A unified latent-space background-error covariance model for midlatitude and tropical atmospheric data assimilation

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Introduction and motivation

Data assimilation (DA) is a process that estimates the current state of the atmosphere $\boldsymbol{x}$ given the previous forecast (*background*; $\boldsymbol{x}_b$) and new observations $\boldsymbol{y}$. If the observations and the background are concurrent, and the problem is addressed variationally, the most probable state of the atmosphere (*analysis*; $\boldsymbol{x}_a$) is obtained by minimising the 3D-Var cost function

$$J(\boldsymbol{x}) = \frac{1}{2}\|\boldsymbol{x}-\boldsymbol{x}_b\|_{\mathbf{B}^{-1}} + \frac{1}{2}\|\boldsymbol{y}-H(\boldsymbol{x})\|_{\mathbf{R}^{-1}}.$$

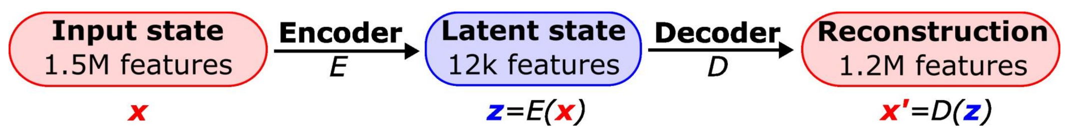
Challenge: In global models, $\boldsymbol{x} \in \mathbb{R}^{10^{10}\times 1} \Rightarrow \mathbf{B} \in \mathbb{R}^{10^{10}\times 10^{10}}$.

- Naively computed background-error covariance matrix $\mathbf{B}$ would be prohibitively large
- **Operational solution**: perform minimisation in a control space with decorrelated variables under assumption that the errors in geostrophically balanced parameters are decoupled from the errors in unbalanced parameters
- **Limitation**: equatorial balances are not adequately represented
- **Proposed solution**: perform cost function minimisation in a reduced (latent) space of an autoencoder, which may encompass all balances

Methods

Autoencoder (AE)

- Reconstruct 20 global atmospheric fields (1° resolution) after severely compressing them to the latent space
- Training data: ERA5 reanalysis



Reconstructed variable	Levels
Geopotential height (Z)	*250 hPa, 500 hPa, 700 hPa, 850 hPa*
Zonal wind (U)	*200 hPa, 500 hPa, 700 hPa, 850 hPa, 10 m*
Meridional wind (V)	*200 hPa, 500 hPa, 700 hPa, 850 hPa, 10 m*
Temperature (T)	*500 hPa, 850 hPa, 2 m, surface*
Mean sea level pressure (MSLP)	-
Total column water vapour (TCWV)	-

Latent-space 3D-Var

- Cost function is now defined (and minimised) in the latent space

$$J_z(\boldsymbol{z}) = \frac{1}{2}\|\boldsymbol{z}-\boldsymbol{z}_b\|_{\mathbf{B}_z^{-1}} + \frac{1}{2}\|\boldsymbol{y}-H(D(\boldsymbol{z}))\|_{\mathbf{R}^{-1}}$$

