to improved sea-ice state performance in both sea-ice concentration and sea-ice thickness

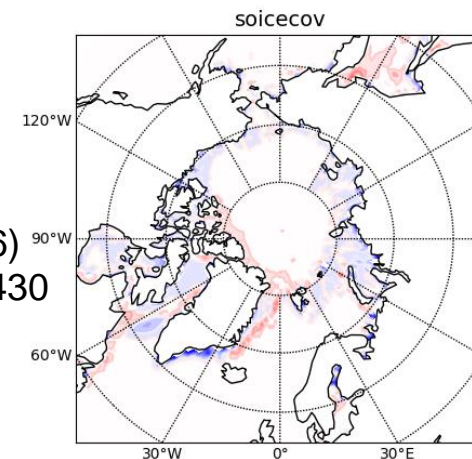
**Without SIC DA**



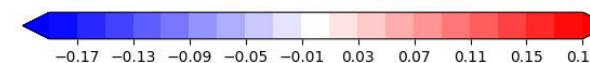
SIC bias (1980-2016)  
Ref data: OSI-SAF 430



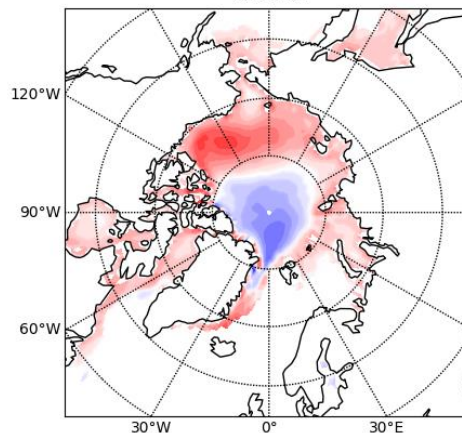
**With SIC DA**



In percent



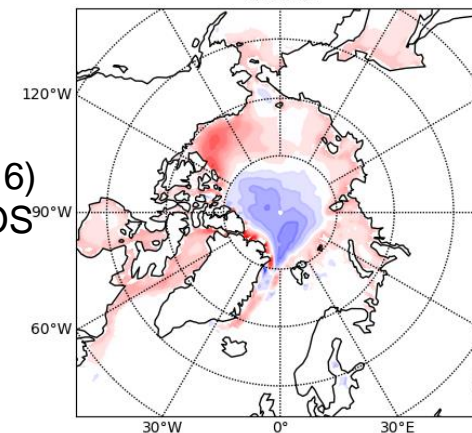
**Without SIC DA**



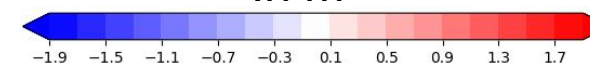
SIT bias (2011-2016)  
Ref data: CS2SMOS  
merged data



**With SIC DA**



In m





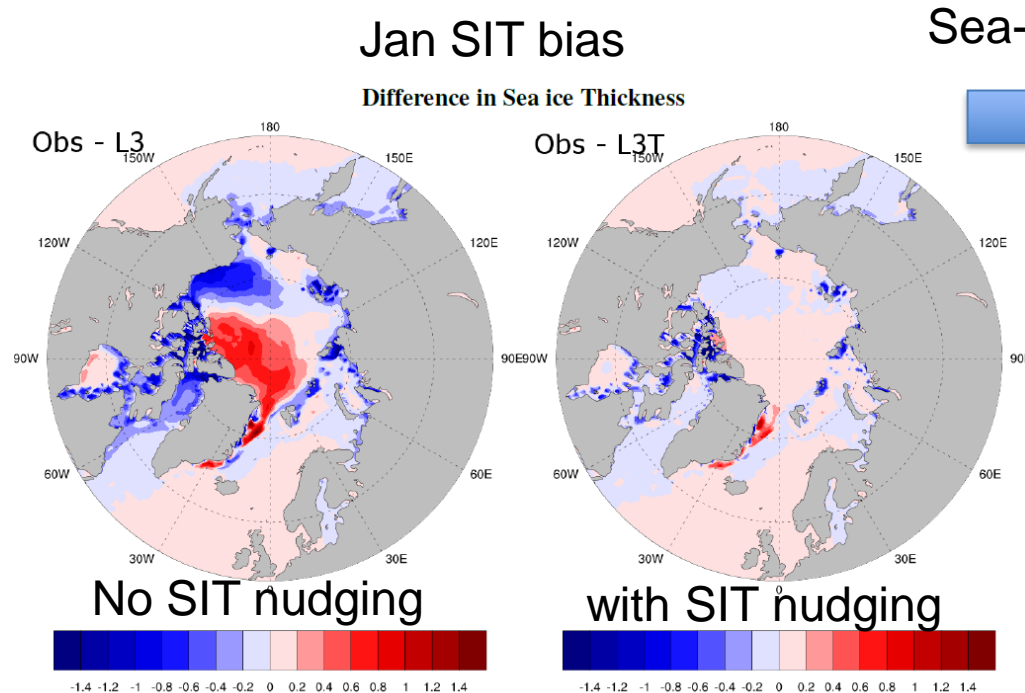
# Nudging sea-ice thickness (SIT)

