Horizontal and vertical resolution, hydrostatic and non-hydrostatic dynamics

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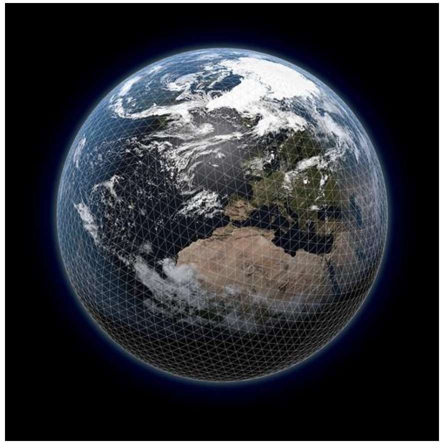
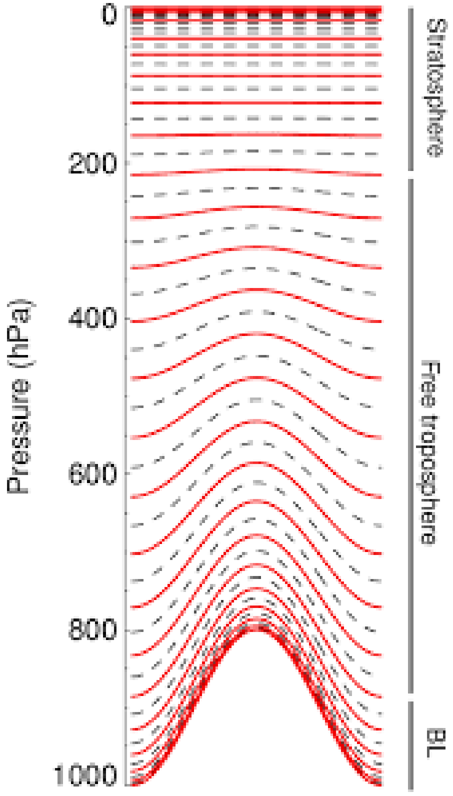
Numerical Methods Training Course, April 2026

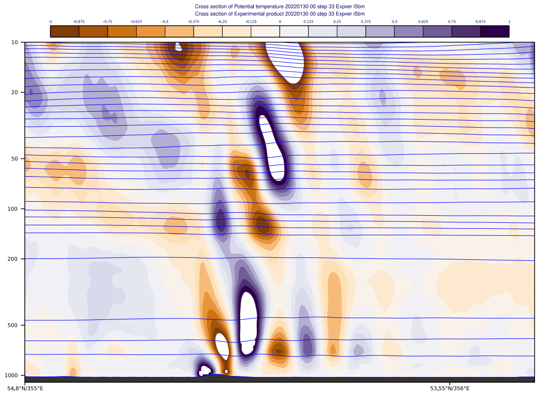
Outline



1) Horizontal resolution at ECMWF



2) Vertical resolution at ECMWF

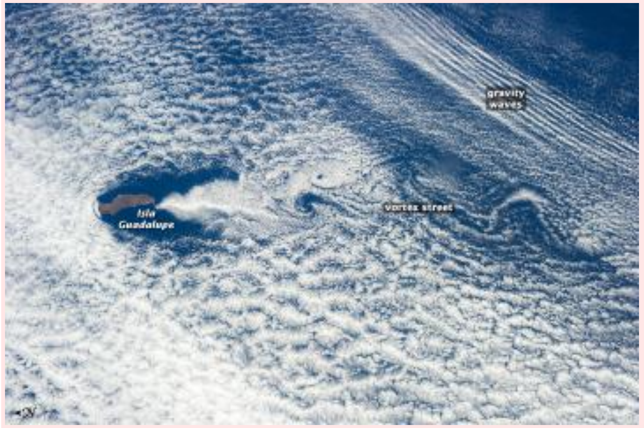


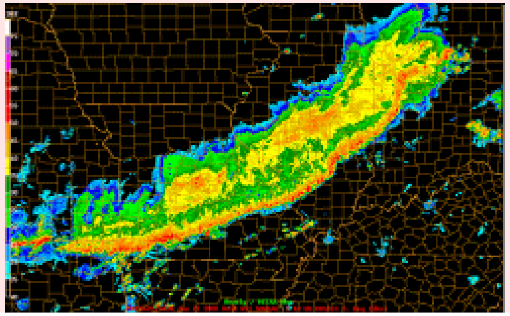
3) Hydrostatic vs. non-hydrostatic dynamics

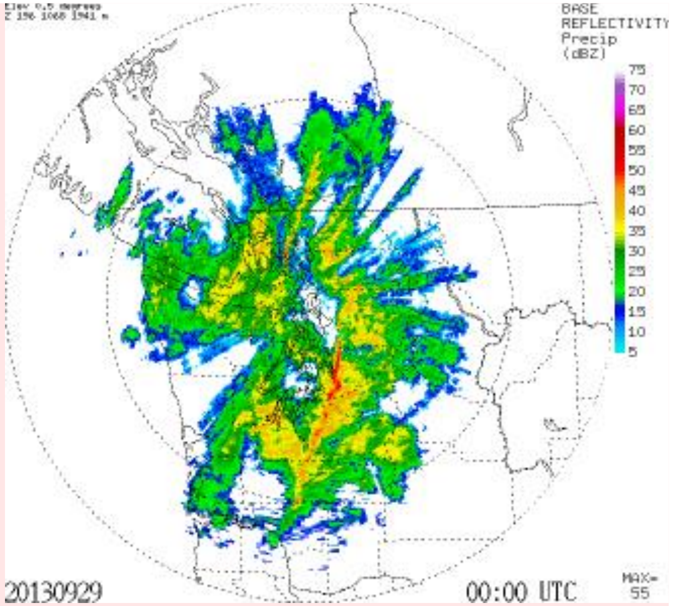
ECMWF

Why strive for higher and higher resolution?

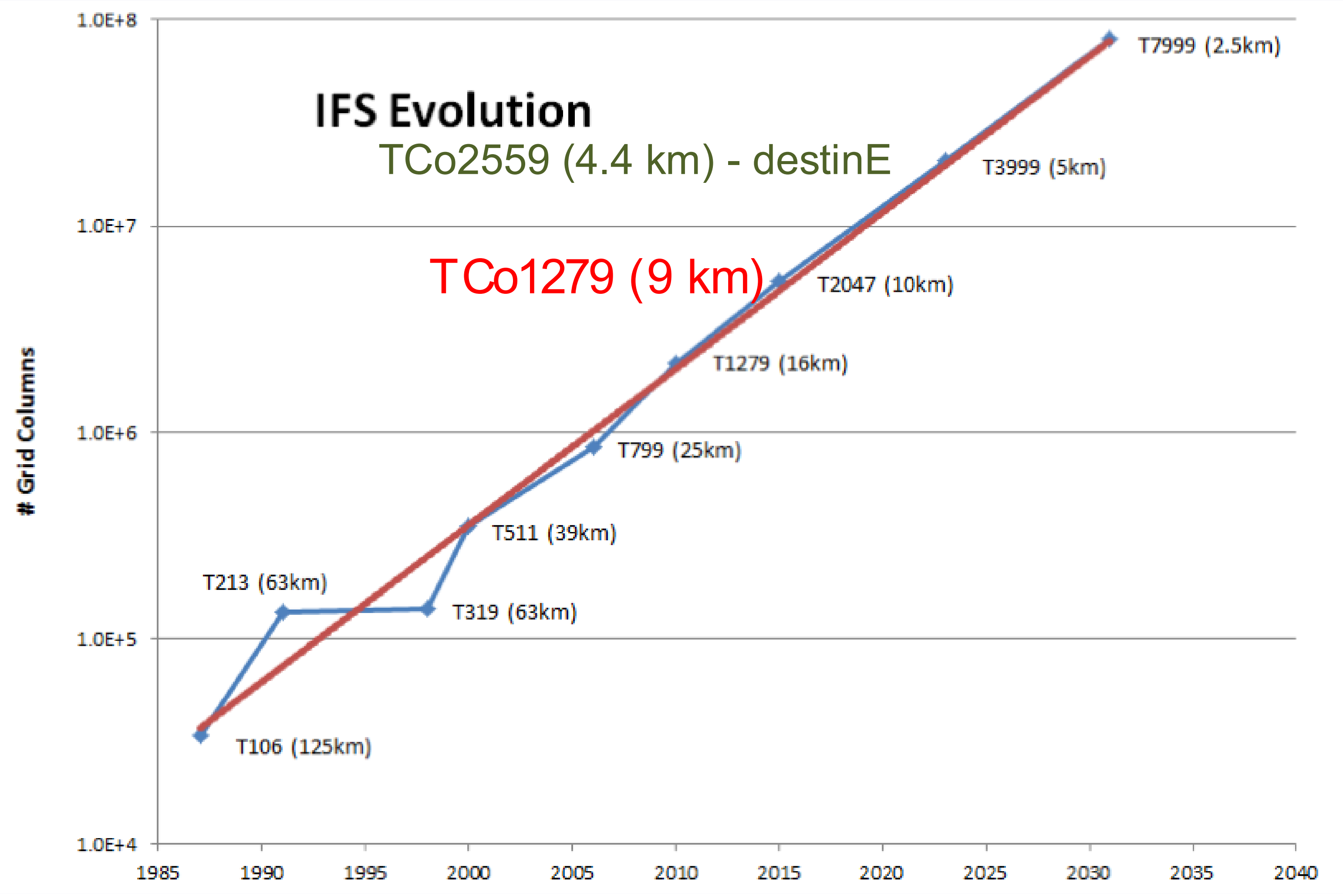
- To better resolve high-impact weather processes, such as mountain waves, tropical cyclones, squall lines, tornadoes etc.







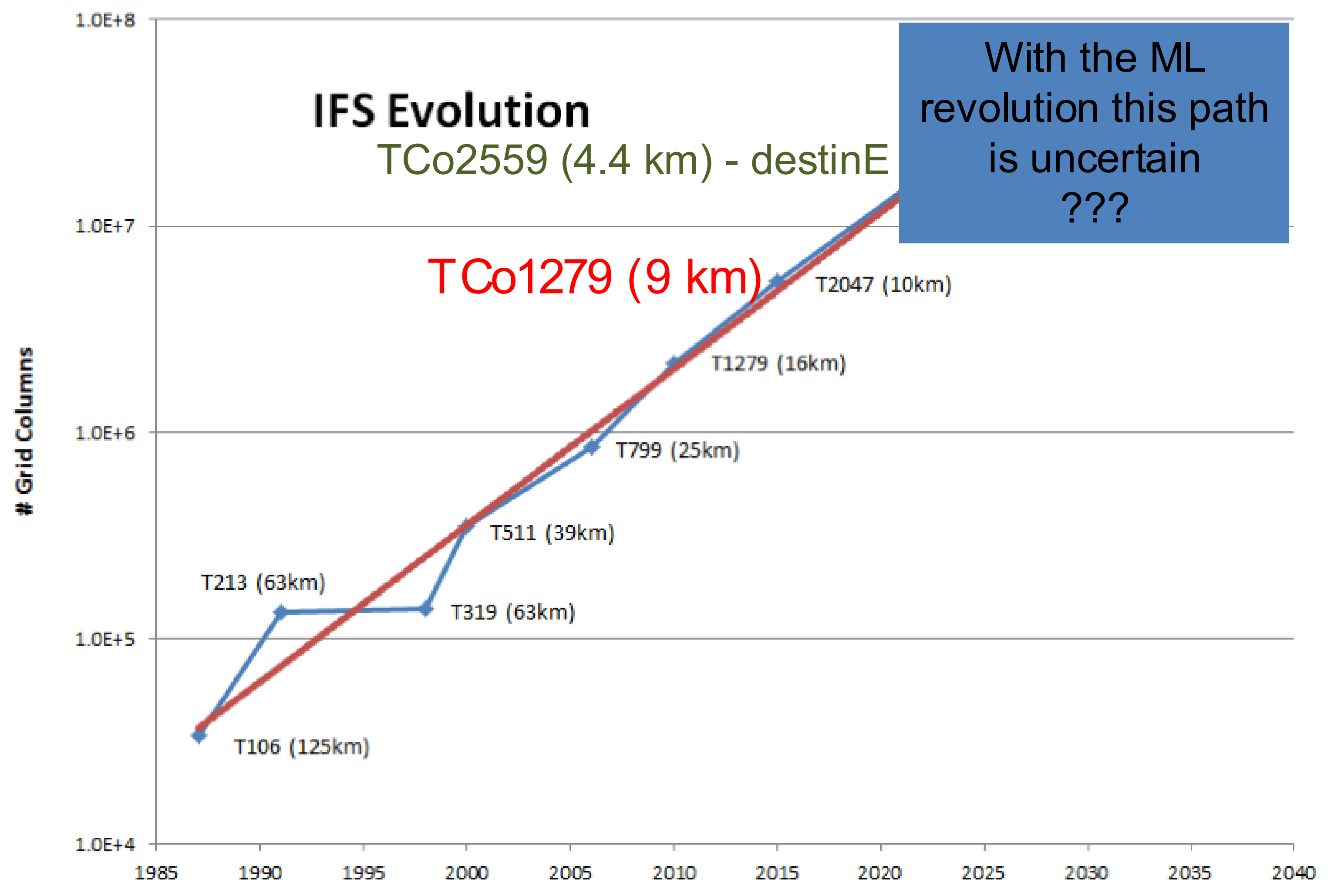
History and future of horizontal resolution at ECMWF



- 'T' is the truncation in the spherical harmonic expansion. The higher the 'T', the higher the horizontal resolution

- Since 2016 ECMWF has used cubic octahedral grid (TCo grid) for grid-point calculation.