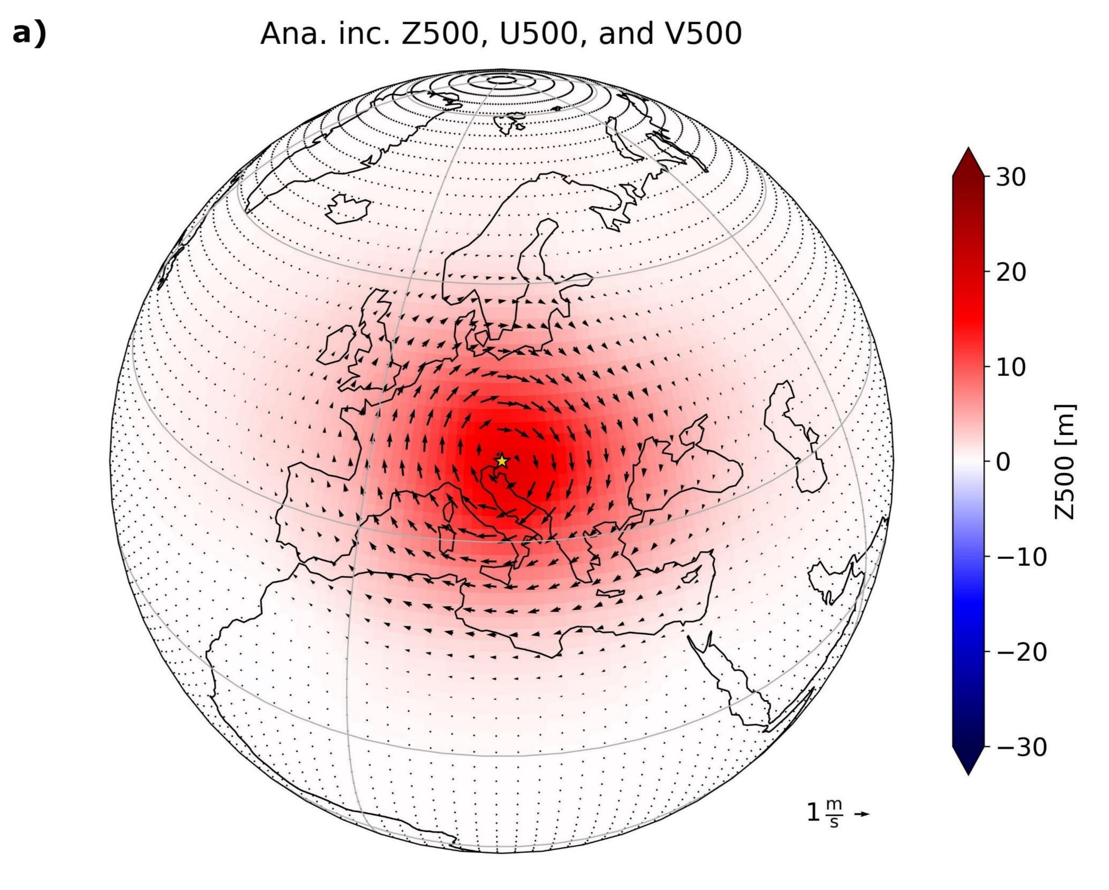
Background-error covariance modelling

- Climatology-based:

$$\mathbf{B}_z = \langle(\boldsymbol{z}_b - \boldsymbol{z}_{\mathrm{ERA5}})(\boldsymbol{z}_b - \boldsymbol{z}_{\mathrm{ERA5}})^\top\rangle$$

  - $\boldsymbol{z}_{\mathrm{ERA5}} = E(\boldsymbol{x}_{\mathrm{ERA5}})$
  - $\boldsymbol{z}_b = E\left(\mathrm{NN}^{24\mathrm{h}}(\boldsymbol{x}_{\mathrm{ERA5}}(t-24\mathrm{h}))\right)$
  - Averaging over validation set (2015-2019)
  - $\mathrm{NN}^{24\mathrm{h}}$ … UNet-based forward model
- $\mathbf{B}_z$ quasi-diagonal: we keep only diagonals for inverse computation and ensemble generation!