$$SIT^n = SIT^m - \left[ \frac{\Delta t}{\tau} (SIT^m - SIT^o) \right]$$

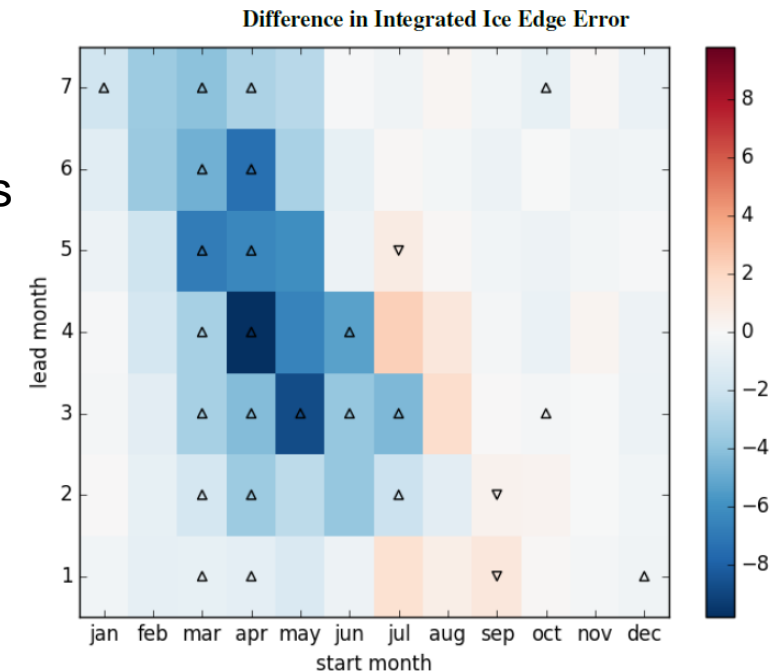
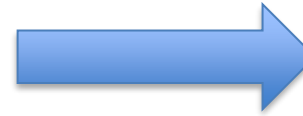
where  $SIT^n$  is the nudged thickness,  $SIT^m$  is the modelled thickness,  $SIT^o$  is the observed thickness,  $\tau$  is the nudging coefficient

Difference in forecast Integrated Ice Edge Error  
(2011-2016, verified against OSI-401b)

**with SIT nudging – No SIT nudging**



Sea-ice forecasts



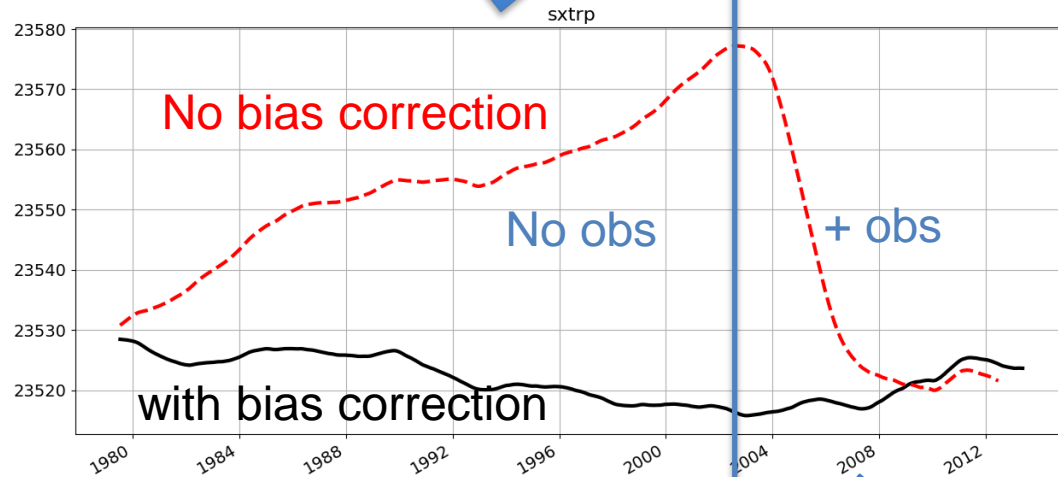
*Balan Sarojini, et al. 2019*

- Ocean system
- Ocean Data Assimilation system - NEMOVAR
- Assimilation of ocean observations (In-situ and satellite data)
- **Bias correction method**
- Coupled Data Assimilation

