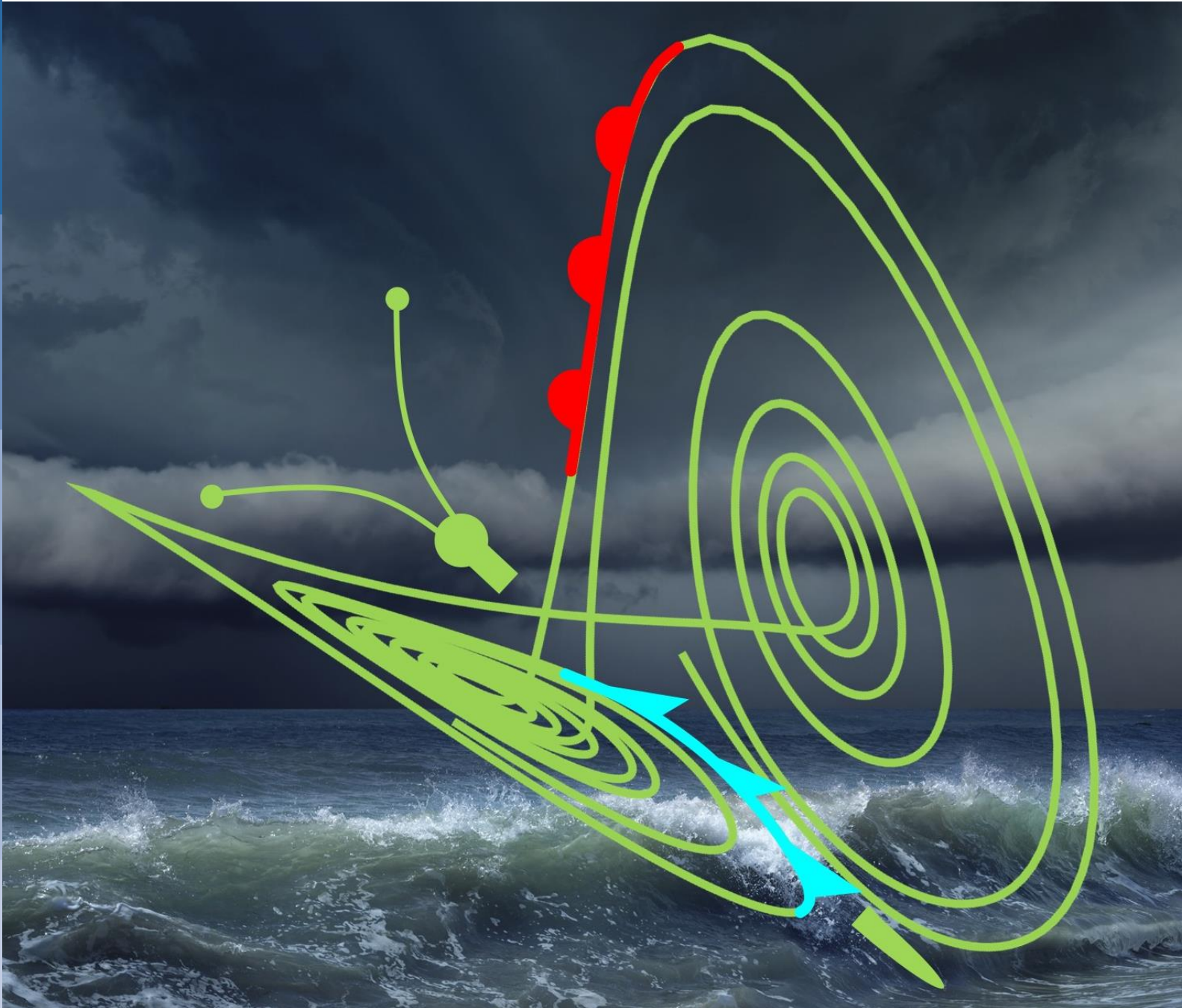


Diagnostics 1

Mark Rodwell

Training Course on Predictability

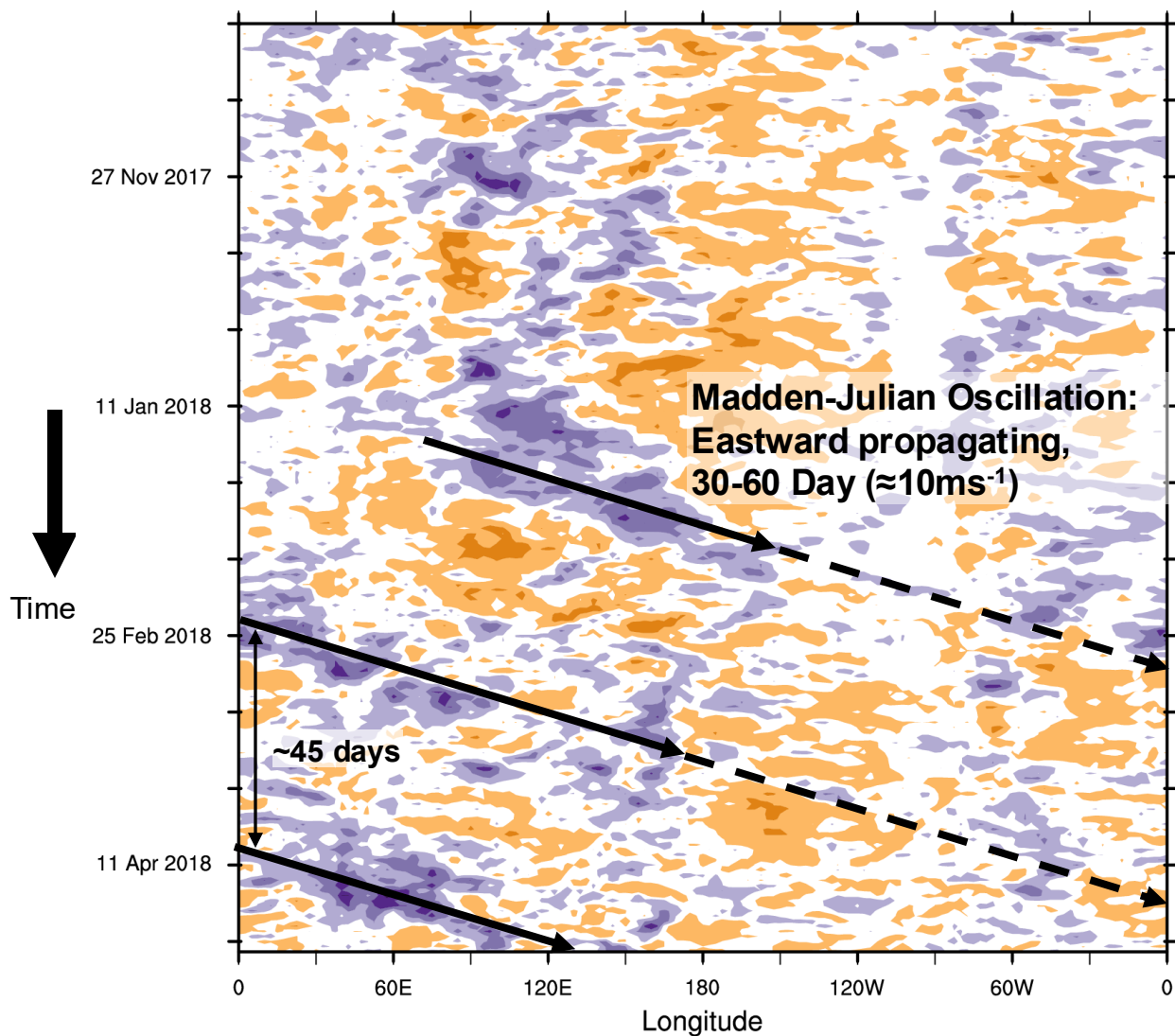
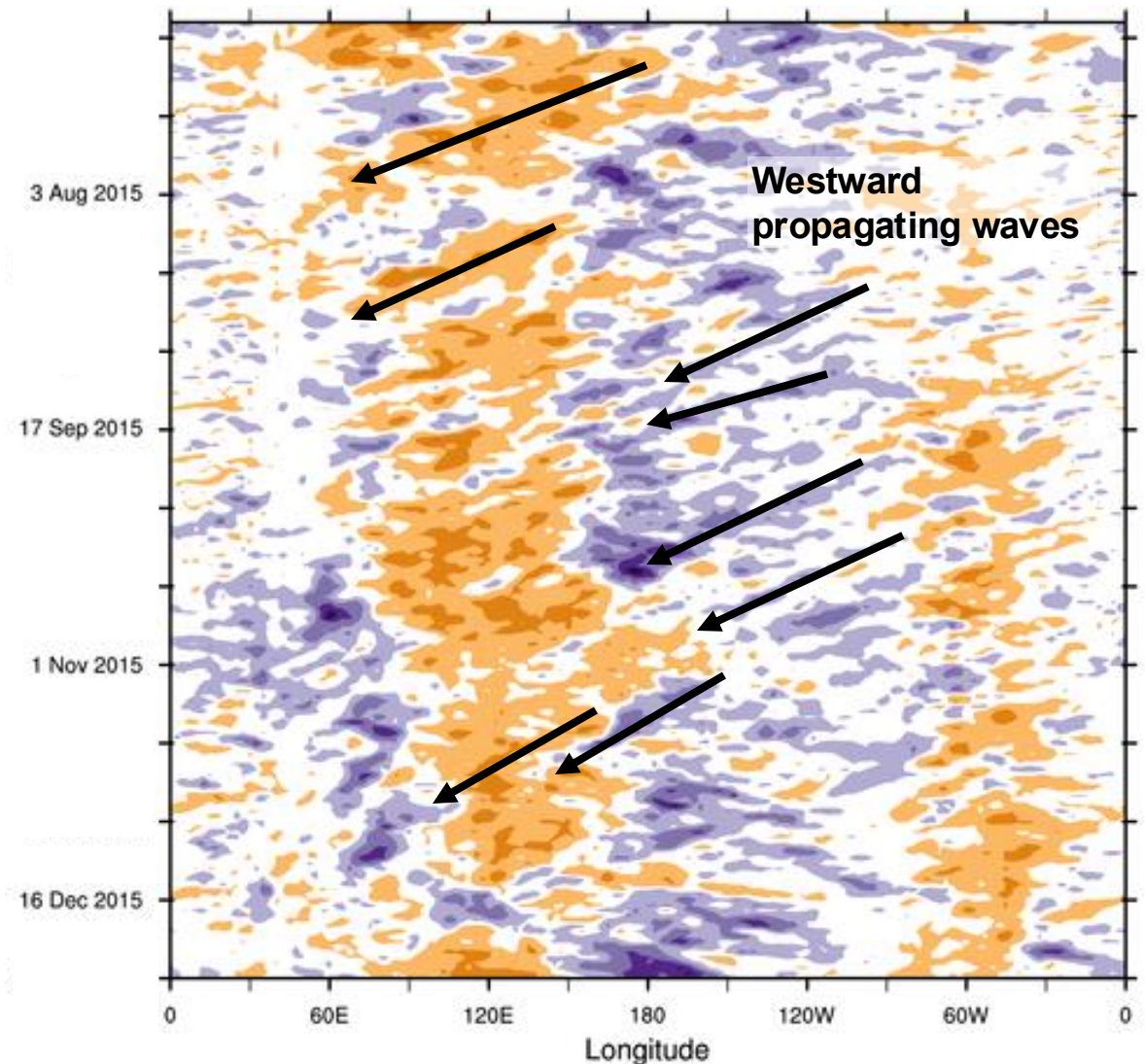
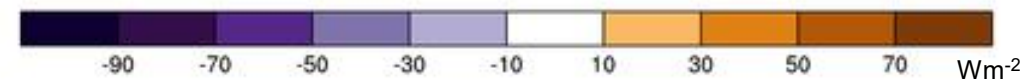
19 October 2026, ECMWF Reading



