

# Observation errors

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NWP SAF Training Course

# Outline

1. What are observation errors?
2. Estimating observation errors
3. Specification of observation errors in practice
4. Accounting for observation error correlations
5. Summary

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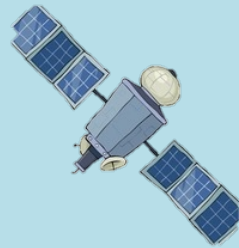
# Errors in observations

- **Every observation has an error vs the truth:**
  - **Systematic error**
    - Needs to be removed through bias correction (previous lecture)
  - **Random error**
    - Topic of this lecture!

# Contributions to observation error

## Measurement error

E.g., instrument noise for satellite radiances

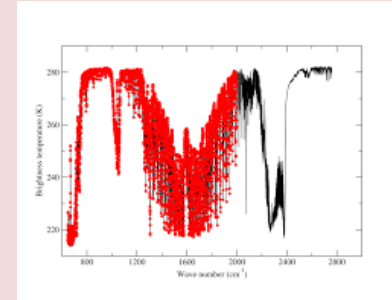


## Representation error

(e.g., Janjić et al 2017)

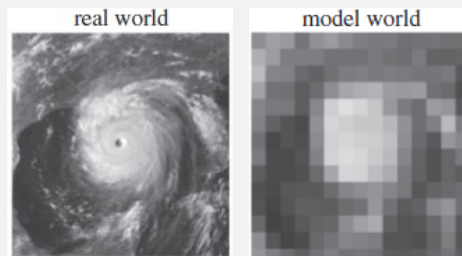
## Forward model (observation operator) error

E.g., radiative transfer error



## Representativeness error

E.g., point measurement vs model representation



## Quality control/pre-processing error

E.g., error due to the cloud detection scheme missing some clouds in clear-sky radiance assimilation



# Contributions to observation error

## Measurement error

E.g., instrument noise for satellite radiances

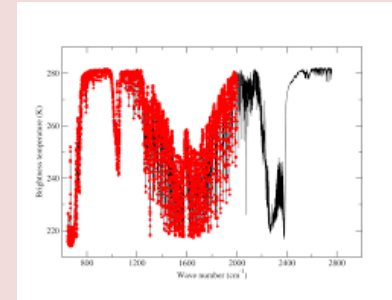


## Representation error

(e.g., Janjić et al 2017)

## Forward model (observation operator) error

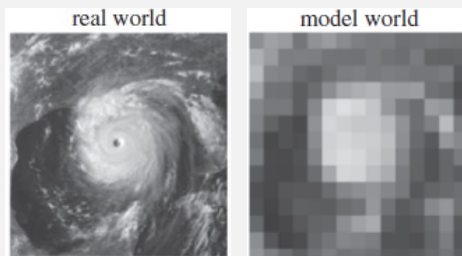
E.g., radiative transfer error



- Are the errors situation-dependent?
- Are the errors correlated (spatially, temporally, between channels)?
- Are the errors systematic (→bias correction)?

## Representativeness error

E.g., point measurement vs model representation



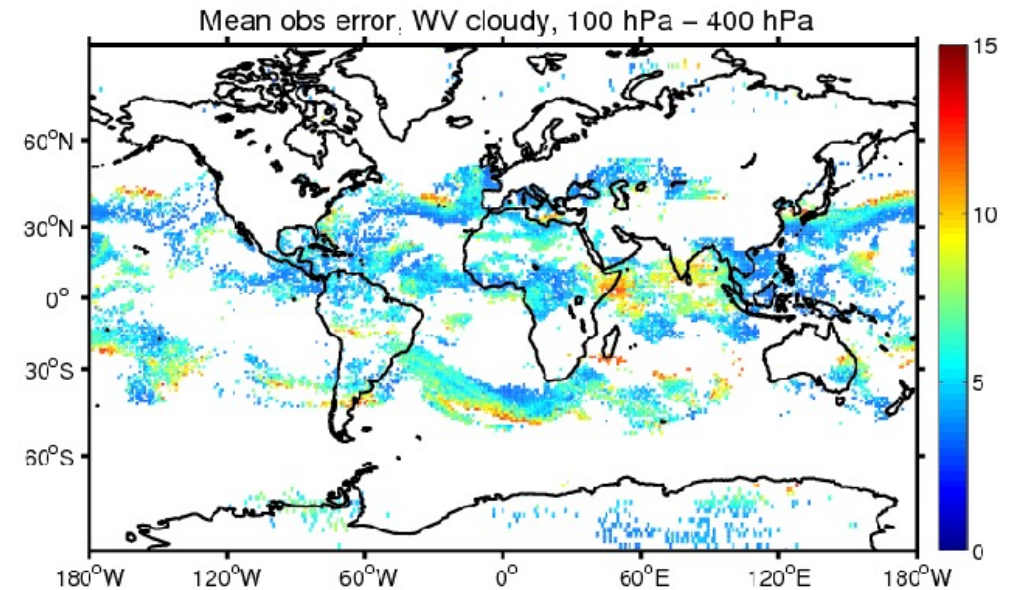
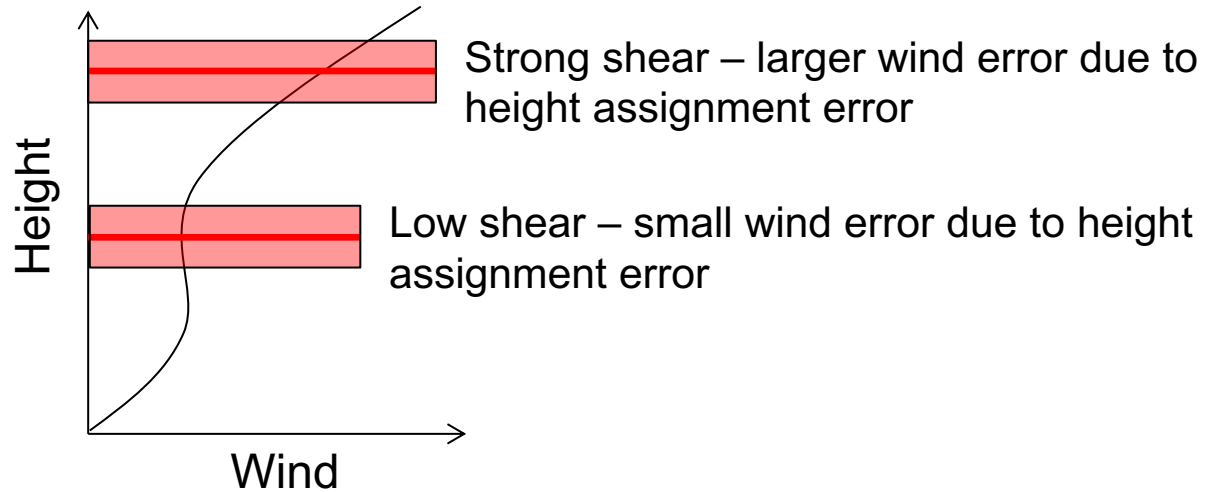
## Quality control/pre-processing error

E.g., error due to the cloud detection scheme missing some clouds in clear-sky radiance assimilation



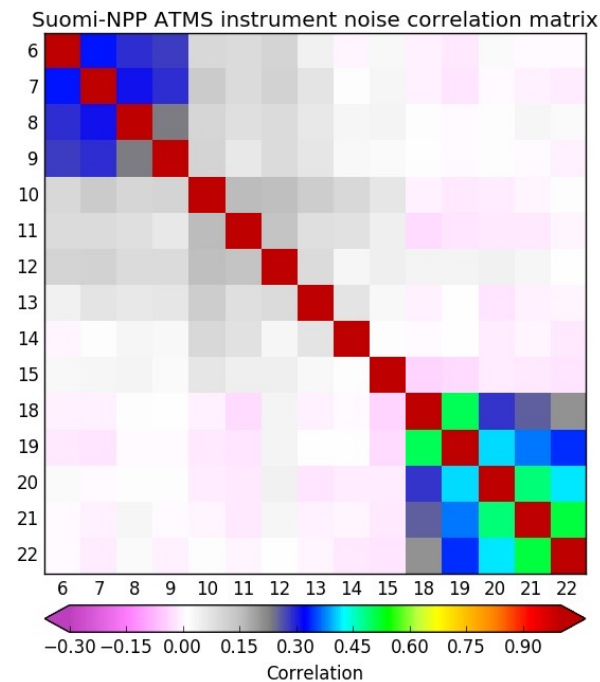
# Examples of situation-dependence of representation error

- Cloud/rain-affected radiances: Representativeness error is much larger in cloudy/rainy regions than in clear-sky regions
- Effect of height assignment error for Atmospheric Motion Vectors:



# Examples of correlated observation error

- Different channels with similar radiative transfer error.
- Different channels with similar error in spatial representativeness.
- Different channels with similar cloud sensitivity in clear-sky assimilation.
- Even instrument noise can be correlated.





## Observation error and the cost function

- In data assimilation, observation errors are commonly assumed Gaussian.
- Denoted by the observation error covariance matrix “**R**” in the observation cost function:

$$J(\mathbf{x}) = \frac{1}{2} (\mathbf{x} - \mathbf{x}_b)^T \mathbf{B}^{-1} (\mathbf{x} - \mathbf{x}_b) + \frac{1}{2} (\mathbf{y} - \mathbf{H}[\mathbf{x}])^T \mathbf{R}^{-1} (\mathbf{y} - \mathbf{H}[\mathbf{x}])$$

- It is often specified through the square root of the diagonals (“ $\sigma_o$ ”) and a correlation matrix (which can be the identity matrix).

## Role of observation error

- $\mathbf{R}$  and the background error  $\mathbf{B}$  together determine the weight of an observation in the assimilation.
- In the linear case, the minimum of the cost function can be found at  $\mathbf{x}_a$ :

$$\underbrace{(\mathbf{x}_a - \mathbf{x}_b)}_{\text{Increment}} = \mathbf{B}\mathbf{H}^T (\mathbf{H}\mathbf{B}\mathbf{H}^T + \mathbf{R})^{-1} \underbrace{(\mathbf{y} - \mathbf{H}\mathbf{x}_b)}_{\substack{\text{Departure, innovation,} \\ \text{"o-b"}}}$$

- “*Large*” *observation error* → *smaller increment*, analysis draws less closely to the observations
- “*Small*” *observation error* → *larger increment*, analysis draws more closely to the observations

# Current observation error specification for satellite data in the ECMWF system

- Globally constant, diagonal:
  - Scatterometer data
- Globally constant fraction, dependent on impact parameter; diagonal:
  - GPS-RO
- Globally constant, inter-channel error correlations taken into account:
  - IASI, CrIS, AIRS, ATMS (with different values for different satellites)
- Situation dependent, diagonal:
  - All-sky treatment of radiances from passive microwave instruments: dependent on satellite, channel and cloud amount
  - AMVs: dependent on level and shear (and satellite, channel, height assignment method)
  - Aeolus: based on physically estimated error for each derived wind

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# How can we estimate observation errors?