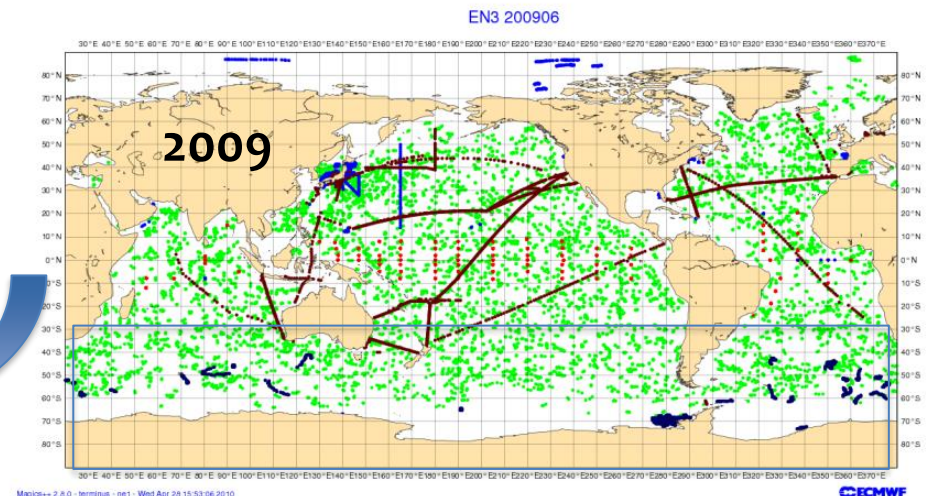
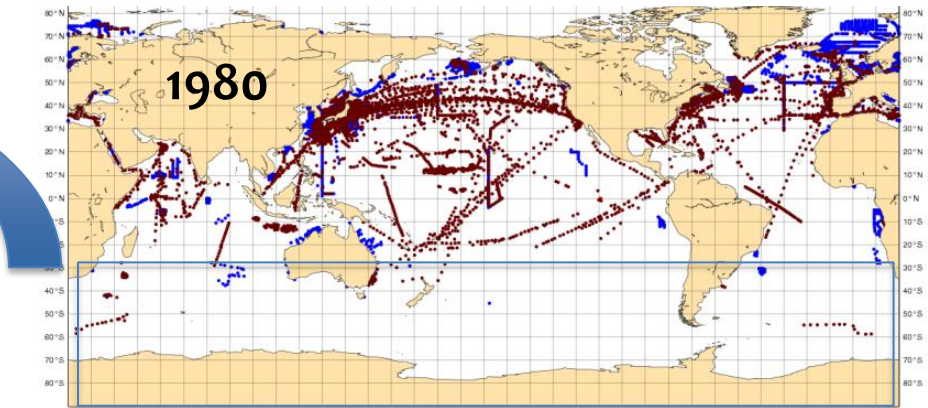
# Why bias correction

To mitigate changes in the observing system. E.g. salt content drift in the Southern Ocean during pre-Argo period due to lack of in-situ observations.

## Salt content (0-700m) in the Southern Ocean



## GOOS

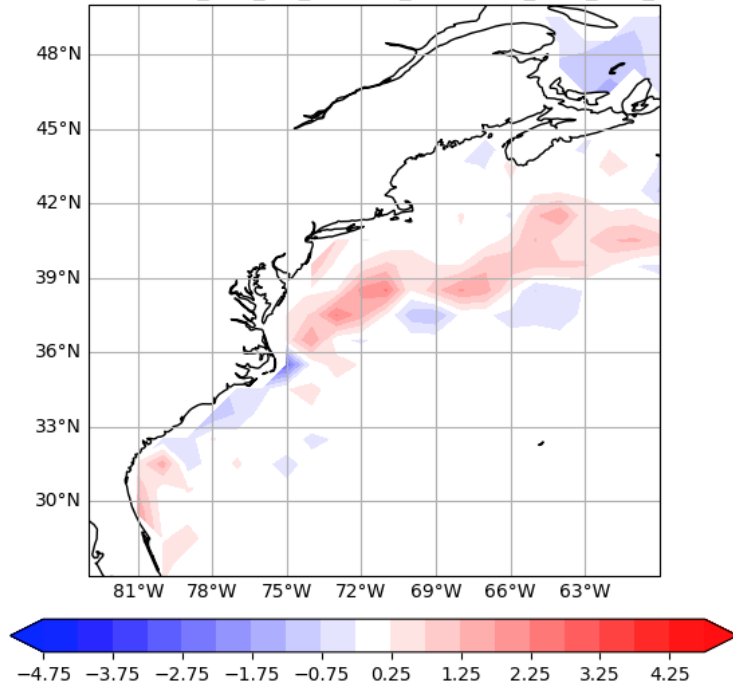


# Why bias correction

To correct systematic errors in models/forcing/boundary conditions. E.g. SST biases in the Gulf Stream regions where the  $\frac{1}{4}$  degree NEMO ocean model has persistent bias.