- Tropical waves and teleconnections
- Seamless approaches to model evaluation

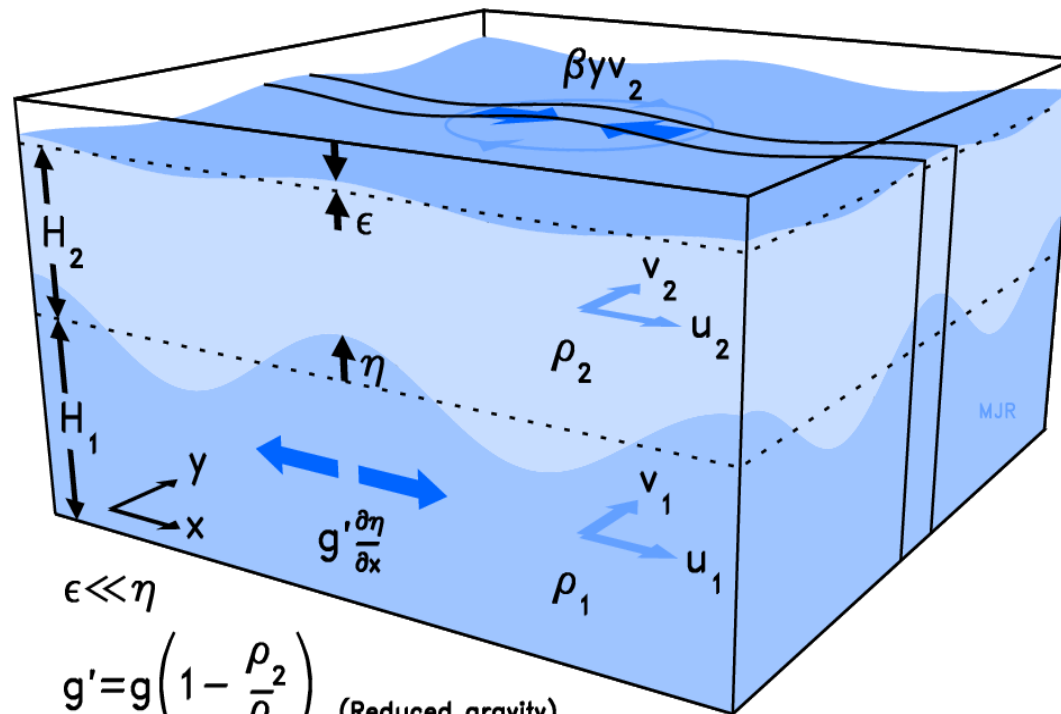
- Tropical waves and teleconnections
- Seamless approaches to model evaluation

Tropical Waves (longitude-time “Hovmöller” diagram)



Plots from Bureau of Meteorology. Based on Outgoing Longwave Radiation (OLR), 15°S - 15°N

Equatorial wave theory – the model



$$\epsilon \ll \eta$$

$$g' = g \left(1 - \frac{\rho_2}{\rho_1} \right) \quad (\text{Reduced gravity})$$

$$u \equiv u_1 - u_2 \quad v \equiv v_1 - v_2 \quad (\text{Baroclinic mode})$$

$$c_e^2 \equiv g' \frac{H_1 H_2}{H_1 + H_2} \equiv g H_e \quad c_e \approx 20 \text{ to } 80 \text{ ms}^{-2}$$

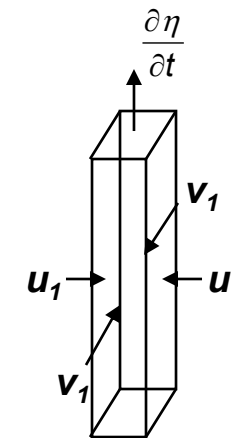
c_e is the propagation speed of a barotropic gravity wave in a single layer of depth H_e

Momentum:

$$\begin{aligned} \frac{\partial u}{\partial t} - \beta y v + g' \frac{\partial \eta}{\partial x} &\approx 0 \\ \frac{\partial v}{\partial t} + \beta y u + g' \frac{\partial \eta}{\partial y} &\approx 0 \end{aligned} \quad (1)$$

Continuity:

$$\frac{\partial \eta}{\partial t} + \frac{c_e^2}{g'} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \approx 0 \quad (2)$$



Solving for v:

$$\frac{\partial}{\partial t} \left\{ \frac{\partial^2 v}{\partial t^2} + \beta^2 y^2 v - c_e^2 \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \right\} - c_e^2 \beta \frac{\partial v}{\partial x} = 0 \quad (3)$$

Use of the shallow water equations on the β -plane ($f=\beta y$) for understanding tropical atmospheric waves. Note: No coupling with convection in this model

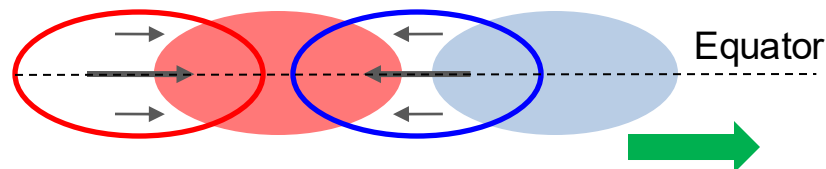
Equatorial wave theory – limiting solutions

Kelvin waves: $v \equiv 0$

$$(K1) \quad \frac{\partial u}{\partial t} = -g' \frac{\partial \eta}{\partial x}$$

$$(K2) \quad \beta y u = -g' \frac{\partial \eta}{\partial y} \Rightarrow u, \eta \text{ in phase}$$

$$(K3) \quad \frac{\partial \eta}{\partial t} = -\frac{c_e^2}{g'} \frac{\partial u}{\partial x} \Rightarrow \text{Eastward propagation} \rightarrow$$

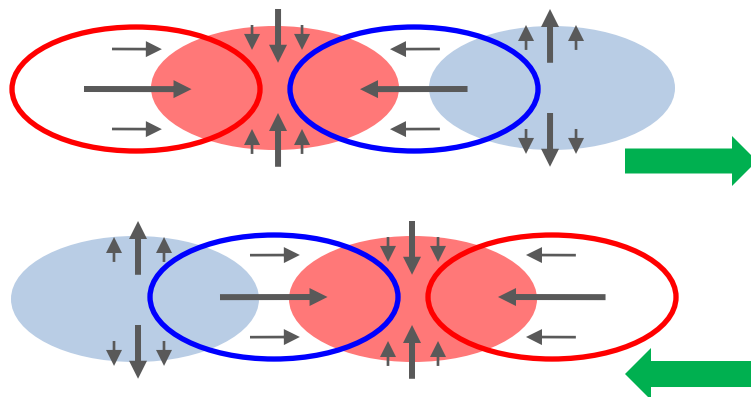


Gravity waves: Fast; pressure gradient force dominates

$$(G1) \quad \frac{\partial u}{\partial t} = -g' \frac{\partial \eta}{\partial x} \Rightarrow \begin{array}{l} u, \eta \text{ in phase: } \rightarrow \\ \text{out of phase: } \leftarrow \end{array}$$

$$(G2) \quad \frac{\partial v}{\partial t} = -g' \frac{\partial \eta}{\partial y} \Rightarrow v \text{ in quadrature with } \eta \text{ and } u$$

$$(G3) \quad \frac{\partial \eta}{\partial t} = -\frac{c_e^2}{g'} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)$$

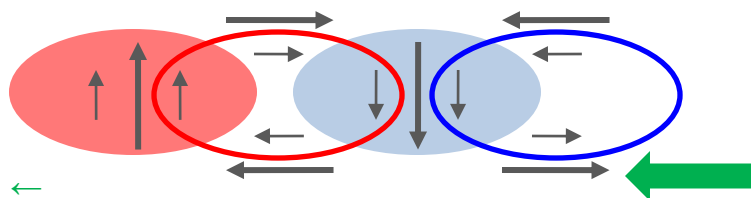


Rossby waves: Slow; Coriolis affect important, less convergence

Curl of (1):

$$(R1) \quad \frac{\partial \xi}{\partial t} = -\beta v - \beta y \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \approx -\beta v$$