- Observation errors are **departures from the truth** – which we don't know.
- We can only **estimate** observation errors. Several methods exist to do this, broadly categorised as:

- **Error inventory:**

- Based on considering all contributions to the error/uncertainty

- **Diagnostics with collocated observations, e.g.:**

- Hollingsworth/Lönnberg on collocated observations
- Triple-collocations/3-cornered hat

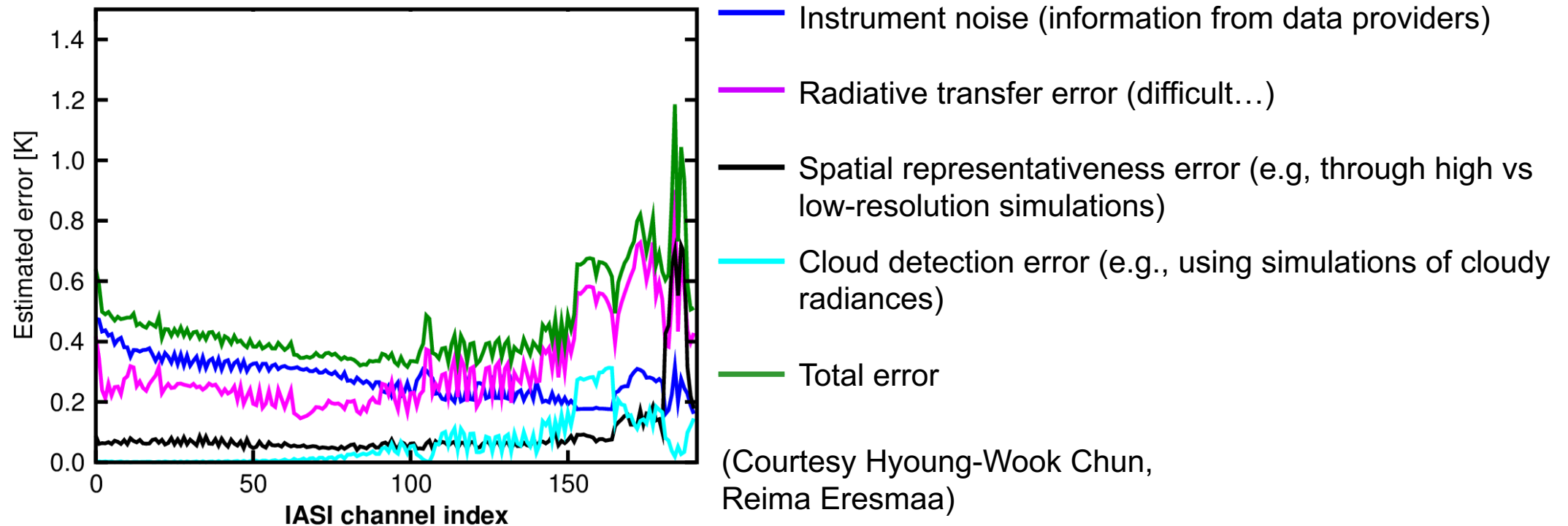
- **Diagnostics based on output from DA systems, e.g.:**

- O-b statistics
- Hollingsworth/Lönnberg
- Desroziers et al 2005
- Methods that rely on an explicit estimate of B

- **Adjoint-based methods**

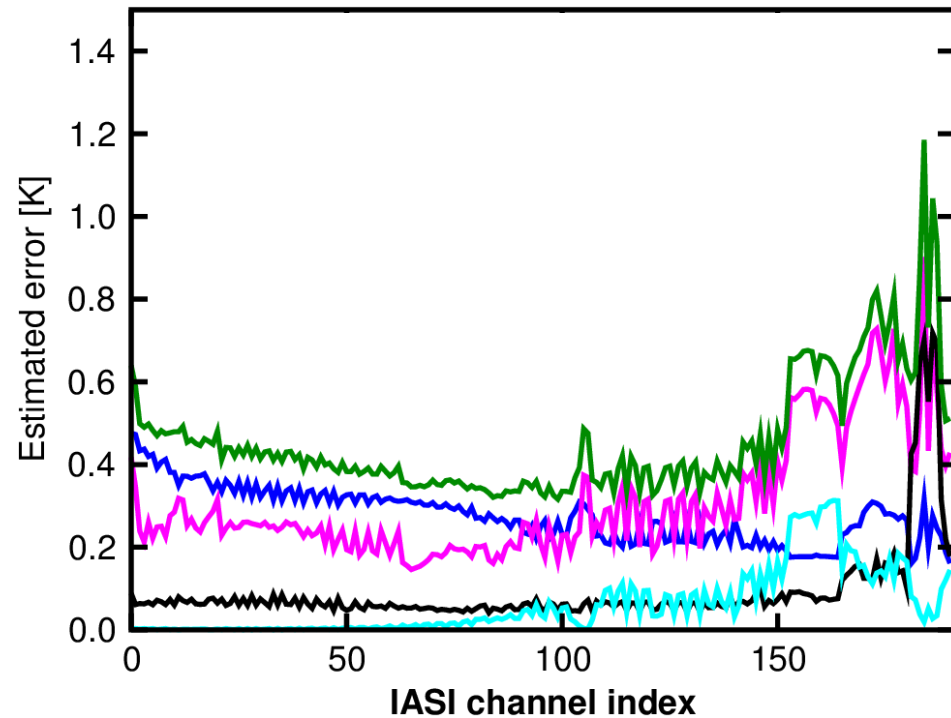
# Error inventory

- Estimate the error from *physical estimates of all uncertainty* contributions.
- Example: error inventory for IASI

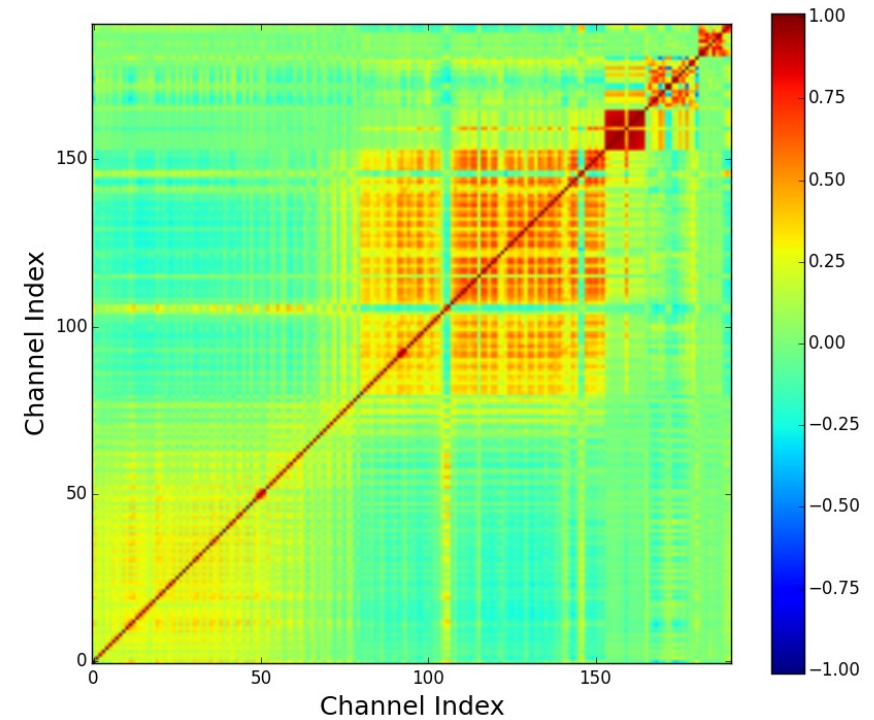


# Error inventory

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**Total error correlation**

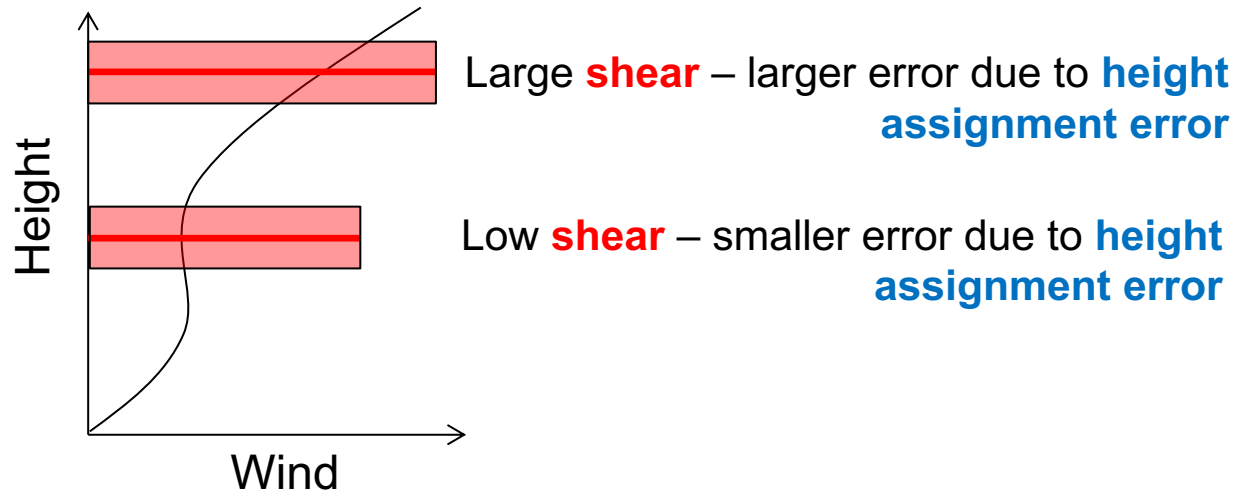


- Very useful to *understand* error contributions.
- *How realistic* is each estimate?

# Error inventory and physical observation error models

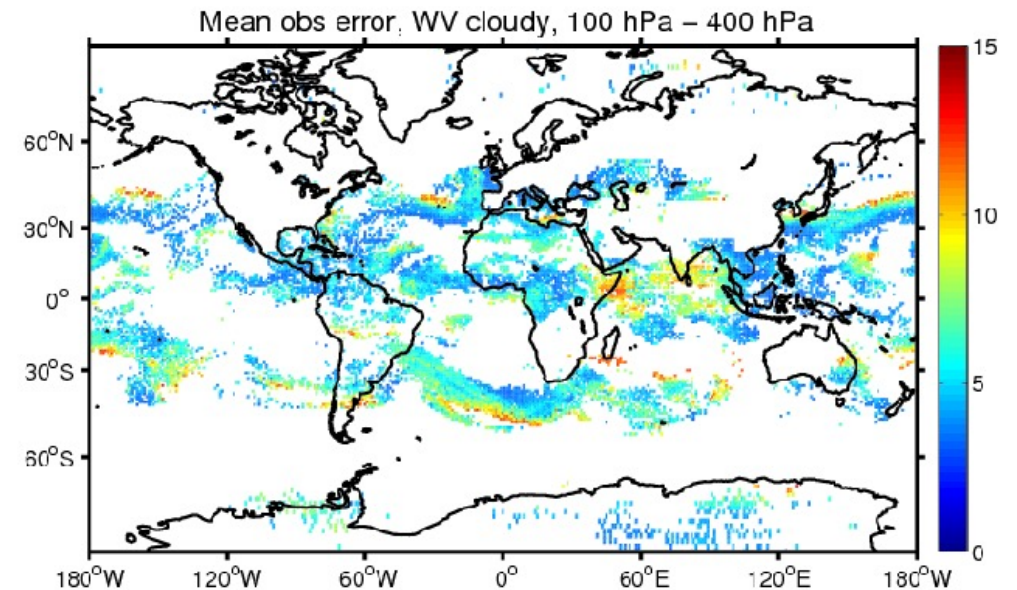
- Other applications of an inventory approach:
  - Physical error models: propagate parameter uncertainty through observation operator/retrieval
  - Useful for identifying leading contributors of observational uncertainty
  - Basis for “observation error models” to capture situation-dependence of observation errors

## Example: Atmospheric Motion Vectors and the error due to height assignment:



An observation error model for the height assignment uncertainty could be:

$$\sigma_{HA} \approx \sigma_p \left( \frac{dv}{dp} \right)$$





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- **Adjoint-based methods**

## Departure-based diagnostics

- Several methods have been developed that are based on departures from data assimilation systems (ie o-b, o-a).

- If observation errors and background errors are **uncorrelated** then:

$$\text{Cov}[(\mathbf{y} - \mathbf{H}[\mathbf{x}_b]), (\mathbf{y} - \mathbf{H}[\mathbf{x}_b])] = \mathbf{H}\mathbf{B}_{true}\mathbf{H}^T + \mathbf{R}_{true}$$

- In this case, stdev(o-b) is an **upper bound for  $\sigma_o$** .
- Statistics of background departures give information on **observation and background error combined**. To separate the two, we need to **make assumptions** (which may or may not be true).