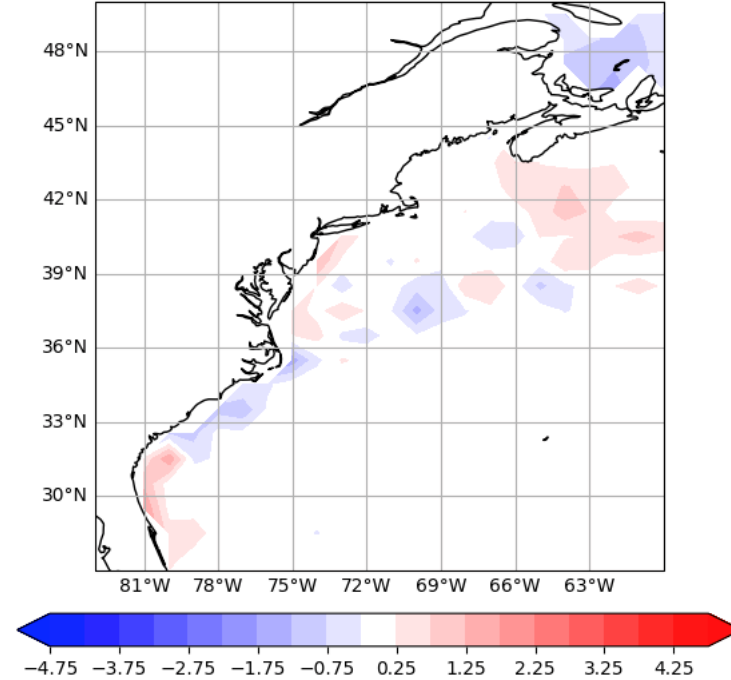
## SST biases (February) in the Gulf Stream Extensions

month=1 hx0k\_cci2\_tos\_sosstsst\_19901994\_r1x1\_bias\_12.nc



+ BC  
➔

month=1 hwec\_cci2\_tos\_sosstsst\_19901994\_r1x1\_bias\_12.nc



# Bias Correction Method

Bias term include two parts, (a) a-priori bias ( $\bar{b}_i$ ) for systematic errors, and (b) a temporal evolution bias term ( $b'_c$ ) for slow evolving signals ([Balmaseda et al, 2007](#), [Zuo et al., 2019](#))

$$b_c = \bar{b}_i + b'_c$$

Seasonal term, estimated offline from data-rich period

Slow varying term, estimated online from assimilation increments

$$b'_c = \alpha b'_{c-1} - A \delta x_{c-1}^a$$

A is a Linear transformation matrix from the state vector increment ( $\delta x_c^a$ ) to bias control vector;  $\alpha$  is the memory factor

$$\bar{b}_i = \bar{b}_{i-1} + \overline{\delta x_i^a}$$

A-priori bias term can be estimated with iterative approach, where  $\bar{b}_0 = 0$ , and  $\overline{\delta x_i^a}$  is averaged increments from the  $i_{th}$  iteration, which should approach to zero with  $i$  increases

The bias correction is used to modify the tendencies of the nonlinear model used in the background and analysis outer loops, so the time evolution of the background and analysis states can be expressed as

$$\begin{aligned} \mathbf{x}_c^b(t_i) &= M(t_i, t_{i-1}) [\mathbf{x}_c^b(t_{i-1}), \mathbf{b}_{c-1}], \\ \mathbf{x}_c^a(t_i) &= M(t_i, t_{i-1}) [\mathbf{x}_c^a(t_{i-1}), \mathbf{b}_{c-1}, F_i \delta \tilde{\mathbf{x}}_c^a] \end{aligned}$$

# Partition of bias correction contributions

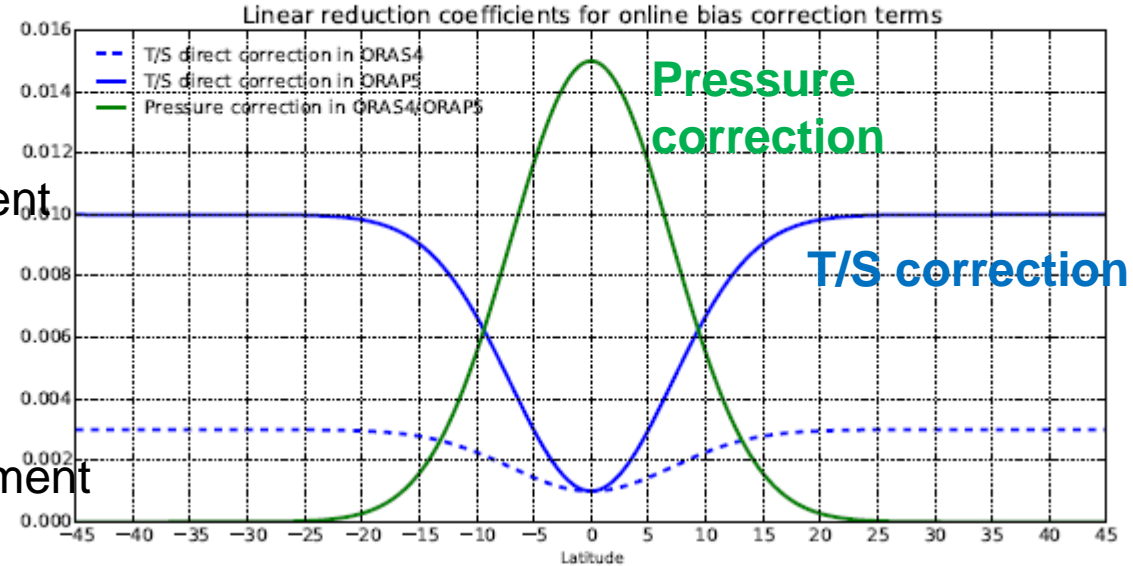
The latitude dependent partition coefficients determine the proportion of online bias corrections applied directly on T/S, and on pressure term. These values ensure that at low latitude the dominant bias term is pressure correction.

