High order discontinuous Galerkin transport in IFS

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Discontinuous Galerkin (DG)

- Discontinuous Galerkin (DG) methods are a class of finite element techniques to solve partial differential equations that allow solutions to be discontinuous across element boundaries, combining features of finite volume and finite element methods.
- They are important for numerical weather prediction (NWP) because they provide high-order accuracy, excellent scalability on modern parallel architectures (including GPUs), great flexibility (adaptivity) and robust handling of complex geometries, leading to more efficient and accurate atmospheric simulations.
- DG minimizes the numerical diffusion inherent to the numerical solution to ensure that strong gradients persist in time. Thus, discontinuous Galerkin (DG) modelling offers an improved accuracy compared to finite difference and finite volume methods at the same resolution.

The CATRINE project

- CATRINE = Carbon Atmospheric Tracer Research to Improve Numerics and Evaluation is a Horizon Europe research project (2024-2026) coordinated by ECMWF.
- It supports the CO2 Monitoring and Verification Support (CO2MVS) capacity of the EU's Copernicus program, which aims to monitor anthropogenic CO_2 (and CH_4) emissions.
- One of its main goals is to improve numerical representation of tracer transport with focus on mass conservation.
- Among novel emerging approaches to improve tracers transport, DG-SL has been investigated for its higher accuracy, scalability and flexibility.

Discontinuous Galerkin Semi-Lagrangian advection (DG-SL)

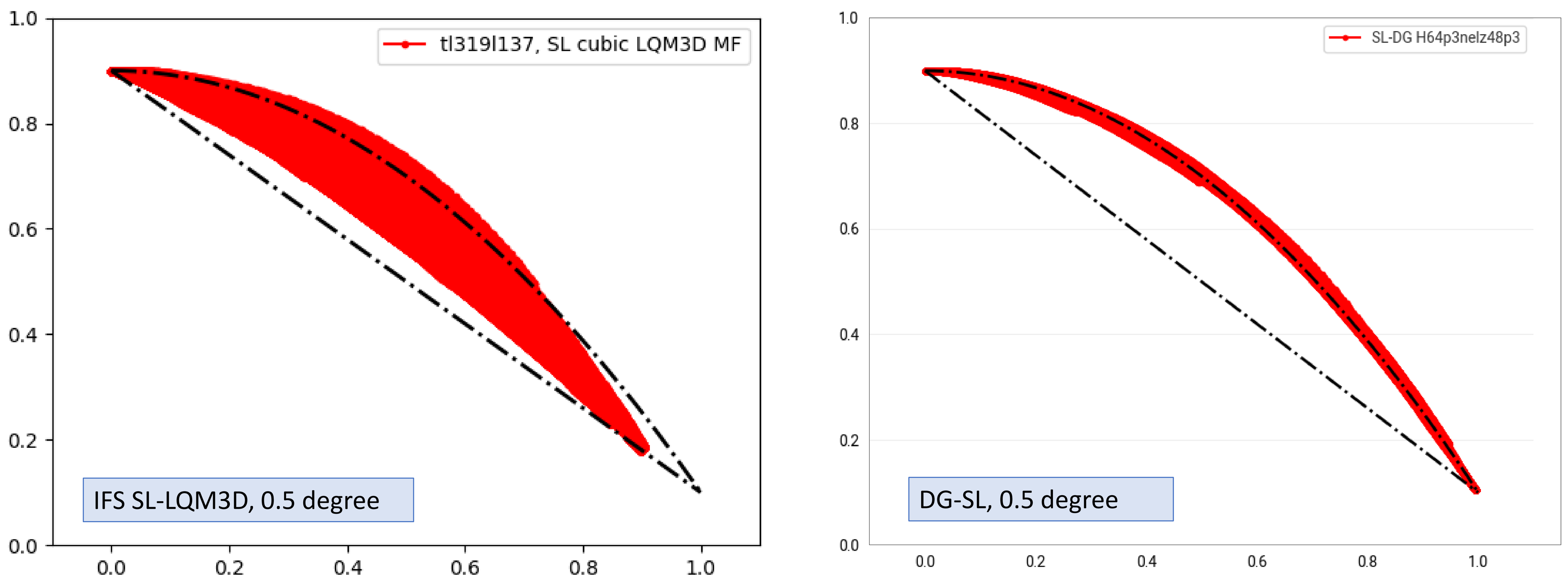
- When used for space discretization of time-dependent evolutionary equations, DG methods lead to very severe stability restrictions on timestep size if standard Eulerian time-stepping techniques are used.
- For this reason, we combined the DG space discretization with the semi-Lagrangian (SL) time-stepping technique in order to get an advection scheme (DG-SL) which is high order accurate and also unconditionally stable.
- The DG-SL technique aims at improving advection via higher accuracy from DG in a way that is efficient for NWP problems.
- DG methods efficiency is maximized when they are implemented on quadrilateral meshes in order to take advantage from the efficiency of tensor products of 1D basis functions. In addition, DG methods by definition have sub-element internal degrees of freedom, related to the local polynomial order used.