Vorticity anomaly strengthened to west: $\leftarrow$



$$\frac{\partial^2 \eta}{\partial t^2} = c_e^2 \frac{\partial^2 \eta}{\partial x^2}$$

$$\eta \propto \sin(kx - \omega t) \hat{\eta}(y)$$

$$\text{phase speed} = \frac{x}{t} = \frac{\omega}{k} = c_e$$

(non-dispersive)

$$\hat{\eta}(y) = e^{-\frac{\beta}{2c_e} y^2}$$

(K1 & K3)

k = zonal wavenumber
 ω = frequency
 $\hat{\eta}(y)$ = meridional structure

(K2 & K3) Decays away from equator

$$\frac{\partial^2 \eta}{\partial t^2} = c_e^2 \left(\frac{\partial^2 \eta}{\partial x^2} + \frac{\partial^2 \eta}{\partial y^2} \right) \quad (G1, G2 \text{ \& } G3)$$

$$\eta \propto \sin(kx - \omega t) \hat{\eta}(y)$$

$$\text{phase speed} = \frac{\omega}{k} = \pm c_e \left(1 - \frac{1}{k^2 \hat{\eta}} \frac{\partial^2 \hat{\eta}}{\partial y^2} \right)^{\frac{1}{2}}$$

(larger $\Rightarrow$ faster, $\frac{\omega}{k} \rightarrow \pm c_e$ as $|k| \rightarrow \infty$)

$\hat{\eta}$ and $\partial^2 \hat{\eta} / \partial y^2$ have opposite sign
 $\hat{\eta}$ multi-node ($v = 0$)

$$\frac{\partial \nabla^2 \psi}{\partial t} = -\beta \frac{\partial \psi}{\partial x}$$

(R1)

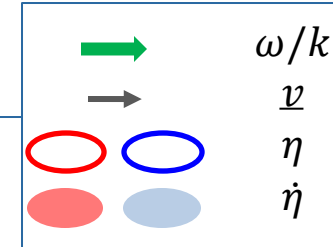
$$\psi \propto \sin(kx - \omega t) \hat{\psi}(y)$$

$$\text{phase speed} = \frac{\omega}{k} = -\beta \left(k^2 - \frac{1}{\hat{\psi}} \frac{\partial^2 \hat{\psi}}{\partial y^2} \right)^{-1}$$

(larger $\Rightarrow$ faster)

ψ = streamfunction
 $\xi = \nabla^2 \psi$, $v = \frac{\partial \psi}{\partial x}$

$\hat{\psi}(y)$ = structure
 $\hat{\psi}$ and $\partial^2 \hat{\psi} / \partial y^2$ have opposite sign
 Multi-node ($v = 0$)



$V=0$:

$$u = u_0 e^{-y^2/2} e^{ik(x-c_e t)}$$

East propagating **Kelvin Wave**

- Non-dispersive
- In geostrophic balance

$V \neq 0$:

$$v = \hat{v}(y) e^{i(kx - \omega t)}$$

Substitute into equation for v

Structures

(Meridional structures are solutions to Schrodinger's simple harmonic oscillator)

$$\hat{v}(y) = \begin{bmatrix} 1 \\ 2y \\ 4y^2 - 1 \\ 8y^3 - 12y \\ \vdots \\ H_n(y) \end{bmatrix} e^{-y^2/2}$$

Hermite Polynomials: $H_n(y)$

- Each successive polynomial ($n=0,1,2,\dots$) has one more node
- Modes alternate asymmetric / symmetric about equator

Dispersion

(How phase speed is related to spatial scale)

$$\left(\frac{\omega^2}{c_e^2} - k^2 - \frac{\beta k}{\omega} \right) = (2n+1) \frac{\beta}{c_e} \quad (n = 0, 1, 2, \dots)$$

For $n \neq 0$: 3 values of ω for each k

- West propagating **Rossby Wave**
- E & W propagating **Gravity Wave**

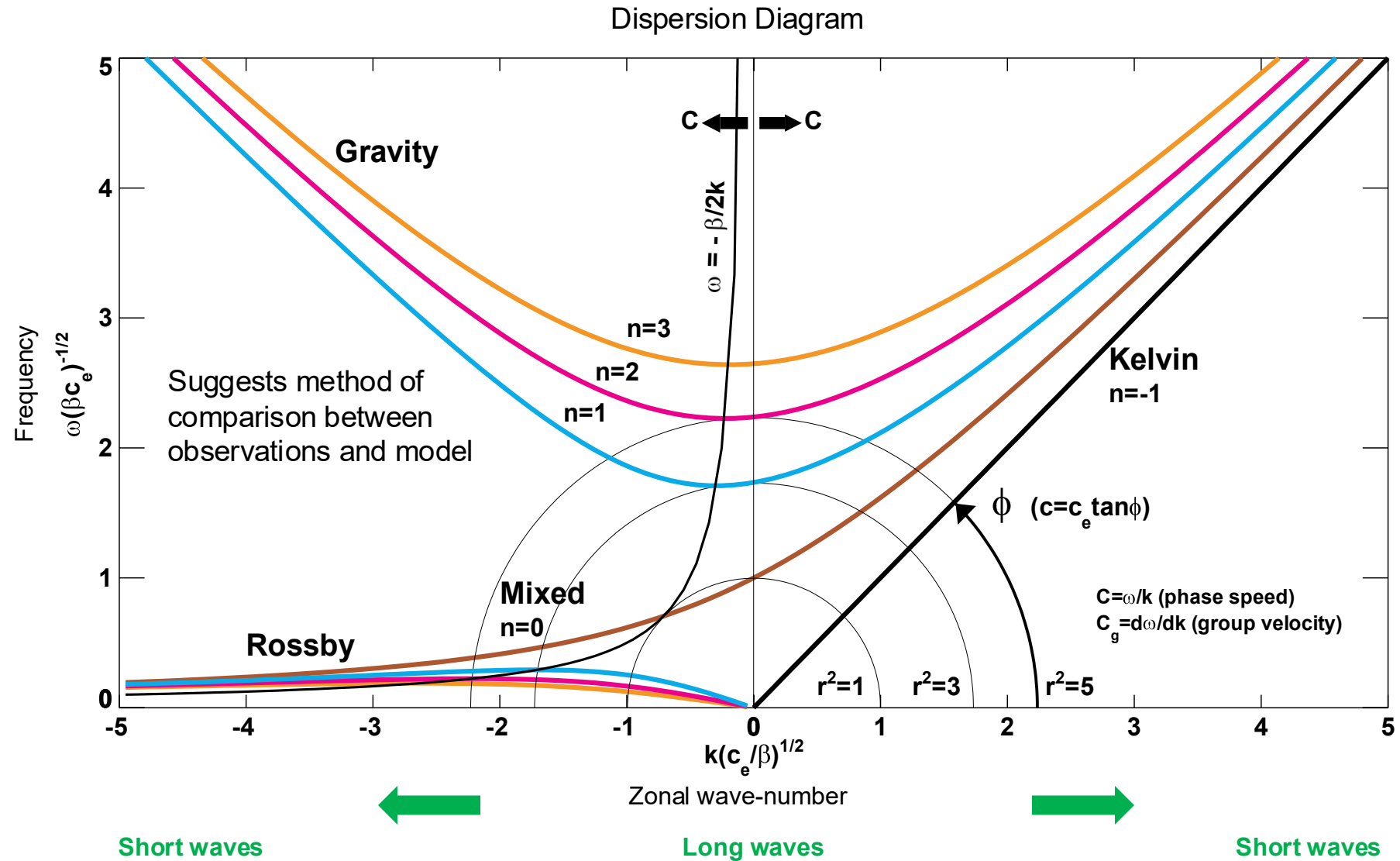
For $n=0$: 2 values of ω for each k

- E & W prop. **Mixed Rossby-Gravity**

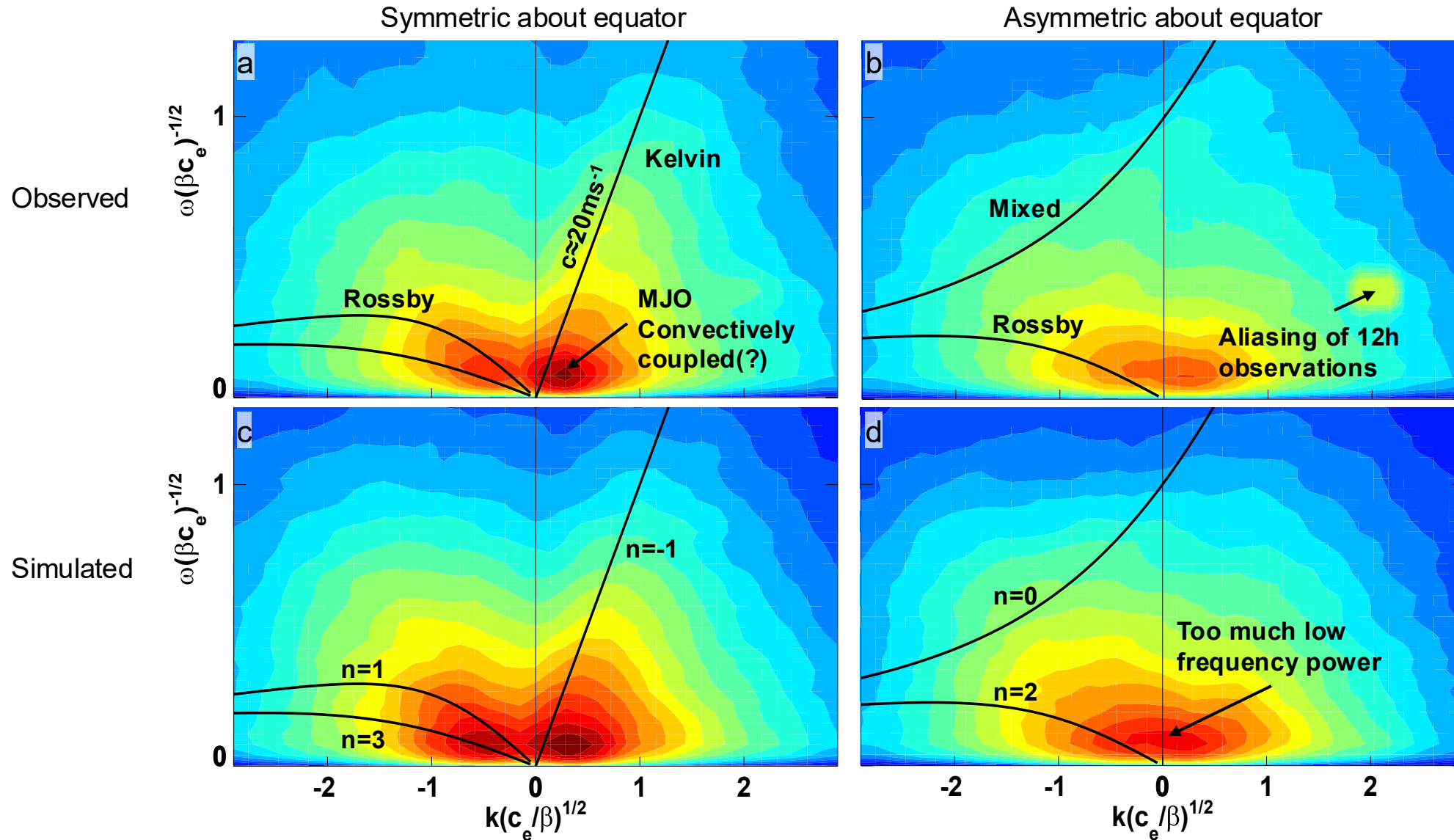
Note: y has been non-dimensionalised by the factor $(\beta/c_e)^{1/2}$