$$\mathbf{A} = \begin{bmatrix} a^{tr,T} & 0 & 0 & 0 & 0 \\ 0 & a^{tr,S} & 0 & 0 & 0 \\ a^{p,T} & 0 & 0 & 0 & 0 \\ 0 & a^{p,S} & 0 & 0 & 0 \end{bmatrix}$$

A: Partition matrix, The coefficients in A is latitude dependent in NEMOVAR

1 % increment

0.1 % increment

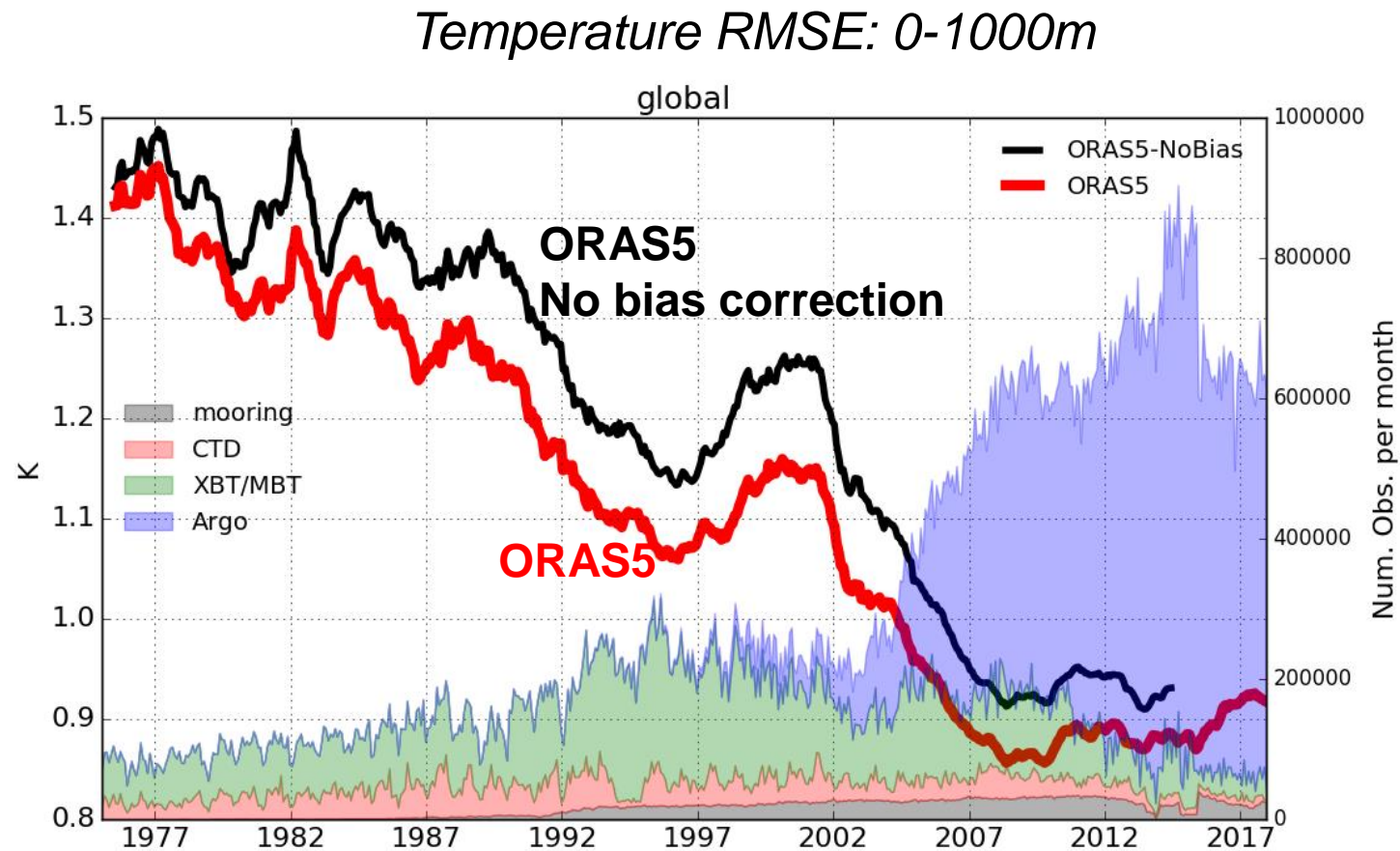


Zuo et al., 2015

Figure 3: Latitude-dependent linear reduction coefficients as applied on online bias correction terms in equations 6 and 7: blue line -  $a^{tr,T/S}$ , reduction coefficients that apply to direct temperature and salinity corrections (different for ORAS4 and ORAP5); and green line -  $a^{p,T/S}$ , reduction coefficients that apply to pressure bias correction.



# Impact of bias correction on ocean reanalysis



Mean: 2005-2014

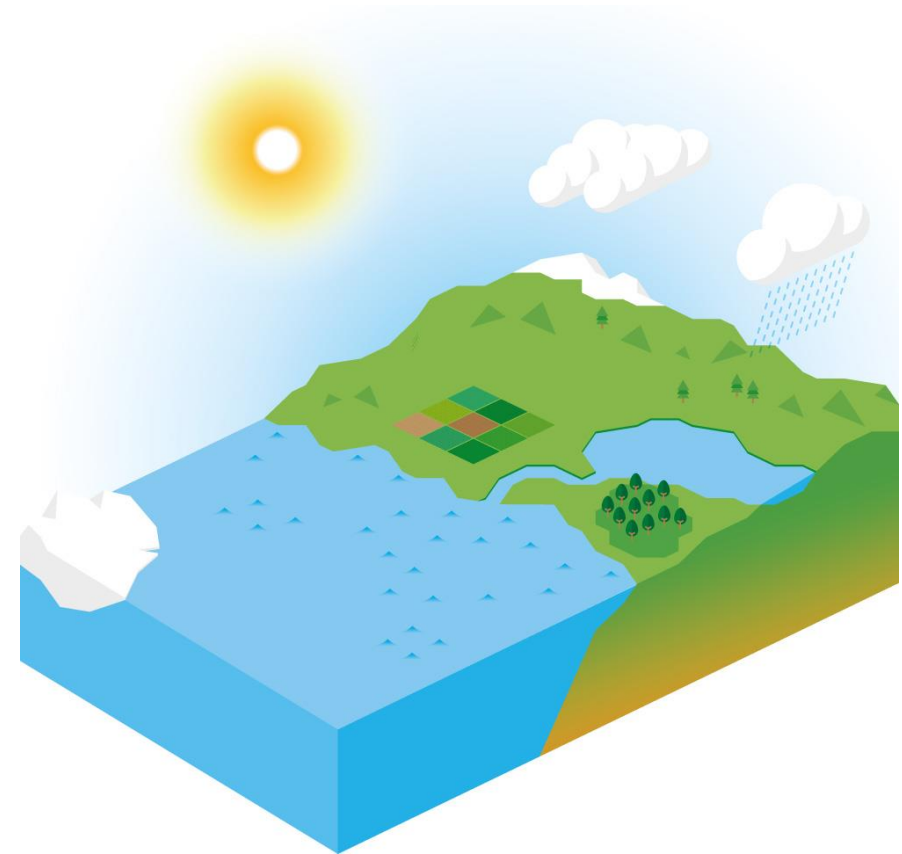
	T RMS reduction	S RMS reduction
In-situ	65%	90%
Bias-corr.	14%	10%

Bias correction in ODA is essential for mitigating spurious signals introduced due to changes in the observing system

- Ocean system
- Ocean Data Assimilation system - NEMOVAR
- Bias correction methods
- Assimilation of ocean observations (In-situ and satellite data)
- **Coupled Data Assimilation**

# Motivation for coupled DA

- ECMWF produces forecasts from 10 days to seasonal timescales
- These are **Earth system** forecasts comprising
  - atmosphere
  - land surface
  - ocean waves
  - ocean
  - sea ice
  - lakes
  - snow
  - chemistry
  - rivers
  - fire
- All of these need initial conditions



# Coupled Data Assimilation

- Coupled DA is the approximations and simplifications we make to data assimilation in a coupled system
  - If we could do it all as per the theory, it would just be Data Assimilation
- Coupled DA aims to:
  - Give the best quality initial conditions for coupled forecasts
  - Extract the maximum information contained in observations
- Coupled DA is a minefield of terms and acronyms:
  - They are largely unhelpful, unstandardized, and confusing
  - One person's coupled system is another person's single component!