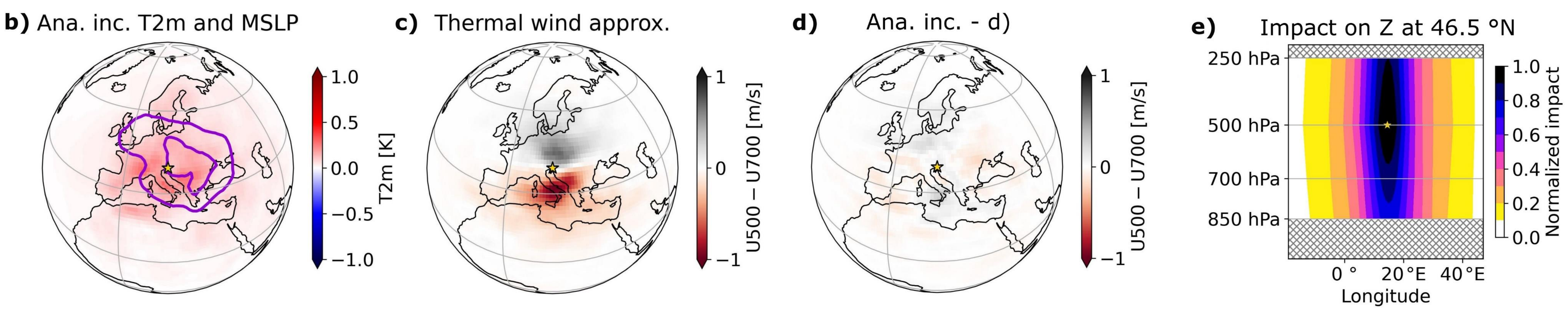
Results – midlatutides



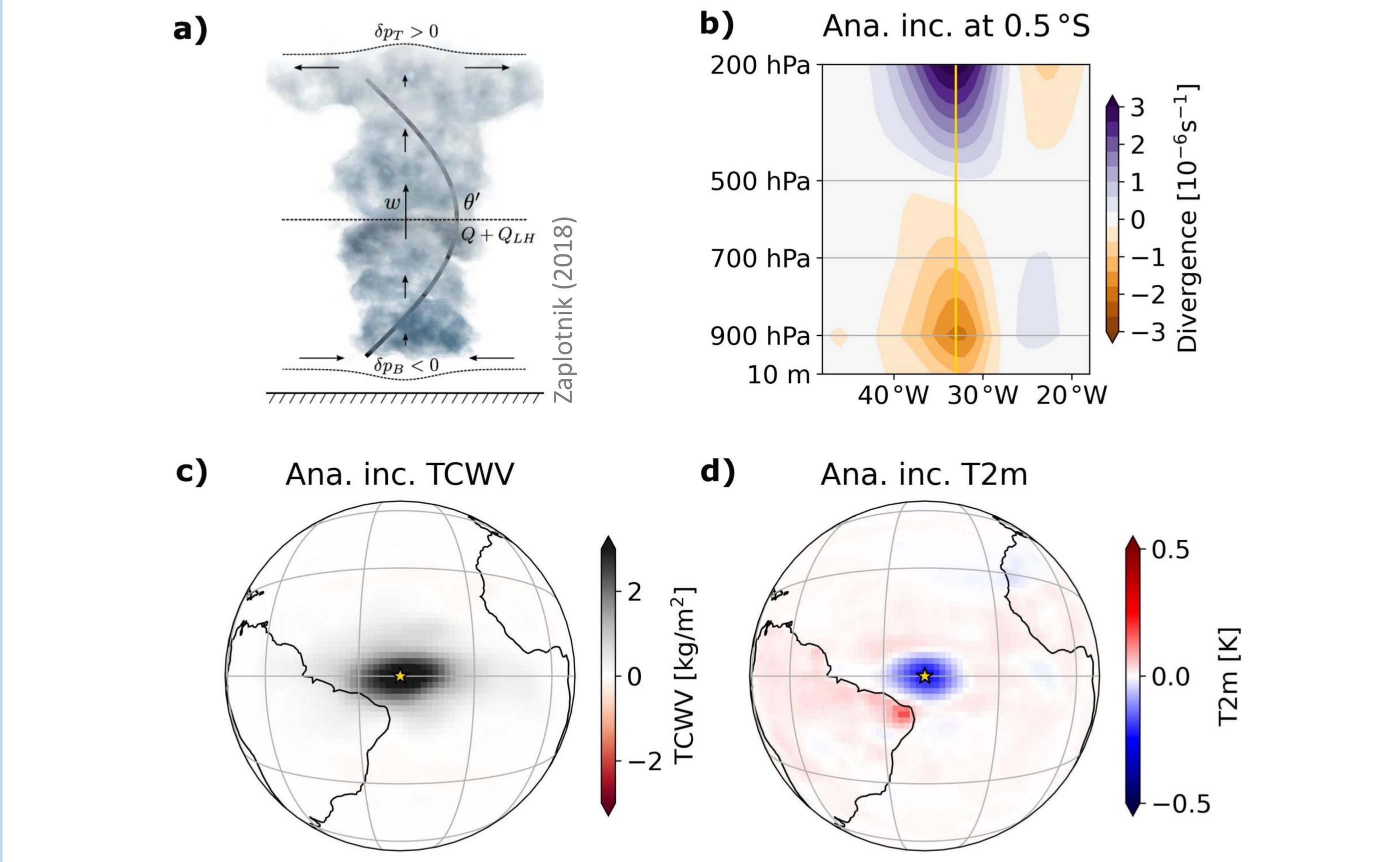
Example: assimilation of simulated Z500 observation at (46.1 °N, 14.5 °E) with + 30 m departure and 10 m standard deviation