In dispersion relation, gravity waves mainly associated with first two terms on lhs, Rossby waves with last two terms on lhs, mixed Rossby-gravity waves with all three terms

Interpretation of Free Equatorial Waves



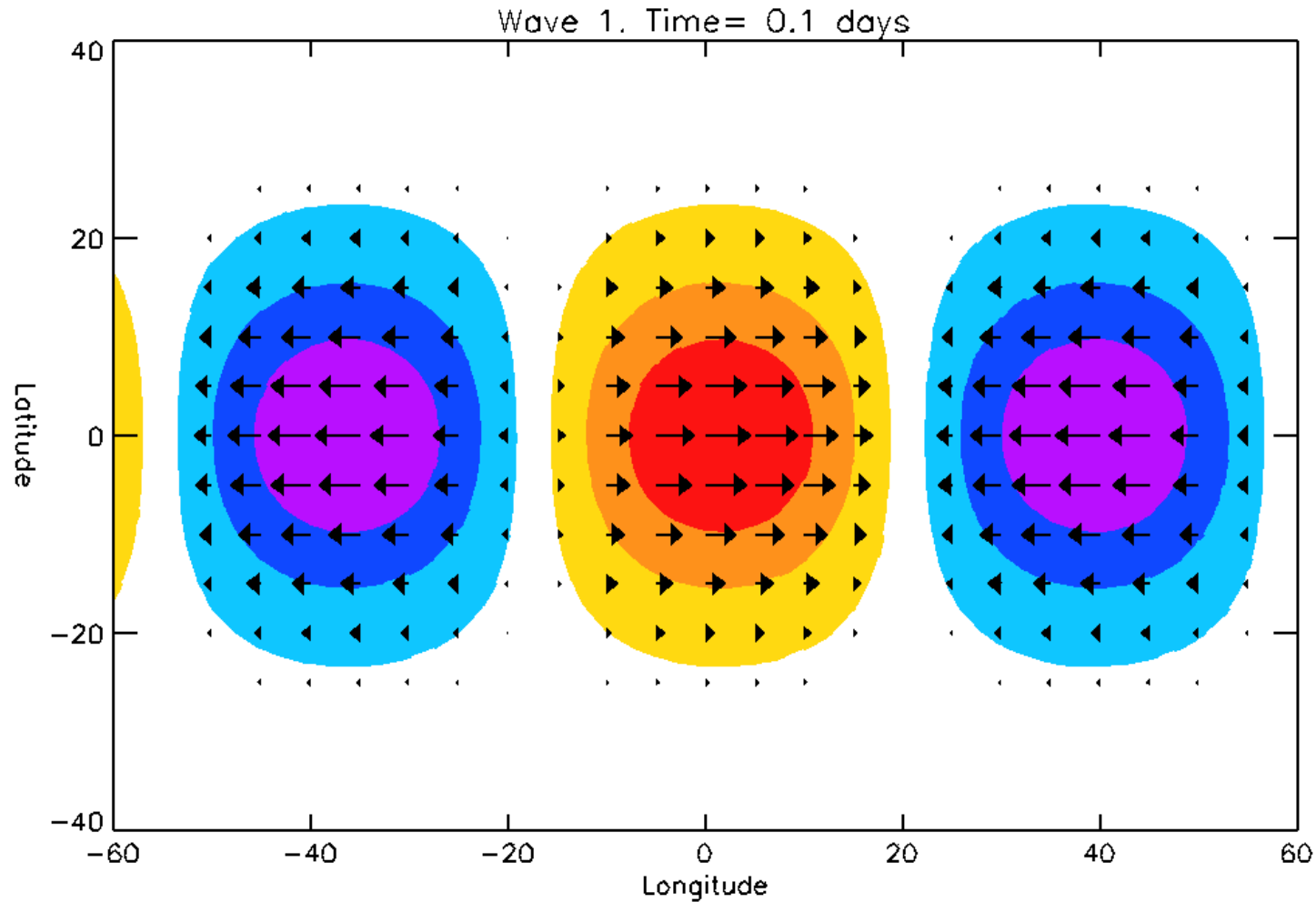
Wave power for OLR, with dispersion relation overlaid



Agreement with shallow water theory if OLR is a 'slave' to the free waves, linearity, etc.

Based on data from NOAA & IFS cycle 32R3 for DJF 1990-2005

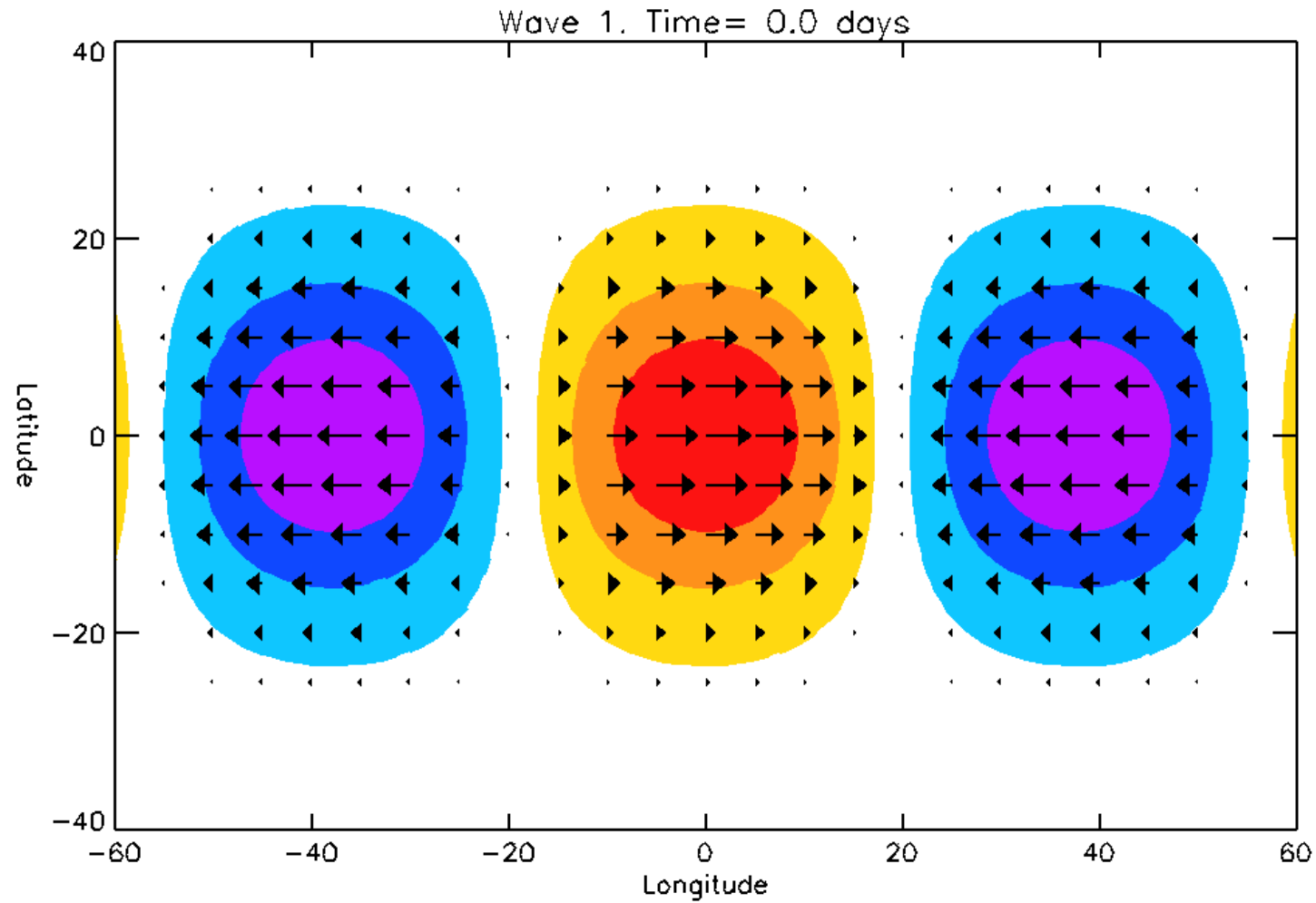
Wave Spotting! – How information (and errors) can propagate within the Tropics