## Alternatives to Coupled DA

- Completely ignore all processes in other components
- Use fixed climatology for other components
- Use someone else's analysis of the other components,
  - e.g. Level 4 products as a boundary condition

### “Benefits of not coupling”

You can blame all your problems on errors in external products

### “Disadvantages of coupling”

You have to correct for errors and biases as they are now internal issues

# Reference for Methodology

Received: 21 December 2021 | Revised: 11 May 2022 | Accepted: 12 May 2022 | Published on: 10 August 2022

DOI: 10.1002/qj.4330

Quarterly Journal of the  
Royal Meteorological Society



## RESEARCH ARTICLE

### Coupled data assimilation at ECMWF: current status, challenges and future developments

Patricia de Rosnay<sup>1</sup> | Philip Browne<sup>1</sup> | Eric de Boissésou<sup>1</sup> | David Fairbairn<sup>1</sup> |  
Yoichi Hirahara<sup>1,2</sup> | Kenta Ochi<sup>1,2</sup> | Dinand Schepers<sup>1</sup> | Peter Weston<sup>1</sup> |  
Hao Zuo<sup>1</sup> | Magdalena Alonso-Balmaseda<sup>1</sup> | Gianpaolo Balsamo<sup>1</sup> |  
Massimo Bonavita<sup>1</sup> | Niels Borman<sup>1</sup> | Andy Brown<sup>1</sup> | Marcin Chrust<sup>1</sup> |  
Mohamed Dahoui<sup>1</sup> | Giovanna Chiara<sup>1</sup> | Stephen English<sup>1</sup> | Alan Geer<sup>1</sup> |  
Sean Healy<sup>1</sup> | Hans Hersbach<sup>1</sup> | Patrick Laloyaux<sup>1</sup> |  
Linus Magnusson<sup>1</sup> | Sébastien Massart<sup>1</sup> | Anthony McNally<sup>1</sup> |  
Florian Pappenberger<sup>1</sup> | Florence Rabier<sup>1</sup>

Patricia de Rosnay et al. (2022). “Coupled data assimilation at ECMWF: current status, challenges and future developments”. In: Quarterly Journal of the Royal Meteorological Society 148.747, pp. 2672–2702



# Working within the Variational Data Assimilation framework

Since 1997, 4DVar has been the operational core of the ECMWF atmospheric DA

Initial conditions are found by minimizing an objective (or cost) function

$$J(\mathbf{x}) = \frac{1}{2}(\mathbf{x}_b - \mathbf{x})^T B^{-1}(\mathbf{x}_b - \mathbf{x}) + \frac{1}{2} \sum_k (\mathbf{y} - \mathcal{H}_k(\mathcal{M}_k(\mathbf{x})))^T R_k^{-1}(\mathbf{y} - \mathcal{H}_k(\mathcal{M}_k(\mathbf{x})))$$

This is solved by linearising to get an expression for the gradient

$$-\nabla J(\mathbf{x}) = B^{-1}(\mathbf{x}_b - \mathbf{x}) + \sum_k M_k^T H_k^T R_k^{-1}(\mathbf{y} - \mathcal{H}_k(\mathcal{M}_k(\mathbf{x})))$$

And finally, once the minimization has completed, we cycle the system to the next assimilation window by running a model forward in time

$$\mathbf{x}_b = \mathcal{M}(\mathbf{x}_a)$$

## First things first - coupled control vectors

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

The components  $x_1$  and  $x_2$  can

- Be on different grids
- Have different representations (eg. spectral vs grid point)
- Stored on different processors
- Stored in very different places in the code (i.e. different modules)
- Not even represented in the same code!

So there can be huge technical challenges to overcome before one can even write down a coupled state vector.