# Departure-based observation error diagnostics: Methods that rely on an estimate of the background error

- **Basic assumptions:**

- Background and observation error are *uncorrelated*.
- We have a *reliable estimate of the background error*, for instance:

- Background error is small:

$$\mathbf{R} = \text{Cov}[(\mathbf{y} - \mathbf{H}[\mathbf{x}_b]), (\mathbf{y} - \mathbf{H}[\mathbf{x}_b])] \quad - \quad \delimit{HBH^T}$$

- Or: we “know”  $\mathbf{H} \mathbf{B}_{\text{true}} \mathbf{H}^T$  from the assimilation system:

$$\mathbf{R} = \text{Cov}[(\mathbf{y} - \mathbf{H}[\mathbf{x}_b]), (\mathbf{y} - \mathbf{H}[\mathbf{x}_b])] \quad - \quad \mathbf{HBH}^T$$

# Departure-based observation error diagnostics: Hollingsworth/Loennberg method

- **Basic assumption:**

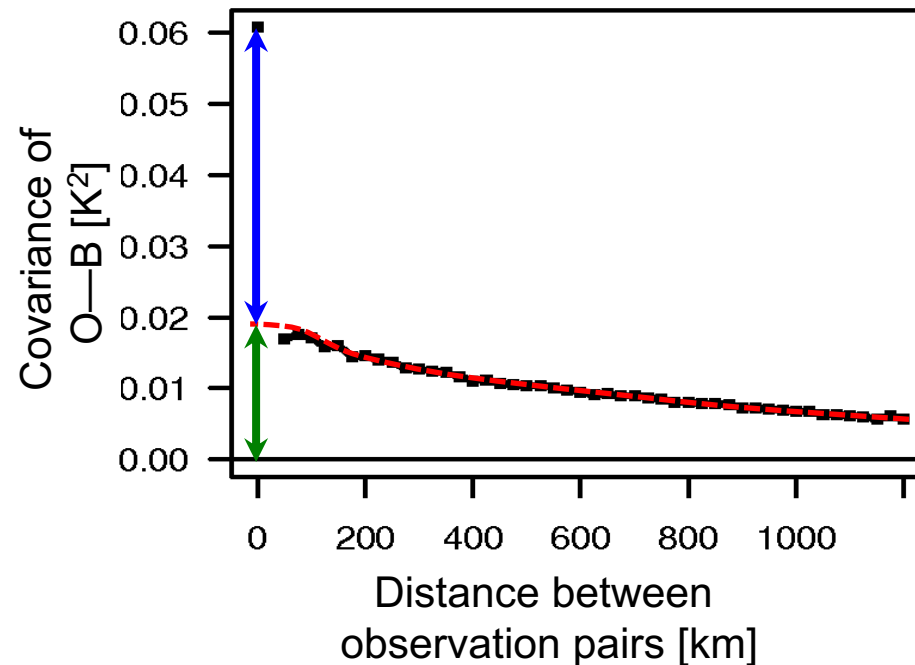
- Background errors are spatially correlated, whereas observation errors are not.
- This allows to separate the two contributions to the variances of background departures.

- **Recipe:**

- Take a large database of pairs of departures and bin by distance between the observations.
- Calculate covariance of departures for each bin.

- **Drawback:**

- Not reliable when observation errors are spatially correlated.



**Spatially uncorrelated variance**  
→ **Observation error**

**Spatially correlated variance**  
→ **Background error**

# Departure-based observation error diagnostics: Desroziers diagnostic (I)

- **Basic assumptions:**

- Assimilation process can be adequately described through linear estimation theory.
- Weights used in the assimilation system are consistent with true observation and background errors.

- Then the following relationship can be derived:

$$\mathbf{R} = \text{Cov}[\mathbf{d}_a, \mathbf{d}_b]$$

with  $\mathbf{d}_a = (\mathbf{y} - \mathbf{H}[\mathbf{x}_a])$  (analysis departure)

$\mathbf{d}_b = (\mathbf{y} - \mathbf{H}[\mathbf{x}_b])$  (background departure)

(see Desroziers et al. 2005, QJRMS)

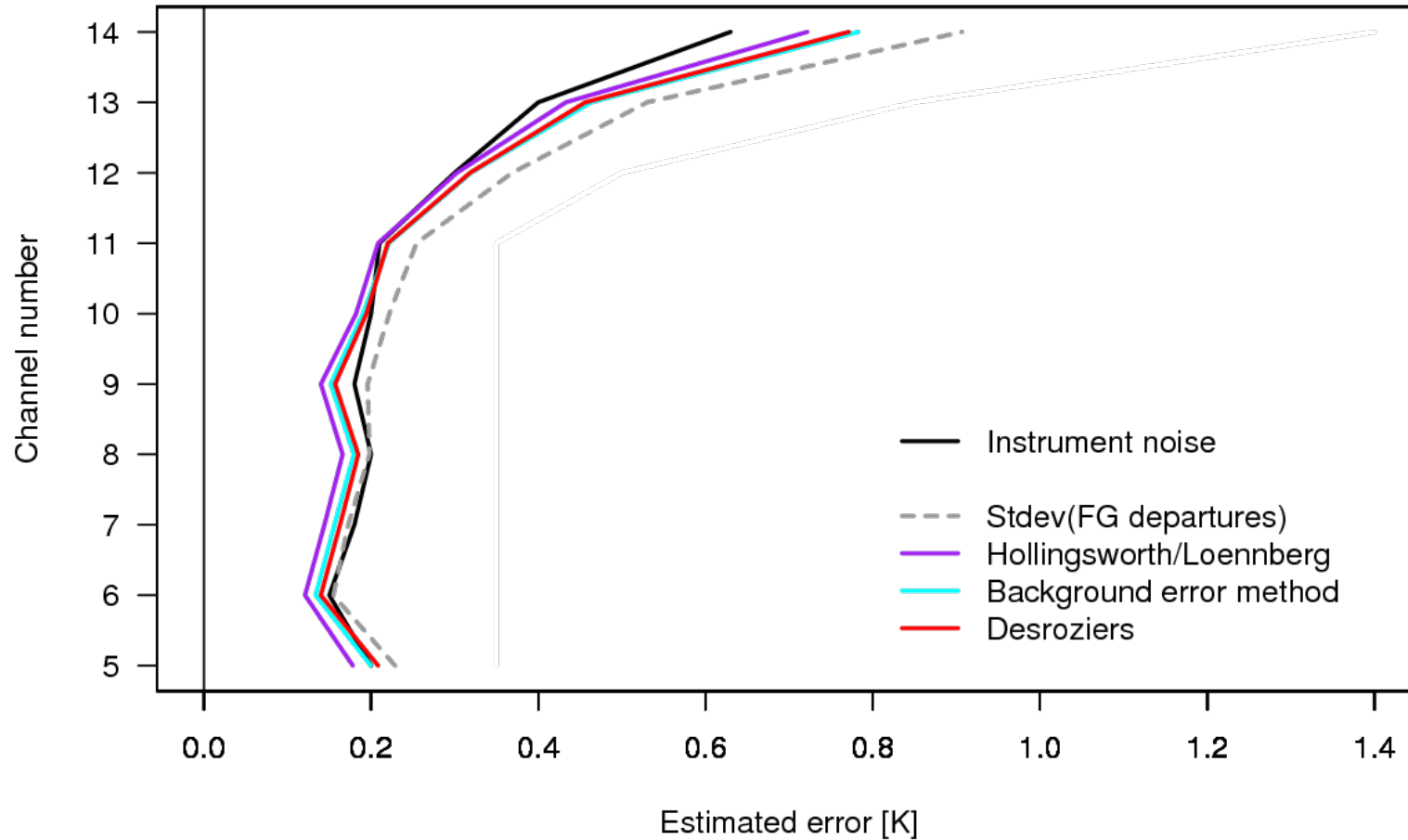
- **Consistency diagnostic** for the specification of  $\mathbf{R}$ . Increasingly used to estimate  $\mathbf{R}$ .

## Some points on departure-based diagnostics

- All departure-based diagnostics rely on assumptions (which may or may not be true):
  - Assume we **know the background error** characteristics → remove B
  - Assume a **certain structure of the errors** → Hollingsworth/Lönnberg
  - Assume **weights** used in the assimilation system are **accurate** → Desroziers diagnostic
- All diagnostics additionally assume that the **error in the observations and background are uncorrelated.**
- **Before applying any diagnostic, think about whether the assumptions are likely to be true.**
- It is best to **use several diagnostics** to avoid misleading estimates due to violated assumptions.
- Diagnostics do not tell you where the error comes from.
  - **Additional physical understanding** of the error sources will be beneficial → **error inventory.**
  - Diagnostics can be used together with physical error models.

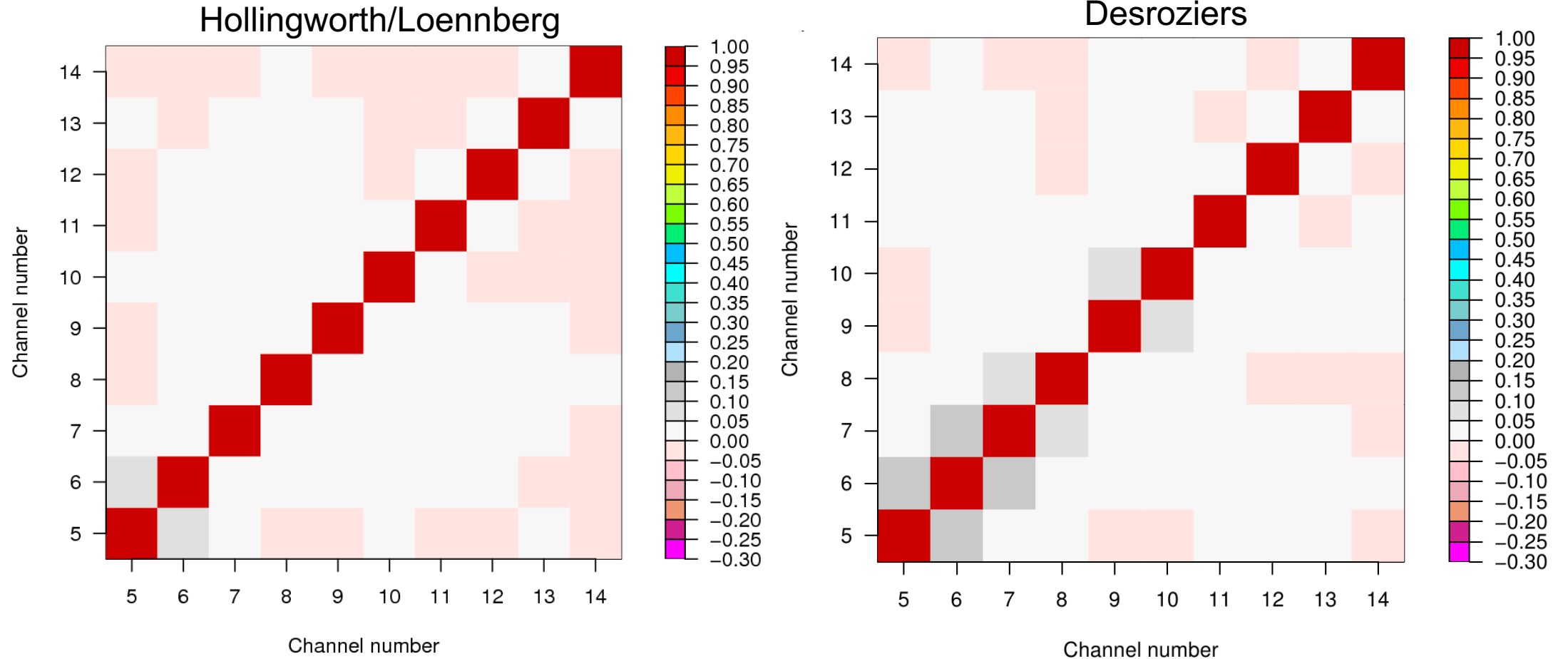
# Examples of applying observation error diagnostics: AMSU-A