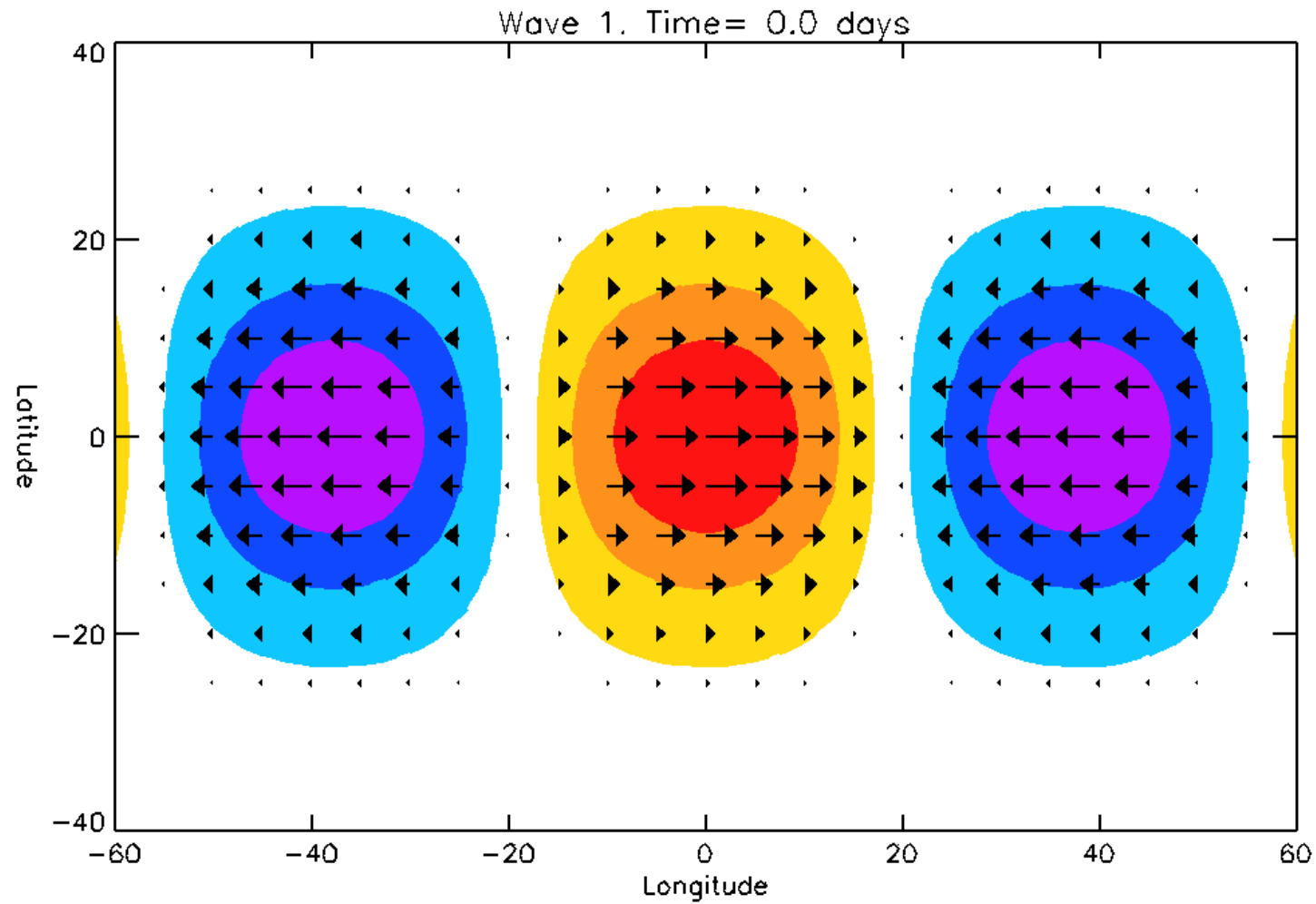
What is the wave?

1. Kelvin
2. Mixed Rossby-Gravity
3. Rossby
4. Westward Gravity
5. Eastward Gravity

Colours show height perturbation (red positive, blue negative), arrows show lower-level winds



Colours show height perturbation (red positive, blue negative), arrows show lower-level winds



Colours show height perturbation (red positive, blue negative), arrows show lower-level winds

Wave spotting: Your Answers

Wave	Kelvin	Mixed Rossby- Gravity	Rossby	Eastward Gravity	Westward Gravity
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					
12					

Animation of tropical waves – analysed then forecast

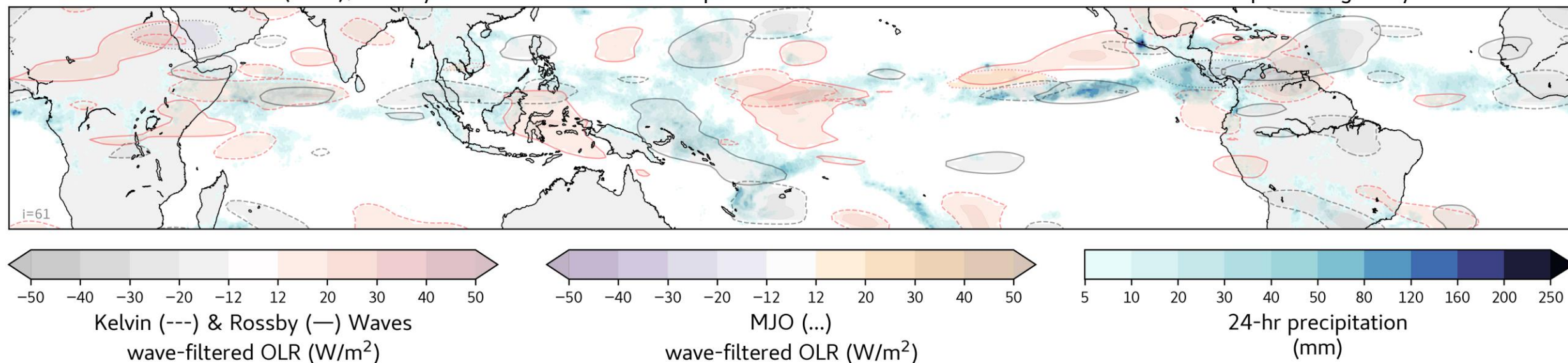
Thanks: Rebecca Emerton

ECMWF TROPICAL WAVE MONITORING

Madden-Julian Oscillation (MJO), Rossby & Kelvin Waves and Precipitation

extended-range control forecast 24-11-2023

preceding analysis 25-10-2023



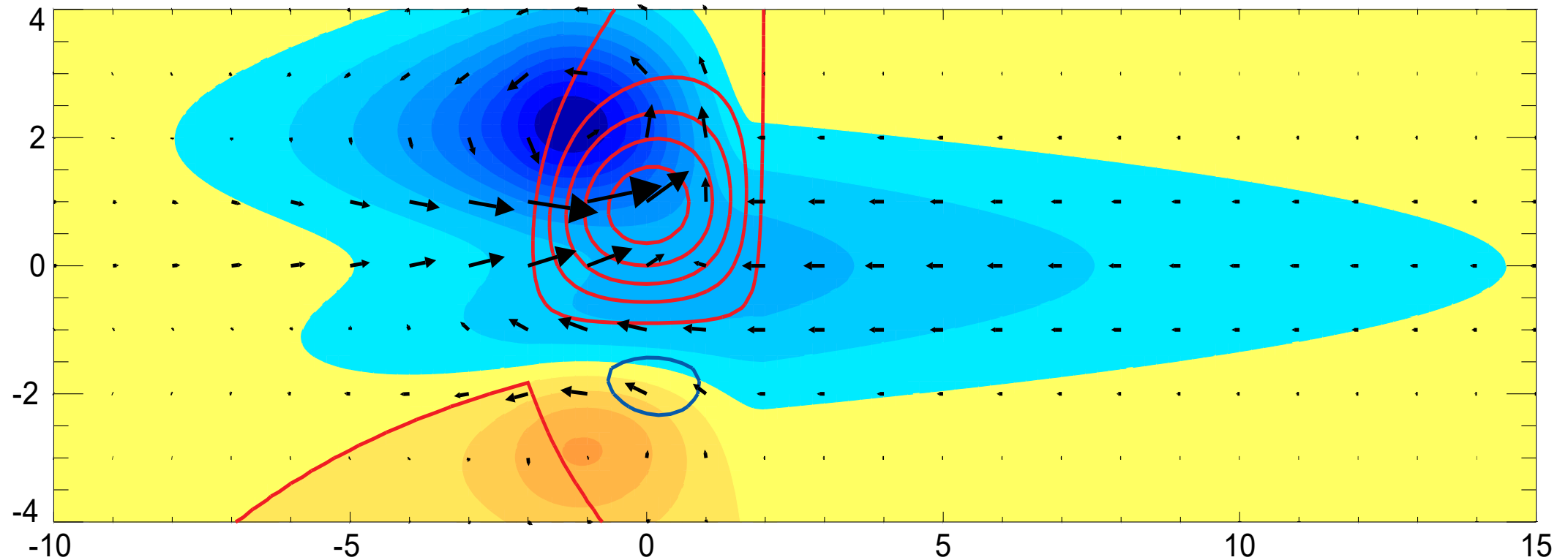
Tropical waves identified using wavenumber-frequency power of OLR, as in the previous dispersion diagram

Gill's steady solution to monsoon heating

Following Gill (1980). See also Matsuno (1966)

Damping/heating terms take the place of the time derivatives. Explicitly solve for the x-dependence

Obtain a Kelvin wave solution to the east (zero meridional flow) and a Rossby wave response to the west (super-position of two Rossby waves: one symmetric and one anti-symmetric about the equator)