## The other pieces which might be coupled

$\mathcal{M}$  to cycle the model

$\mathcal{M}$  within the cost function

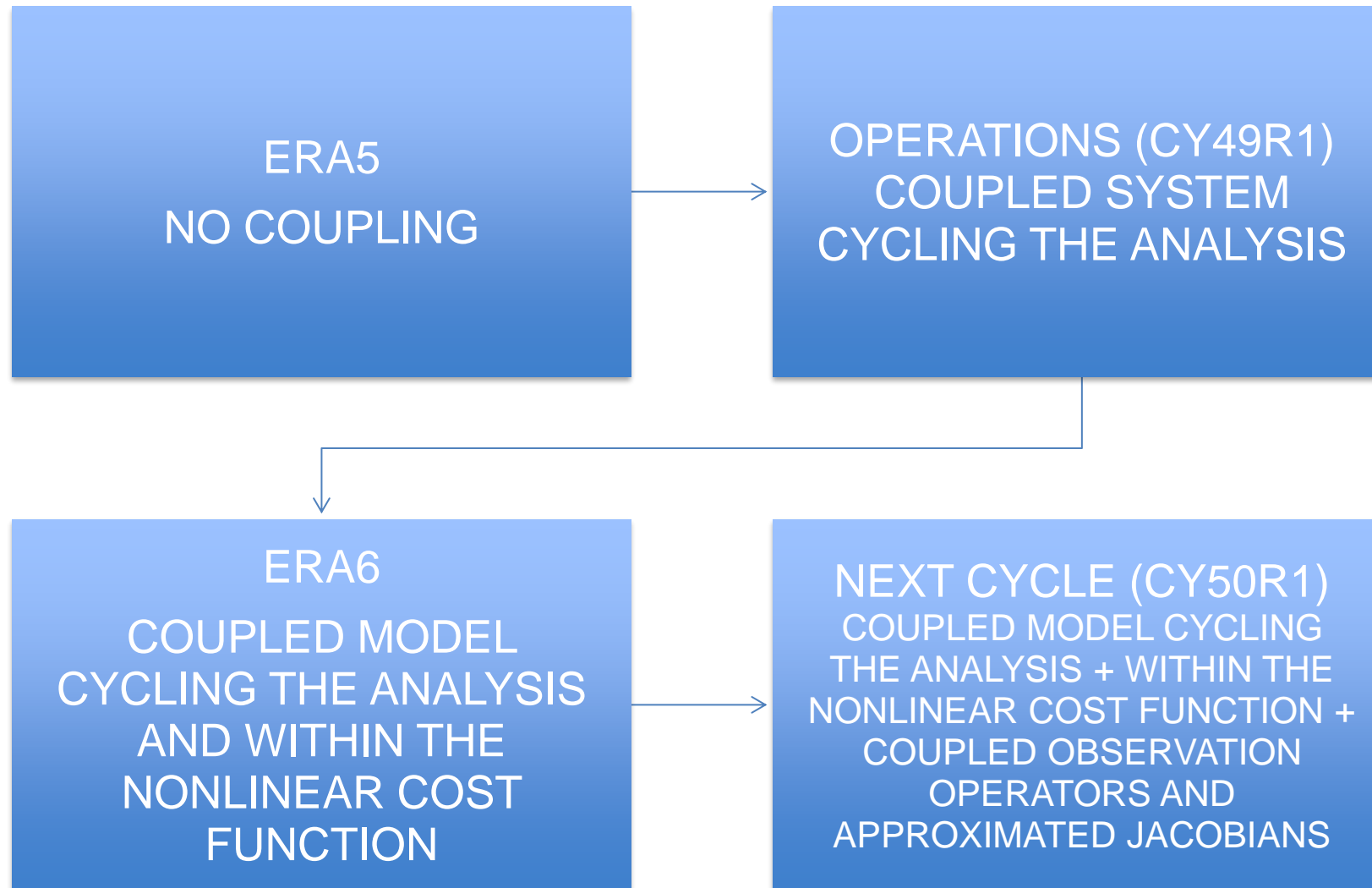
B – background error covariance coupling

H – coupled observation operators

$H^T$  – coupled Jacobians

$M^T$  – coupled TL/AD models

# ECMWF ocean-atmosphere coupled systems



# Summary

- **Data assimilation in the ocean serves a variety of purposes**, from climate monitoring to initialization of coupled model forecasts and ocean mesoscale prediction.
- ECMWF ocean analysis relies on NEMOVAR for data assimilation, which uses an incremental **3DVar-FGAT** configuration and linearized cost function. The BGE covariance is modelled use **balance** operator and **diffusion** operator.
- Assimilation of ocean observations reduces the **large uncertainty** due to model and forcing errors. It improves the initialization of coupled forecasts in NWP and provides **calibration and initialization** for reforecast for seasonal forecasts and decadal forecasts.
- **Coupled data assimilation** can provide consistent initial conditions for the ECMWF coupled forecasting system, by gradually introducing coupling in the cost function components (model trajectories; obs oper and jacobians; TD/AL and B).

# Further Readings

## Ocean data assimilation

- Balmaseda, M. A., Dee, D., Vidard, A., & Anderson, D. L. T. (2007). A multivariate treatment of bias for sequential data assimilation: Application to the tropical oceans. *Quarterly Journal of the Royal Meteorological Society*, 133(622), 167–179.
- Mogensen, K., Alonso Balmaseda, M., & Weaver, A. (2012). The NEMOVAR ocean data assimilation system as implemented in the ECMWF ocean analysis for System 4. Technical Memorandum (Vol. 668).
- Weaver, A. T., Deltel, C., Machu, É., Ricci, S., & Daget, N. (2005). A multivariate balance operator for variational ocean data assimilation. *Quarterly Journal of the Royal Meteorological Society*, 131(613), 3605–3625.
- Zuo, H., Balmaseda, M. A., Tietsche, S., Mogensen, K., & Mayer, M. (2019). The ECMWF operational ensemble reanalysis-analysis system for ocean and sea-ice : a description of the system and assessment. *Ocean Science*, (January), 1–44.

## Coupled data assimilation

- Patricia de Rosnay et al. (2022). “Coupled data assimilation at ECMWF: current status, challenges and future developments”. In: *Quarterly Journal of the Royal Meteorological Society* 148.747, pp. 2672–2702