## Diagnostics for $\sigma_0$



# Examples of applying observation error diagnostics: AMSU-A

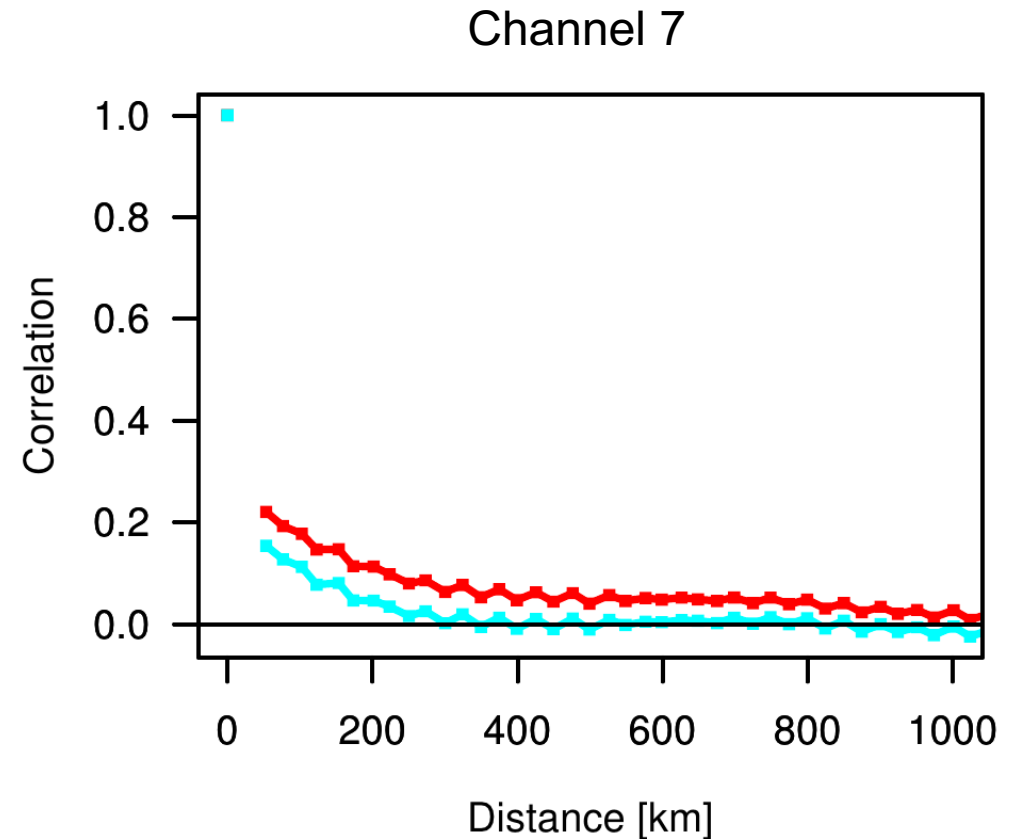
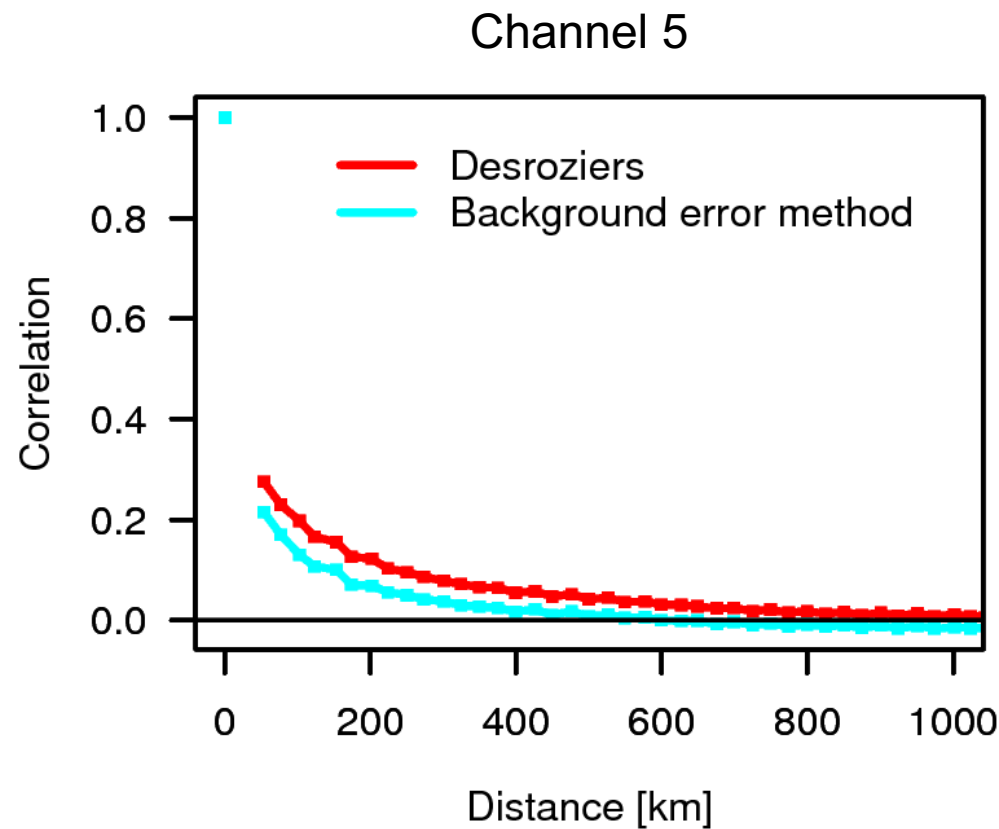
## Inter-channel error correlations:





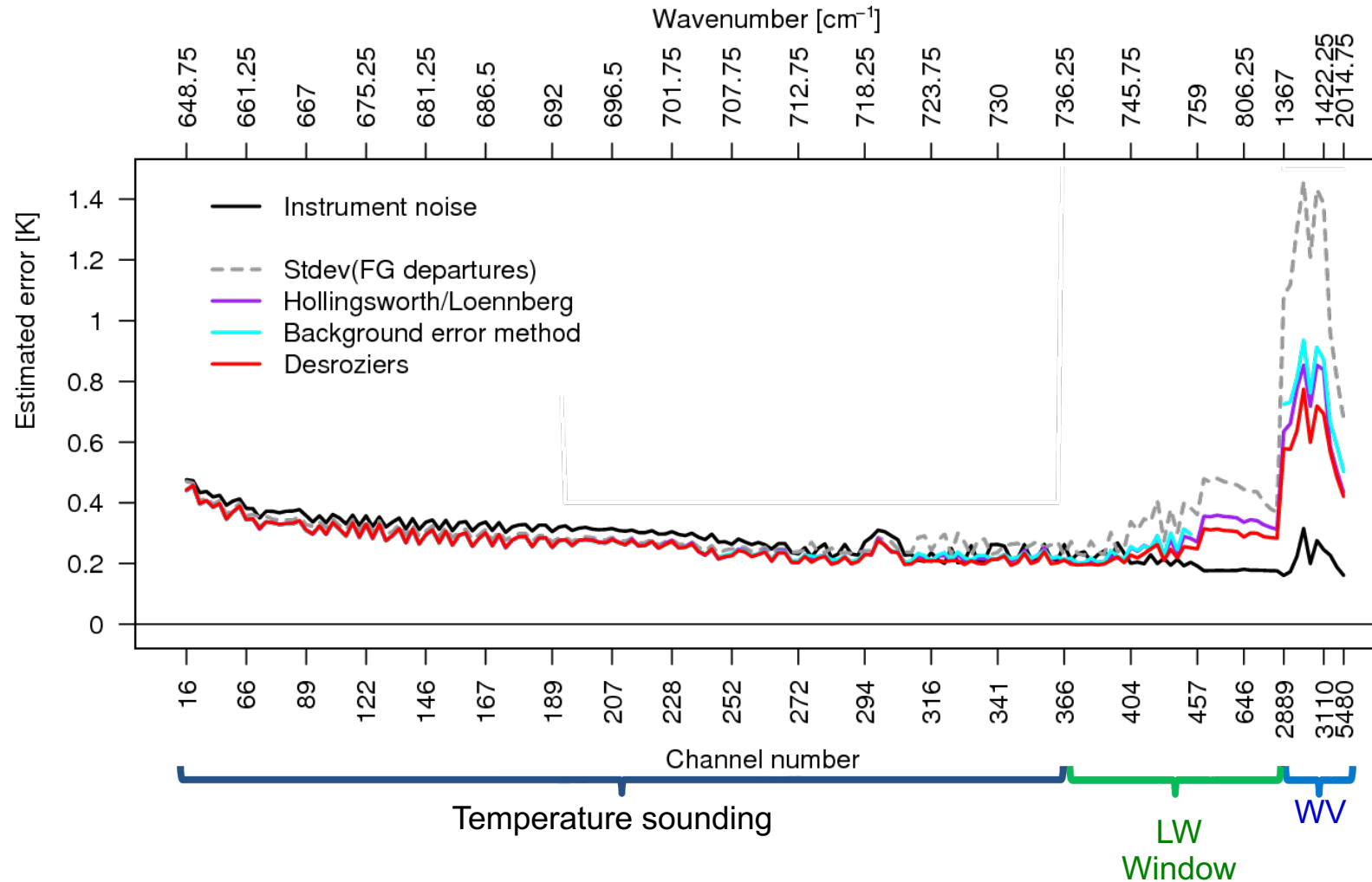
# Examples of applying observation error diagnostics: AMSU-A

## Spatial error correlations:



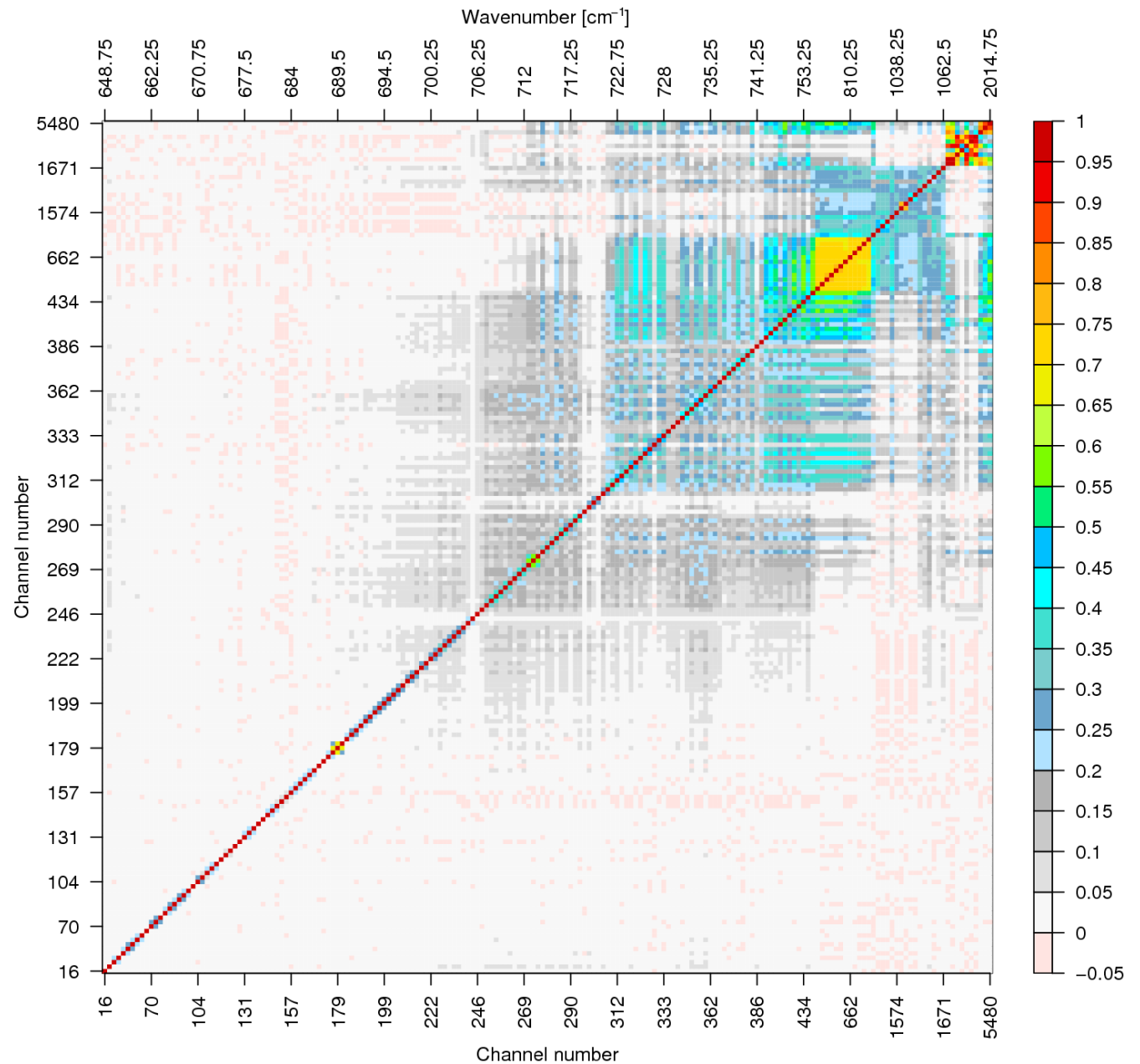
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## Diagnostics for $\sigma_o$