History and future of horizontal resolution at ECMWF



- 'T' is the truncation in the spherical harmonic expansion. The higher the 'T', the higher the horizontal resolution
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ECMWF

Integrated Forecast System (IFS) dynamical core

- Operational IFS is a **semi-Lagrangian, semi-implicit, hydrostatic** dynamical core. **Spectral** in the horizontal, **vertical finite elements** with hybrid eta-coordinate in the vertical. Unresolved processes represented by the IFS ‘physics’ package.

$$\frac{D\mathbf{V}}{Dt} + f\mathbf{k}\times\mathbf{V} + \nabla_h\Phi + R_d T_V \nabla_h \ln p = P_V + K_V,$$

$$\frac{DT}{Dt} - \frac{\kappa T_V \omega}{[1+(\delta-1)q]p} = P_T + K_T,$$

$$\frac{\partial}{\partial t}\left(\frac{\partial p}{\partial \eta}\right) + \nabla_h\cdot\left(\mathbf{V}\frac{\partial p}{\partial \eta}\right) + \frac{\partial}{\partial \eta}\left(\dot{\eta}\frac{\partial p}{\partial \eta}\right) = 0,$$

$$\frac{Dq}{Dt} = P_q,$$

$$\frac{\partial \Phi}{\partial \eta} = -R_d T_V \frac{\partial \ln p}{\partial \eta},$$

$$T_V = T\left[1 + \left(\frac{R}{R_d} - 1\right) q\right]$$

Spectral vs. grid-point representation in the IFS dynamical core

- **Derivatives, dissipation and semi-implicit solver** calculated in **spectral** space. Only VOR, DIV, T_v, Φ and p_s have spectral representations.

- **Nonlinear terms and semi-Lagrangian advection** calculated in **grid-point** space. Physical parametrizations applied in grid-point space → Need a grid to convert from spectral space.

- For every spectral truncation there is a **physical space grid**.

$$\frac{D\mathbf{V}}{Dt} + f\mathbf{k} \times \mathbf{V} + \nabla_h \Phi + R_d T_V \nabla_h \ln p = P_V + K_V,$$

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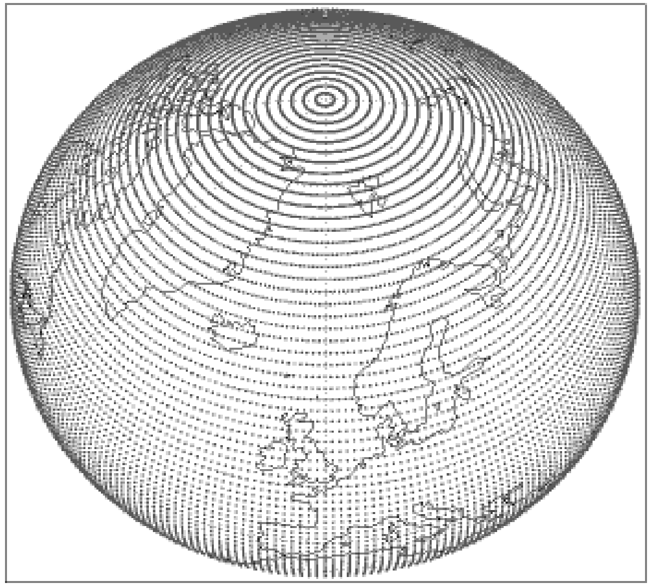
IFS Gaussian grids

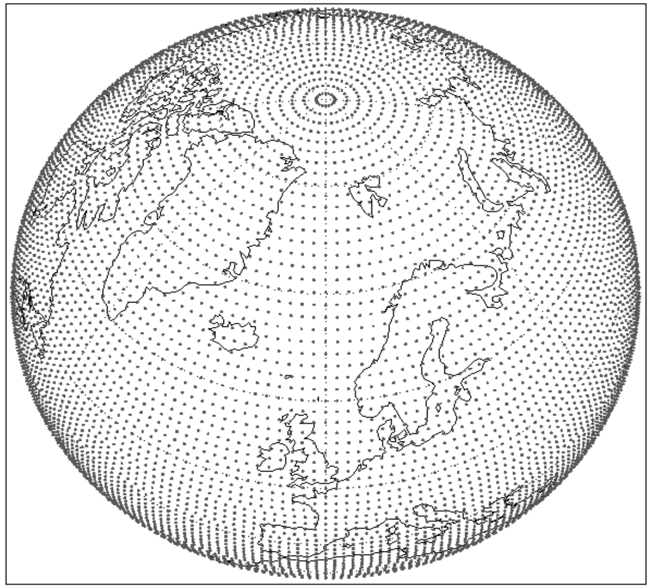
- **Regular (full)** grid: Same number of points around each latitude circle (i.e., crowded near the poles). No aliasing of quadratic terms (i.e. product of 2 variables).

- **Reduced:** Number of points per latitude circle decreases towards the pole, such as to maintain quasi-regular grid spacing dx≃dy.
  Cubic: No aliasing of cubic terms (i.e., product of 3 variables)

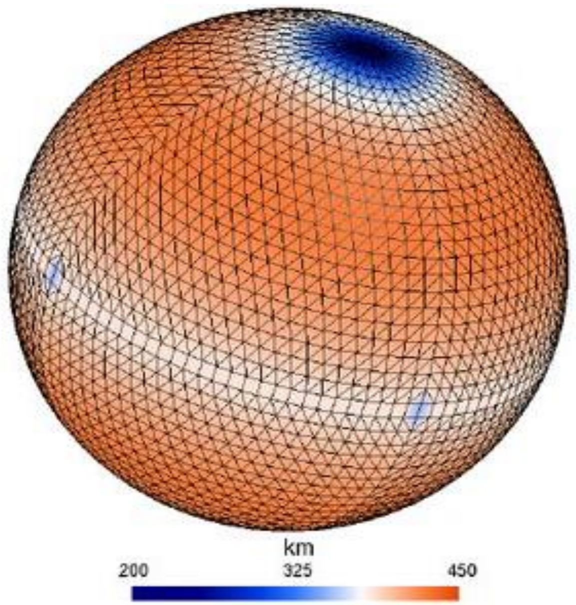
Full: Regular quadratic grid



Reduced: linear grid



Reduced: Cubic octahedral grid



IFS grid to spectral truncation (NSMAX) pairing

Pairing grid/truncation

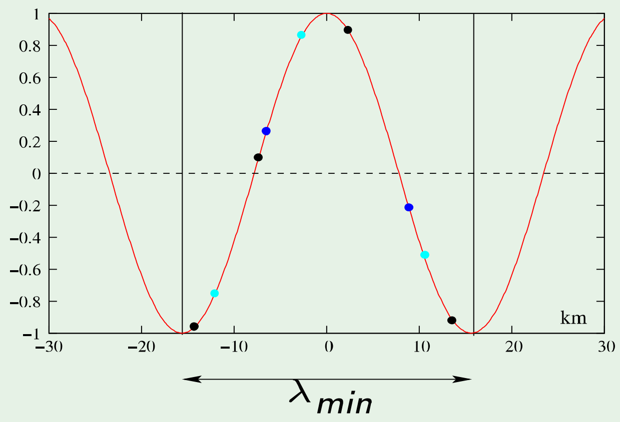
linear: the smallest wavelength $\lambda_{min} = (2 * \pi * RA)/NSMAX$ is sampled on the grid, along the equator, by 2 points $\Rightarrow NDLON_{lin} \simeq 2 * NSMAX$

quadratic: by 3 points $\Rightarrow NDLON_{quad} \simeq 3 * NSMAX$

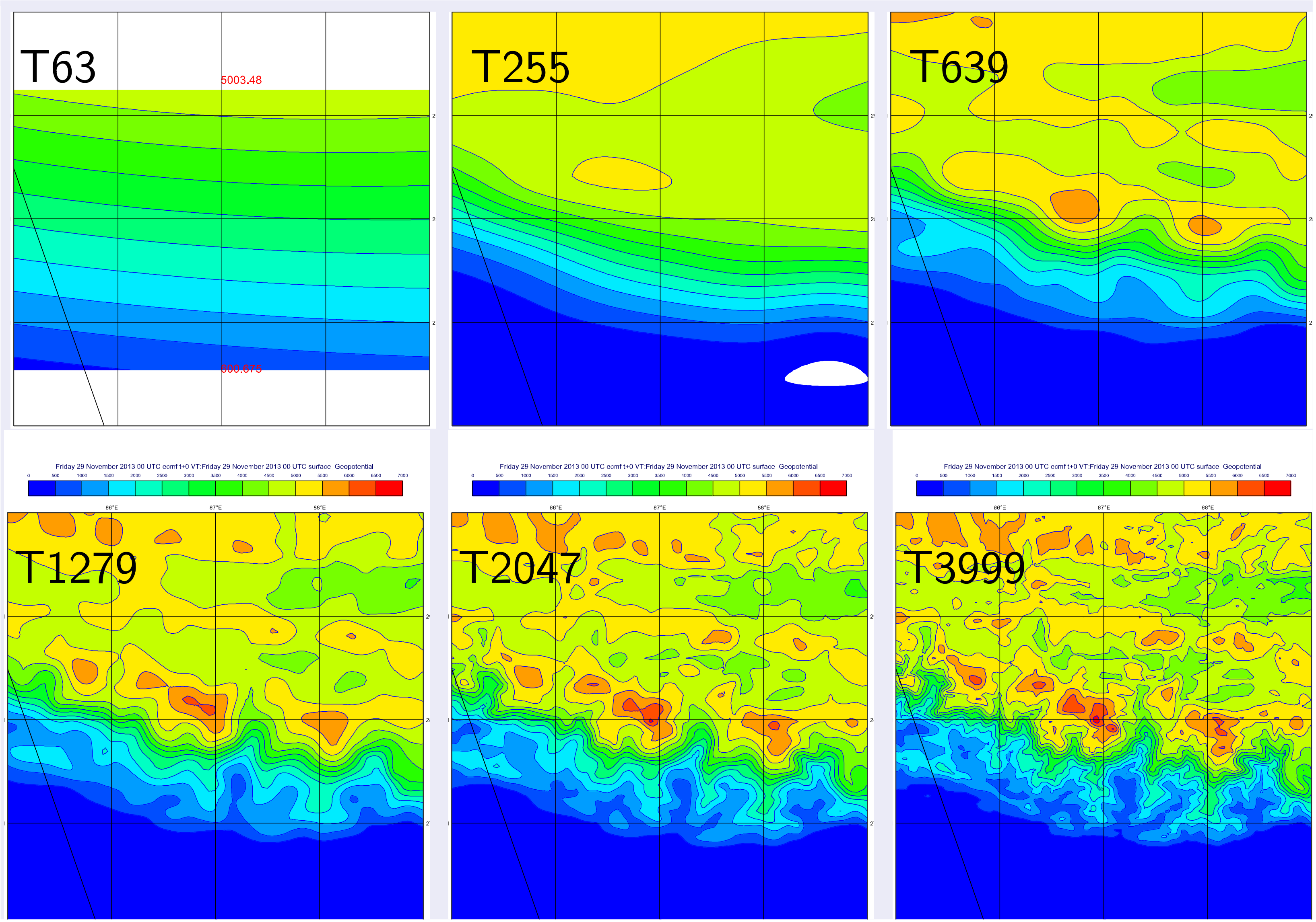
cubic: by 4 points $\Rightarrow NDLON_{cub} \simeq 4 * NSMAX$

$$T1279 \Rightarrow \begin{array}{l} NDLON_L = 2560 \\ NDLON_Q = 3840 \\ NDLON_C = 5120 \end{array}$$

$$NDLON = 2560 \Rightarrow TL1279 \text{ or } TC639$$



Example of increasing horizontal resolution



Surface geopotential height (i.e., resolved orography) around Mount Everest.

As the horizontal resolution is increased, more realistic details are represented by the model.

Different ways to increase horizontal resolution at ECMWF

- Increase the spectral truncation (i.e., the amount of waves retained in the spherical harmonic expansion) but keep the same grid: What we did in 1999.
- Increase the spectral truncation as well as the grid-point resolution: What we did from 1999 to 2016.
- Keep the same spectral truncation, but resolve better in grid-point space by increasing the grid-point resolution: This is what we did in 2016.