- Positive Z500 analysis increment, surrounded by anticyclonic wind increment (panel **(a)**) → geostrophic balance
- Local features: T2m increment confined to land, MSLP increment distorted by orography (panel **(b)**)
- If both geostrophic and hydrostatic balance are preserved, thermal wind approximation can be implied → Panel **(c)**: expected U500-U700 increment from Z increment gradient; Panel **(d)**: obtained increment minus **(c)**
- Realistic spatial propagation of observation impact (panel **(e)**)

$$I = \frac{\text{mean ana. inc.}}{\text{ana. std.}}$$



Results – tropics



Example: assimilation of simulated TCWV observation in nearly saturated area (0 °N, 33 °E) with + 10 kg/m² departure and 3 kg/m² standard deviation

- Theory in case of saturation (panel **(a)**; Gill, 1980; Davey and Gill, 1987): condensation → latent heat release → updraft → *lower tropospheric convergence and upper tropospheric divergence*
- Resulting wind increment complies with theory (panel **(b)**)
- TCWV increment zonally elongated due to background lower tropospheric zonal wind (panel **(c)**)
- Negative T2m increment consistent with precipitation (panel **(d)**)

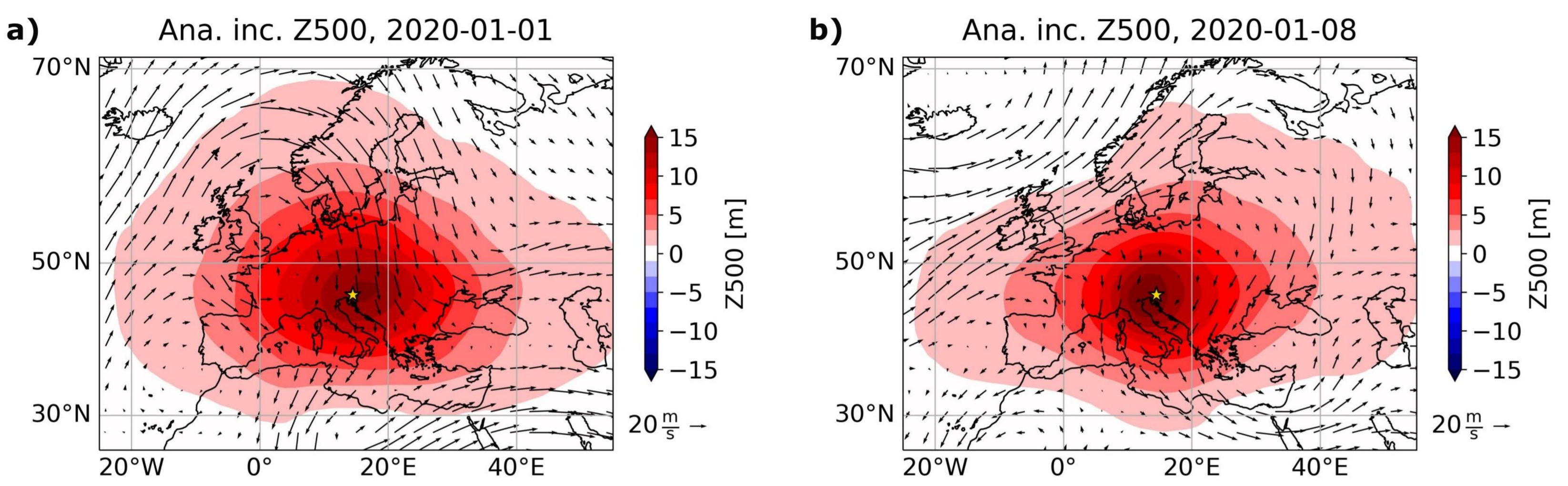
Results – flow dependence and operational ensembles