# Examples of applying observation error diagnostics: IASI

## Inter-channel error correlations

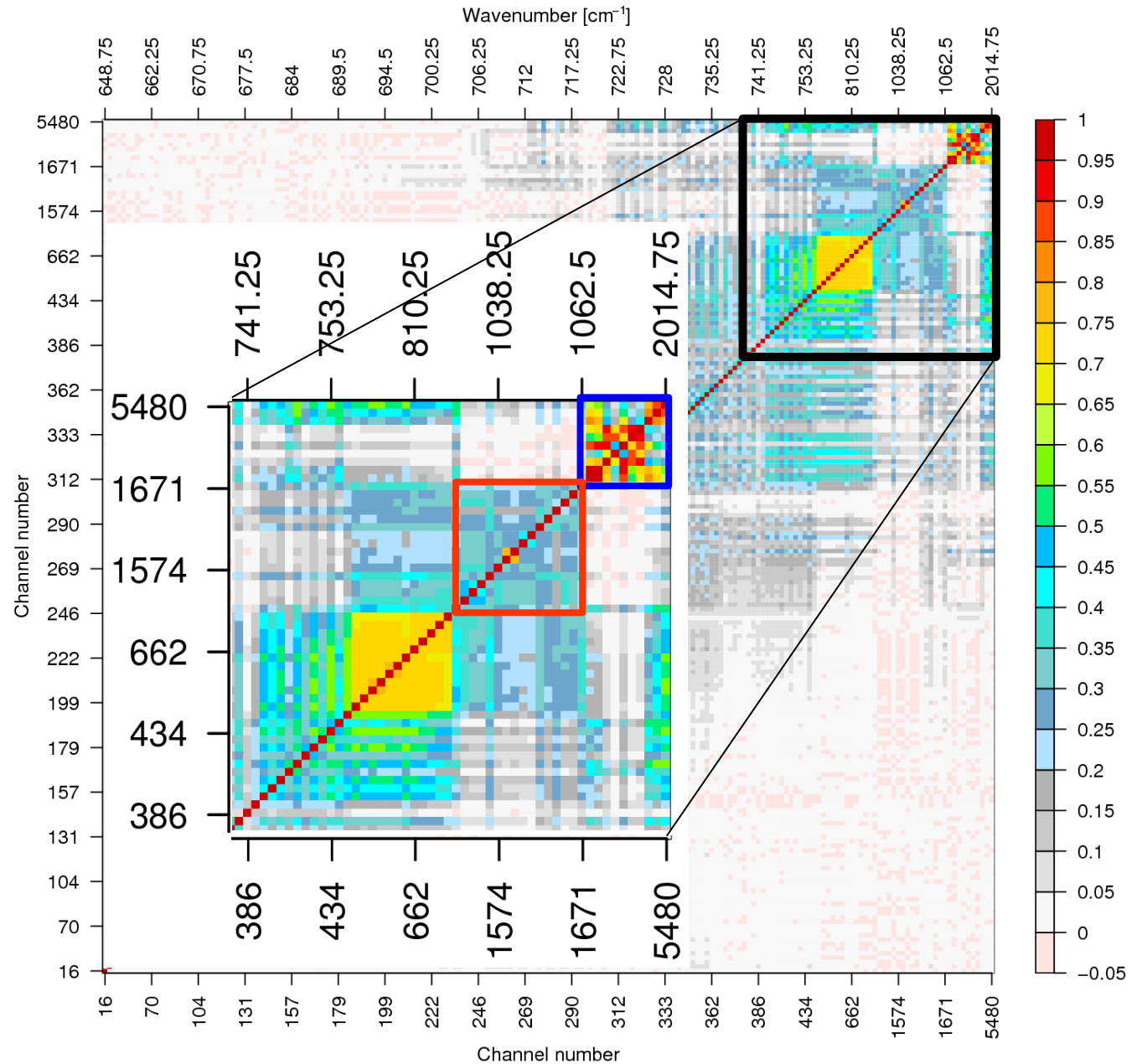


# Examples of applying observation error diagnostics: IASI

## Inter-channel error correlations

Humidity

Ozone



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# How do I specify observation errors in practice?

- Observation error diagnostics or error inventories can provide guidance for observation error specification in DA, including on:

- **Relative size** of observation and background errors
- Presence of observation **error correlations**
- **Situation-dependence** of observation errors

- **But:**

- Estimates might have short-comings (violated assumptions).
- Observation errors specified in assimilation systems often need to be **simplified**:
  - Observation error covariance is often **assumed to be diagonal or globally constant**.
- **Assumed** observation errors may need **adjustments** compared to estimated ones.

# Too large assumed observation errors tend to be safer than too small ones. Why?

Consider a linear combination of two estimates  $x_b$  and  $y$ :

$$x_a = \alpha x_b + (1 - \alpha)y$$

The error variance of the linear combination is:

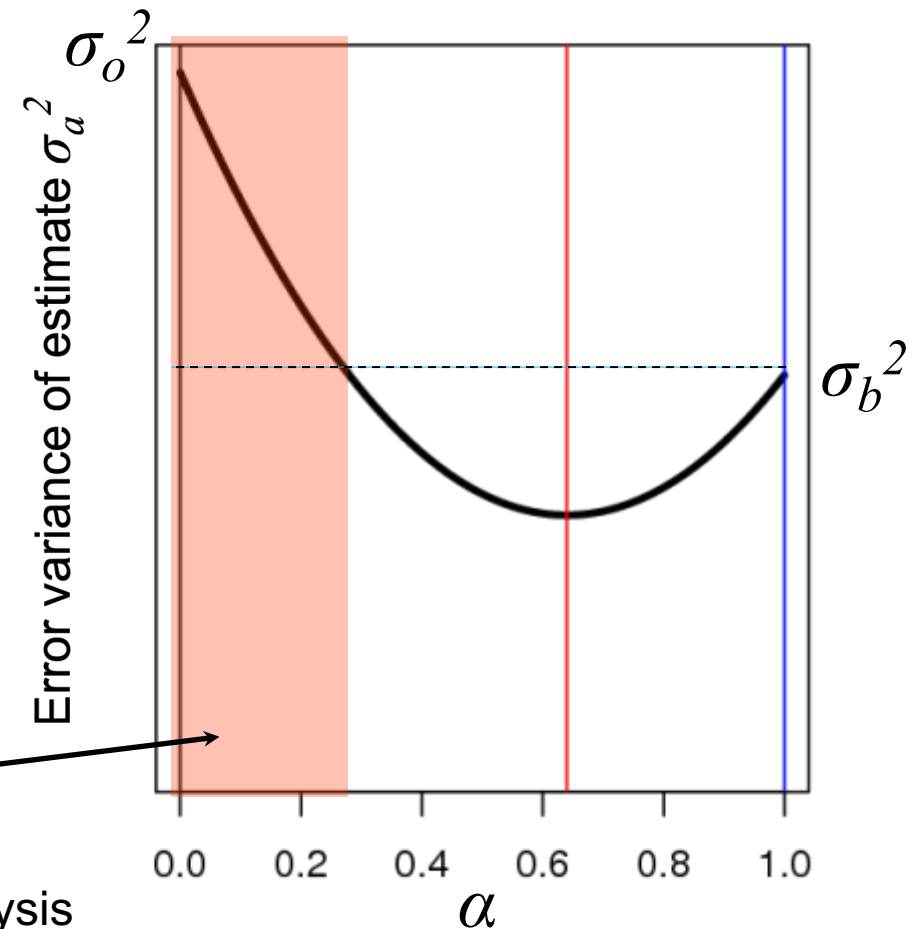
$$\sigma_a^2 = \alpha^2 \sigma_b^2 + (1 - \alpha)^2 \sigma_o^2$$

The optimal weighting (ie minimum  $\sigma_a$ ) is:

$$\alpha = \frac{\sigma_o^2}{\sigma_b^2 + \sigma_o^2}$$

**Danger zone:** Too small *assumed*  $\sigma_o$  will lead to an analysis worse than the background when the (true)  $\sigma_o > \sigma_b$ .

**Assuming an inflated  $\sigma_o$  will never result in deterioration.**



# What to do when there are error correlations?

## Option 1: Thinning

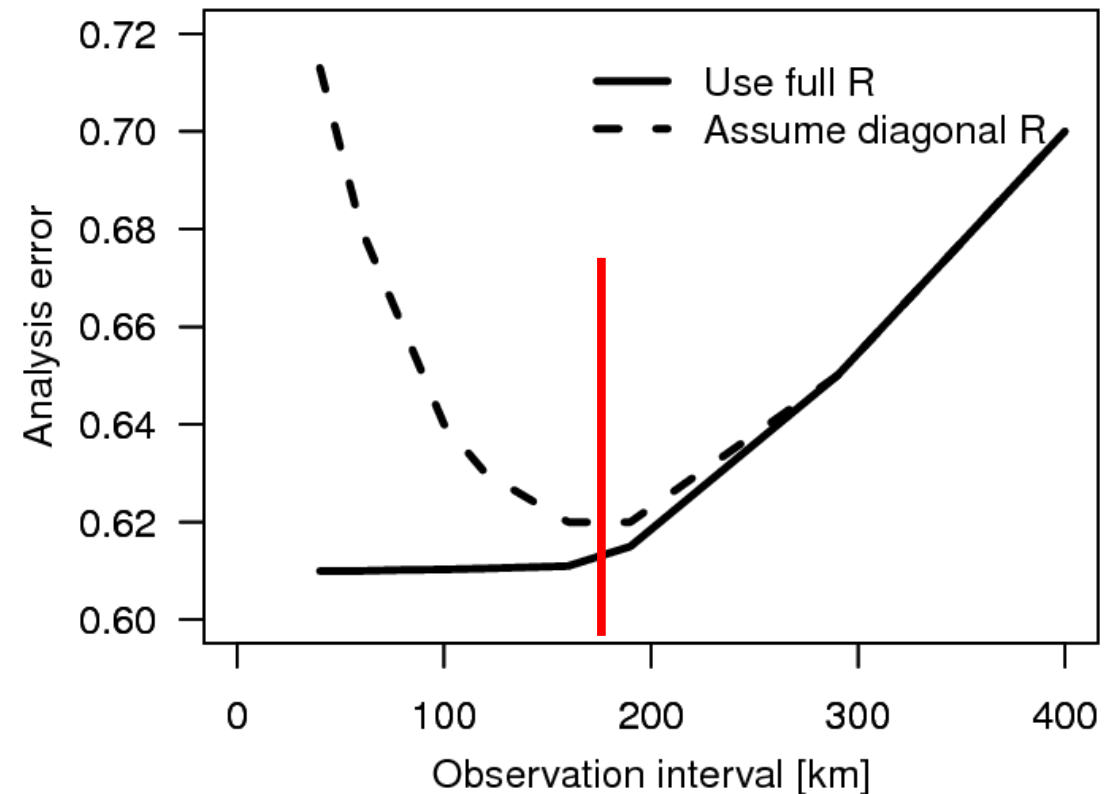
- If the observations have **spatial error correlations, but these are neglected** in the assimilation system, assimilating these observations too densely can have a **negative effect**.

- **Pragmatic solution 1:** Select one observation within a “thinning box”.

- See Liu and Rabier (2003), QJRMS: “Optimal” thinning when  $r \approx 0.15-0.2$

- Using **fewer** observations gives **better** results!

- (But we lose out on information on smaller scales.)