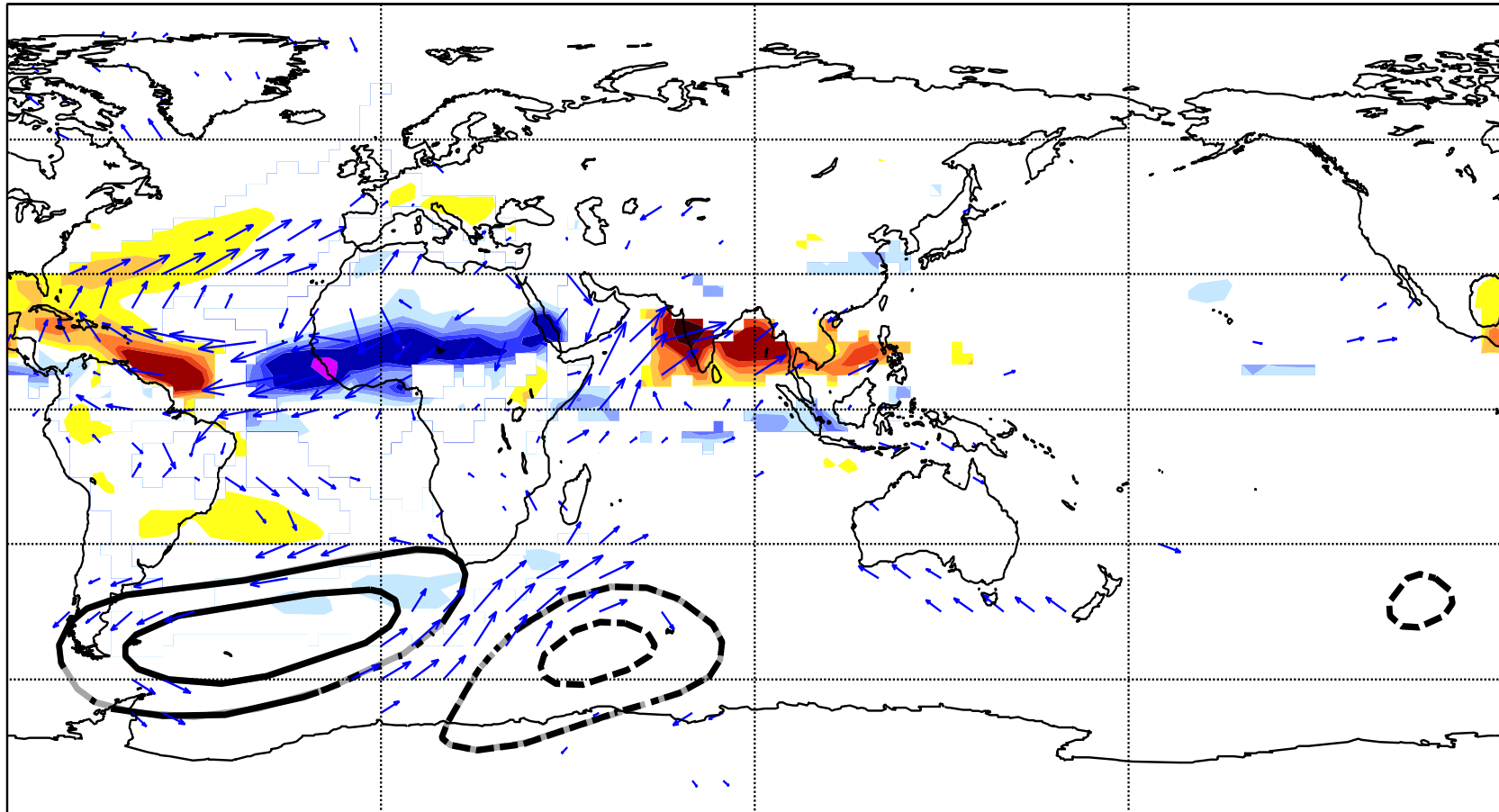


Colours show perturbation pressure, vectors show velocity field for lower level, contours show vertical motion (blue = -0.1, red = 0.0, 0.3, 0.6, ...)

Model climate response to a change in aerosol climatology

Change in JJA precipitation (shaded), $\bar{u}_{925}$ (vectors) and Z500 (contours)

mm day⁻¹. 10% Sig.



Gill's steady solution helps explain the model's climate response in the tropics and subtropics, but could we have identified aerosol as a problem a priori?

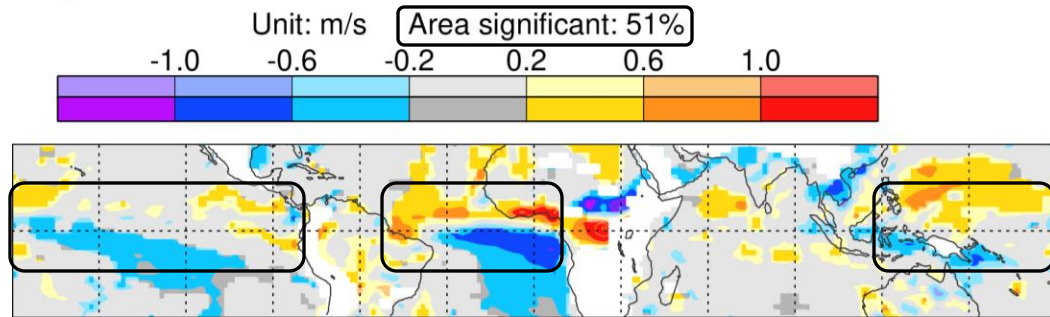
The extratropical response will be discussed later

- Tropical waves and teleconnections
- Seamless approaches to model evaluation

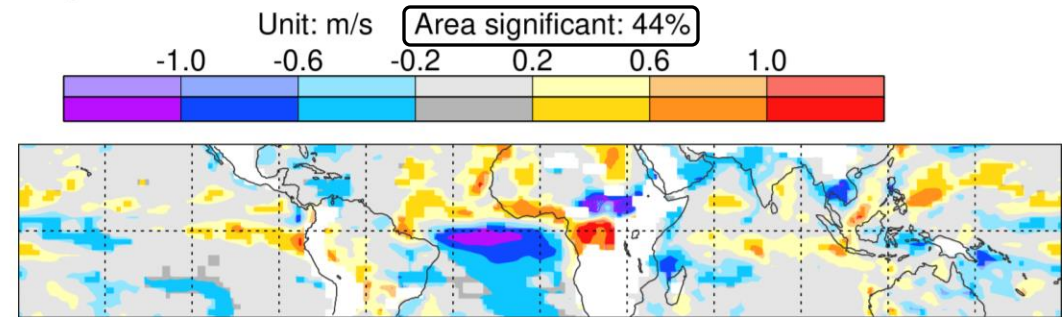
Mean forecast error for meridional wind (v925) at different lead-times

In general, at short lead-times, mean errors are more coherent, statistically significant and linked to model deficiencies. Here they indicate a lack of convergence into the Hadley Circulation. Note this is only $\sim 0.6 \text{ ms}^{-1}$. Can we be sure it is a model error and not an analysis error? (see next slide)

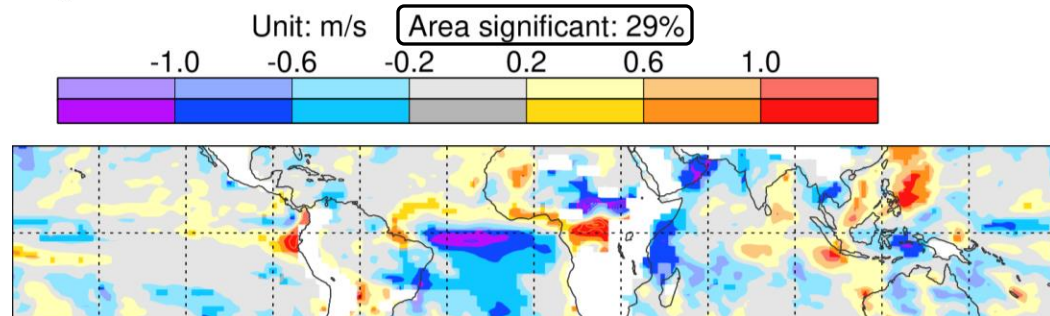
Day 1



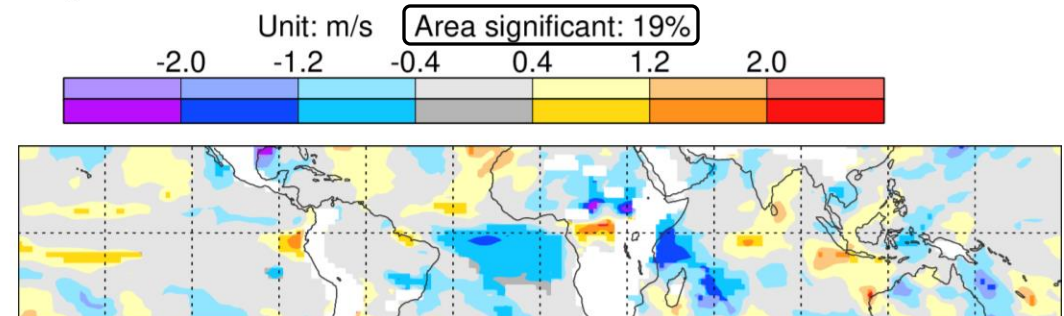
Day 2



Day 5



Day 10

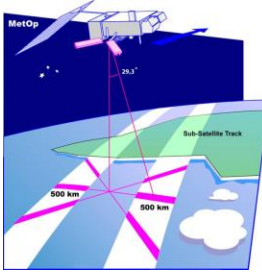


December 2022 – February 2023. Saturated colours highlight mean errors (forecast minus analysis) which are statistically significant at the 5% level ($\sim 5\%$ of points would pass the test by chance)

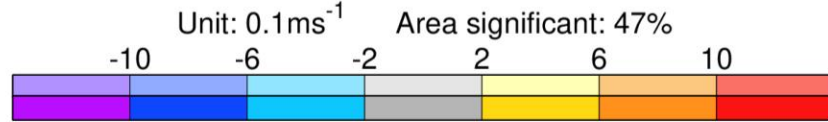
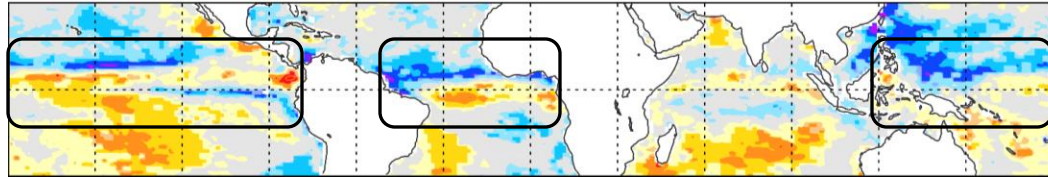
At longer lead-times, errors are more associated with lack of predictability (of equatorial waves, etc.)