Latest resolution upgrade

FROM:		TO:
4DV: TCo1279/TL255-319-399	→	**4DV:** TCo1279/TL255-319-399-511
HRES: TCo1279		**DestinE:** TCo2559
EDA: TCo639		**EDA:** TL255
ENS: TCo639/TCo319 (d1-10/d11-30)		**ENS:** TCo1279/TCo319 (d1-15/d1-45)

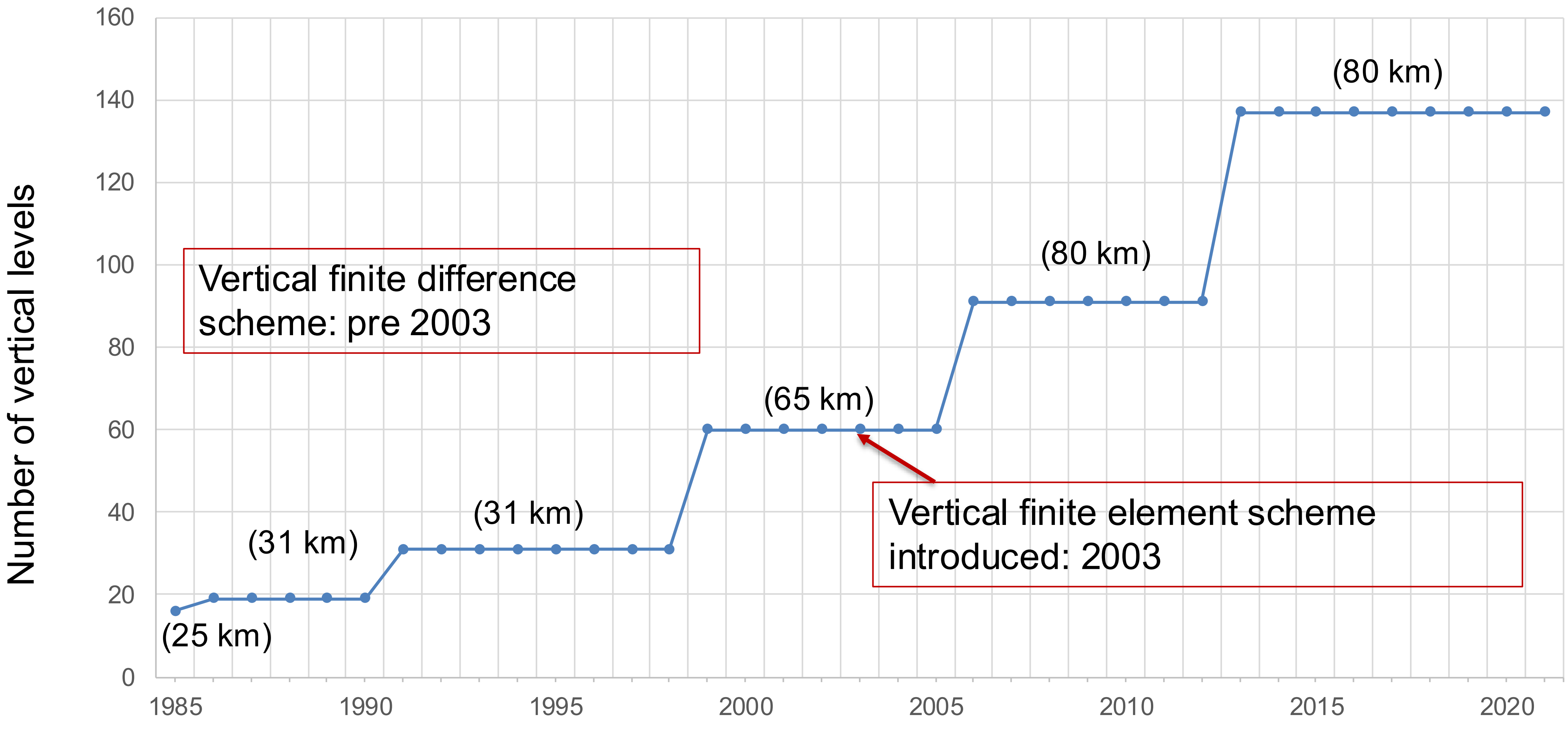
Part I: Recap

- High horizontal resolution is needed to better resolve high-impact weather events.
- Horizontal resolution at ECMWF can be increased by:
  1. Increasing the truncation wavenumber in the spherical harmonic expansion, keeping grid resolution unchanged.
  2. Increasing both the truncation wavenumber in the spherical harmonic expansion and the grid resolution.
  3. Increasing the grid resolution, keeping the truncation wavenumber unchanged.
- Currently, a cubic-octahedral grid is used at ECMWF.

  Prevents aliasing of products of 3 variables.

  Represents the shortest resolved wavelength by 4 points along the equator.

History of vertical resolution at ECMWF (model top in parentheses)



ECMWF

Vertical Finite Differences vs. Vertical Finite Element

- Pre 2003, IFS used the Simmons & Burridge (1981) vertical FD discretization.

- **Idea:** All the vertical derivatives calculated as **centered finite differences**.

  All the vertical integrals performed by the **mid-point rule**.

$$\partial_x f \approx \frac{f(x+dx) - f(x-dx)}{2dx}$$

MIDPOINT RULE

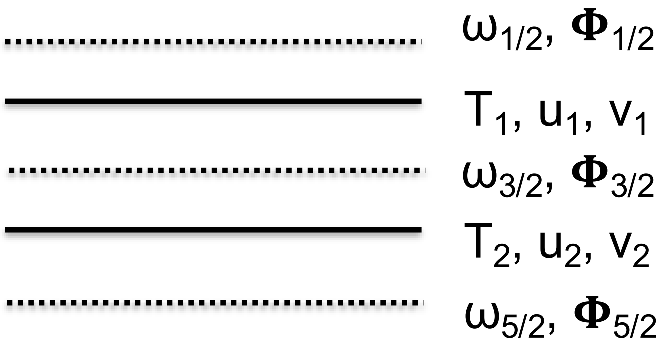
$$\int_a^b f(x)\,dx \approx M_n = \Delta x\,[f(\bar{x}_1) + f(\bar{x}_2) + \cdots + f(\bar{x}_n)]$$

where $\Delta x = \frac{b-a}{n}$

and $\bar{x}_i = \frac{1}{2}(x_{i-1} + x_i) = \text{midpoint of } [x_{i-1}, x_i]$

- Order of accuracy: 2

- Staggering of variables: Yes, **Lorenz staggering** → introduces a computational 2Δz mode. A solution to the 2Δz mode is to use Charney-Phillips staggering (e.g. the UKMO)

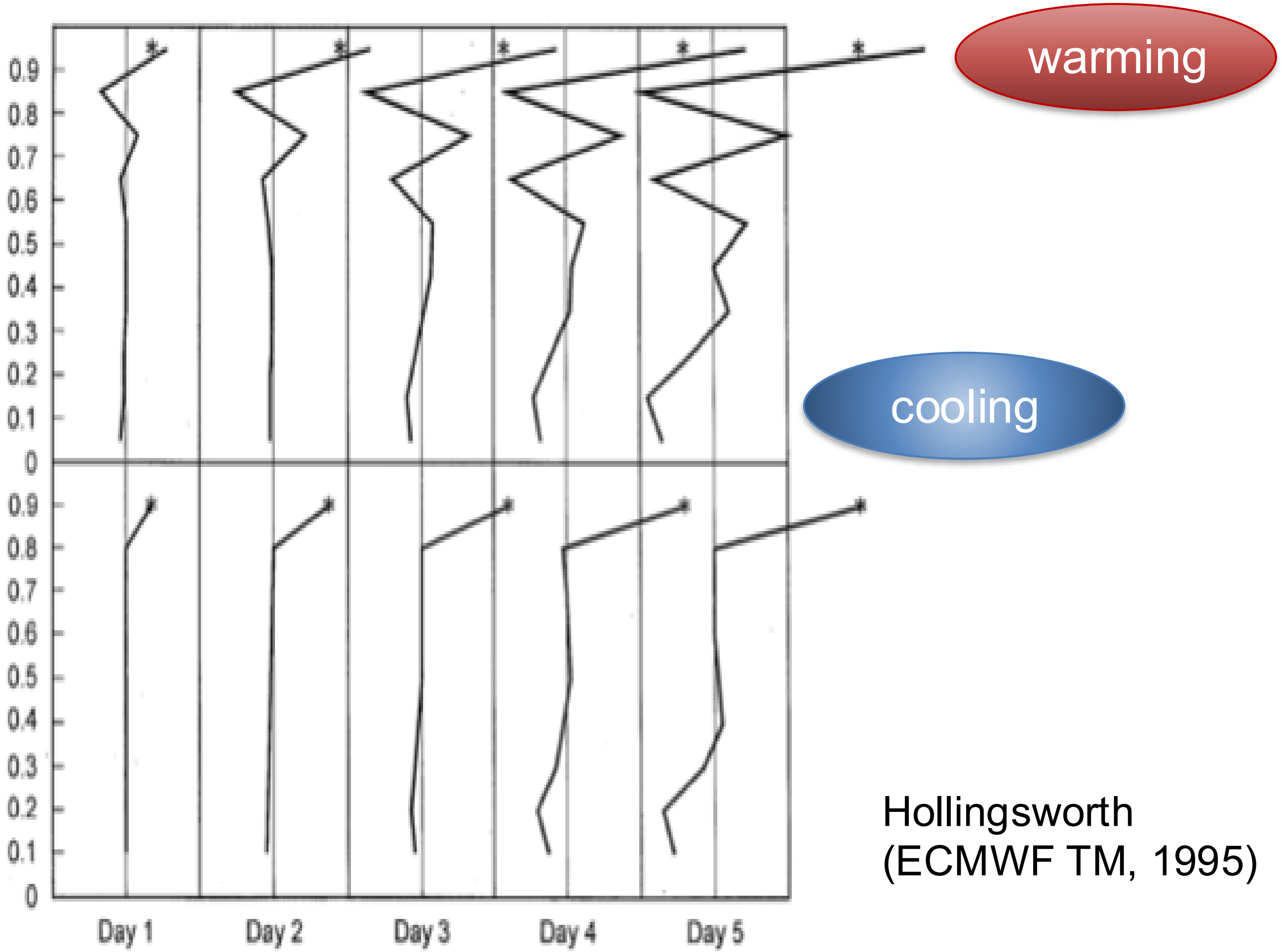


Impact of the 2Δz computational mode

Lorenz staggering permits 2Δz computational mode **in temperature.** If excited, can generate spurious cooling/heating.

Lorenz staggering produces spurious heating/cooling, due to 2Δz noise

Charney-Phillips staggering, which eliminates 2Δz noise, produces correct response



Hollingsworth (ECMWF TM, 1995)

ECMWF

Vertical Finite Differences vs. Vertical Finite Element

- Post 2003, IFS uses Untch & Hortal (2004) vertical FE discretization

- **Idea:** Equations discretized without vertical derivatives.
  Vertical integrals

$$F(\eta) = \int_0^{\eta} f(x)\,\mathrm{d}x$$

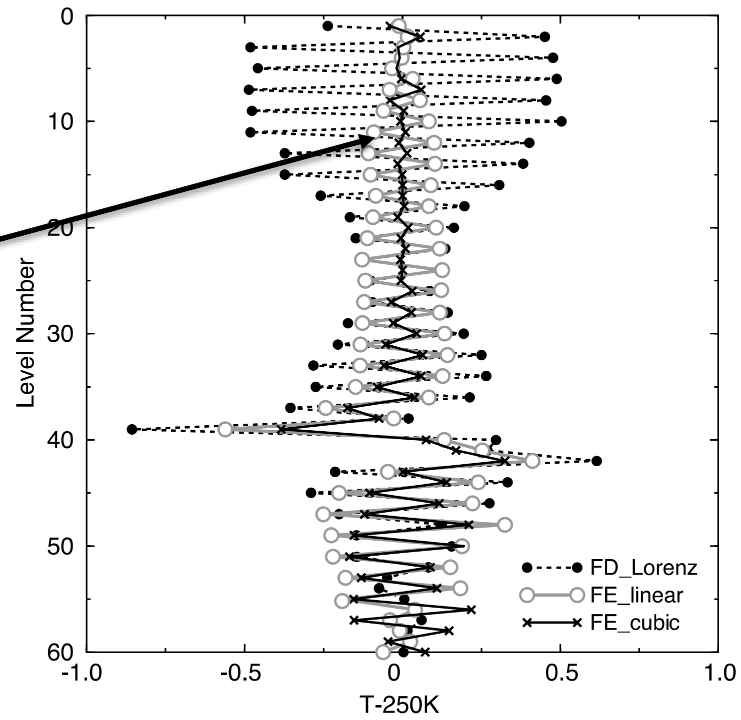
  approximated as

$$\sum_{i=K_1}^{K_2} C_i d_i(\eta) = \sum_{i=M_1}^{M_2} c_i \int_0^{\eta} e_i(x)\,\mathrm{d}x,$$

C_i: coeff. of expansion of F(η)
$d_i(\eta)$: basis functions
*c_i: coeff. of the expansion of f(η) as a linear combination of basis functions $e_i(\eta)$, which are **cubic splines** in IFS*

- Order of accuracy: 8

- Staggering of variables:
  No. Smooths the 2Δz computational mode.