# What to do when there are error correlations?

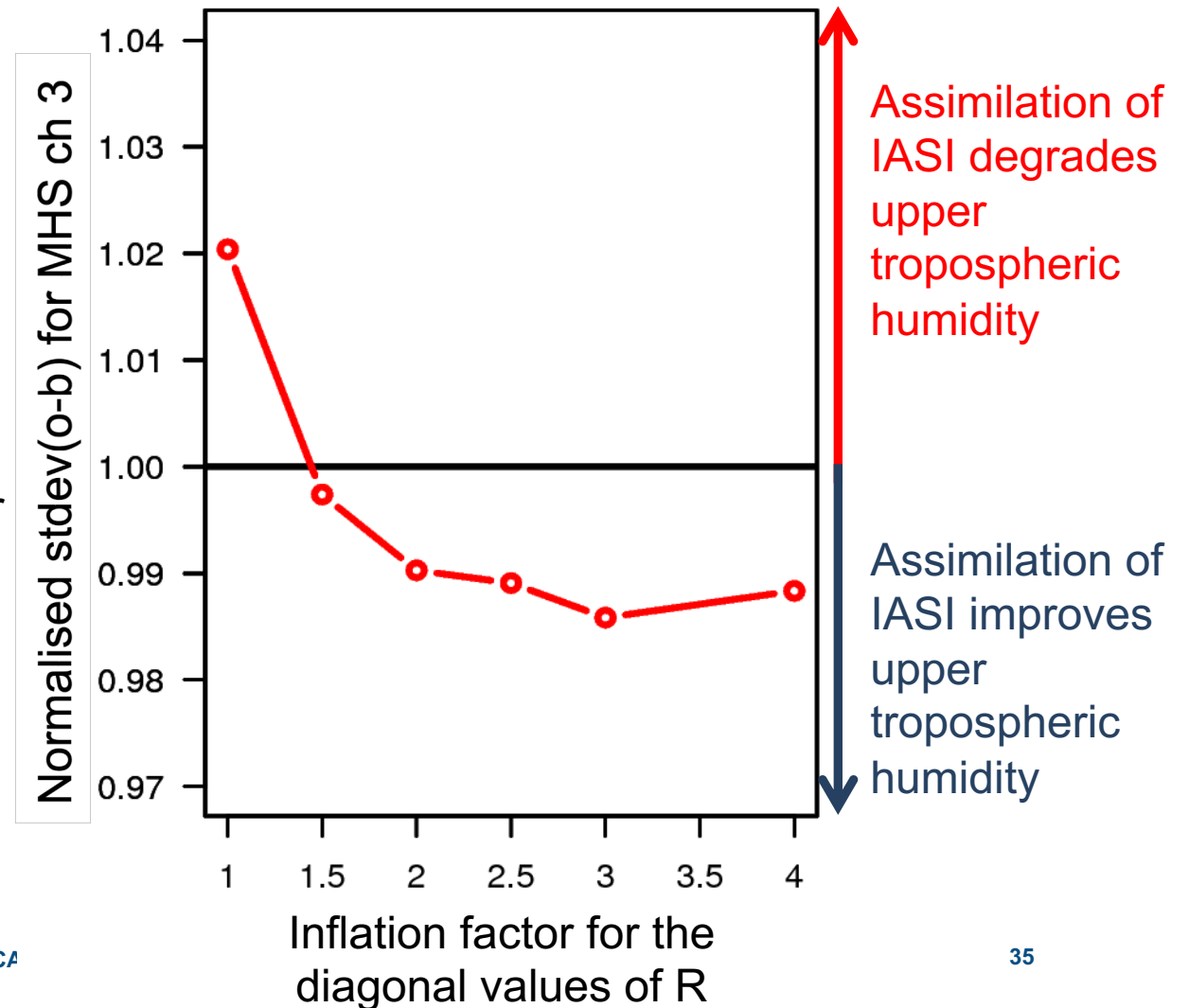
## Option 2: Inflation

- If the observations have **error correlations, but these are neglected** in the assimilation system, assimilating them can have a **negative effect**.

- **Pragmatic solution 2:** Use larger  $\sigma_o$  than expected (“**Error inflation**”).

- **Neglecting error correlation with no inflation** can result in an analysis that is **worse** than the background!

- Note: Background departure statistics for other observations are a useful indicator to tune observation errors.



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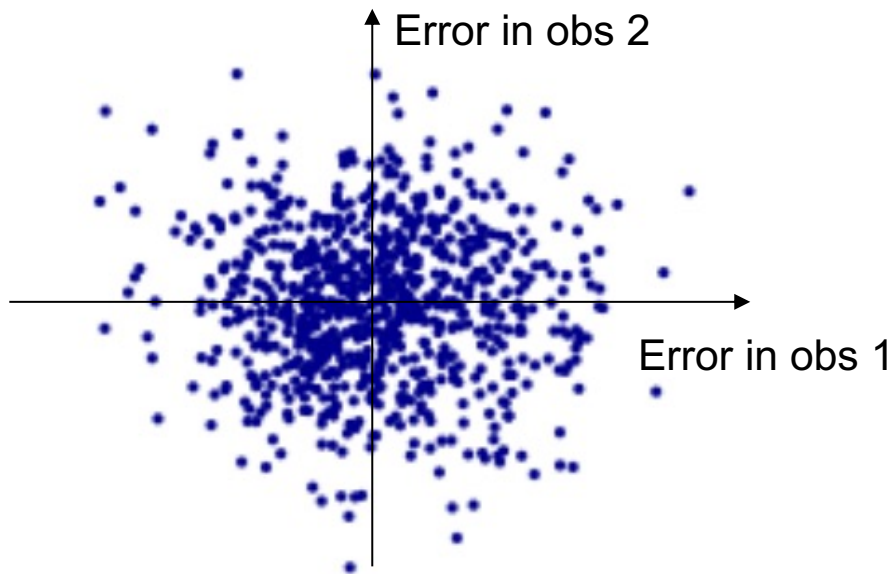
# Accounting for error correlations

- Accounting for observation error correlations is an area of active research.
- Efficient methods exist if the error correlations are restricted to small groups of observations (e.g., **inter-channel error correlations**).
  - E.g., calculate  $R^{-1}(y - H(x))$  without explicit inversion of  $R$ , by using Cholesky decomposition (algorithm for solving equations of the form  $Az = b$ ).
  - Used operationally for IASI, CrIS, AIRS and ATMS at ECMWF and many other centres
- Accounting for **spatial error correlations** is technically more difficult in variational algorithms, though methods are being developed.
  - Met Office is taking spatial error correlations into account in the operational assimilation of Doppler radar data in their limited area model (Simonin et al 2019)

# What is the effect of error correlations?

Uncorrelated error

$$\mathbf{R} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$



Correlated error

$$\mathbf{R} = \begin{pmatrix} 1 & 0.8 \\ 0.8 & 1 \end{pmatrix}$$



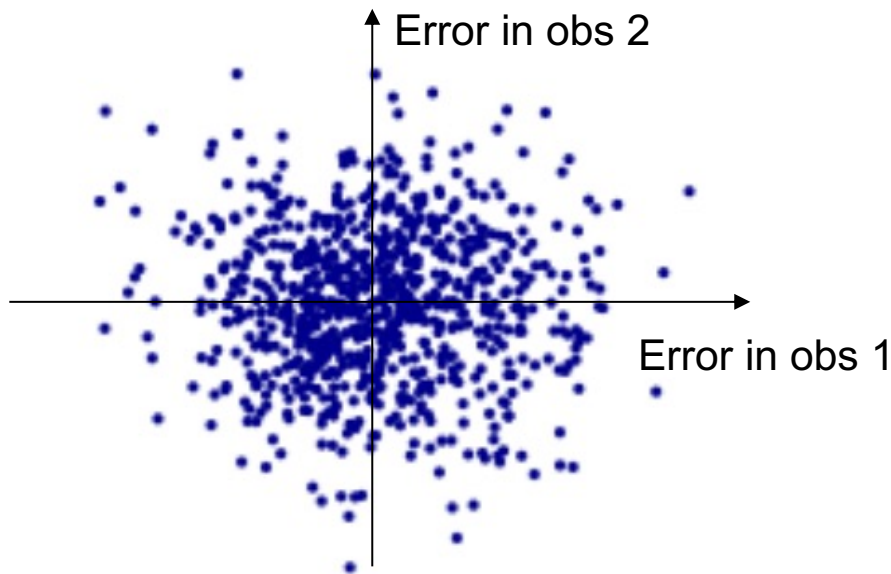
If errors **are correlated** and we **assume no error correlations**, we assign...

- ... an error that is **too small** for features along the blue direction (mean-like features), leading to over-weighting of the observations. Hence inflation helps.
- ... an error that is **too large** for features along the red direction (gradient-type features).

# What is the effect of error correlations?

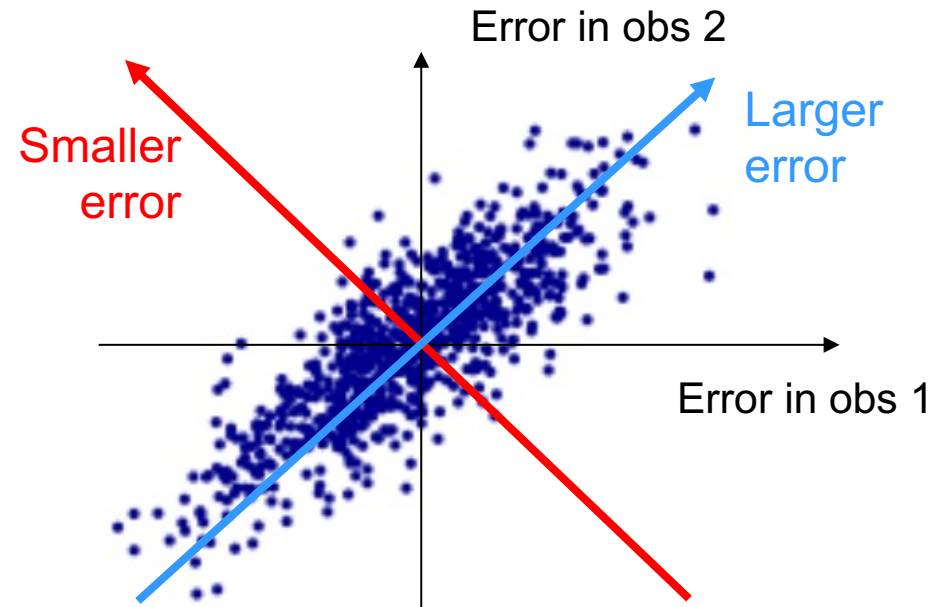
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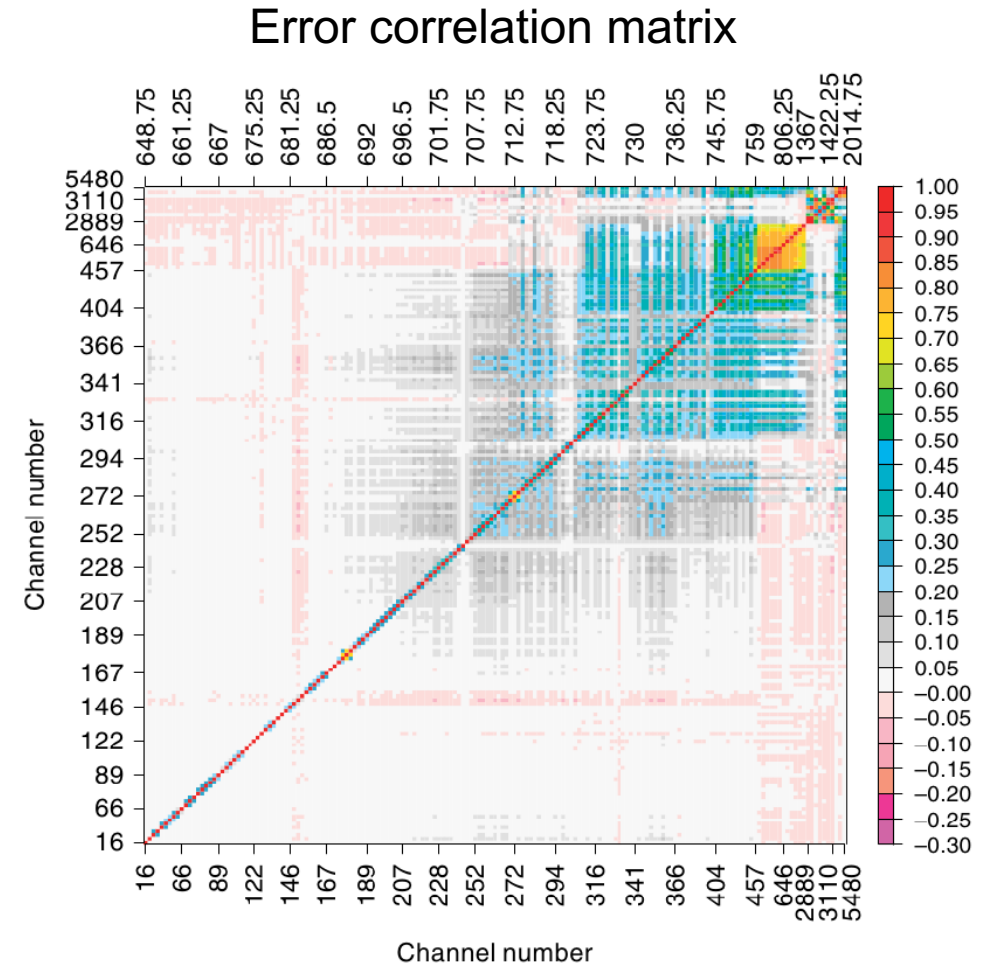
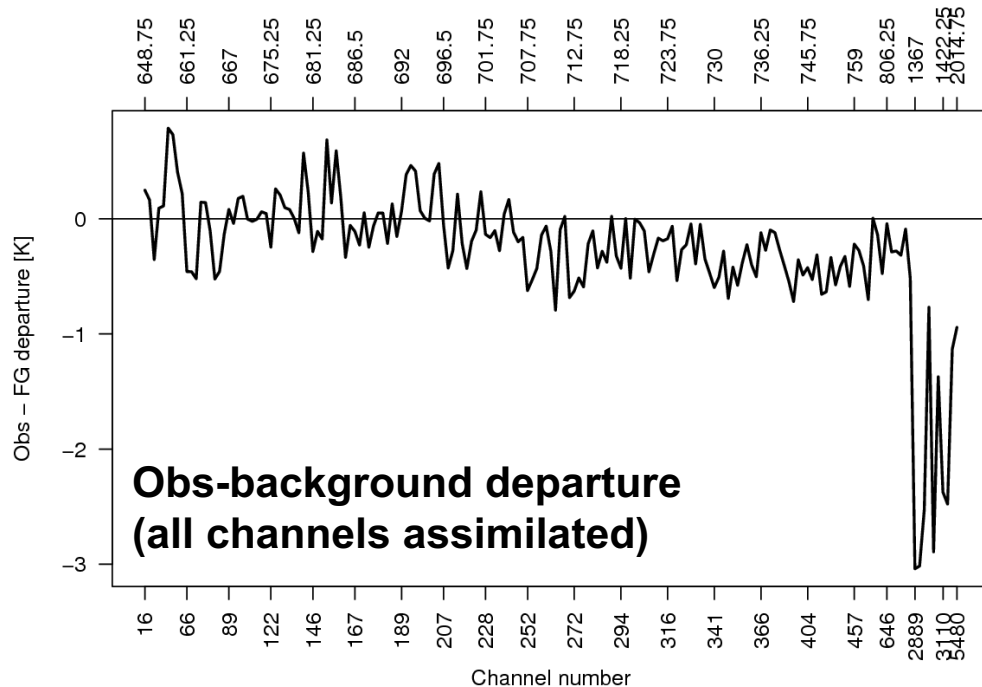
$$\mathbf{R} = \begin{pmatrix} 1 & 0.8 \\ 0.8 & 1 \end{pmatrix}$$