Using data assimilation to better determine the root cause of the problem

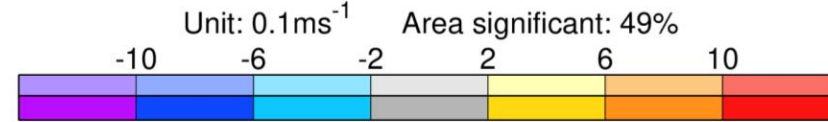
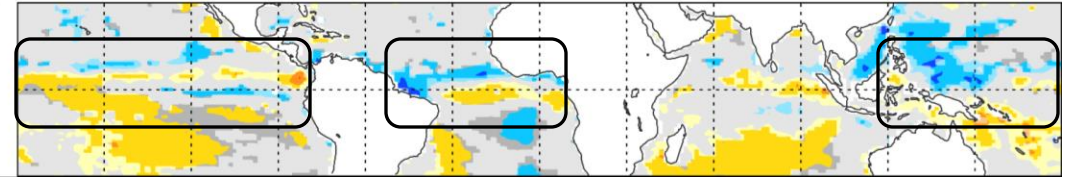
Here the Scatterometer wind 'observations' show a consistent story with weaker convergence in the model (background). The analysis increments correct this mean departure



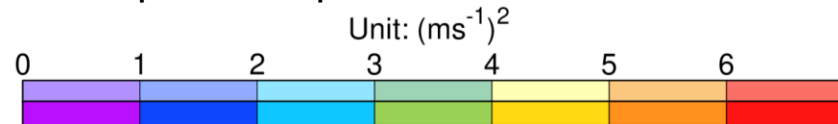
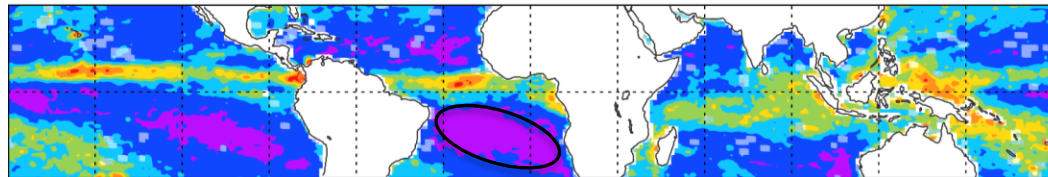
Observation – Background (mean “departure”)



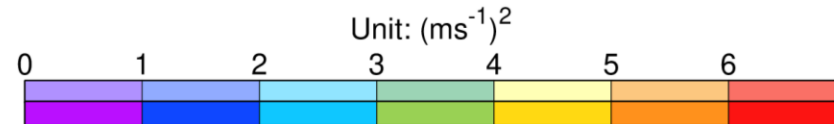
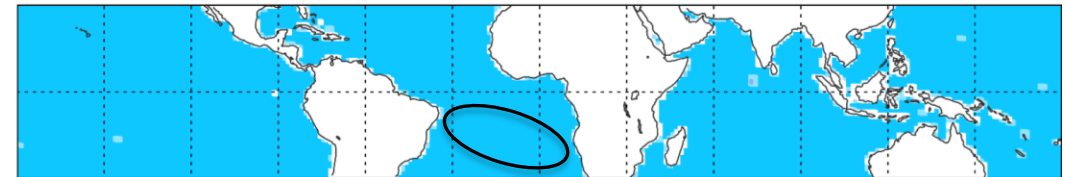
Analysis – Background (mean “increment”)



Mean-squared departure of EDA-mean



Modelled observation error variance



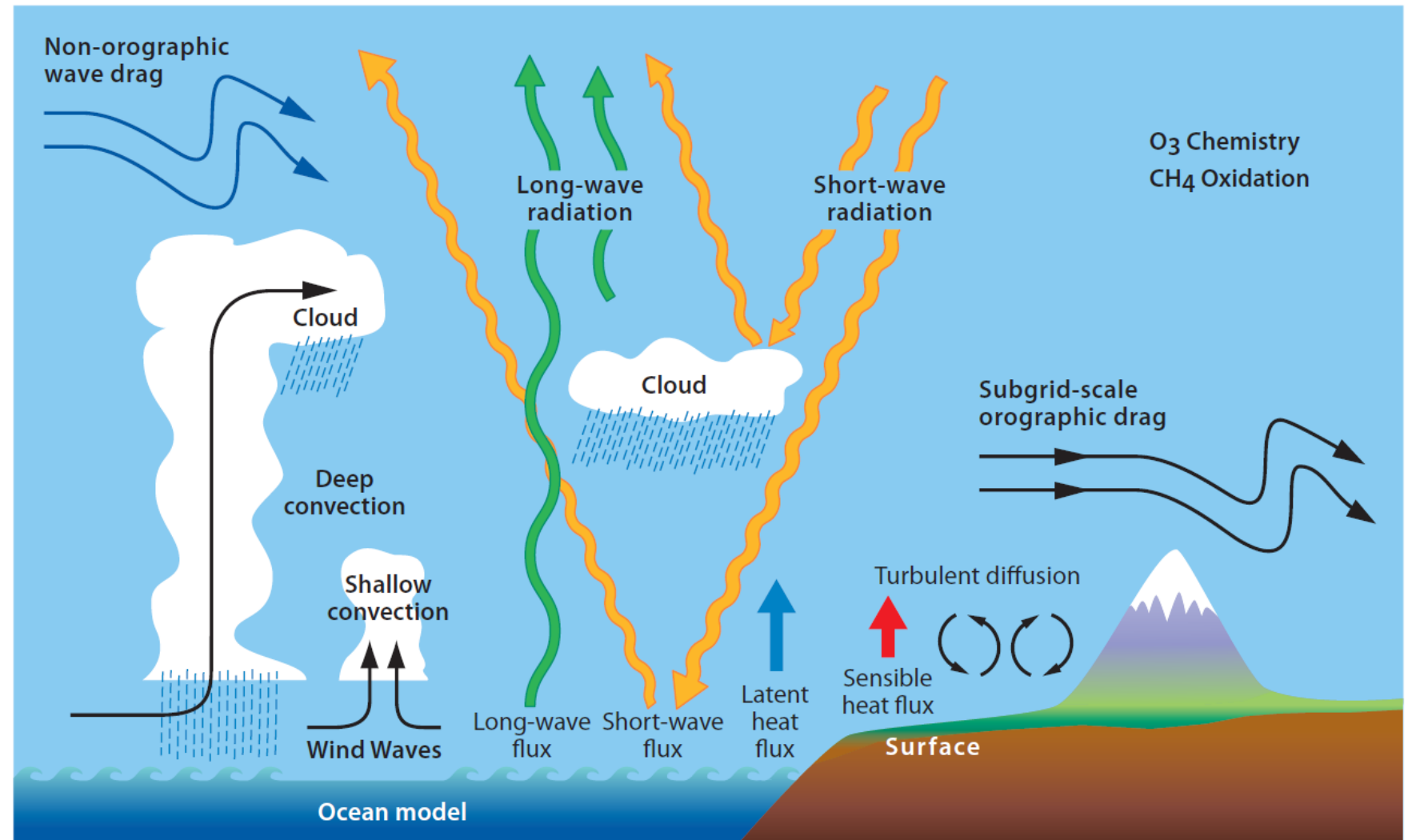
December 2022 – February 2023. Saturated colours highlight mean errors which are statistically significant at the 5% level (~5% of points would pass the test by chance)

The Ensemble of Data Assimilations (EDA) suggests we over-estimate the scatterometer errors (their variance is bigger than the squared departures in some regions) hence the mean analysis increments could be even stronger. This is evidence that the problem is with the model