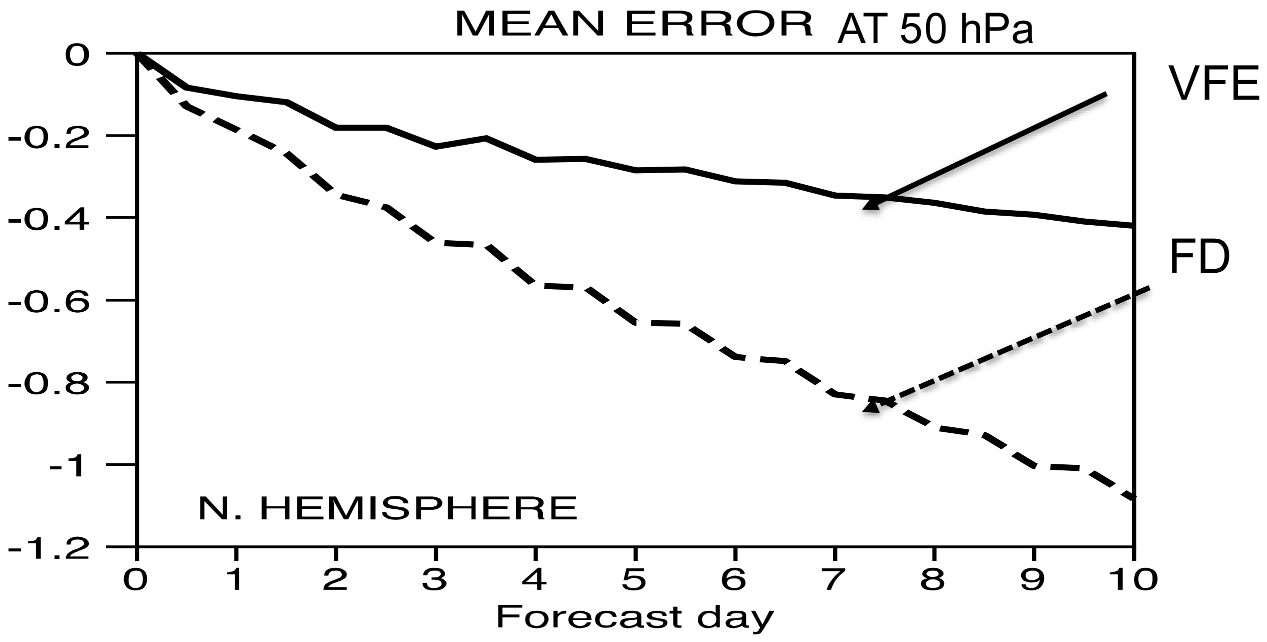


ECMWF

Impact of eliminating 2Δz computational mode

- Vertical finite element discretization damps the 2Δz noise, but does not **fully eliminate it →** aliasing of vertically unresolved waves onto the 2Δz mode can still generate a spurious thermal response.

- Damping of the computational mode via VFE reduces spurious global-mean cooling in the lower to mid-stratosphere in IFS, compared to FD with Lorenz staggering.

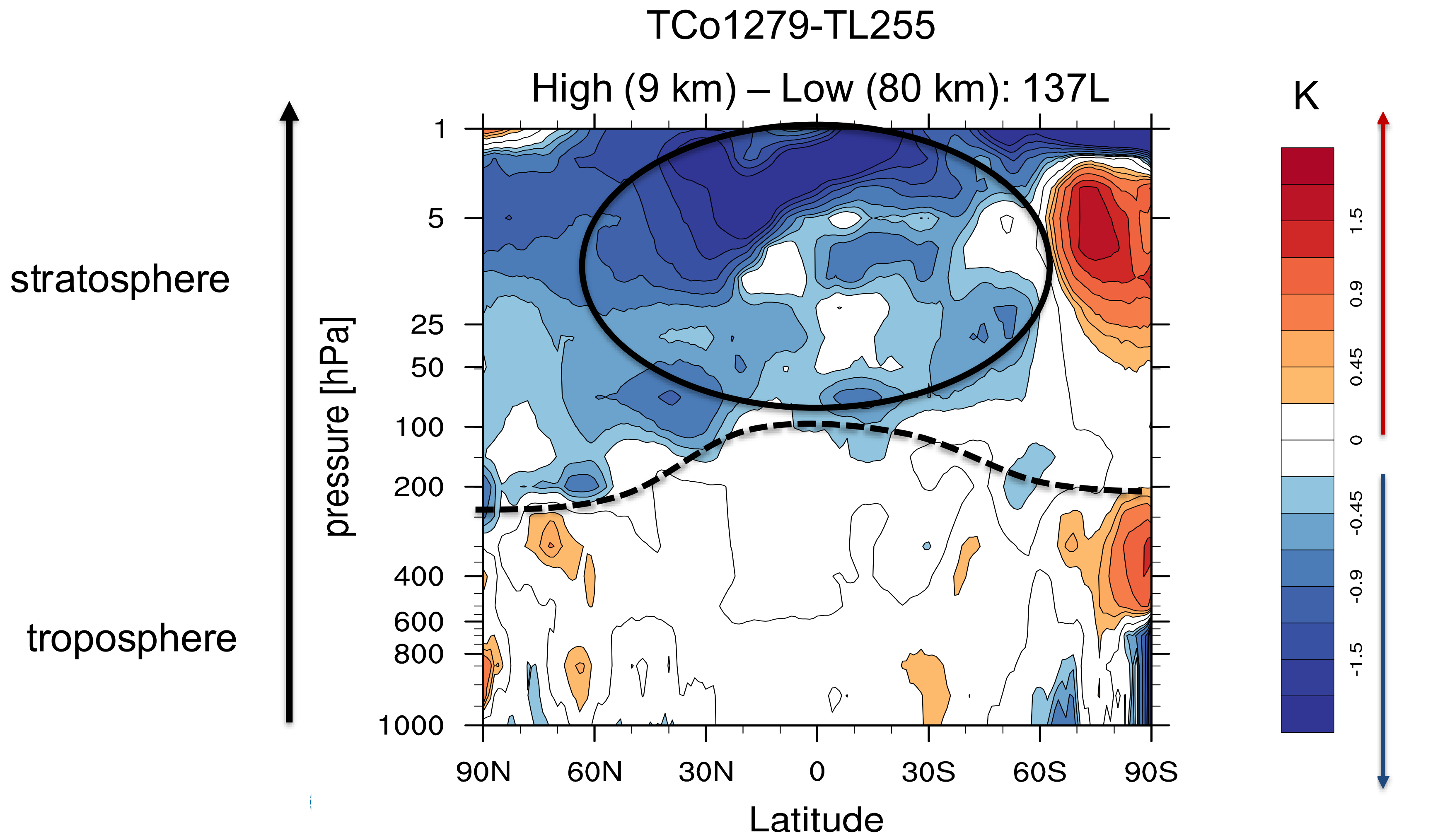


Untch & Hortal (QJRMS, 2004)

ECMWF

Stratosphere at ECMWF: The 2Δz problem returns at high horizontal resolution

- **Question:** Does the cooling at higher horizontal resolution arise due to inadequate **horizontal to vertical resolution aspect ratio**? OR: Is the vertical resolution **too coarse**?



Average over 31 forecasts in July at 10-day lead time.

Theoretical considerations: Horizontal to vertical resolution aspect ratio

- Aspect ratio relevant for balanced **quasi-geostrophic** dynamics (Lindzen & Fox-Rabinovitz, 1989) and **inertia-gravity** wave dispersion relation

$$\frac{\Delta z}{\Delta x} \sim \frac{f}{N}$$

$\Delta z \sim$ 200 m, for Δx ~18 km

- **Stratified turbulence** develops shear layer of thickness (e.g., Waite & Bartello, 2004)

$$L_b \equiv 2\pi U / N$$

In the stratosphere, L_b ~ 1 km, need 4-6 Δz to resolve → Δz ~ 200 m

- **Gravity wave propagation**:

  Dispersion relation (medium-frequency)

$$|\lambda_z| \sim |c\text{-}U| / N$$

If vertical resolution not adequate to resolve λ_z, discretization errors occur

Stratosphere in the IFS: Horizontal to vertical resolution aspect ratio

- Perform **10-day forecast** experiments at **high (9 km)** and **low (80 km)** horizontal resolution and gradually increasing **vertical resolution** in the lower to mid-stratosphere.

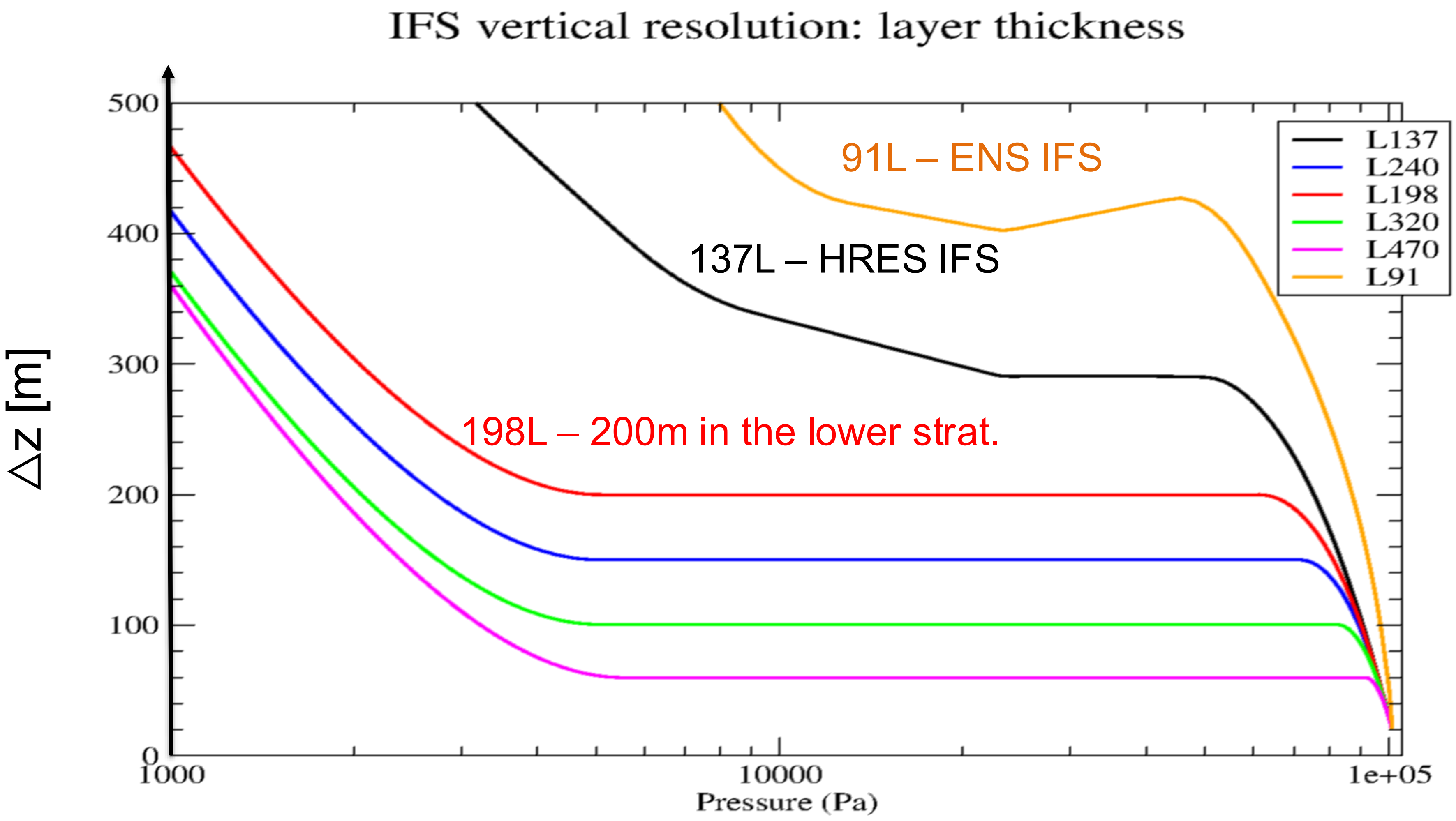
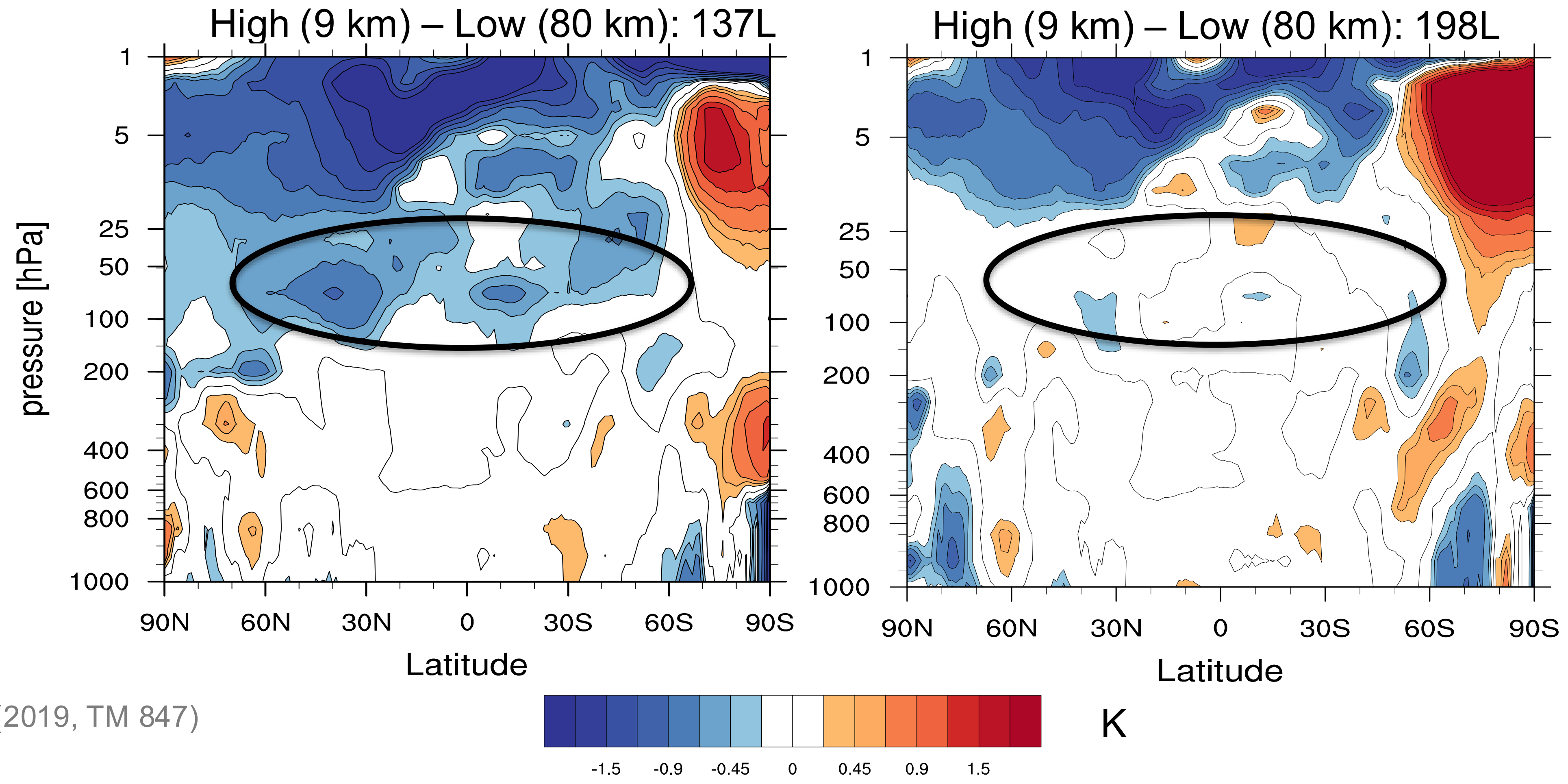