DG-SL coupling to IFS (IFS-DG-SL)

- DG methods are implemented on meshes which typically differ from the IFS grid.
- To couple the DG-SL advection scheme to IFS we used the so called Multiple-Grids approach, based on Atlas interpolations, here called IFS-DG-SL, which consists of three steps:

1. ***Preprocessing***: the IFS solution is interpolated at DG points, then DG expansion coefficients are calculated
2. ***Time-stepping***: a timestep of DG-SL is executed in the DG space
3. ***Postprocessing***: the solution advanced in time in the DG space is remapped back to the IFS grid point space

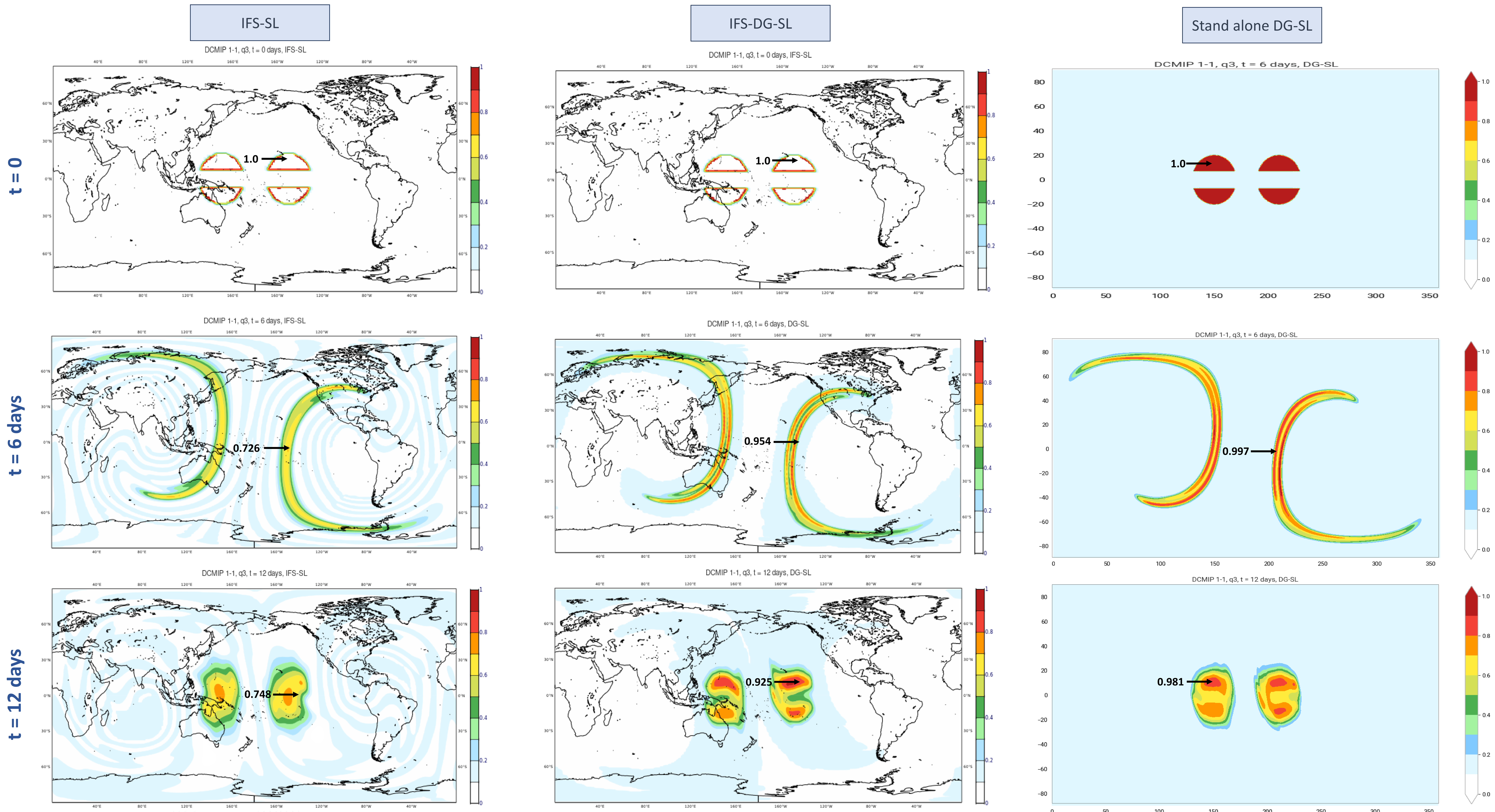
Impact of high order DG on tracers correlation preservation: an example



- We want to assess the ability of the advection scheme to maintain nonlinear correlations between tracers.
- We consider here two tracers, q1 and q2. At the initial time q2 is defined as a quadratic function of q1, i.e. the initial scatter plot obtained from plotting mixing ratios of q2 against q1 is a piece of parabola. They are then transported by DCMIP 1-1 deformational flow.
- As the simulation progresses in time, the nonlinear correlation between tracers will be lost due to numerical errors and so the scatter plot will drift from its initial parabolic distribution.
- The correlation plot after 6 days of numerical advection by DCMIP 1-1 deformational flow at 0.5 degree resolution is shown: left IFS-SL (tl319l137), and right right DG-SL (H64 p3).

Impact of high order DG on accuracy in highly deformational flows: an example

DCMIP1-1 slotted cylinder tracer advection case, 2D deformational velocity field. From top to bottom the solution is shown at time 0, 6days and 12 days respectively. From left to right the solution is shown for standard IFS-SL scheme, IFS-DG-SL scheme and for stand alone DG-SL scheme respectively, all at the same resolution of 1 degree (tco79 for IFS, H32 p3 for DG-SL). The difference in accuracy is evident both qualitatively (sharp filaments are not reproduced by IFS-SL and final solution peaks are in the wrong place for IFS-SL) and quantitatively (more than 25% of the peak height lost by IFS-SL, less than 10% lost by IFS-DG-SL). The difference between the solution in the central and in the right column measures the errors of interpolation between the IFS grid and the DG points.



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