- Climatology-based (static) $\mathbf{B}$-matrices typically impose weather-situation-unaware analysis increment patterns
- However, climatological $\mathbf{B}_z$ already leads to flow-dependent analysis increments due to the decoder's nonlinearity (panels **(a)**, **(b)**; arrows indicate background wind)
- In leading operational systems using variational DA, flow-dependent variances and covariances are introduced by computing $\mathbf{B}$ from an ensemble of backgrounds (Bannister, 2017)
- Can we enhance this flow dependence using operational ensembles and make this method suitable for operational use? We tested our method for ECMWF IFS ensemble of backgrounds-based $\mathbf{B}_z$ computation

$$\mathbf{B}_z^{\mathrm{IFS}} = \frac{1}{N-1}\sum_{i=1}^{N}\left(\boldsymbol{z}_{b,i}^{\mathrm{IFS}} - \langle\boldsymbol{z}_b^{\mathrm{IFS}}\rangle\right)\left(\boldsymbol{z}_{b,i}^{\mathrm{IFS}} - \langle\boldsymbol{z}_b^{\mathrm{IFS}}\rangle\right)^\top,$$

where $N=50$, $\boldsymbol{z}_{b,i}^{\mathrm{IFS}} = E\left(\boldsymbol{x}_{b,i}^{\mathrm{IFS}}\right)$, and $\langle\boldsymbol{z}_b^{\mathrm{IFS}}\rangle = \frac{1}{N}\sum_{i=1}^{N}\boldsymbol{z}_{b,i}^{\mathrm{IFS}}$

- Despite significant variance loss after encoding the operational ensemble (figures available in corresponding paper):
  - $\mathbf{B}_z^{\mathrm{IFS}}$ remains quasi-diagonal
  - The analysis increment after observing positive Z500 departure gives physically plausible, flow-dependent increment pattern, similar to the one obtained using the same ensemble, but climatological $\mathbf{B}_z$



Discussion and conclusions

- We propose a neural-network-based method for background error covariance estimation for variational data assimilation of atmospheric observations in a reduced-dimension latent space discovered by an autoencoder (AE)
- We define a 3D-Var cost function in the latent space
- Climatology-based background-error covariance matrix $\mathbf{B}_z$ is quasi-diagonal, vastly simplifying the assimilation procedure
- Assimilation of a simulated Z500 observation in midlatitudes yields multivariate analysis increments consistent with both large-scale (geostrophic and hydrostatic) balances and local phenomena (land-sea contrast, orography), and realistic spatial propagation of the observation impact
- Assimilation of a simulated TCWV observation with a positive departure in a nearly saturated area in the tropics results in analysis increments corresponding to emerging convective clusters despite convection, condensation and latent heat release not being explicitly included in AE

  ⇒ **Presented $\mathbf{B}_z$ provides the first unified representation of both tropical and extratropical covariances**
- Even though $\mathbf{B}_z$ is static in the latent space, it encompasses flow-dependent covariances in the physical space due to the decoder's nonlinearity
- Operational ensemble of backgrounds-based $\mathbf{B}_z$ retains the balances and flow-dependence similar to those in climatological $\mathbf{B}_z$ in spite of severe variance reduction in some variables, highlighting the method's potential for operational use with a more sophisticated AE

Corresponding paper:

Melinc, B., Perkan, U. & Zaplotnik, Ž. (2026). A Unified Neural Background-Error Covariance Model for Midlatitude and Tropical Atmospheric Data Assimilation. JAMES, 18(1), e2025MS005360.

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