Figure courtesy:
Tim Stockdale

Stratosphere in the IFS: Horizontal to vertical resolution aspect ratio

- **Vertical resolution** of **200** m in the lower to mid stratosphere eliminates global mean cooling there at high horizontal resolution.

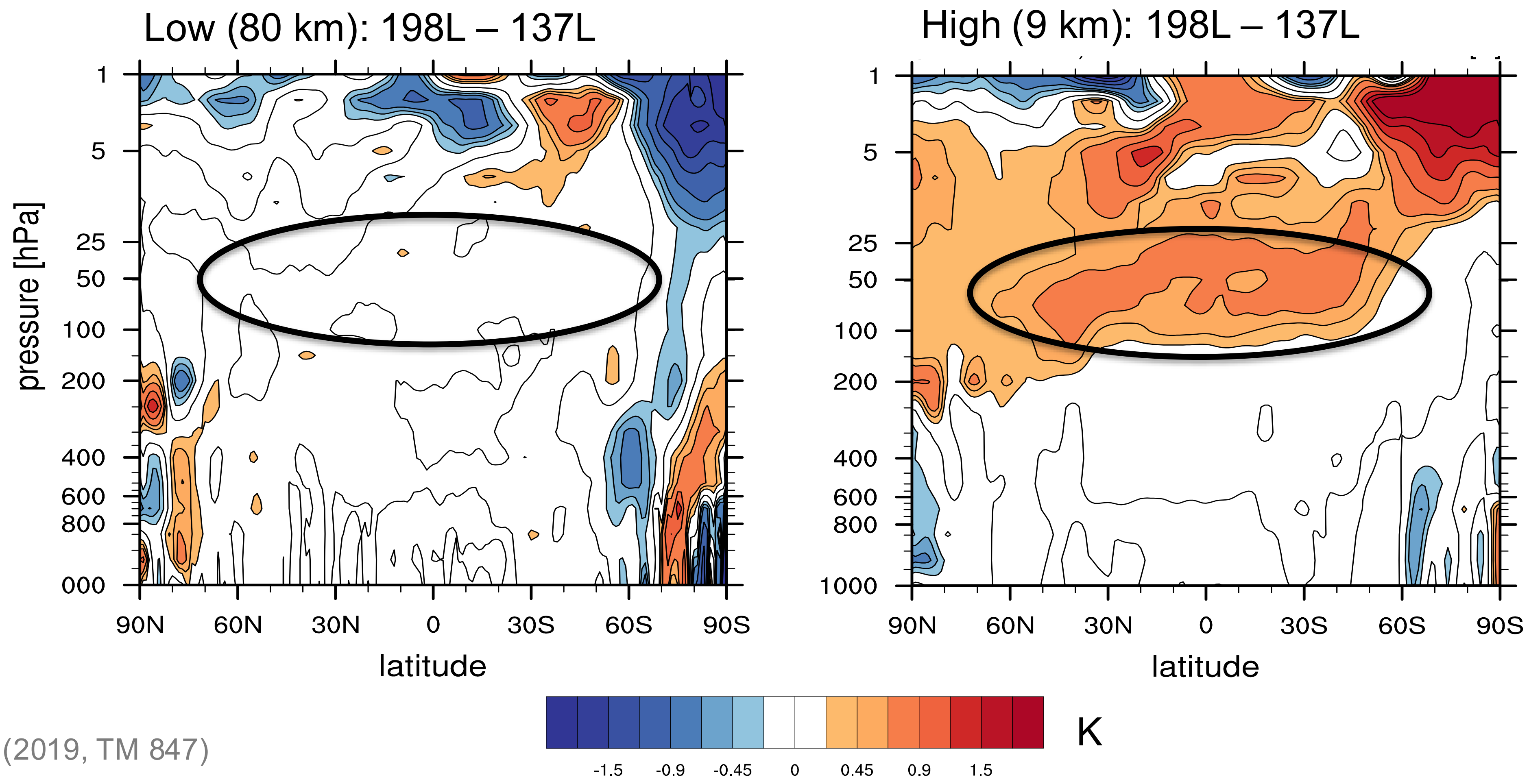
Average over 31 forecasts in July at 10-day lead time.



Polichtchouk et al. (2019, TM 847)

Stratosphere in the IFS: Horizontal to vertical resolution aspect ratio

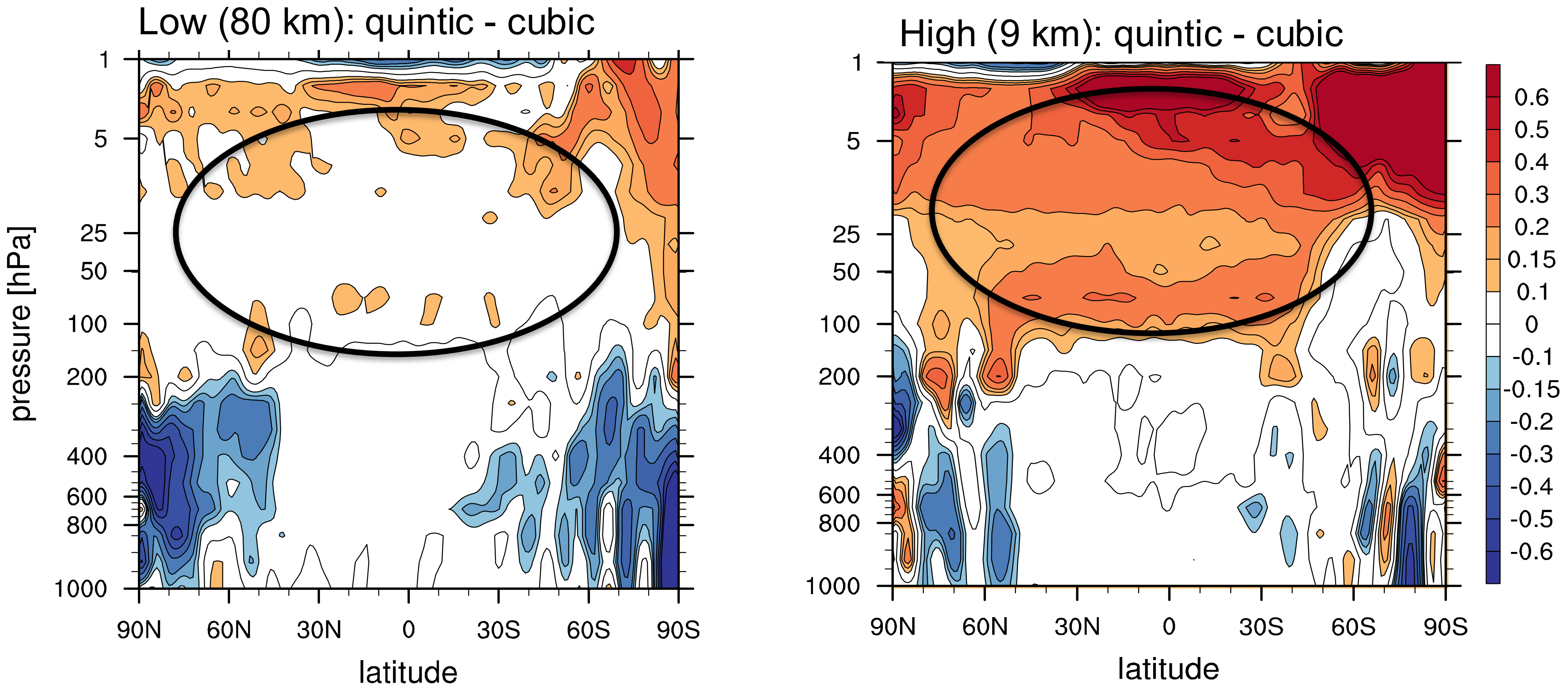
- Increase in the **vertical resolution** leads to warming in the stratosphere at **high horizontal resolution. No impact at low horizontal resolution**.



Polichtchouk et al. (2019, TM 847)

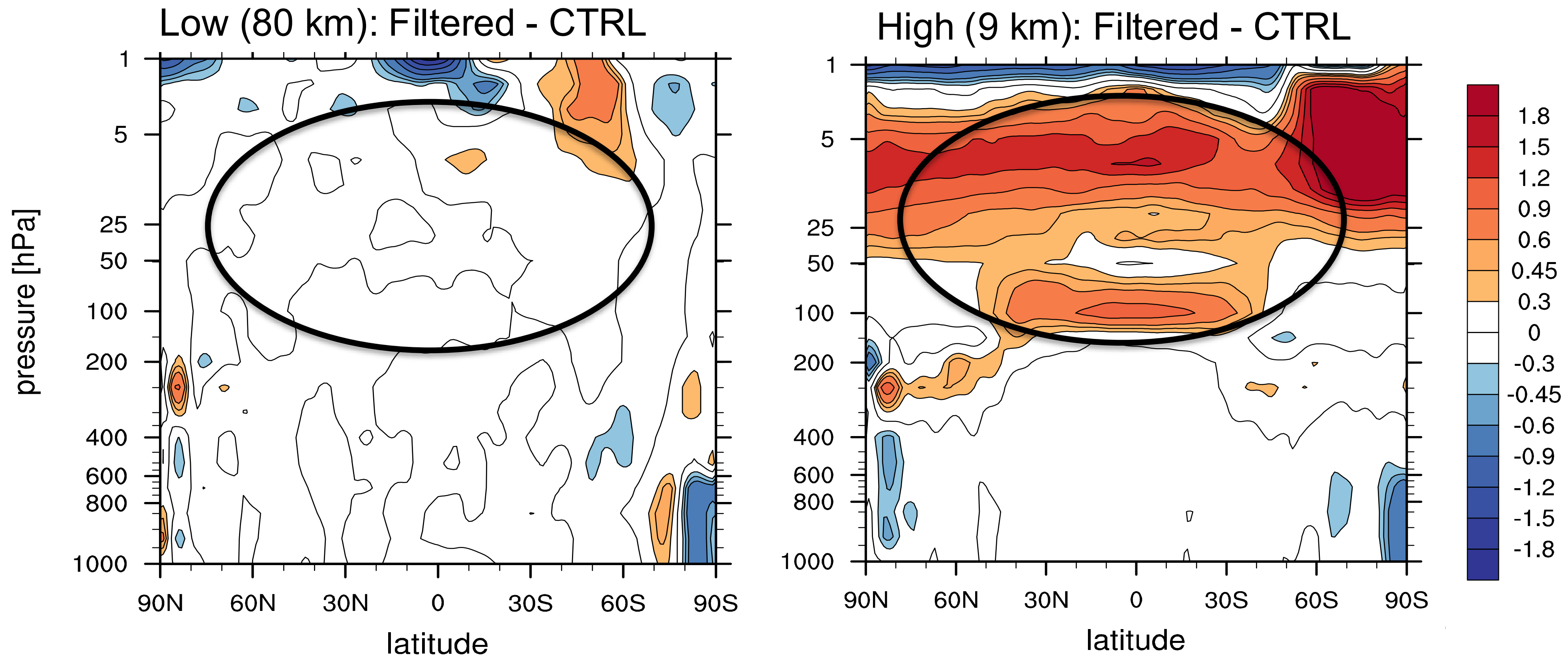
Higher order vertical Semi-Lagrangian interpolation

- Increasing vertical resolution is expensive → seek cheaper solutions to solve the problem.
- **Question:** Does improving the accuracy of the vertical semi-Lagrangian interpolation help?
- Going from **3rd** to **5th** order vertical interpolation helps → Stratosphere warms with higher order interpolation at high horizontal resolution.