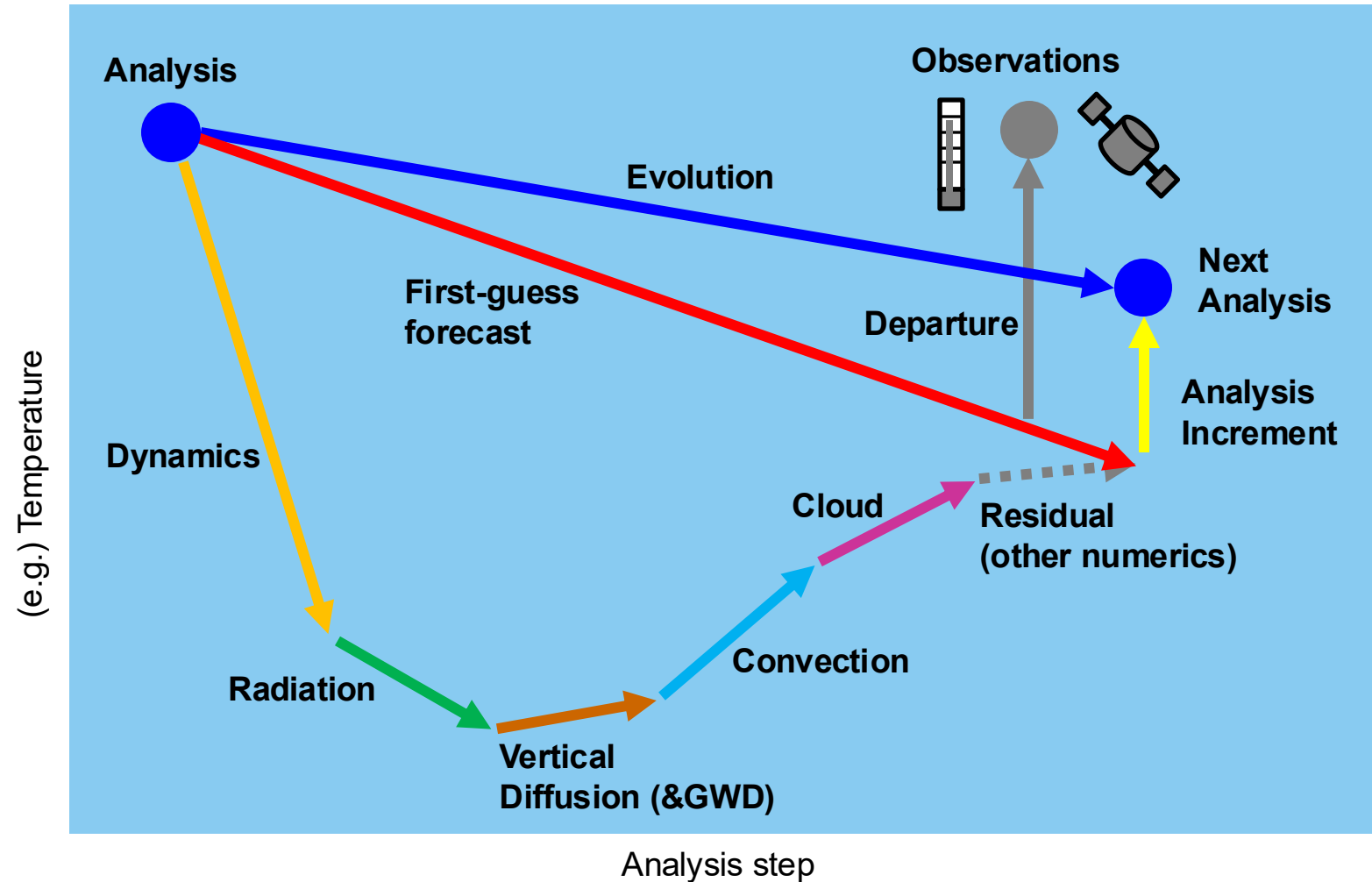
The complexity of present-day model physics

Figure from Peter Bechtold

Ideally, we wish to identify deficiencies at the process level. Again, this should be easier at short timescales since interactions between physical processes and the resolved flow (including teleconnections) are minimised



Schematic of the data assimilation process – a diagnostic perspective

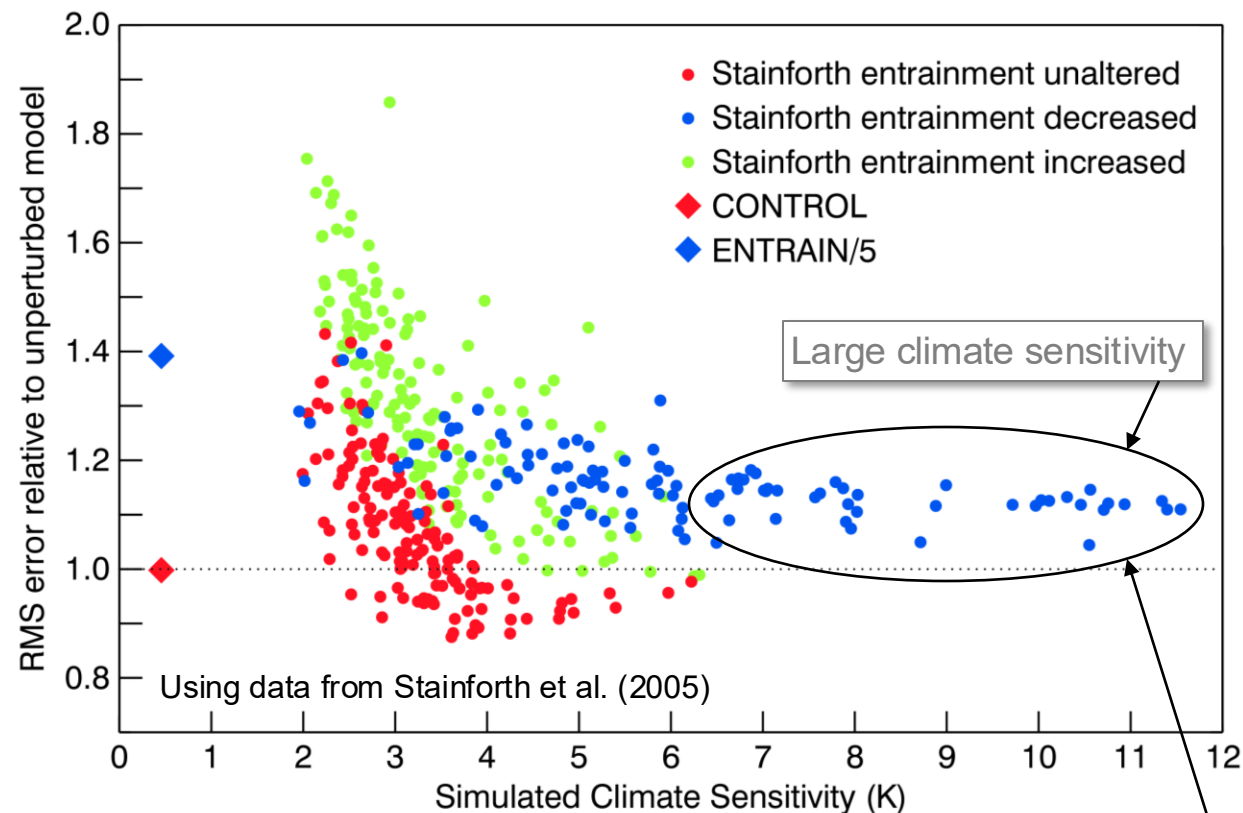


Analysis increment corrects first-guess error, and draws next analysis closer to observations.
First-guess = sum of all processes
Relationship between increment and individual process tendencies can help identify key errors.

“Initial Tendency” approach discussed by Klinker & Sardeshmukh (1992). Refined by Rodwell & Palmer (2007)

Data assimilation as a means of constraining climate sensitivity

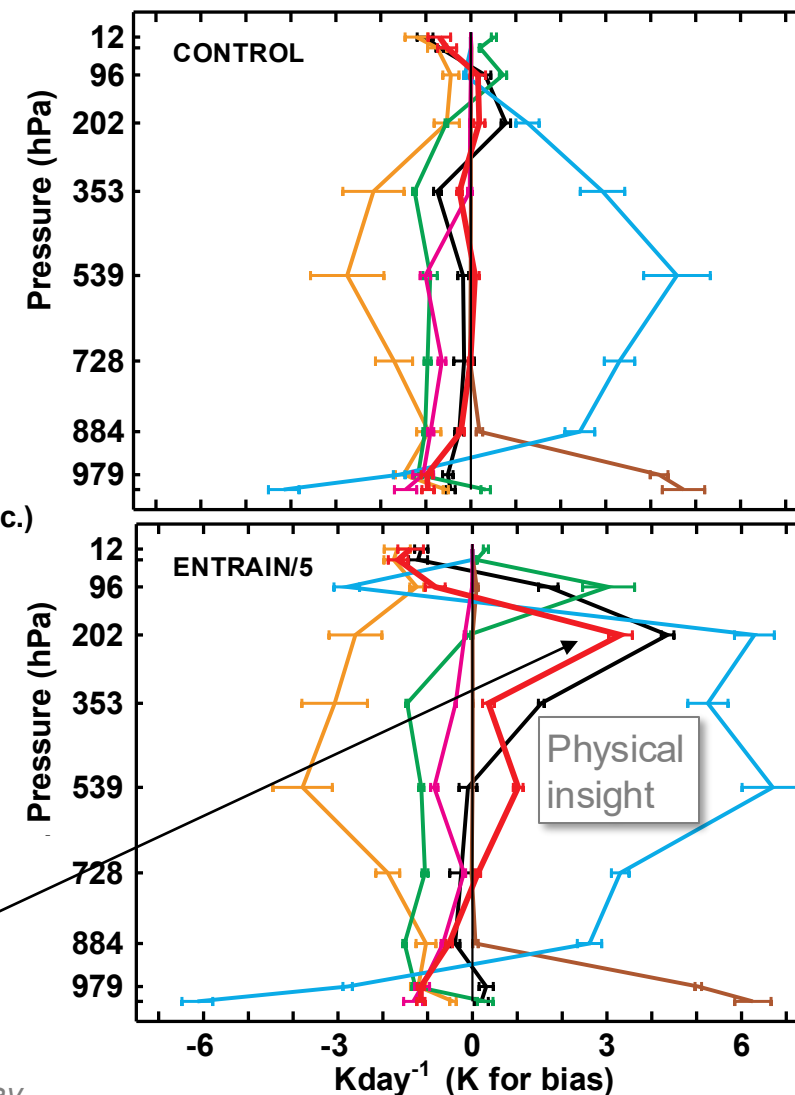
Present-day climate error vs climate sensitivity for perturbed parameter ensemble (HadSM3)



The initial tendency approach was used to investigate the Saharan aerosol result shown earlier

Small present-day climate error: **Accept** perturbation
Inconsistent with assimilated observations: **Reject**

Amazonia mean T initial tendencies (IFS 29r1)



Rodwell and Palmer (2007). Amazonia (300-320°E, 20°S-0°N) 6hr tendencies 31d (Jan 2005) X 4 forecasts per day. 70% conf.int. T159, L60,30min. See also Sexton et al. (2019), Klocke and Rodwell (2014)

- Tropical waves and teleconnections
 - Important for predictability
 - Can complicate the diagnosis of forecast system deficiencies
- Seamless approaches to model evaluation
 - Diagnosis at short leadtimes (associated with data assimilation) can localise errors (geographically, process-wise, model versus observation) before errors and uncertainties have had time to propagate and interact
 - Don't need mean error patterns to agree at short-range and long ranges (although sometimes bias patterns do simply grow in magnitude)
- Next lecture: Ensemble aspects, Processes driving uncertainty growth