Similarly, when we **account for observation error correlations** we tell the assimilation system that...  
... **departures** that are **similar** for different observations are **more likely** due to errors in the observations.  
... **departures** that are **different** for different observations are **less likely** due to errors in the observations.

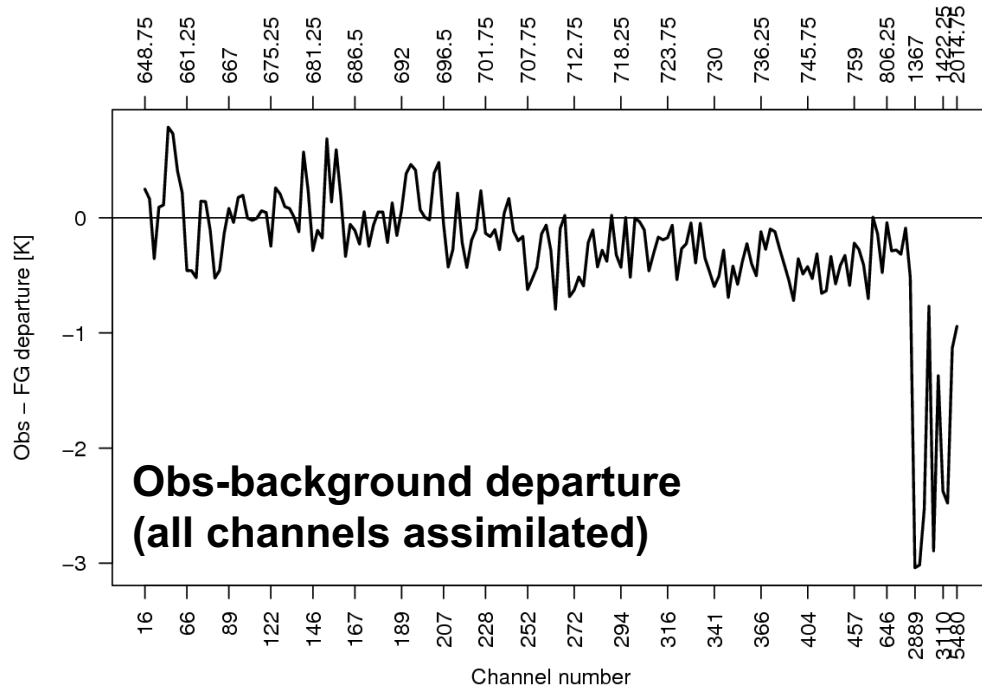
# Example: Assimilation of a IASI spectrum (I)

- Assimilate a single IASI spectrum,
- assuming **no error correlations**,
  - assuming **diagnosed error correlations** ( $\sigma_o$  unchanged in both cases).

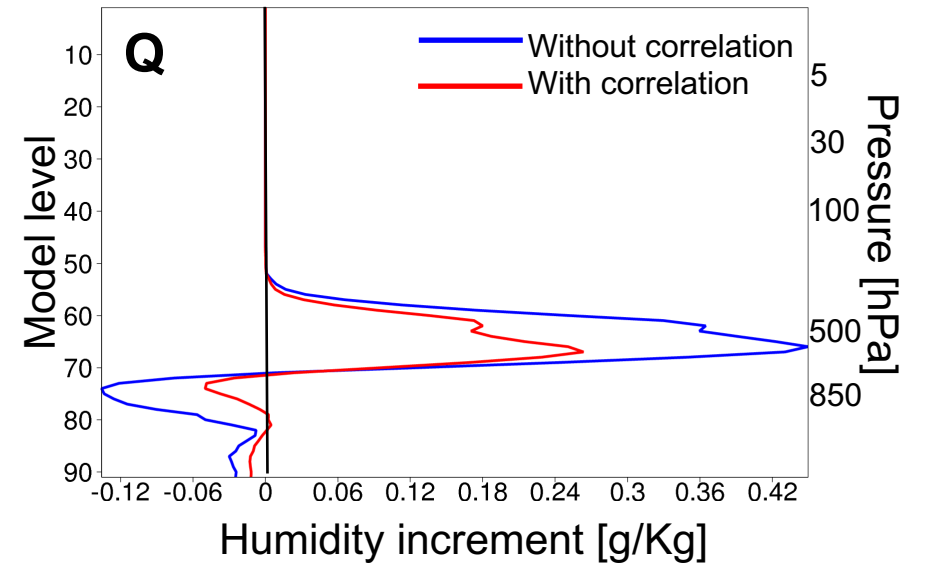
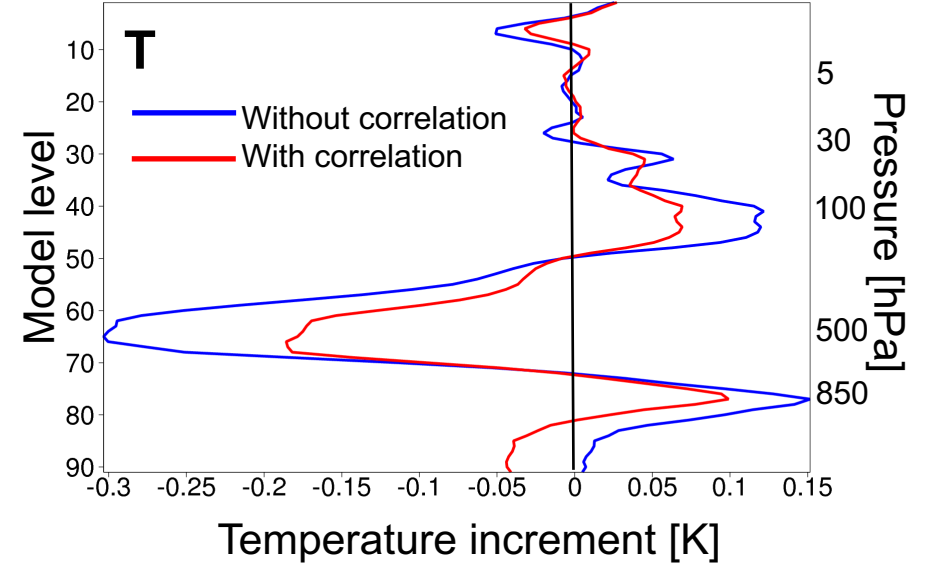


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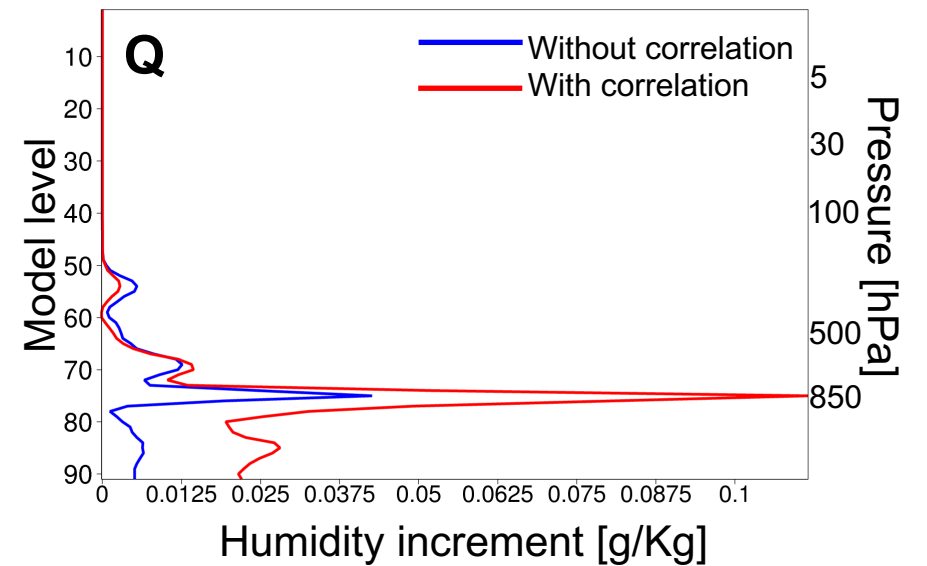
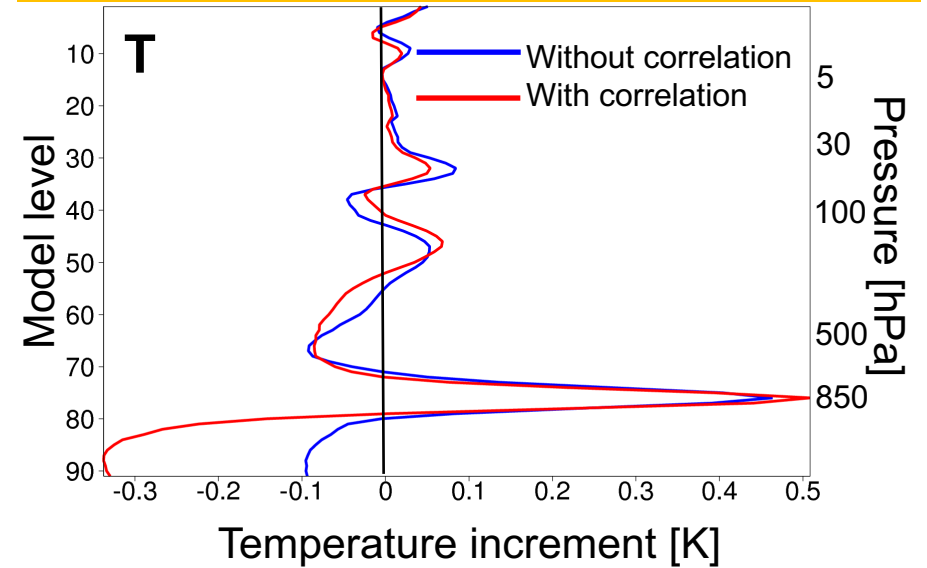
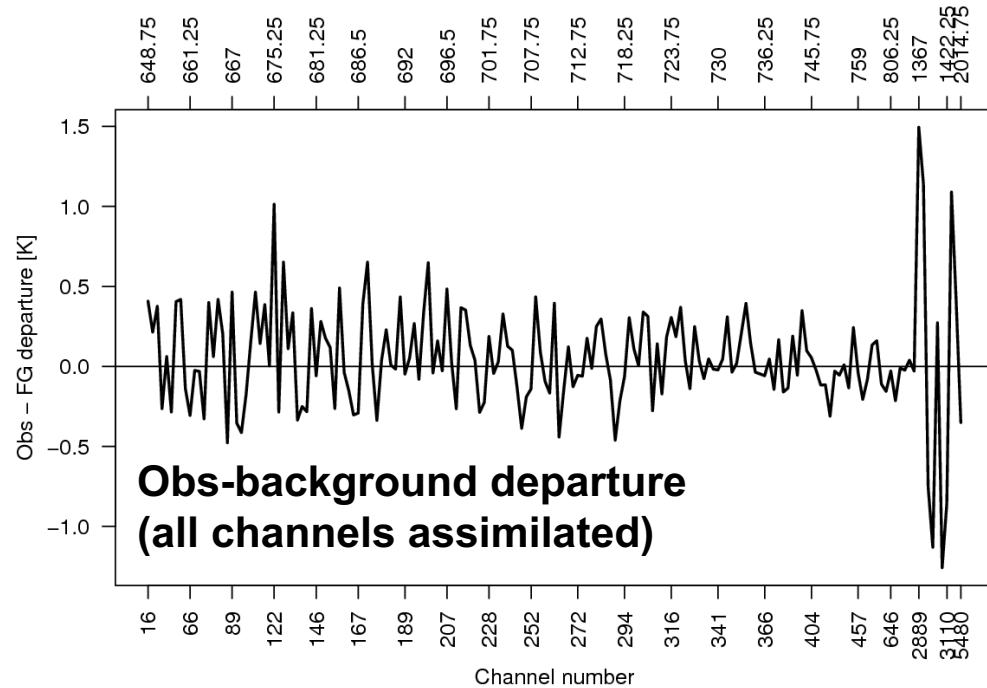
Similar departures → increments reduced with error correlations taken into account



# Example: Assimilation of a IASI spectrum (II)

- Assimilate a single IASI spectrum,
  - assuming **no error correlations**,
  - assuming **diagnosed error correlations** ( $\sigma_o$  unchanged in both cases).