Filtering 2Δz noise during semi-Lagrangian interpolation

- **Question:** Does **filtering 2Δz**-noise in temperature via semi-Lagrangian vertical filter help horizontal resolution sensitivity?
- Filtering warms high horizontal resolution forecasts, **no impact** on **low horizontal resolution**.



Part 2: Recap

- Current vertical resolution at ECMWF is **137 levels**. Vertical discretization by **vertical finite element scheme (VFE)**.
- VFE scheme suppresses **$2\Delta z$** noise inherently present in the vertical finite difference scheme using Lorenz staggering. **$2\Delta z$** noise detrimental for temperature forecasts, especially in the stratosphere leading to unphysical global-mean cooling.
- At high horizontal resolution, **$2\Delta z$** noise returns in the VFE scheme. Due to inconsistent horizontal to vertical resolution aspect ratio for gravity waves → need to increase the vertical resolution together with the horizontal resolution.
- Increasing the **vertical resolution** to **200 m** eliminates the global mean cooling at **higher horizontal resolution**.
- **Filtering out $2\Delta z$-noise** or increasing the **order of vertical SL interpolation** also alleviate the global mean cooling at **high horizontal resolution**.

Hydrostatic vs non-hydrostatic model

- **Question:** As we increase the horizontal and vertical resolutions, do we need to relax the hydrostatic approximation?

- Hydrostatic approximation: $\frac{Dw}{Dt} \ll \left[-\frac{1}{\rho}\frac{\delta p}{\delta z} - g\right] \Rightarrow w$ diagnostic

  ➔Adjustment to the hydrostatic equilibrium faster than a time step.

- Validity of the hydrostatic approximation: $\left|\frac{Dw/Dt}{\frac{1}{\rho}\delta p/\delta z}\right| \ll 1$

$$\frac{Dw}{Dt} \sim \frac{UW}{L} = \frac{U^2 H}{L^2}, \quad \frac{1}{\rho}\frac{\delta p}{\delta z} \sim \frac{U^2}{H} \quad \Rightarrow \left|\frac{Dw/Dt}{\frac{1}{\rho}\delta p/\delta z}\right| \sim \frac{U^2 H/L^2}{U^2/H} \sim (H/L)^2 \ll 1$$

H: vertical length scale (e.g., scale height 10 km)

L : horizontal length scale

W~U H/L

T~L/U: Advective time-scale

dP~ ρU^2 (scaling from considering horiz. momentum eqn.)

ECMWF

Hydrostatic vs non-hydrostatic model

- Validity of hydrostatic approximation

$$(H/L)^2 \ll 1$$

➔ For H=10 km, hydrostatic approximation valid for $L \gg$ 10 km.

- **Common interpretation:** Hydrostatic approximation valid for Δx > 10 km.

- **Recall,** current horizontal resolution at ECMWF is TCo1279 or Δx ≈ 9 km. Do we need a non-hydrostatic (NH) model?

Non-hydrostatic model of the IFS

- A **non-hydrostatic fully compressible** set of Euler equations has been developed for the limited area version of the IFS dynamical core ALADIN/AROME/HARMONIE (Bubnova et al., 1995), and adapted for the global dynamical core (Wedi et al. 2009):

$$\frac{d\mathbf{v}}{dt} = -\frac{RT}{\pi}\nabla\pi - \nabla\phi - \left[RT\nabla\hat{q} + \frac{1}{m}\frac{\partial(p-\pi)}{\partial\eta}\nabla\phi\right] - 2\Omega\times\mathbf{v} + \mathbf{P}_{\mathbf{v}},$$

$$\frac{dw}{dt} = \frac{g}{m}\frac{\partial(p-\pi)}{\partial\eta} + P_w,$$

$$\frac{dT}{dt} = \kappa T\frac{\omega}{\pi} - \kappa T\left[\frac{\omega}{\pi} + \frac{C_p}{C_v}(D+d)\right] + P_T,$$

$$\frac{d\hat{q}}{dt} = -\left[\frac{\omega}{\pi} + \frac{C_p}{C_v}(D+d)\right] + P_{\hat{q}},$$

$$\frac{\partial q_s}{\partial t} = -\frac{1}{\pi_s}\int_0^1 \nabla(m\mathbf{v})d\eta',$$

$$\frac{dq}{dt} = P_q,$$

$$\frac{dq_l}{dt} = P_{q_l},$$

Non-hydrostatic terms

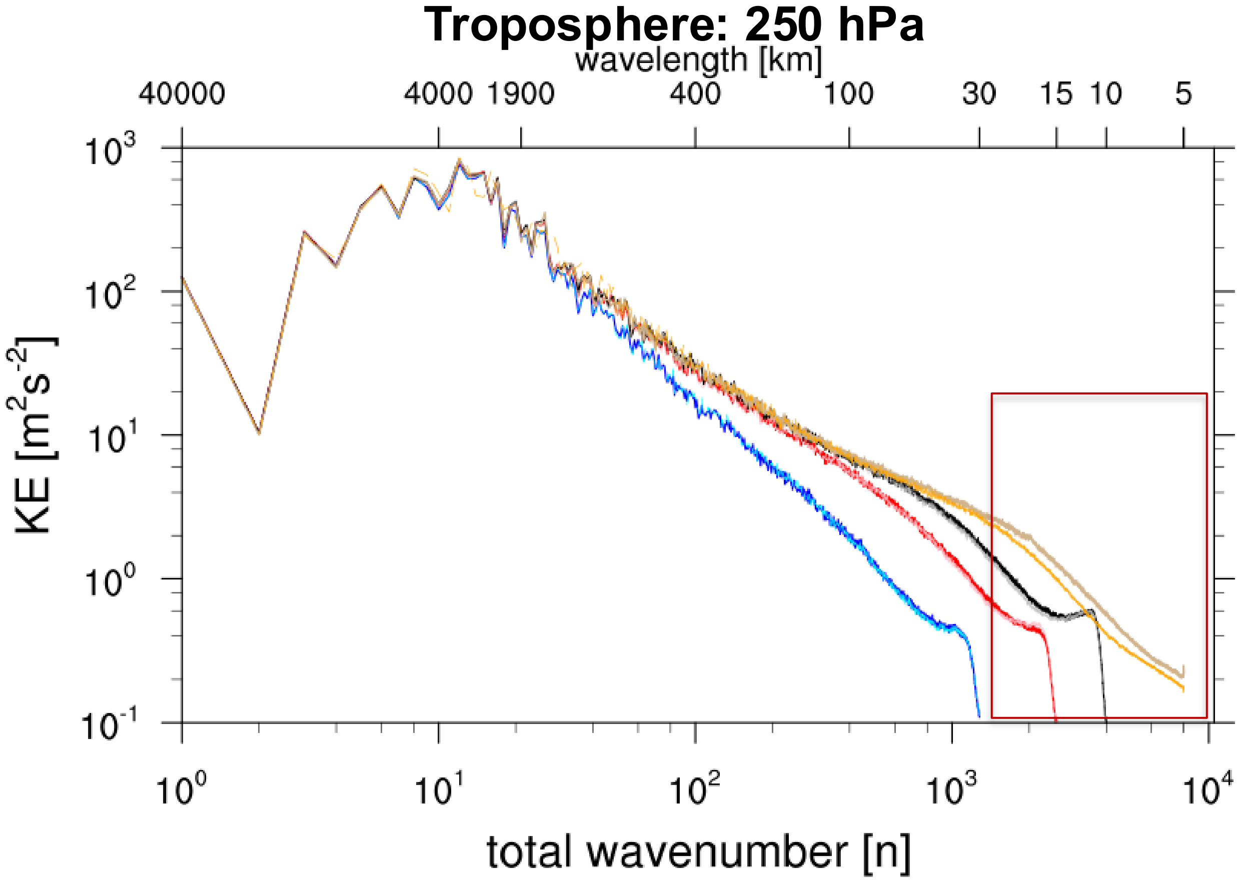
Hydrostatic and non-hydrostatic solvers have otherwise identical numerics:

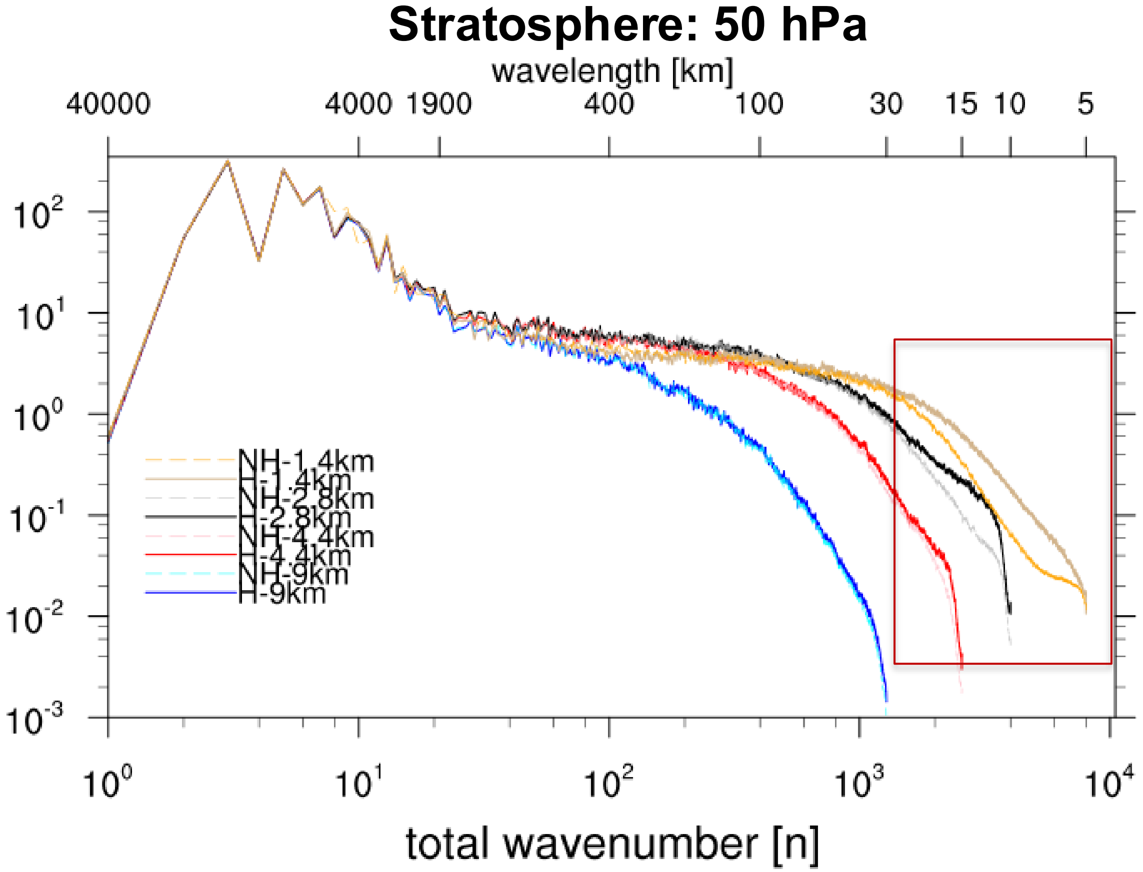
- spectral, semi-Lagrangian, reduced Gaussian grid, hybrid vertical levels $p(\eta) = A(\eta) + B(\eta)\Pi_s$ where Π is the hydrostatic part of the true pressure + IFS physics package.
- 2 more prognostics variables: w and the NH pressure departure $\hat{q}=ln(\frac{p}{\Pi})$.
- Predictor/corrector scheme: Doubles the cost of dynamics.

ECMWF

Non-hydrostatic vs. hydrostatic model of the IFS: Kinetic energy spectra

- Many recent improvements to NH solver mean we can run NH-IFS stably down to 1.4 km grid-spacing. See Vivoda, Polichtchouk, Diamantakis, Vana (2026, QJ)
- Differences between NH and H emerge more clearly in the stratosphere at at grid-spacing of 4.4 km and finer for horizontal wavelengths < 30 km.

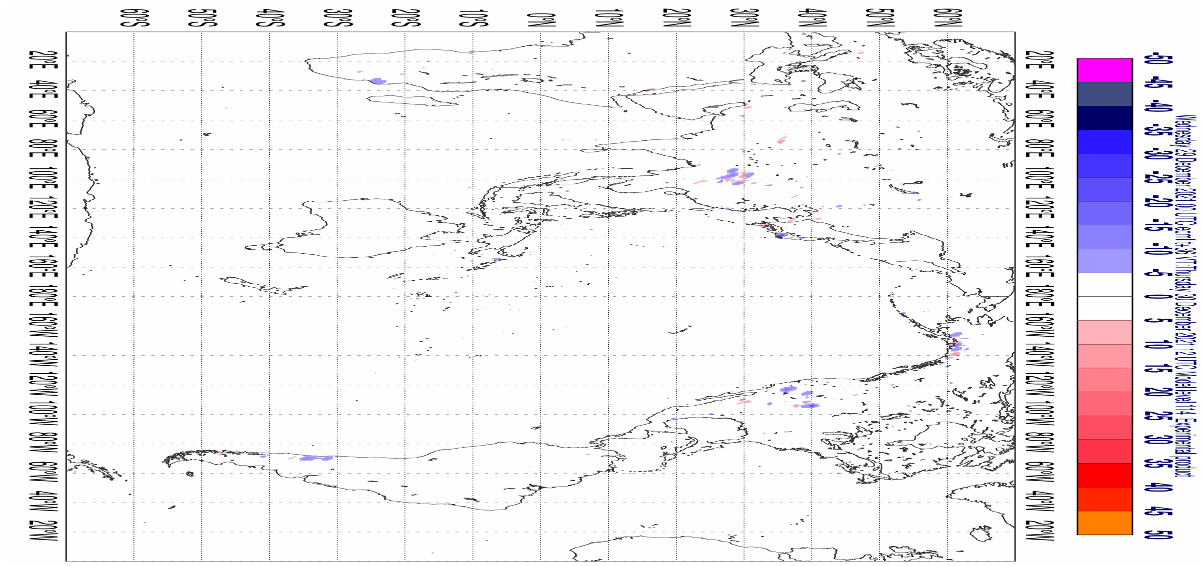




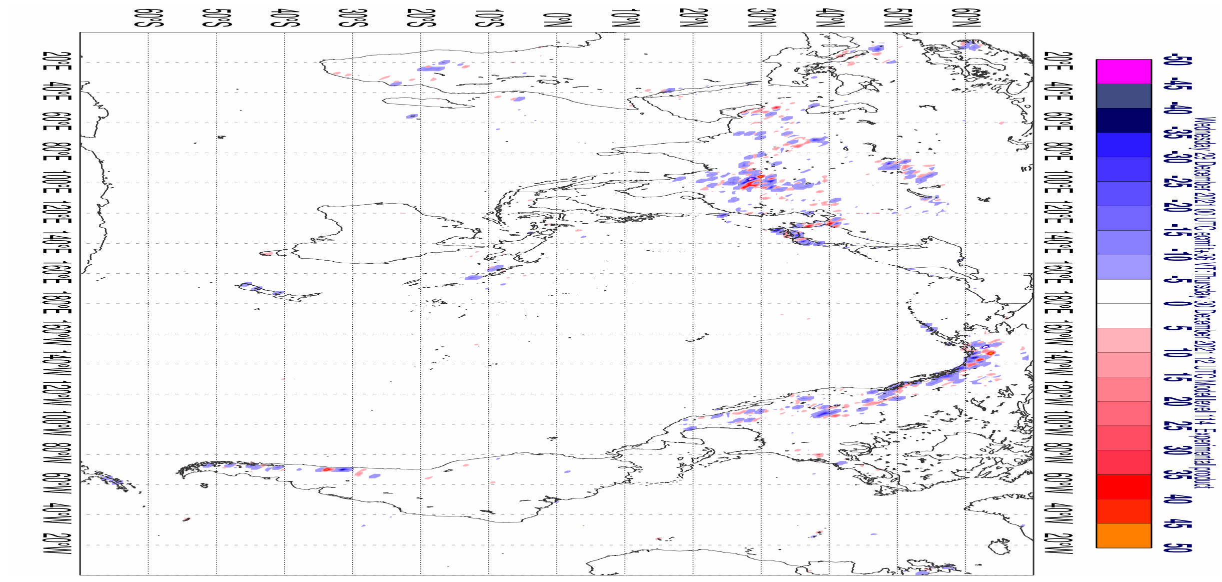
Vivoda, Polichtchouk, Diamantakis, Vana (2026, QJ)

Non-hydrostatic pressure departure @850hPa: NH vs H differences over orography

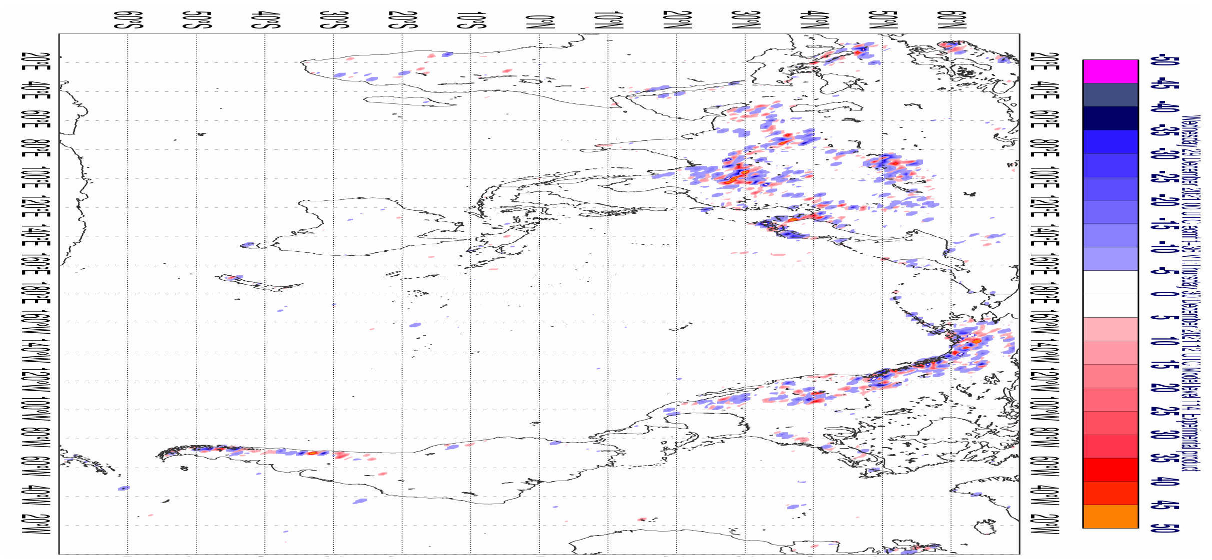
9 km



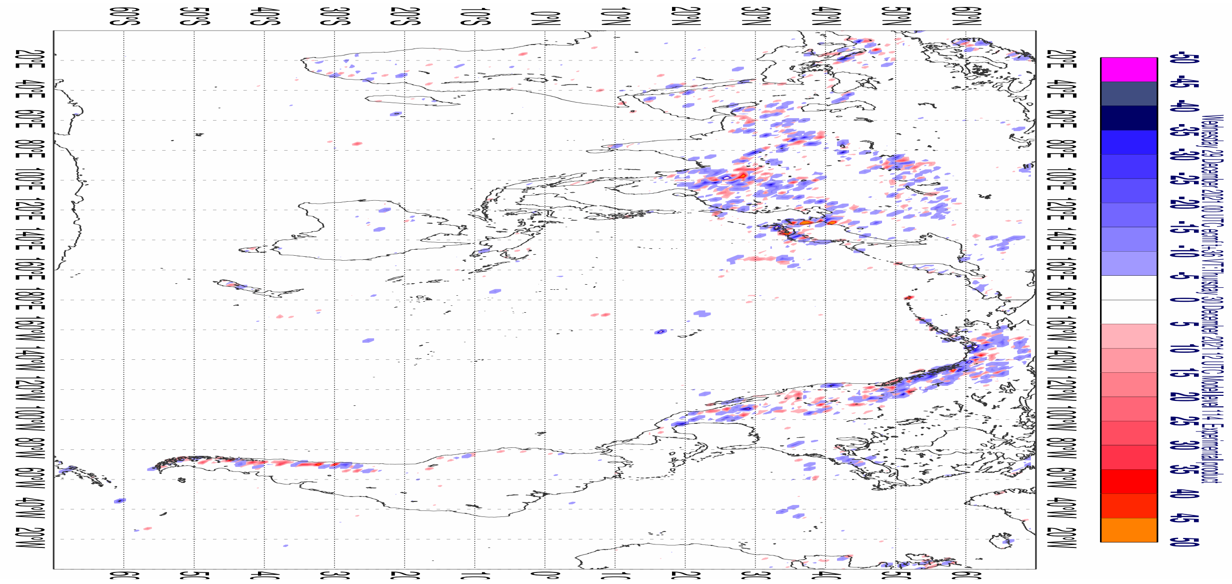
4.4 km



2.8 km



1.4 km



Why do differences emerge over orography?

- Dispersion relation for non-hydrostatic and hydrostatic mountain waves is different.

Non-hydrostatic

$$m = \left[\frac{N^2 k_h^2}{(Uk + Vl)^2} - k_h^2\right]^{1/2}$$

Hydrostatic

$$m = \frac{N k_h}{Uk + Vl}$$

m,k,l are vertical, meridional and zonal wavenumbers.
U, V are typical zonal and meridional flow speed;
N is typical stratification; $k_h=\sqrt{k^2 + l^2}$

ECMWF

Key differences:

- k_h dependence means NH waves are more dispersive.
- NH waves smaller than $\frac{2\pi U}{N}$ do not propagate vertically, whereas H waves always do.
- If $U \sim 10\ m\ s^{-1}$; $N \sim 10^{-2}\ s^{-1}$ waves with wavelength of 6 km are evanescent.
  If $U \sim 40\ m\ s^{-1}$ 24 km wavelength waves are evanescent.

Implications: If hydrostatic approximation made erroneously should have more waves entering the stratosphere. This is what we see.

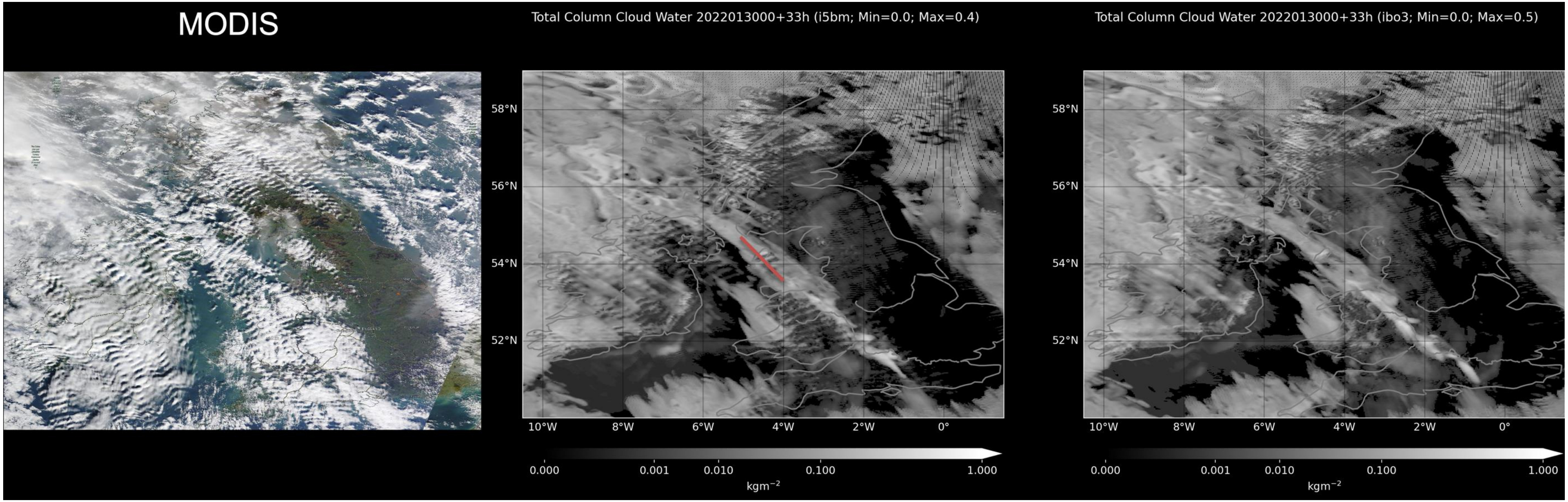
Case study of non-hydrostatic lee wave over the UK

Hydrostatic IFS does not correctly capture non-hydrostatic lee waves over the UK.

observations

Non-hydrostatic
(1.4 km grid-spacing)

Hydrostatic
(1.4 km grid-spacing)