Different departures  $\rightarrow$  increments **increased** with error correlations taken into account

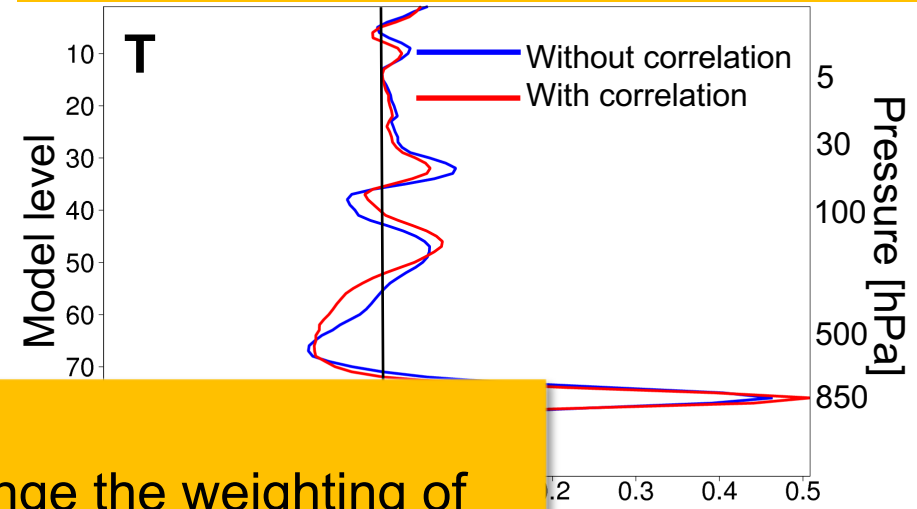




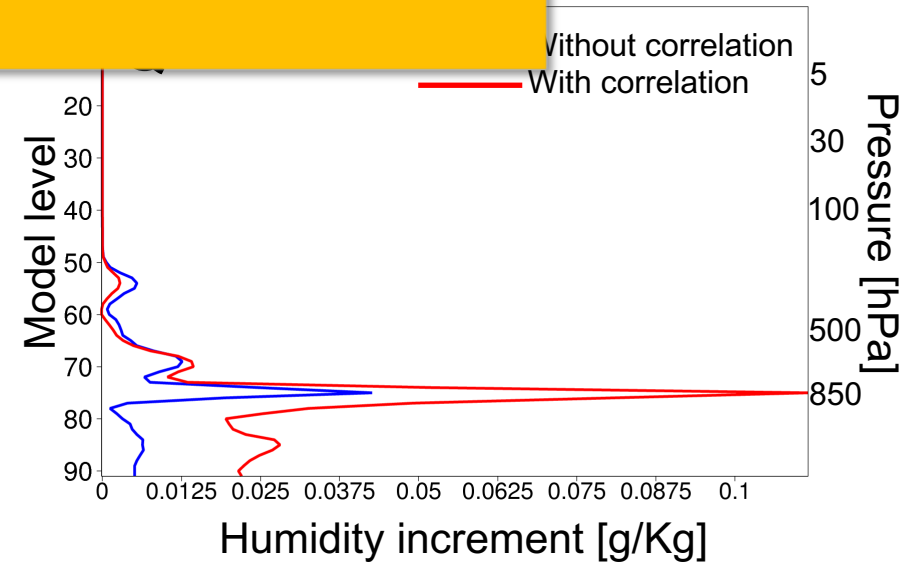
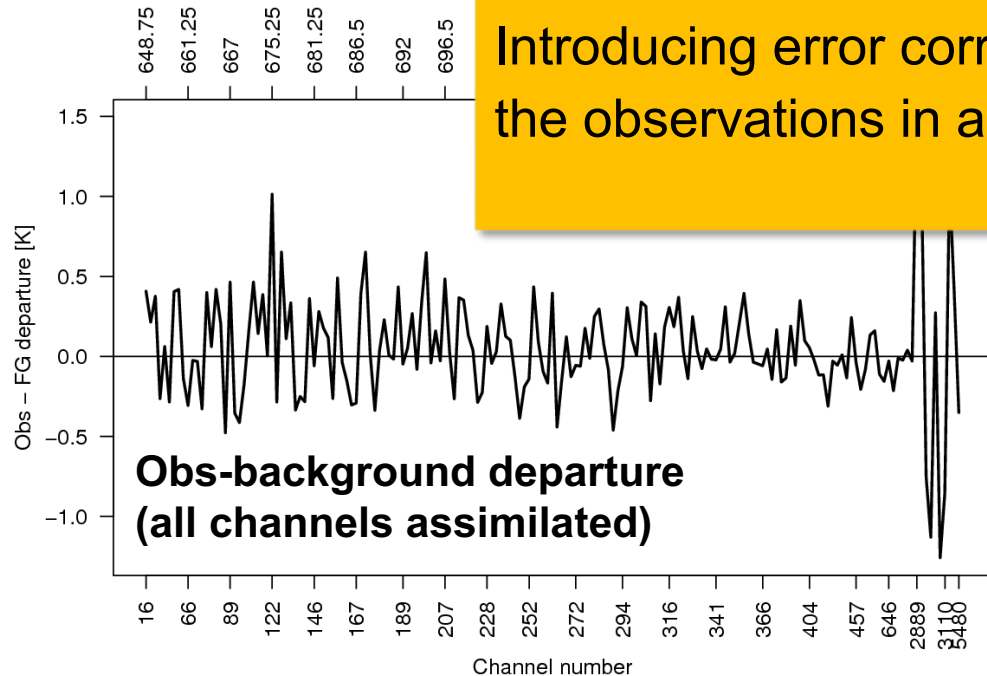
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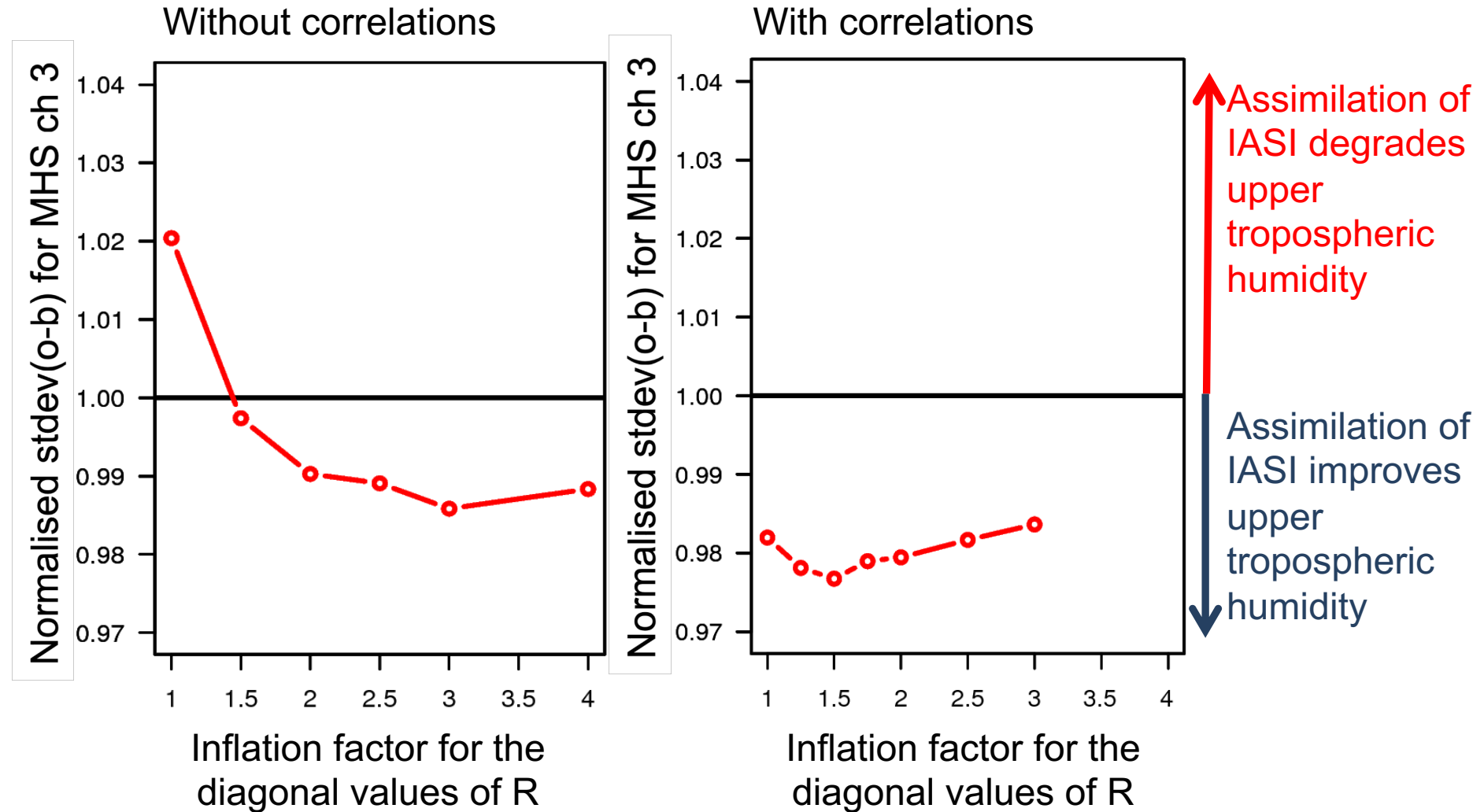
Different departures → increments **increased** with error correlations taken into account



Introducing error correlations will change the weighting of the observations in a situation(/departure)-dependent way.



# Effect of accounting for error correlations in the assimilation of IASI



Most centres now take inter-channel error correlations into account for the assimilation of hyperspectral IR data.

## Some points on accounting for observation error correlations

- Accounting for observation error correlations is an **active area of research**.
- **Benefits** have been **demonstrated** at many centres for accounting for inter-channel error correlation; used widely operationally.
- Note:
  - Assuming error correlations puts **more weight on differences between observations**. Are these differences reliable? How reliable are **inter-channel calibration/bias correction**?
  - Are the **estimates of error correlations reliable**?
  - Accounting for observation error correlations can affect the **conditioning** of the assimilation and lead to slower convergence.
  - Error correlation matrices **may need adjustments** (“re-conditioning”, inflation).
- How important it is to account for error correlations may additionally depend on the structure of the background error.

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# Summary

- Assigned observation and background errors determine how much **weight** an observation receives in the assimilation.
- For satellite data, “true” **observation errors are often correlated** (spatially, in time, between channels, etc) and **situation-dependent**.
- Careful use of **departure-based diagnostics** can provide **guidance** on the setting of observation errors.
- Diagonal observation errors are still widely assumed for many observations, and **thinning and error inflation** are used to counter-act the effects of error correlations.
- Areas of active research:
  - Development of “**observation error models**” to account for situation-dependence of observation errors.
  - **Accounting for observation error correlations** (inter-channel, spatial).
  - **Estimation of observation errors**.

## Further reading

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