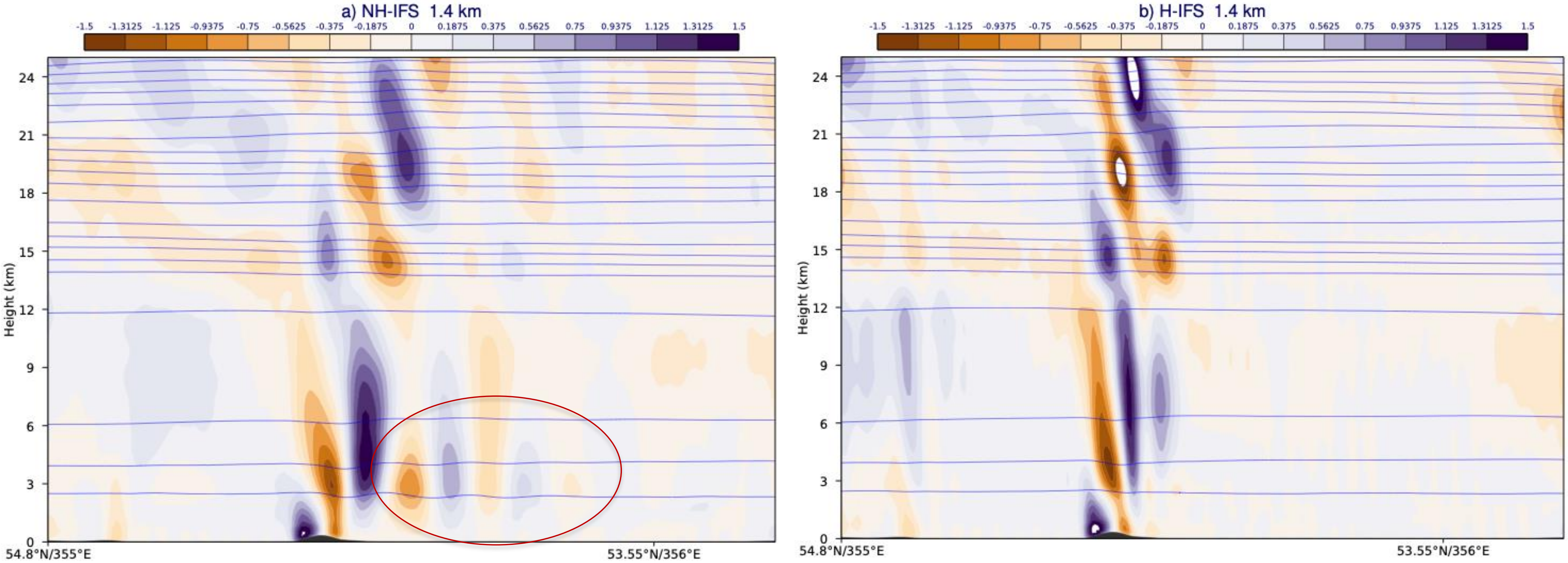


ECMWF

Vivoda, Polichtchouk, Diamantakis, Vana (2026, QJ)

Case study of non-hydrostatic lee wave over the UK

- Hydrostatic IFS does not correctly capture non-hydrostatic trapped lee waves over the UK.
- In hydrostatic model gravity waves propagate erroneously more strongly vertically into the stratosphere.

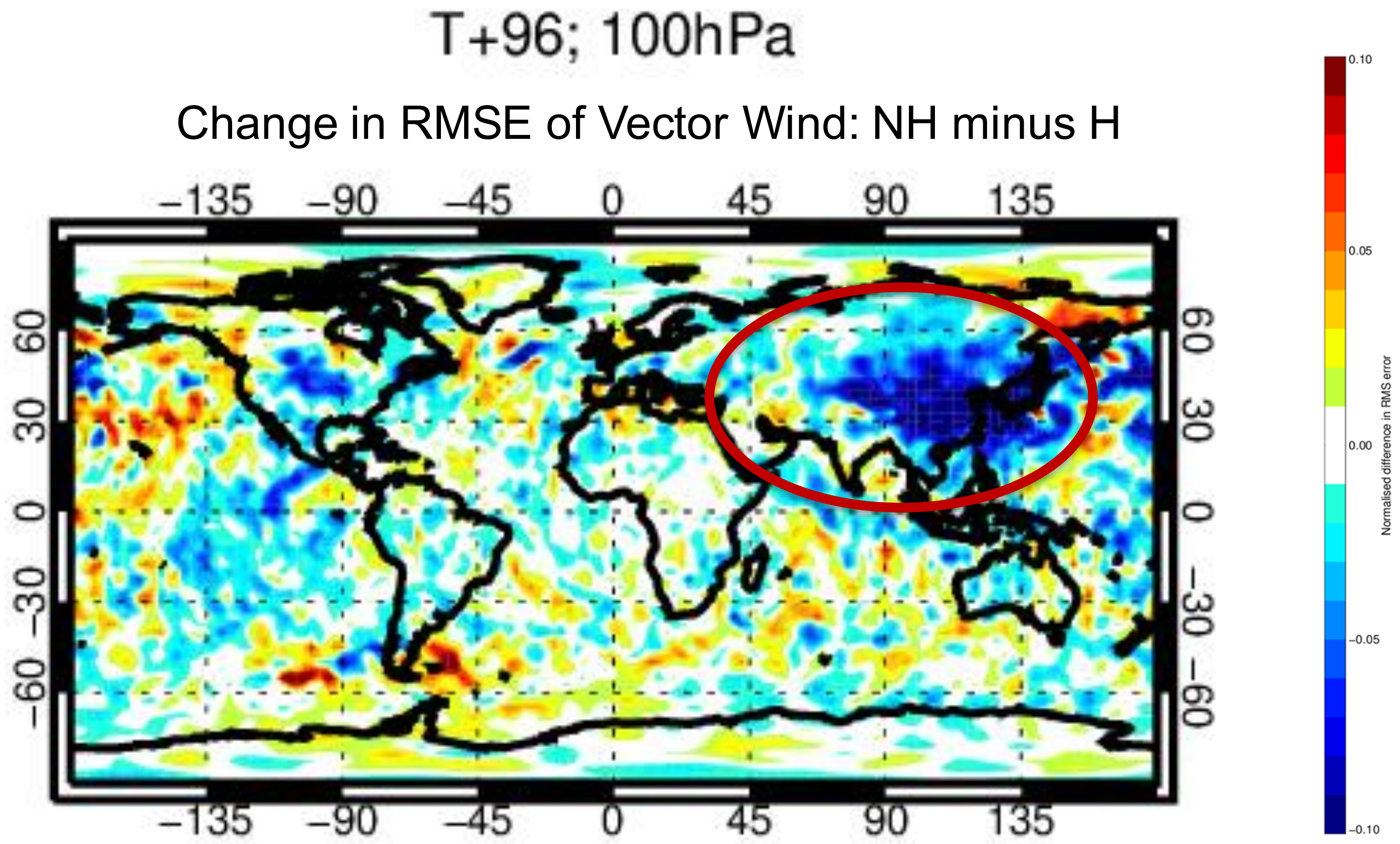


Vivoda, Polichtchouk, Diamantakis, Vana (2026, QJ)

Forecast performance: NH-IFS versus H-IFS

- Generally, if using H-IFS over complex orography at grid-spacings of 4.4 km and finer, too many gravity waves enter the stratosphere.

- This leads to worse forecast skill in H-IFS in comparison to NH-IFS in the stratosphere.



H better

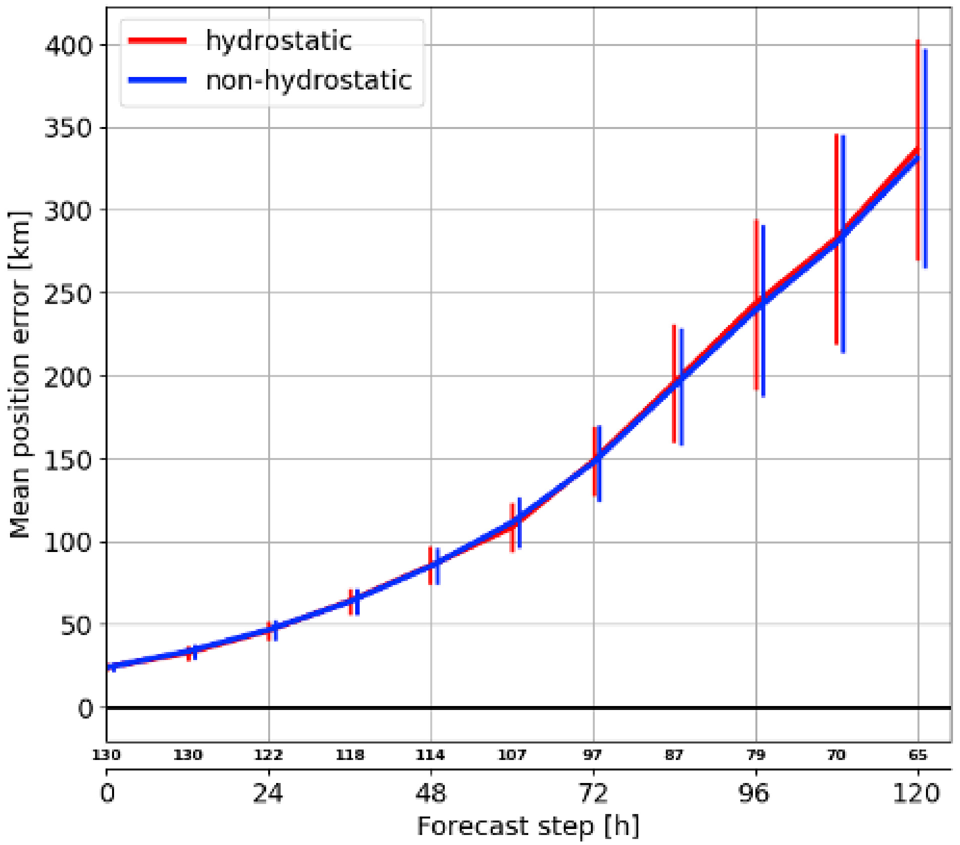
NH better

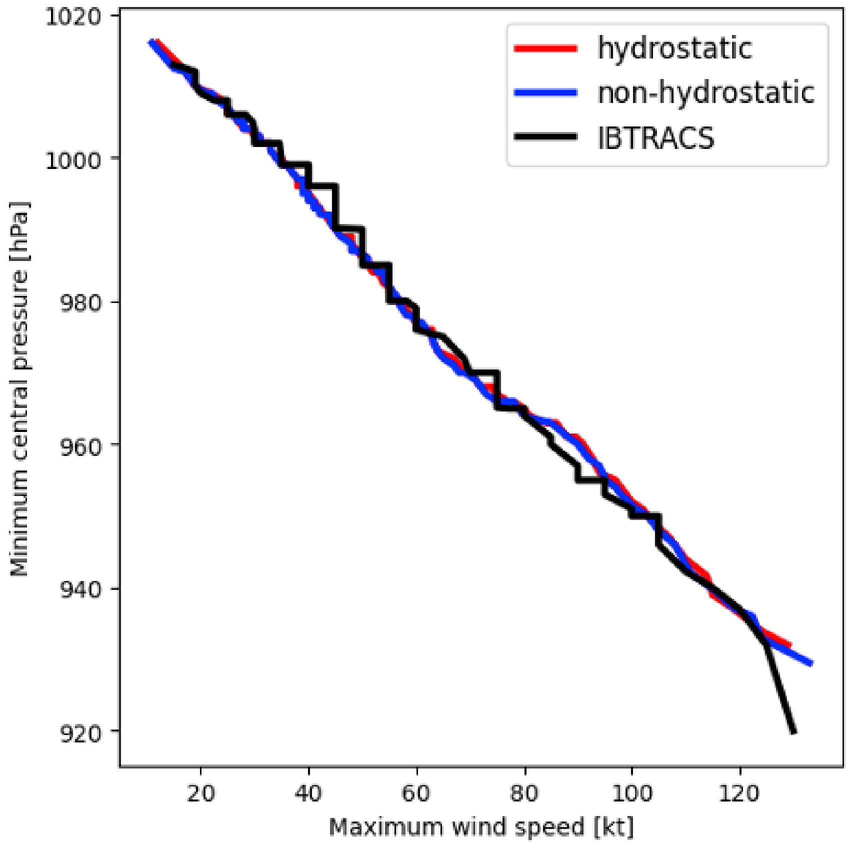
Vivoda, Polichtchouk, Diamantakis, Vana (2026, Q

How about convection?

- Thus far no clear impact of relaxing hydrostatic approximation for deep convection found at grid-spacings down to 1.4 km

- Tropical cyclone evaluation for the whole season at 2.8km grid-spacing shows no impact of NH dynamics.





Vivoda, Polichtchouk, Diamantakis, Vana (2026, QJ)

Part 3: Recap

- Non-hydrostatic effects believed to become important for horizontal resolution finer that 10 km.

- Non-hydrostatic effects are particularly important for resolved convection as well as small-scale gravity waves.

- There exists a non-hydrostatic equivalent of the IFS dynamical core, but it is expensive due to having two additional prognostic equations AND the predictor-corrector numerics.

- In practice, non-hydrostatic effects emerge in medium-range weather forecasts for horizontal grid-spacings <4.4km and mostly over regions dominated by steep orography only.